Digital Signatures from Strong RSA without Prime Generation

David Cash
Rafael Dowsley
Eike Kiltz

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Problem: In practice non-random hash functions like SHA-256 are used, which implies some theoretical limitations of these results [CGH98,DOP05].

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In common: the signing algorithm generates primes.

Generating primes is expensive and it is not an intrinsic step for the signing algorithm, thus it is desirable to avoid it.

Our Work

Strong RSA-based signature without prime generation for signing.

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Conceptual contribution towards the goal of practical schemes from conservative hardness assumptions without random oracles.

Strong RSA Problem and Signatures

Challenger

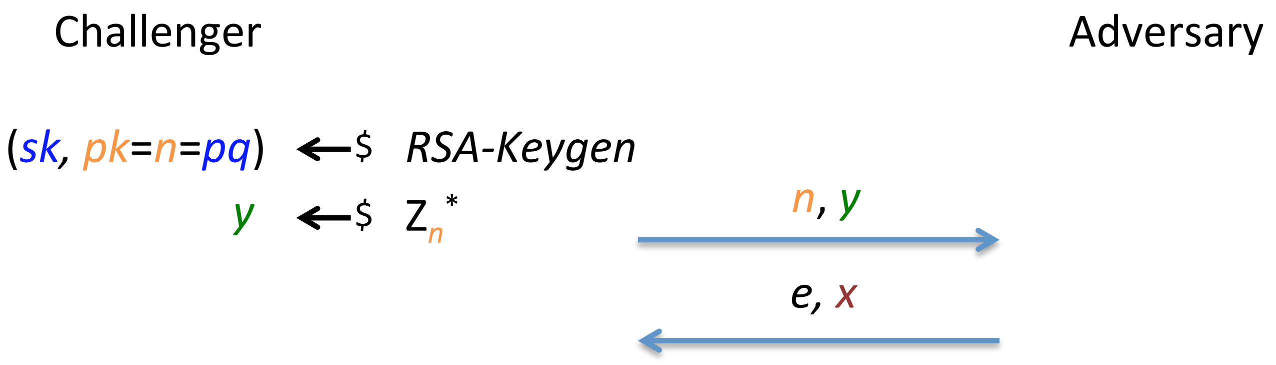
Adversary

$(sk, pk = n = pq) \leftarrow\$ \ \mathit{RSA\text{-}Keygen}$

$y \leftarrow\$ \ Z_n^*$

n, y

Strong RSA Problem and Signatures



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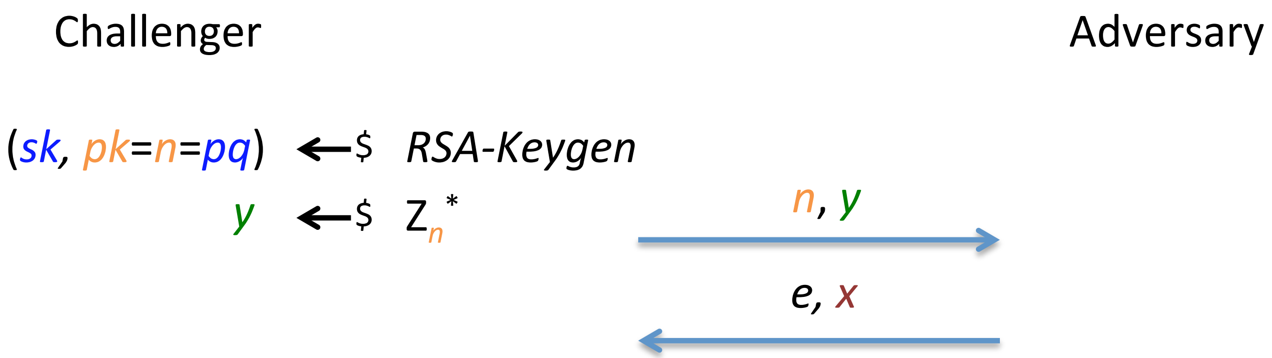

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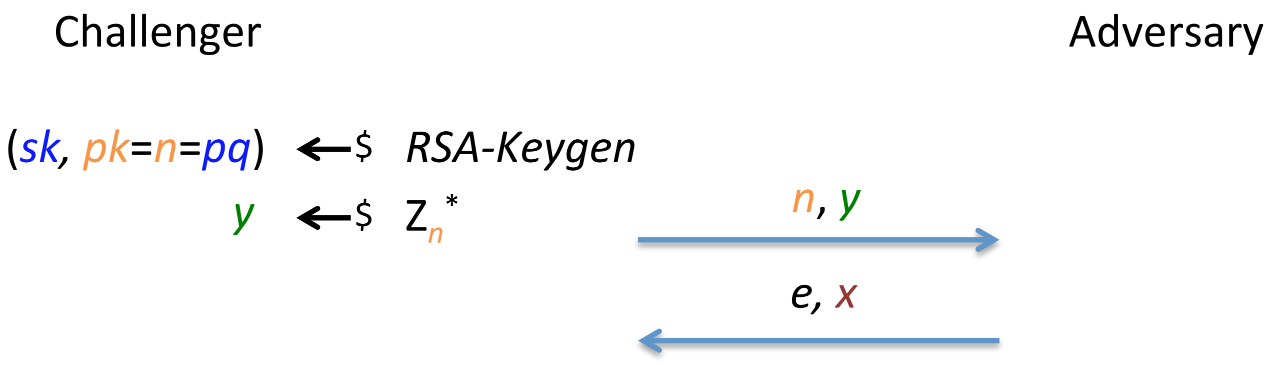

