Chosen-Ciphertext Security from Subset Sum

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Outline

Our Contribution

State of the Art

- CPA-secure Public Key Encryption (PKE) from Subset Sum [LPS10].

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- We construct a CCA-secure PKE from Subset Sum (using [MP12]).
- The security of our PKE does not decrease with the message length.

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3. CCA secure PKE
4. Tag-Based Encryption

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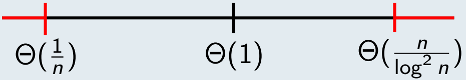
Hardness of Subset Sum

$\delta := \frac{n}{\log \mu}$:



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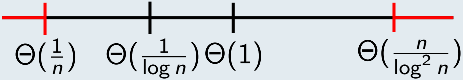

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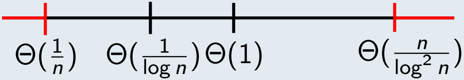

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Decisional Subset Sum [IN96]:
$(A, \mathbf{t})$ is hard to distinguish from uniform.

“LWE” form of Subset Sum [LPS10]

$(A, \mathbf{t}) \in \mathbb{Z}_\mu^n \times \mathbb{Z}_\mu \to \mathbb{Z}_q^{m \times n} \times \mathbb{Z}_q^m$

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$$\mathbf{a} = a^m \cdot q^{m-1} + \ldots + a^1 \cdot q^0 \widehat{=} (a^m, \ldots, a^1)^T \in \mathbb{Z}_q^m$$

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Therefore

$$A = (\mathbf{a}_1, \ldots, \mathbf{a}_n) \,\widehat{=}\, \begin{pmatrix} a_1^m & \cdots & a_n^m \\ \vdots & \ddots & \vdots \\ a_1^1 & \cdots & a_n^1 \end{pmatrix} \in \mathbb{Z}_q^{m \times n}$$

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where $e(A, s)$ is a vector of carries.

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From now on, $(A, \mathbf{t} = As + e(A, s)) \in \mathbb{Z}_q^{m\times n} \times \mathbb{Z}_q^m$ (m samples).

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$\mu = q^m$

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  If $(A, \mathbf{t})$ is uniform $\Rightarrow$ $(A, \mathbf{t}, RA, R\mathbf{t})$ is uniform.

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- $(RA, R\mathbf{t})$ is not Subset Sum distributed $(Re(A, s) \neq e(RA, s))$.

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Given a One-Time Signature (OTS),
[CHK04]:
$TBE + OTS \rightarrow$ CCA-secure PKE.

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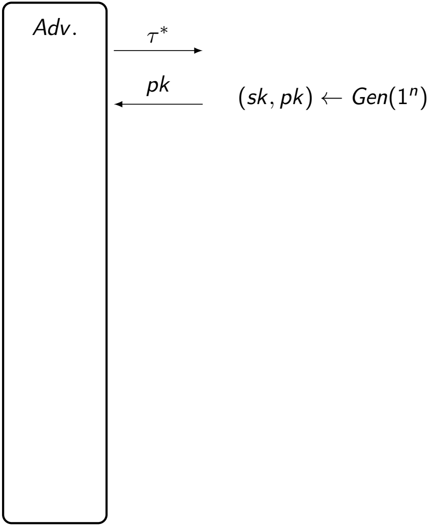

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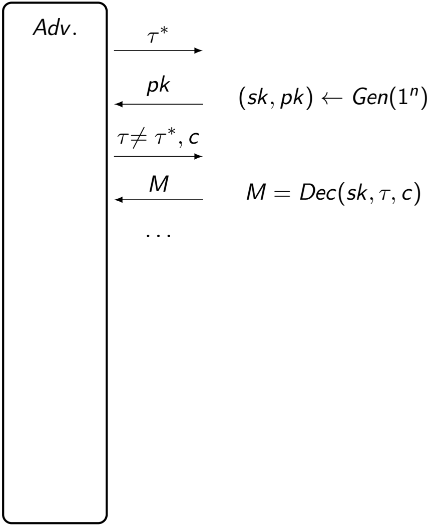

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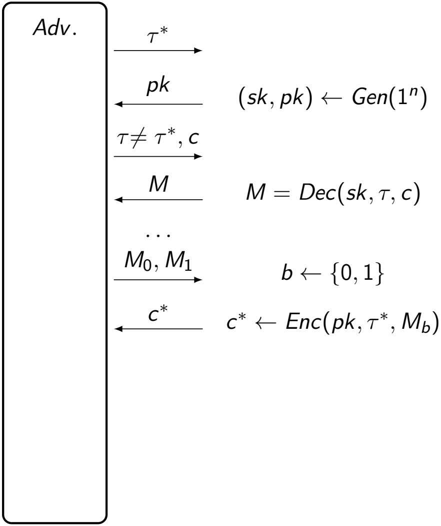

