# Low Noise LPN: KDM Secure Public Key Encryption and Sample Amplification

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- Solve noisy linear decoding problem for random linear codes over $\mathbb{F}_2$
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- Goal: Given a *few* LPN samples, generate more samples
- In [Lyu05]: Given instance $(\mathbf{A}, \mathbf{y} = \mathbf{A}\mathbf{s} + \mathbf{e})$, set

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- Dual Formulation:

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- Matrix Form

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# LPN with bounded samples vs. LPN with unbounded samples

**Theorem**

DLPN$(n, \rho')$ *is as hard as* DLPN$(n, 2n, \rho)$ *whenever* $\rho' \geq \rho^2 2n$

# Proof Idea

| | static | volatile | samples |
|---|---|---|---|
| 1. | $\mathbf{s} \leftarrow_\$ \mathbb{F}_2^n$ | $\mathbf{a} \leftarrow_\$ \mathbb{F}_2^n$<br>$e \leftarrow_\$ \mathrm{Ber}(\rho')$ | $(\mathbf{a}, \langle \mathbf{a}, \mathbf{s} \rangle + e)$ |
| | | | |
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# Proof Idea

eDLPN: $(\mathbf{A}, \mathbf{r}^\top\mathbf{A}, \mathbf{z}, \mathbf{r}^\top\mathbf{z}) \approx_c (\mathbf{A}, \mathbf{a}, \mathbf{z}, \mathbf{r}^\top\mathbf{z}) \equiv (\mathbf{A}, \mathbf{a}, \mathbf{z}, e)$

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# KDM-CPA Security

**Challenger**

$(pk, sk) \leftarrow \mathsf{KeyGen}(1^\lambda)$ $\xrightarrow{pk}$

$\mathcal{A}$

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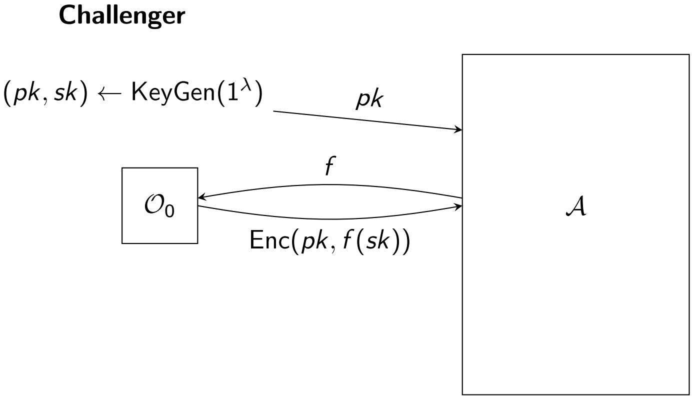

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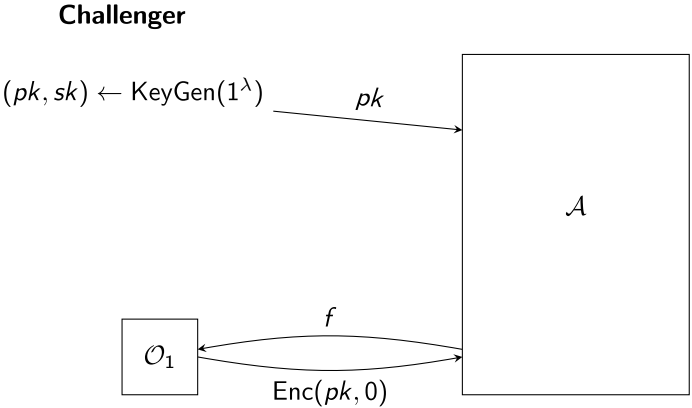

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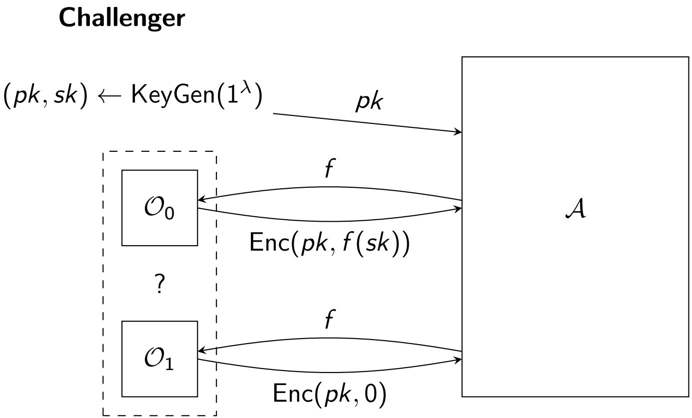

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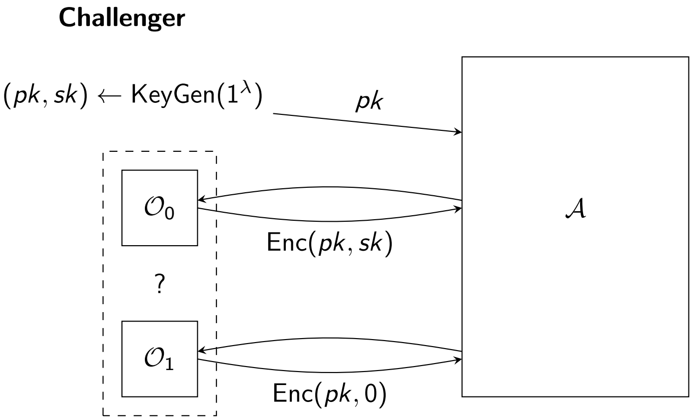
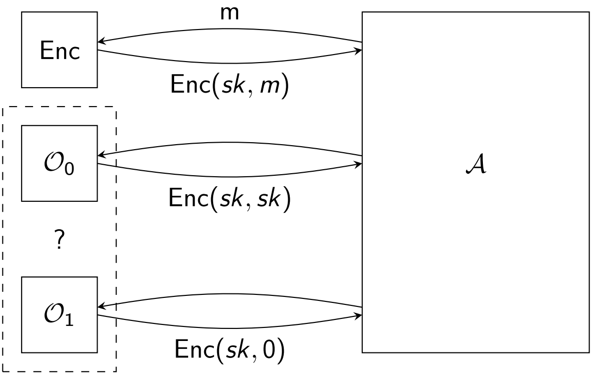