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Natural approach for embedding it into digital signatures: the signature is (e, x) where x is the e-th root of a value y that depends on the message.

In order to apply known techniques that prevent an adversary from assembling several signatures into a new signature, e is typically required to be prime or a product of primes.

Current Schemes

Many existing schemes [CS99,F03,HK08,Z01,Z03,CL04] use

$$Sign(sk, m) = (H(m)^{1/e} \text{ mod } n, e)$$

where e is a random prime and is H some (algebraic) hash function.

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Other schemes [GHR99,HW09,HJK11] based on the strong/standard RSA problem use

$$Sign(sk, m) = g^{1/\prod h_i(m)} \bmod n$$

where h_i are independent hash functions that hash into primes.

Our Scheme

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The signature on a message m of L-bits is

$$Sign(sk, m) = g^{1/e} \bmod n$$

$$e = \prod_{i=1}^{L} F(k,m[1 \ldots i]) \prod_{i=1}^{u} F(k,m||i)$$

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To verify a signature σ on a message m first compute

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We show that this scheme is secure against weak chosen message attacks.

Weak CMA Security

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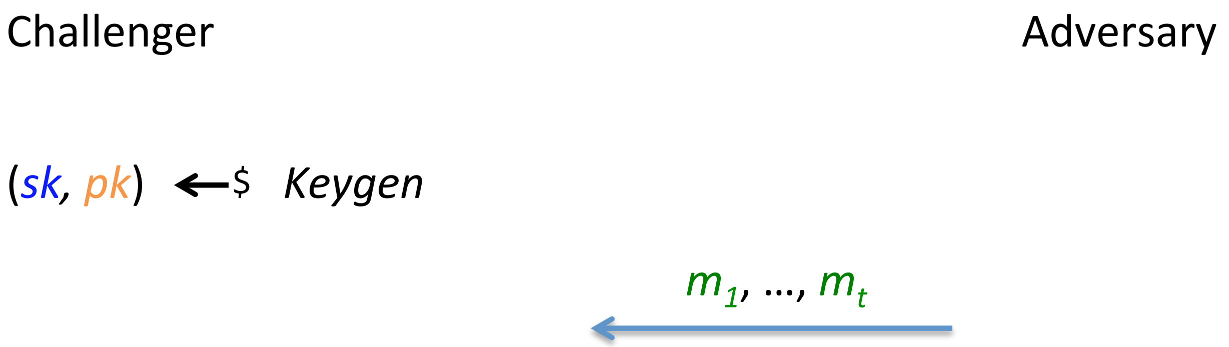
Weak CMA Security



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$m_1, \ldots, m_t$

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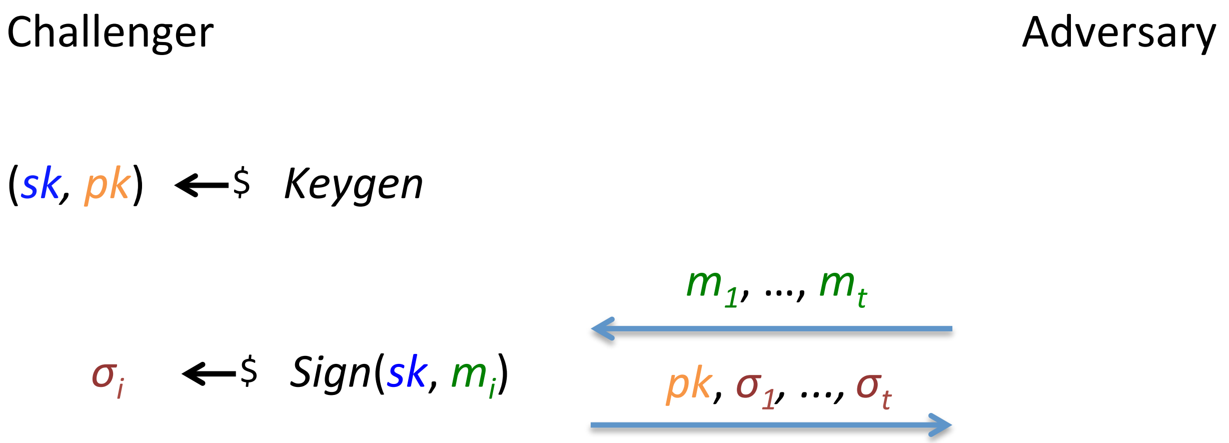