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M_0, M_1 — $b \leftarrow \{0, 1\}$

c^* — $c^* \leftarrow Enc(pk, \tau^*, M_b)$

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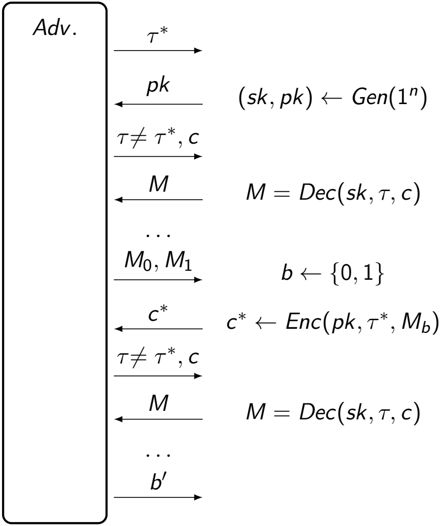

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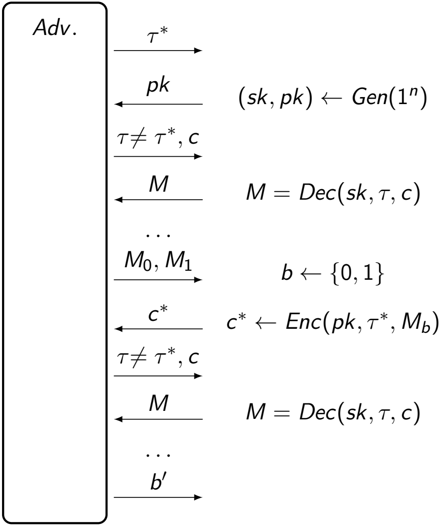
For all ppt $Adv.$: $\Pr[b' = b] = 1/2$.



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$2 \mid q$. Let $H_\tau \in \mathbb{Z}_2^{n \times n}$ represent τ.
For $\tau \neq \tau'$, $H_\tau - H_{\tau'}$ is invertible for [ABB10].

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For $M \in \{0, 1\}^\ell$:

$\mathit{Gen}(1^n) :\ A \leftarrow \mathbb{Z}_q^{m \times n},\ C \leftarrow \mathbb{Z}_q^{\ell \times n},\ R \leftarrow \mathcal{D}^{m \times n}.$

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$Gen(1^n):$ $A \leftarrow \mathbb{Z}_q^{m \times n}$, $C \leftarrow \mathbb{Z}_q^{\ell \times n}$, $R \leftarrow \mathcal{D}^{m \times n}$.
Output $sk = R$, $pk = (A, B := RA, C)$.

Tag-Based Encryption, *Enc*

For $M \in \{0,1\}^{\ell}$:

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$Enc(pk, H_\tau, M)$: Sample $R' \leftarrow \mathcal{D}^{m \times n}$, $R'' \leftarrow \mathcal{D}^{\ell \times n}$, $s \leftarrow \{0,1\}^n$

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$$c_0 := As + e(A,s);$$

- (A, c_0) is a Subset Sum instance for secret s.

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$$\begin{aligned} c_0 &:= As + e(A,s); \\ c_1 &:= (B + q/2 \cdot H_\tau)s + R'e(A,s); \end{aligned}$$

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$$\begin{aligned} c_0 &:= As + e(A,s); \\ c_1 &:= (B + q/2 \cdot H_\tau)s + R'e(A,s); \\ c_2 &:= Cs + R''e(A,s) + q/2 \cdot M. \end{aligned}$$

- (A, c_0) is a Subset Sum instance for secret s.
- s can be recovered from (c_0, c_1).
- c_2 encrypts M under secret s.

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$Dec(sk, H_\tau, c_0, c_1, c_2):\ s = H_\tau^{-1}\lfloor c_1 - Rc_0 \rceil_2$, output $M = \lfloor c_2 - Cs \rceil_2$.

$(\lfloor \cdot \rceil_2 : \mathbb{Z}_q \to \{0,1\})$

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$(\lfloor\cdot\rceil_2 : \mathbb{Z}_q \to \{0,1\})$

Correctness: Since $RAs = Bs$:

$H_\tau^{-1}\lfloor c_1 - Rc_0\rceil_2$

Tag-Based Encryption, *Dec*

For $M \in \{0,1\}^{\ell}$:

$Gen(1^n):\ sk = R,\ pk = (A, B := RA, C)$.

$$\begin{aligned} Enc(pk, H_\tau, M):\ & c_0 := As + e(A,s), \\ & c_1 := (B + q/2 \cdot H_\tau)s + R' e(A,s), \\ & c_2 := Cs + R'' e(A,s) + q/2 \cdot M. \end{aligned}$$

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$(\lfloor \cdot \rceil_2 : \mathbb{Z}_q \to \{0,1\})$

Correctness: Since $RAs = Bs$:

$$H_\tau^{-1} \lfloor c_1 - Rc_0 \rceil_2 = H_\tau^{-1} \lfloor q/2 \cdot H_\tau s + (R' - R)e(A,s) \rceil_2$$

Tag-Based Encryption, *Dec*

For $M \in \{0,1\}^{\ell}$:

$Gen(1^n) :\ sk = R,\ pk = (A, B := RA, C)$.

$Enc(pk, H_\tau, M) :\ c_0 := As + e(A,s),$

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$$H_\tau^{-1}\lfloor c_1 - Rc_0\rceil_2 = H_\tau^{-1}\lfloor q/2 \cdot H_\tau s + (R' - R)e(A,s)\rceil_2$$

Tag-Based Encryption, *Dec*

For $M \in \{0,1\}^{\ell}$:

$Gen(1^n)$: $sk = R$, $pk = (A, B := RA, C)$.