# CIRC-CPA Security (Private Key)

**Challenger**



# Private Key Scheme of [ACPS12]

Let $\mathbf{G}$ be the generator of an asymptotically good $[k, n]$ code that can efficiently decode from a constant fraction of errors.

- KeyGen: $\mathbf{s} \leftarrow_{\$} \mathbb{F}_2^n$

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- KeyGen: $\mathbf{s} \leftarrow_\$ \mathbb{F}_2^n$
- Enc$(\mathbf{s}, m)$: $\mathbf{C}_1 \leftarrow_\$ \mathbb{F}_2^{k \times n}$, $\mathbf{e} \leftarrow_\$ \text{Ber}(k, \rho)$, $\mathbf{c}_1 = \mathbf{C}_1\mathbf{s} + \mathbf{e} + \mathbf{Gm}$
  $c = (\mathbf{C}_1, \mathbf{c}_2)$

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  $c = (\mathbf{C}_1, \mathbf{c}_2)$
- $\mathsf{Dec}(\mathbf{s}, c)$: $(\mathbf{C}_1, \mathbf{c}_2) = c$, $\mathbf{z} = \mathbf{c}_2 - \mathbf{C}_1 \cdot \mathbf{s}$
  $m = \mathsf{Decode}(\mathbf{z})$

## Correctness

Let $(\mathbf{C}_1, \mathbf{c}_2)$ be a valid ciphertext. Consider the value $\mathbf{z}$ computed during decryption.

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## Correctness

Let $(\mathbf{C}_1, \mathbf{c}_2)$ be a valid ciphertext. Consider the value $\mathbf{z}$ computed during decryption.

$$\begin{aligned} \mathbf{z} &= \mathbf{c}_2 - \mathbf{C}_1 \cdot \mathbf{s} \\ &= \mathbf{Gm} + \mathbf{C}_1\mathbf{s} + \mathbf{e} - \mathbf{C}_1\mathbf{s} \\ &= \mathbf{Gm} + \underbrace{\mathbf{e}}_{\text{weight} \approx \rho m} \end{aligned}$$

Scheme is correct if decoding corrects $\rho n$ errors.

# Circular Security

Circular secure under standard (high noise) LPN

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- KeyGen: $\mathbf{s} \leftarrow_{\$} \mathbb{F}_2^n$, $\mathbf{A} \leftarrow_{\$} \mathbb{F}_2^{m \times n}$,$\mathbf{e} \leftarrow_{\$} \mathsf{Ber}(\rho)^m$
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- $\mathrm{Dec}(sk, c)$: $(\mathbf{C}_1, \mathbf{c}_2) = c$, $\mathbf{z} = \mathbf{c}_2 - \mathbf{C}_1 \cdot \mathbf{s}$
  $m = \mathrm{Decode}(\mathbf{z})$

# Circular Security

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$$\text{eDLPN: } (\mathbf{A}, \mathbf{RA}, \mathbf{e}, \mathbf{Re}) \approx_c (\mathbf{A}, \mathbf{U}, \mathbf{e}, \mathbf{Re})$$

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# Circular Security

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| 7. | $(\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e})$ | $(\mathbf{R}\mathbf{A} - \mathbf{G}, \mathbf{R}(\mathbf{A}\mathbf{s} + \mathbf{e}))$ | |

## Circular Security

$$\text{DLPN: } (\mathbf{A}, \mathbf{As}+\mathbf{e}) \approx_c (\mathbf{A}, \mathbf{u})$$

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|---|---|---|---|
| 1. | $(\mathbf{A}, \mathbf{y}=\mathbf{As}+\mathbf{e})$ | $(\mathbf{RA}, \mathbf{Ry}+\mathbf{Gs})$ | |
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| 6. | $(\mathbf{A}, \mathbf{As}+\mathbf{e})$ | $(\mathbf{RA}-\mathbf{G}, \mathbf{RAs}+\mathbf{Re})$ | eDLPN |
| 7. | $(\mathbf{A}, \mathbf{As}+\mathbf{e})$ | $(\mathbf{RA}-\mathbf{G}, \mathbf{R}(\mathbf{As}+\mathbf{e}))$ | |
| 8. | $(\mathbf{A}, \mathbf{u})$ | $(\mathbf{RA}-\mathbf{G}, \mathbf{Ru})$ | DLPN |

# Circular Security

eDLPN: $(\mathbf{A}, \mathbf{RA}, \mathbf{u}, \mathbf{Ru}) \approx (\mathbf{A}, \mathbf{U}, \mathbf{u}, \mathbf{Ru}) \approx_s (\mathbf{A}, \mathbf{U}, \mathbf{u}, \mathbf{u}')$

| | public key | challenge ciphertext | remark |
|---|---|---|---|
| 1. | $(\mathbf{A}, \mathbf{y} = \mathbf{As} + \mathbf{e})$ | $(\mathbf{RA}, \mathbf{Ry} + \mathbf{Gs})$ | |
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# Thank You!