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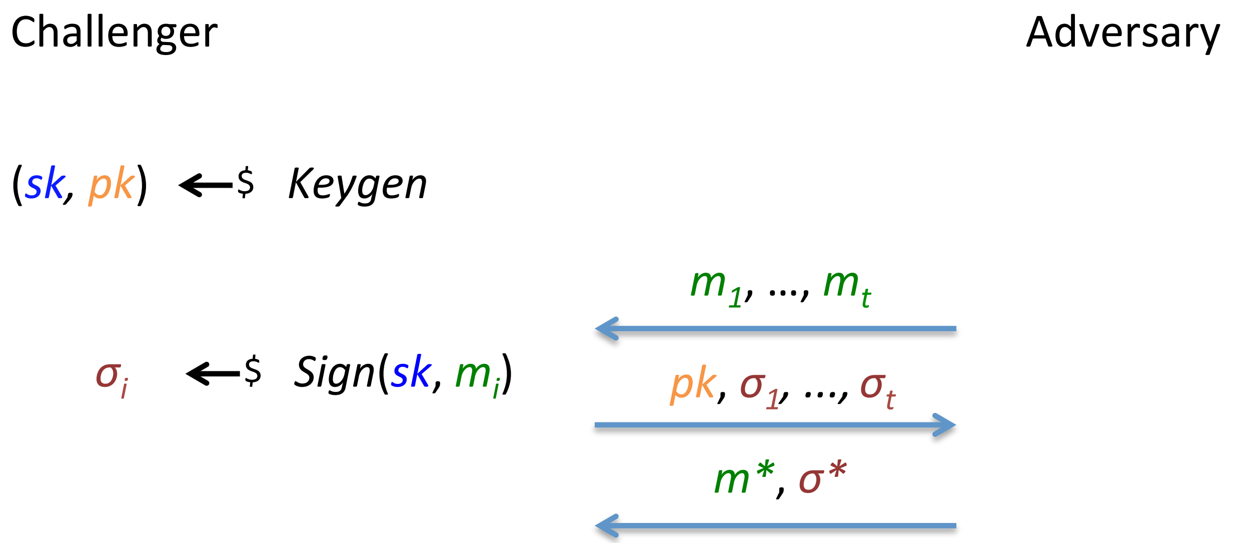

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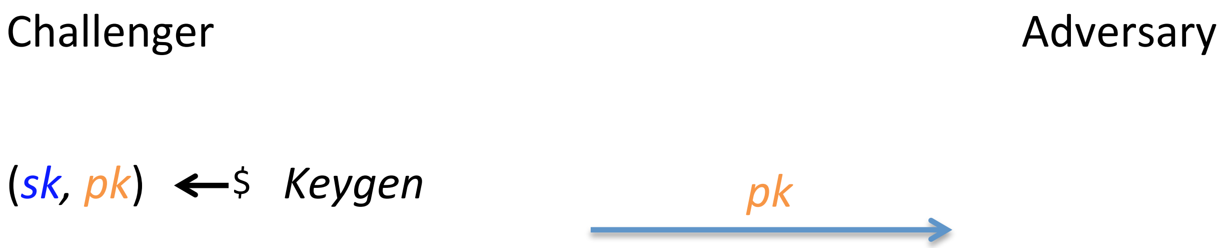
$pk, \sigma_1, \ldots, \sigma_t$

m^*, σ^*

Wins if $\mathit{Verify}(pk, m^*, \sigma^*)$=1 and m^* was not queried.