$Enc(pk, H_\tau, M)$: $c_0 := As + e(A,s)$,

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$$H_\tau^{-1}\lfloor c_1 - Rc_0 \rceil_2 = H_\tau^{-1}\lfloor q/2 \cdot H_\tau s + (R' - R)e(A,s) \rceil_2 = H_\tau^{-1} H_\tau s = s,$$

Tag-Based Encryption, *Dec*

For $M \in \{0,1\}^{\ell}$:

$Gen(1^n)$: $sk = R$, $pk = (A, B := RA, C)$.

$Enc(pk, H_\tau, M)$: $c_0 := As + e(A,s)$,

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$$\lfloor c_2 - Cs \rceil_2 = \lfloor R''e(A,s) + q/2 \cdot M \rceil_2$$

Tag-Based Encryption, *Dec*

For $M \in \{0,1\}^{\ell}$:

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$$H_\tau^{-1}\lfloor c_1 - Rc_0 \rceil_2 = H_\tau^{-1}\lfloor q/2 \cdot H_\tau s + (R' - R)e(A,s) \rceil_2 = H_\tau^{-1} H_\tau s = s,$$

$$\lfloor c_2 - Cs \rceil_2 = \lfloor R''e(A,s) + q/2 \cdot M \rceil_2$$

Tag-Based Encryption, *Dec*

For $M \in \{0,1\}^{\ell}$:

$Gen(1^n):\ sk = R,\ pk = (A, B := RA, C).$

$Enc(pk, H_\tau, M):\ c_0 := As + e(A,s),$

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$(\lfloor \cdot \rceil_2 : \mathbb{Z}_q \to \{0,1\})$

Correctness: Since $RAs = Bs$:

$$H_\tau^{-1}\lfloor c_1 - Rc_0 \rceil_2 = H_\tau^{-1}\lfloor q/2 \cdot H_\tau s + (R' - R)e(A,s) \rceil_2 = H_\tau^{-1}H_\tau s = s,$$

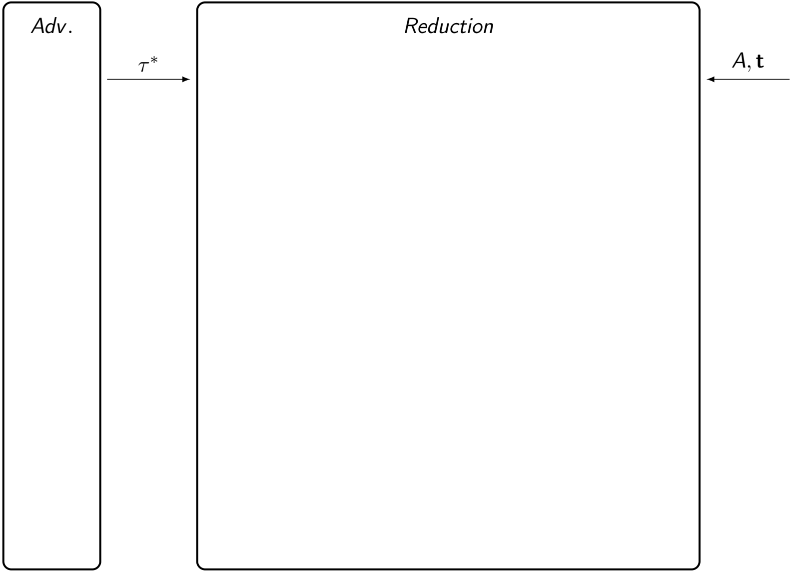
$$\lfloor c_2 - Cs \rceil_2 = \lfloor R''e(A,s) + q/2 \cdot M \rceil_2 = M.$$

Proof Sketch

Adv.

Reduction

Proof Sketch



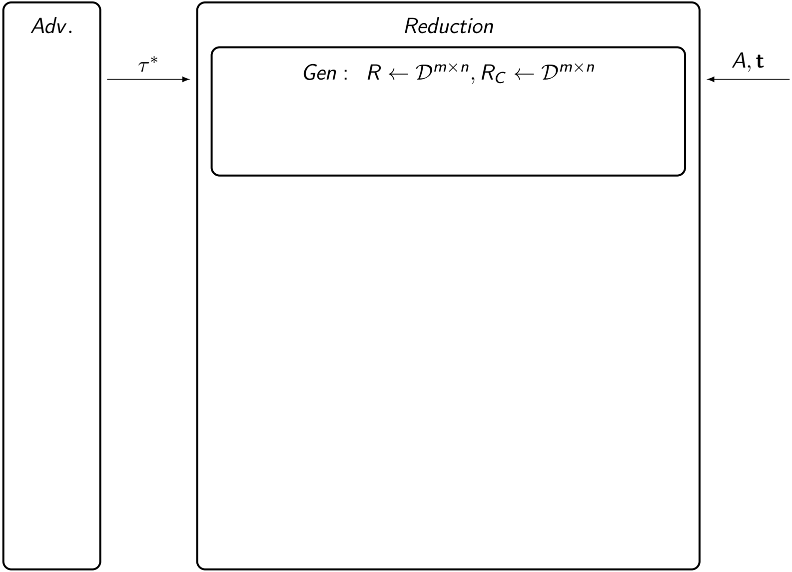
Adv.

τ^*

Reduction

$A, \mathbf{t}$

Proof Sketch



Adv.

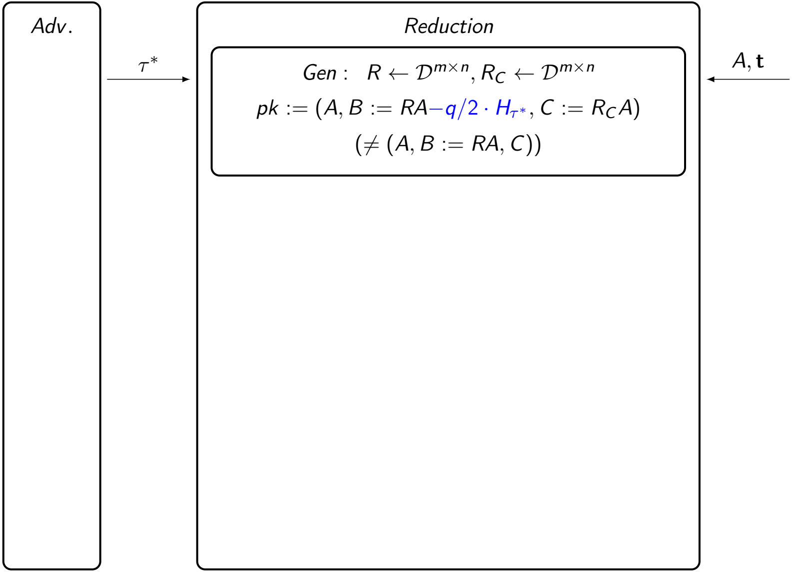
Reduction

τ^*

$Gen: \quad R \leftarrow \mathcal{D}^{m \times n}, R_C \leftarrow \mathcal{D}^{m \times n}$

$A, \mathbf{t}$

Proof Sketch



Adv.

Reduction

τ^*

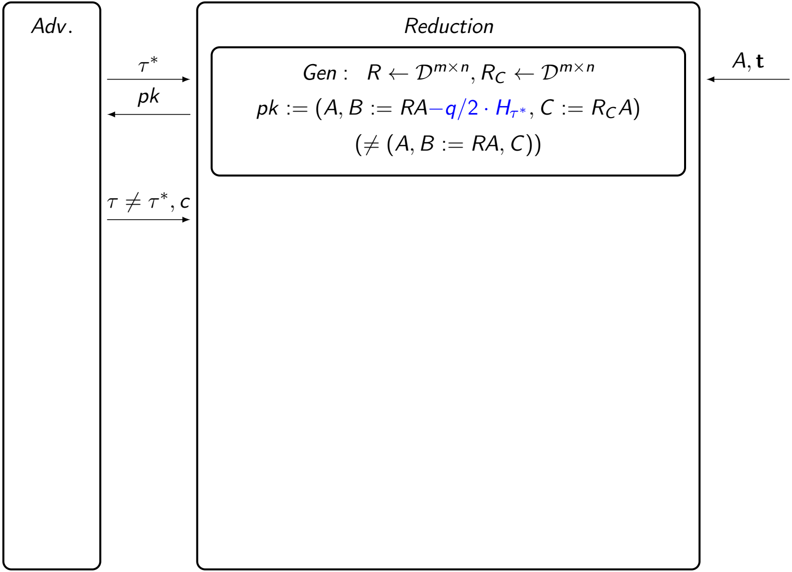
$A, \mathbf{t}$

$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

Proof Sketch



Adv.

Reduction

$A, \mathbf{t}$

τ^*

pk

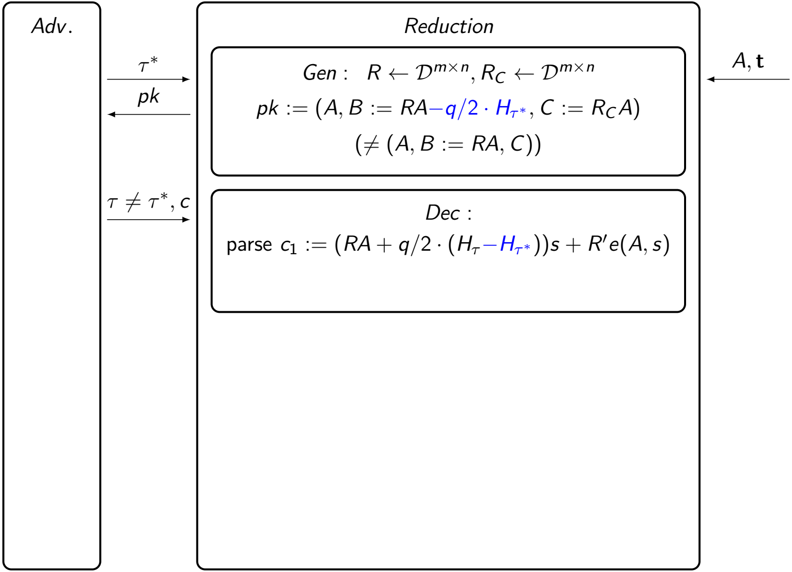
$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

$\tau \neq \tau^*, c$

Proof Sketch



Adv.