CMA Security



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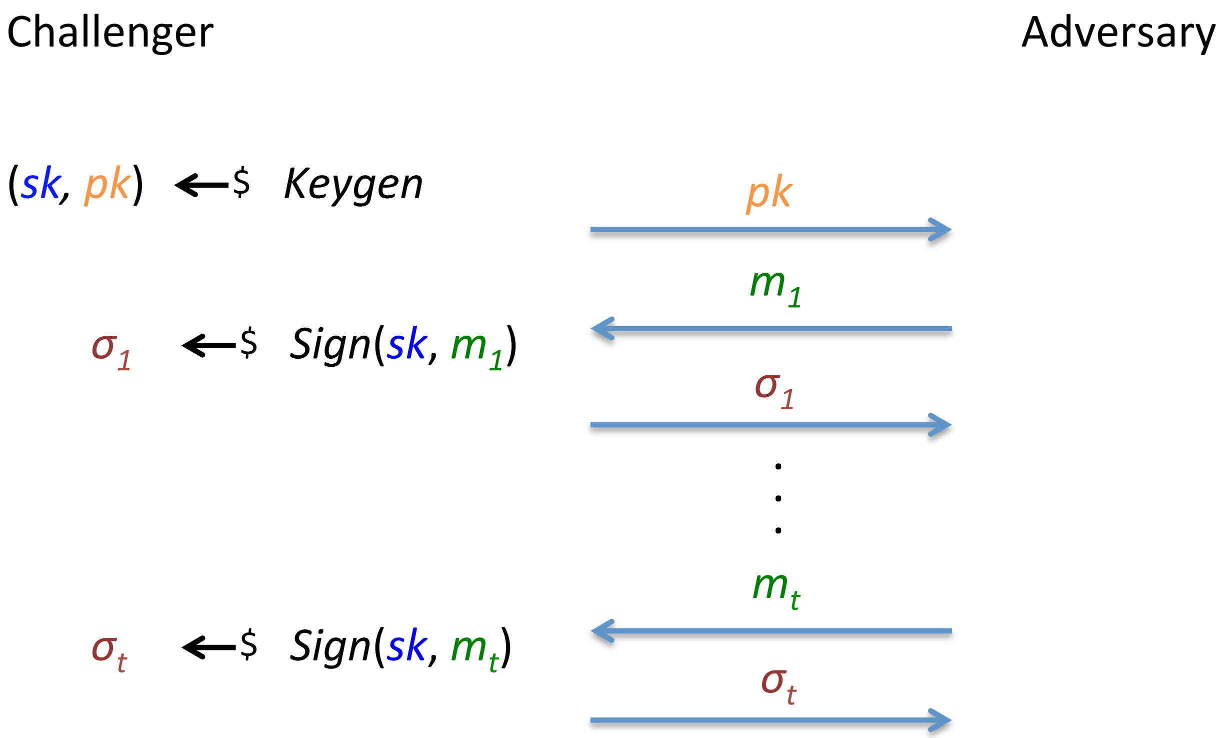

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$(sk, pk) \xleftarrow{\$} Keygen$

pk

m_1

$\sigma_1 \xleftarrow{\$} Sign(sk, m_1)$

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$\vdots$

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$\sigma_t \xleftarrow{\$} Sign(sk, m_t)$

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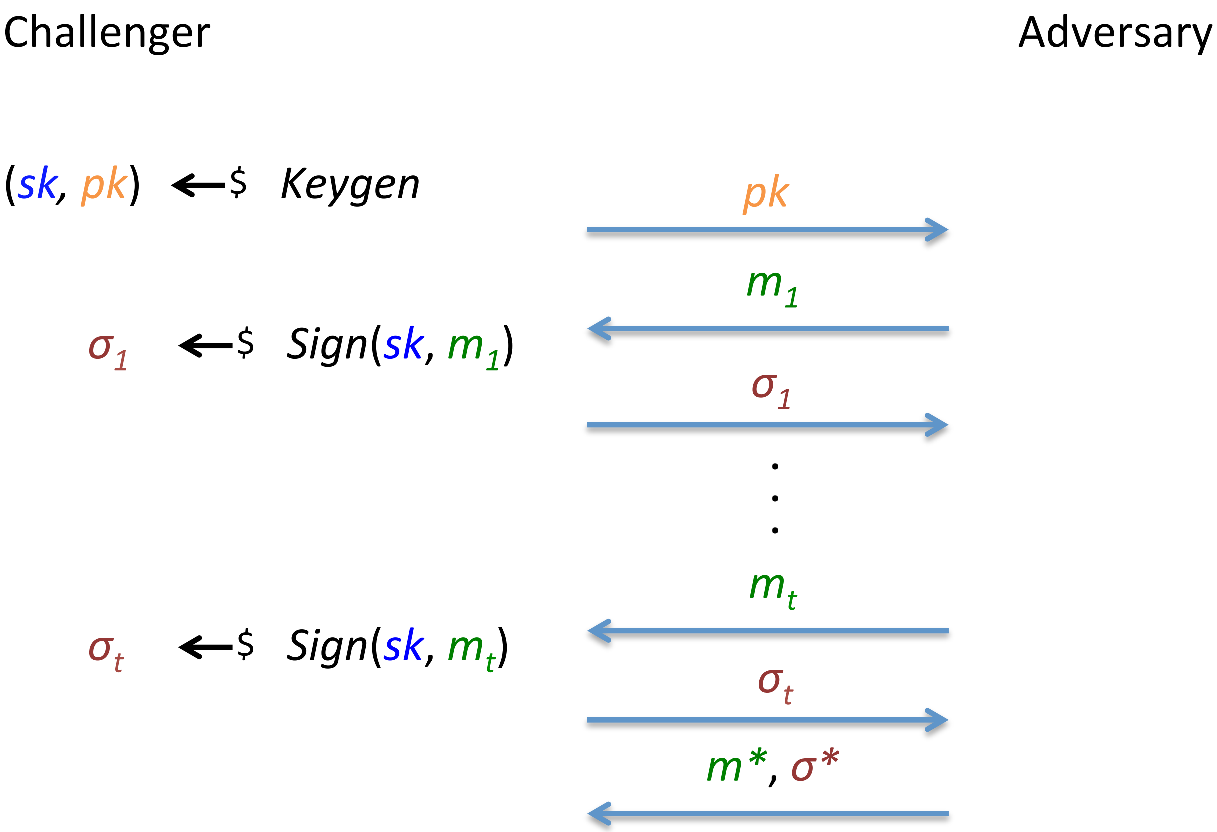