Reduction

τ^*

pk

$A, \mathbf{t}$

$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

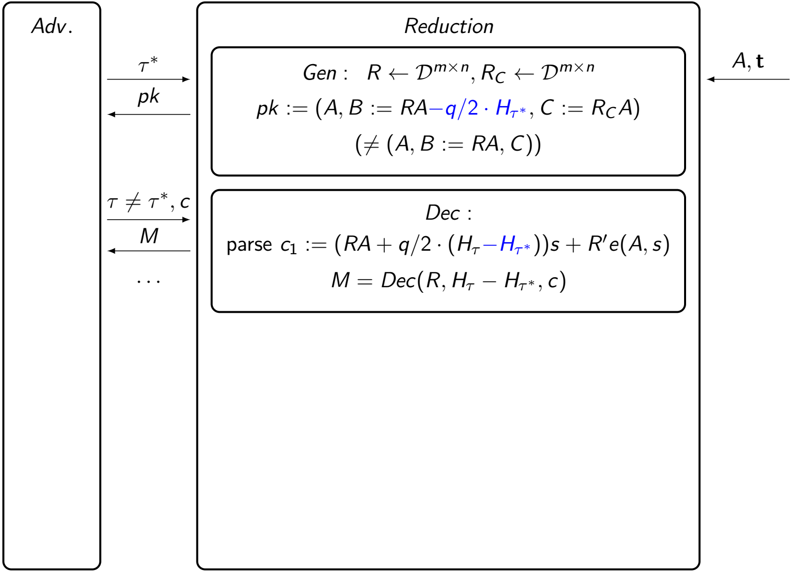
$(\neq (A, B := RA, C))$

$\tau \neq \tau^*, c$

$Dec:$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

Proof Sketch



Adv.

τ^*

pk

$\tau \neq \tau^*, c$

M

$\ldots$

Reduction

$$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$$

$$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$$

$$(\neq (A, B := RA, C))$$

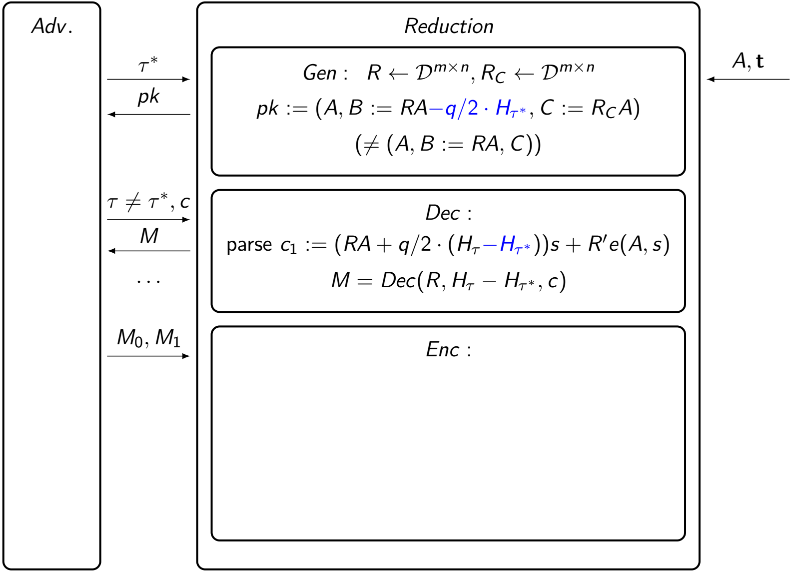
$$Dec:$$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

$$M = Dec(R, H_\tau - H_{\tau^*}, c)$$

$A, \mathbf{t}$

Proof Sketch



Adv.

τ^*

pk

$\tau \neq \tau^*, c$

M

$\cdots$

M_0, M_1

Reduction

$A, \mathbf{t}$

$$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$$

$$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$$

$$(\neq (A, B := RA, C))$$

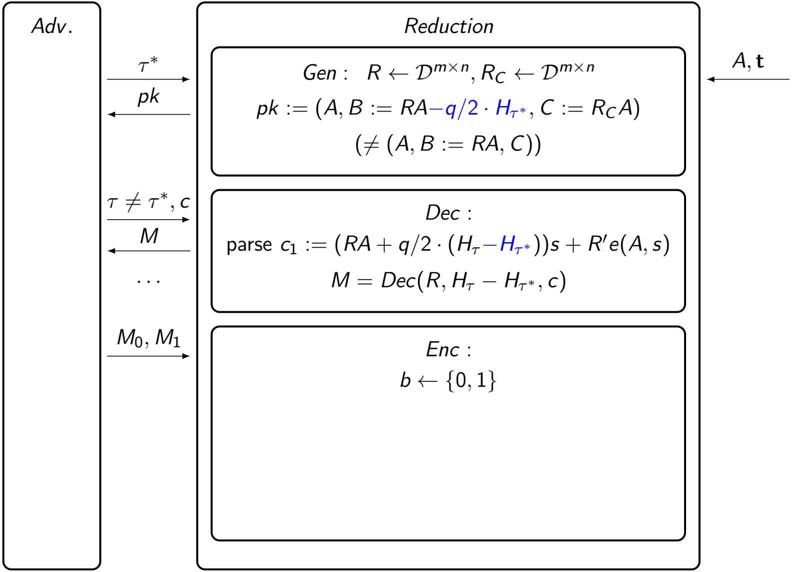
$Dec:$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

$$M = Dec(R, H_\tau - H_{\tau^*}, c)$$

$Enc:$

Proof Sketch



Adv.

Reduction

$A, \mathbf{t}$

τ^*

pk

$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

$\tau \neq \tau^*, c$

M

$\cdots$

$Dec:$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

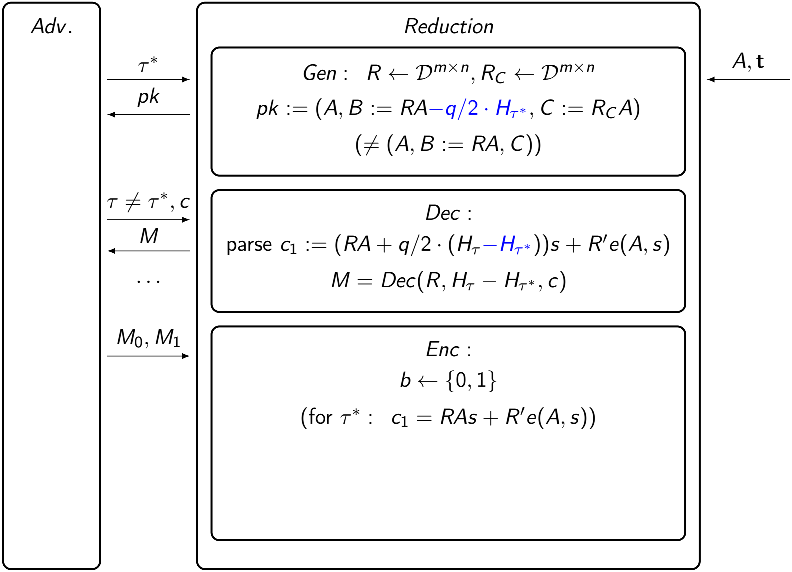
$M = Dec(R, H_\tau - H_{\tau^*}, c)$

M_0, M_1

$Enc:$

$b \leftarrow \{0, 1\}$

Proof Sketch



Adv.

Reduction

τ^*

pk

$\tau \neq \tau^*, c$

M

$\cdots$

M_0, M_1

$A, \mathbf{t}$

$$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$$

$$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$$

$$(\neq (A, B := RA, C))$$

$$Dec:$$

$$\text{parse } c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$$

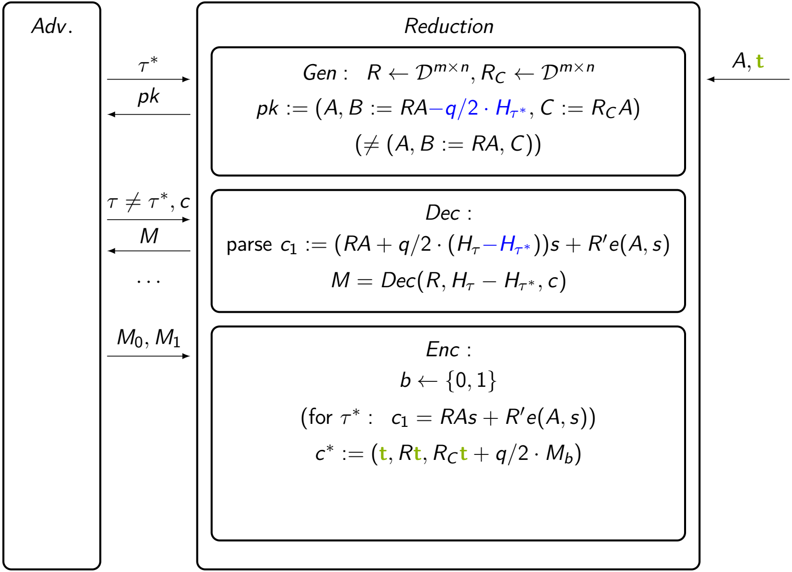
$$M = Dec(R, H_\tau - H_{\tau^*}, c)$$

$$Enc:$$

$$b \leftarrow \{0, 1\}$$

$$(\text{for } \tau^*: \quad c_1 = RAs + R'e(A, s))$$

Proof Sketch



Adv.