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pk →

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σ_1 →

⋮

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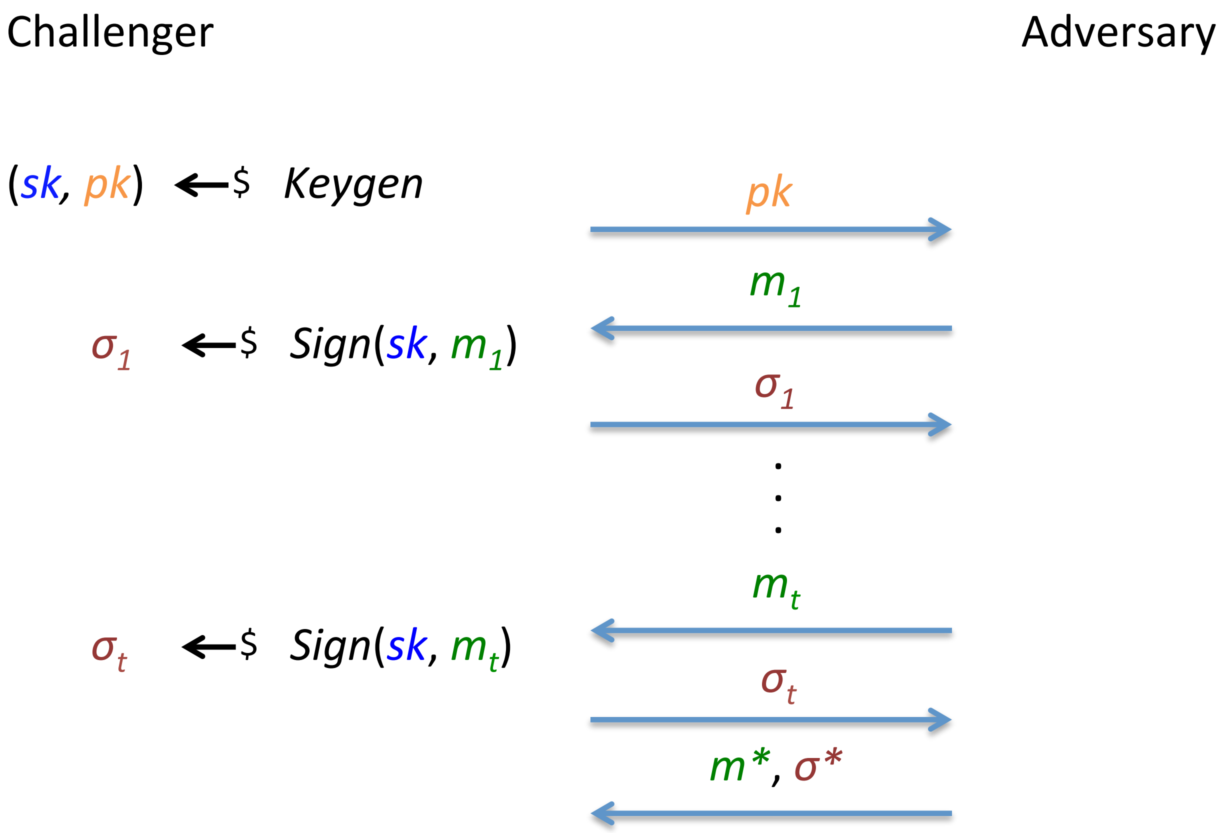

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Wins if $\mathit{Verify}(pk, m^*, \sigma^*)=1$ and m^* was not queried.

A chameleon hash function can be used to get from weak CMA to CMA [KR00,HW09].