$\xrightarrow{\tau^*}$

$\xleftarrow{pk}$

$\xrightarrow{\tau \neq \tau^*, c}$

$\xleftarrow{M}$

$\dots$

$\xrightarrow{M_0, M_1}$

Reduction

$\xleftarrow{A, \mathbf{t}}$

$Gen: \quad R \leftarrow \mathcal{D}^{m \times n}, R_C \leftarrow \mathcal{D}^{m \times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

$Dec:$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

$M = Dec(R, H_\tau - H_{\tau^*}, c)$

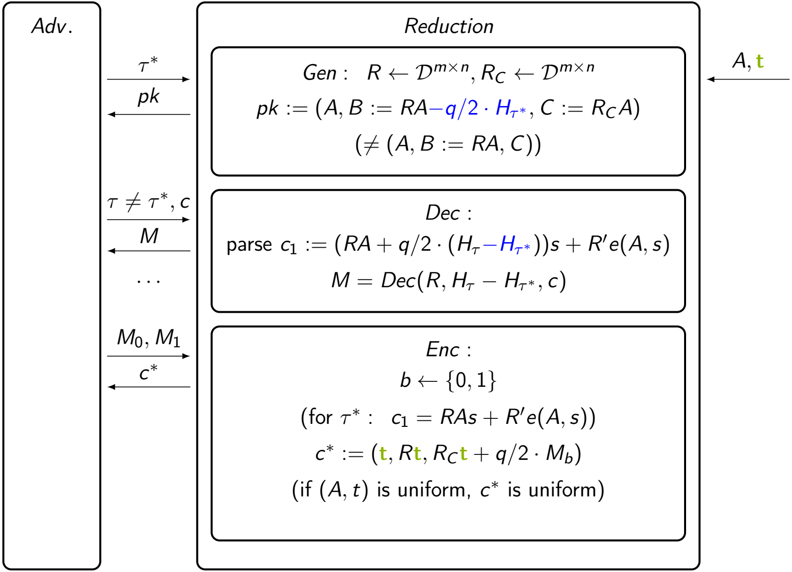
$Enc:$

$b \leftarrow \{0, 1\}$

(for $\tau^*: \quad c_1 = RAs + R'e(A, s)$)

$c^* := (\mathbf{t}, R\mathbf{t}, R_C\mathbf{t} + q/2 \cdot M_b)$

Proof Sketch



Reduction

$Gen: \quad R \leftarrow \mathcal{D}^{m \times n}, R_C \leftarrow \mathcal{D}^{m \times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

$Dec:$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

$M = Dec(R, H_\tau - H_{\tau^*}, c)$

$Enc:$

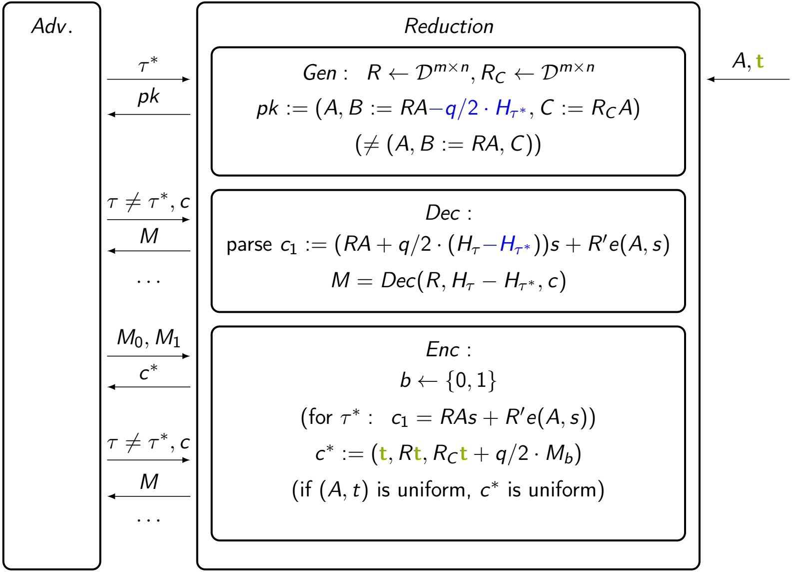
$b \leftarrow \{0, 1\}$

(for $\tau^*: \quad c_1 = RAs + R'e(A, s)$)

$c^* := (\mathbf{t}, R\mathbf{t}, R_C\mathbf{t} + q/2 \cdot M_b)$

(if (A, t) is uniform, c^* is uniform)

Proof Sketch



Adv.

$\xrightarrow{\tau^*}$

$\xleftarrow{pk}$

$\xrightarrow{\tau \neq \tau^*, c}$

$\xleftarrow{M}$

$\dots$

$\xrightarrow{M_0, M_1}$

$\xleftarrow{c^*}$

$\xrightarrow{\tau \neq \tau^*, c}$

$\xleftarrow{M}$

$\dots$

Reduction

$\xleftarrow{A, \mathbf{t}}$

$Gen: \quad R \leftarrow \mathcal{D}^{m \times n}, R_C \leftarrow \mathcal{D}^{m \times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