Prefix Signing Technique [HW09]

$$Sign(sk, m) = g^{1/e} \bmod n$$

$$e = \prod_{i=1}^{L} F(k, m[1 \ldots i])$$

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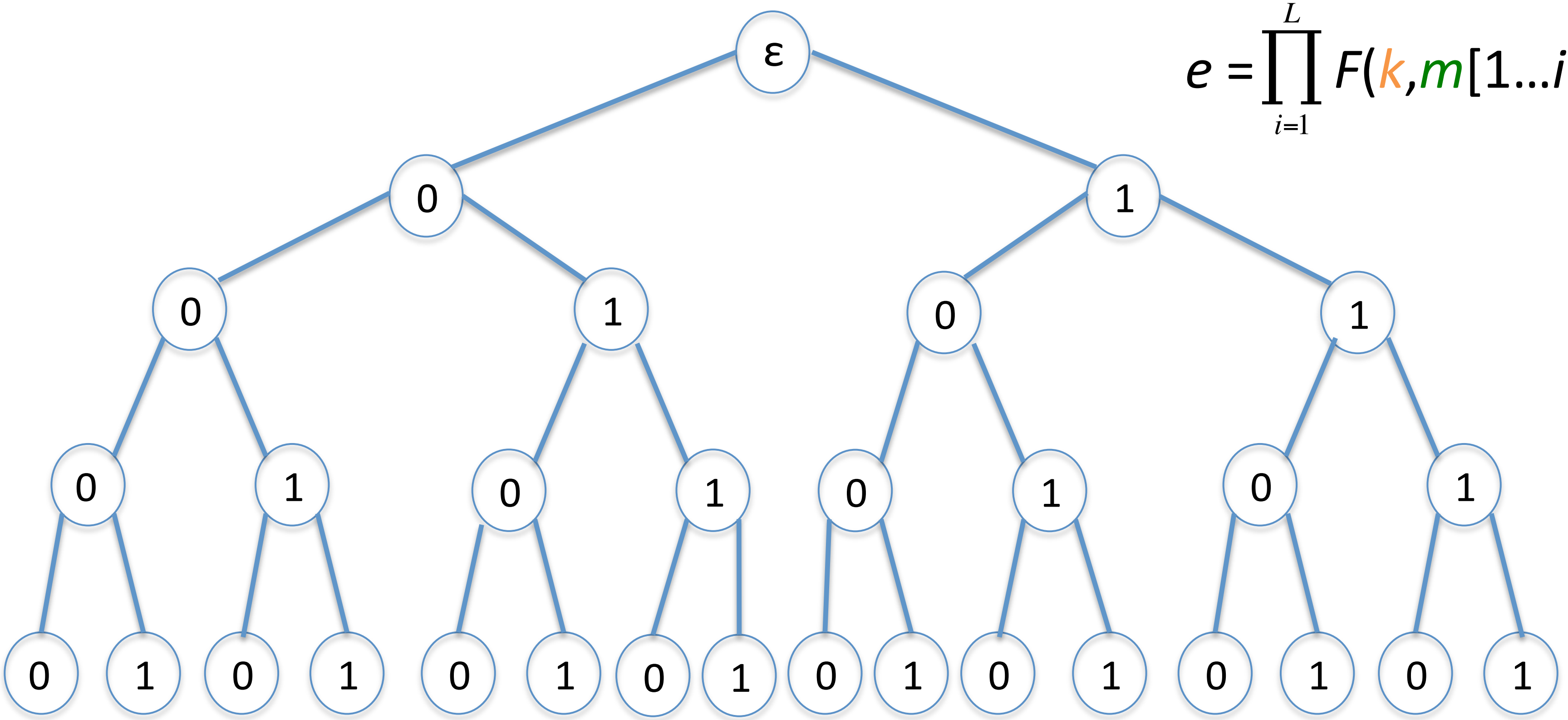

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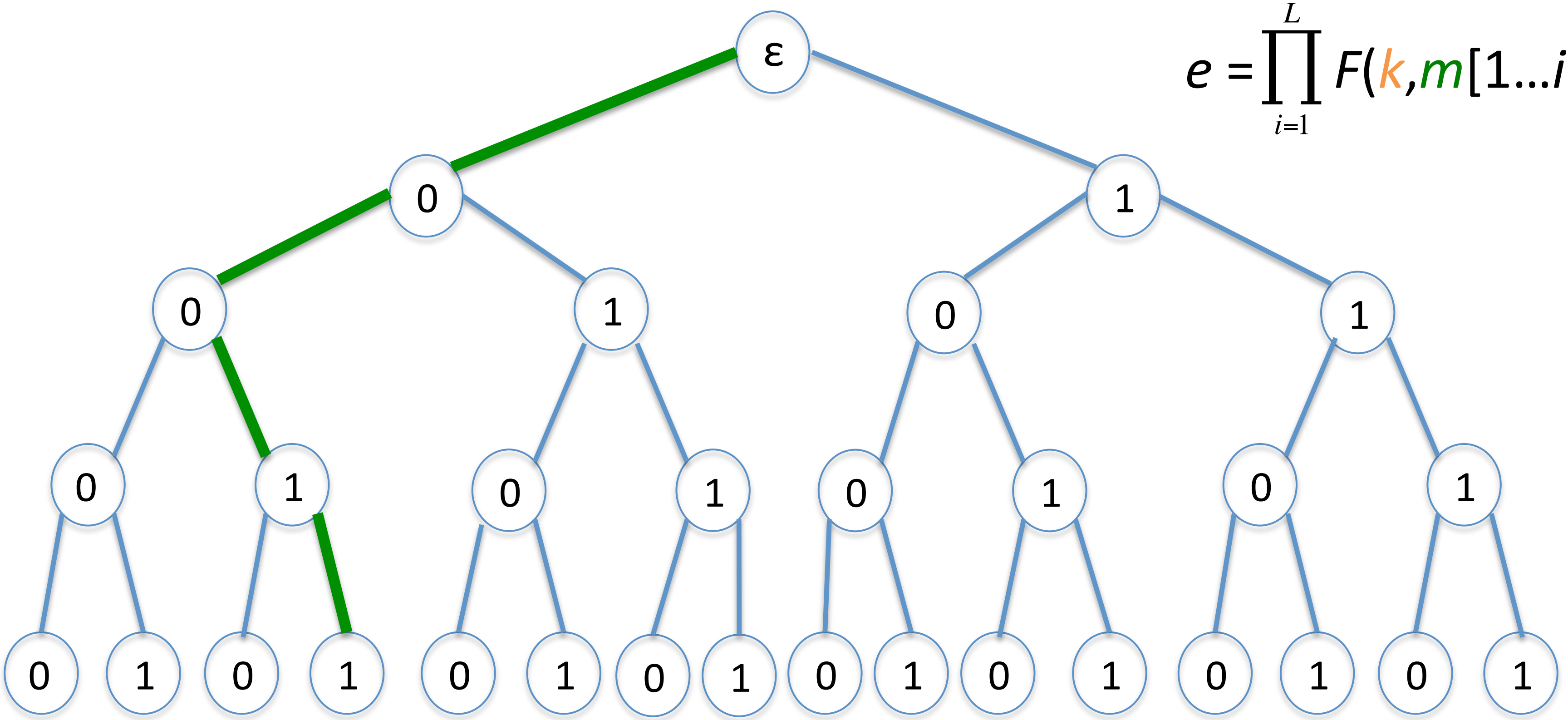

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To sign a message *m*=0011, for instance, compute *e* as the product of the numbers associated with the branches in green.

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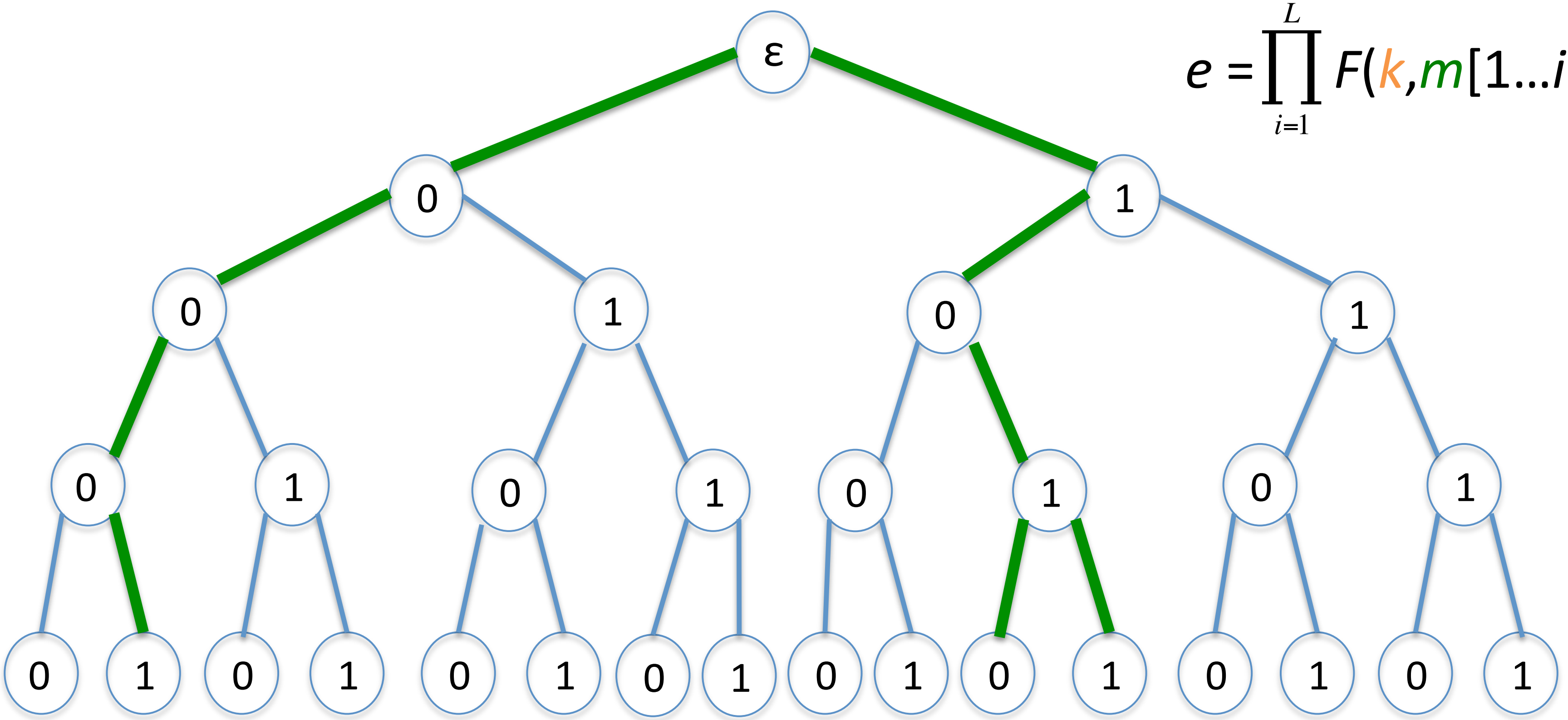