$Dec:$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

$M = Dec(R, H_\tau - H_{\tau^*}, c)$

$Enc:$

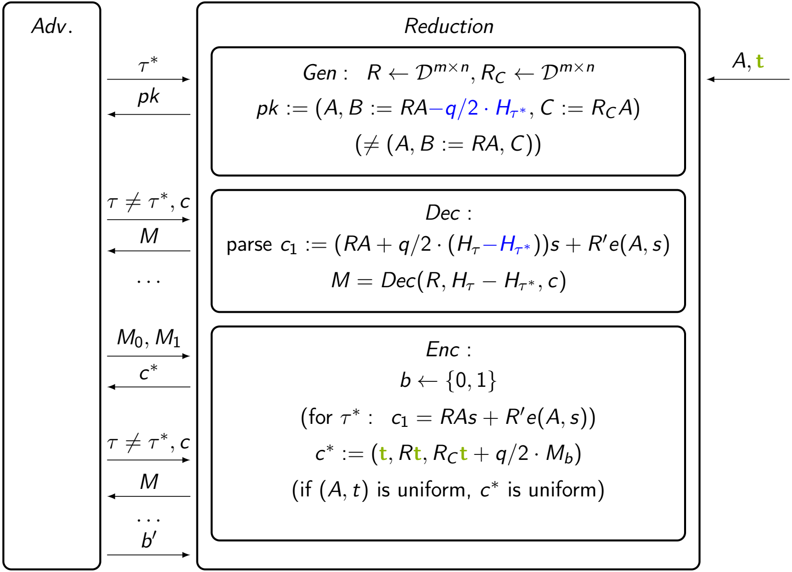
$b \leftarrow \{0, 1\}$

(for $\tau^*: \quad c_1 = RAs + R'e(A, s)$)

$c^* := (\mathbf{t}, R\mathbf{t}, R_C\mathbf{t} + q/2 \cdot M_b)$

(if (A, t) is uniform, c^* is uniform)

Proof Sketch



Adv.

Reduction

$A, \mathbf{t}$

τ^* →

pk ←

Gen : $R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

$\tau \neq \tau^*, c$ →

M ←

$\cdots$

Dec :

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

$M = Dec(R, H_\tau - H_{\tau^*}, c)$

M_0, M_1 →

c^* ←

$\tau \neq \tau^*, c$ →

M ←

$\cdots$

b' →

Enc :

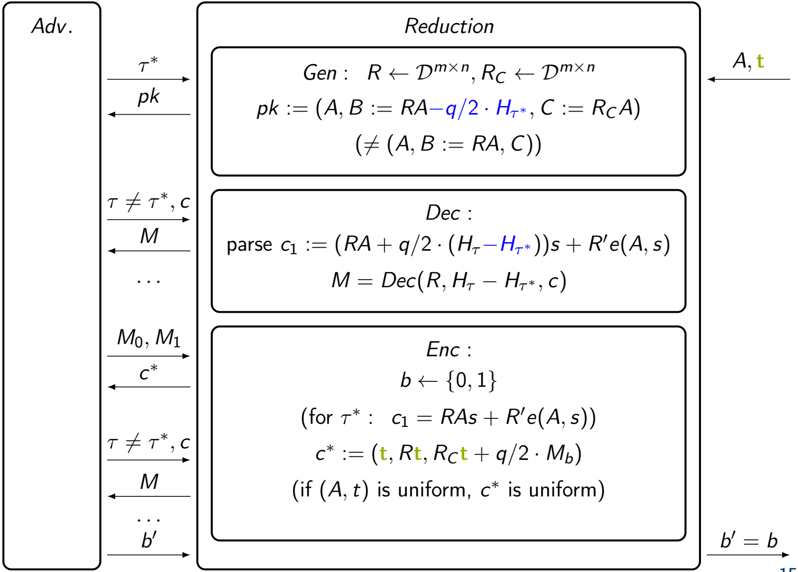
$b \leftarrow \{0, 1\}$

(for τ^* : $c_1 = RAs + R'e(A, s)$)

$c^* := (\mathbf{t}, R\mathbf{t}, R_C\mathbf{t} + q/2 \cdot M_b)$

(if (A, t) is uniform, c^* is uniform)

Proof Sketch



Adv.

$\xrightarrow{\tau^*}$

$\xleftarrow{pk}$

$\xrightarrow{\tau \neq \tau^*, c}$

$\xleftarrow{M}$

$\dots$

$\xrightarrow{M_0, M_1}$

$\xleftarrow{c^*}$

$\xrightarrow{\tau \neq \tau^*, c}$

$\xleftarrow{M}$

$\dots$

$\xrightarrow{b'}$

Reduction

$Gen: \quad R \leftarrow \mathcal{D}^{m\times n}, R_C \leftarrow \mathcal{D}^{m\times n}$

$pk := (A, B := RA - q/2 \cdot H_{\tau^*}, C := R_C A)$

$(\neq (A, B := RA, C))$

$Dec:$

parse $c_1 := (RA + q/2 \cdot (H_\tau - H_{\tau^*}))s + R'e(A, s)$

$M = Dec(R, H_\tau - H_{\tau^*}, c)$

$Enc:$

$b \leftarrow \{0, 1\}$

(for τ^* : $\quad c_1 = RAs + R'e(A, s)$)

$c^* := (\mathbf{t}, R\mathbf{t}, R_C\mathbf{t} + q/2 \cdot M_b)$

(if (A, t) is uniform, c^* is uniform)

$\xleftarrow{A, \mathbf{t}}$

$\xrightarrow{b' = b}$

Conclusion

Our Results

- “LWE" form of Subset Sum [LPS10] + LWE trapdoor [MP12] $\Rightarrow$ IND-CCA-secure PKE from Subset Sum.
- Unlike the CPA-secure PKE of [LPS10], the security of our scheme does not decrease with the message length ℓ.

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