Let the sub-tree in green be the branches associated with the parallel signing query of the adversary.

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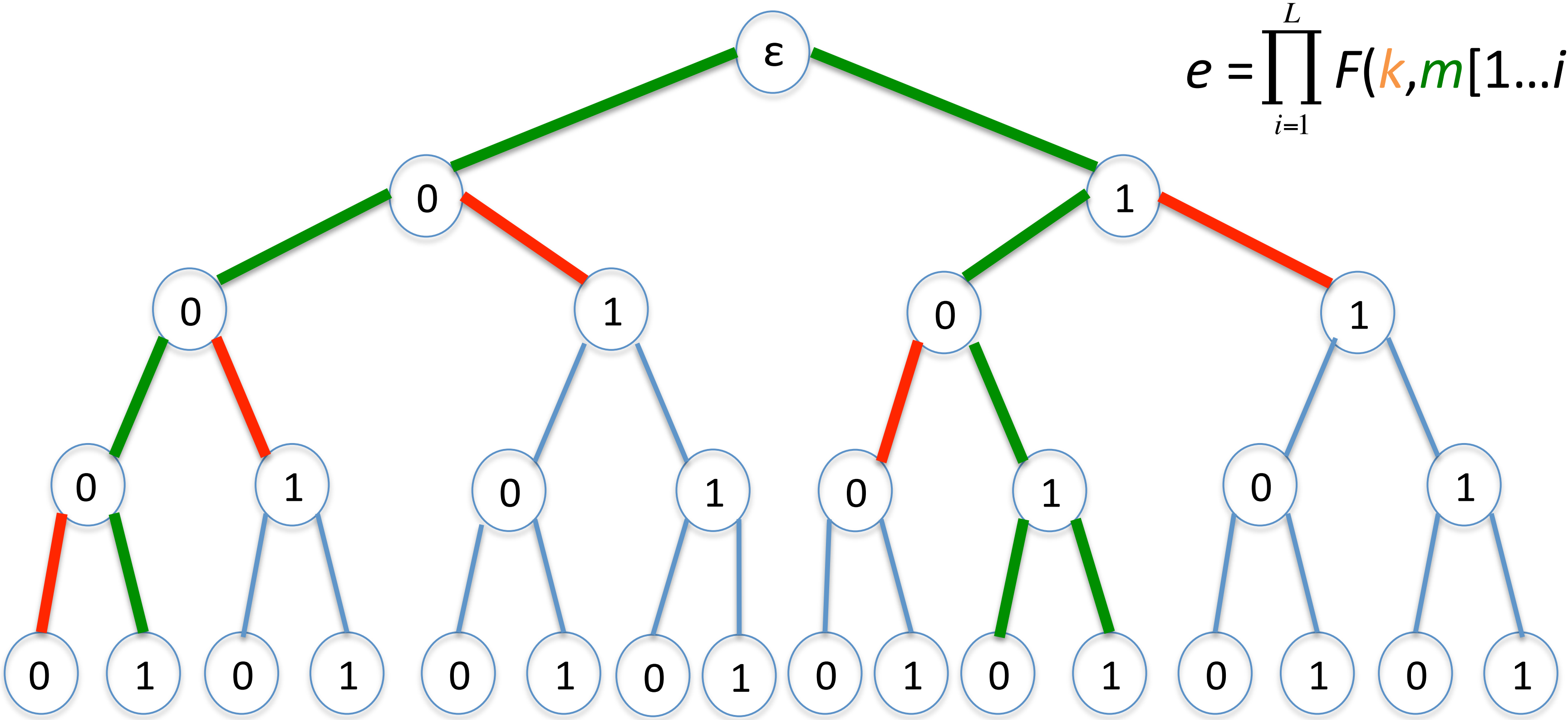

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Exit branch: non-green, but only has green branches on path from root until it.

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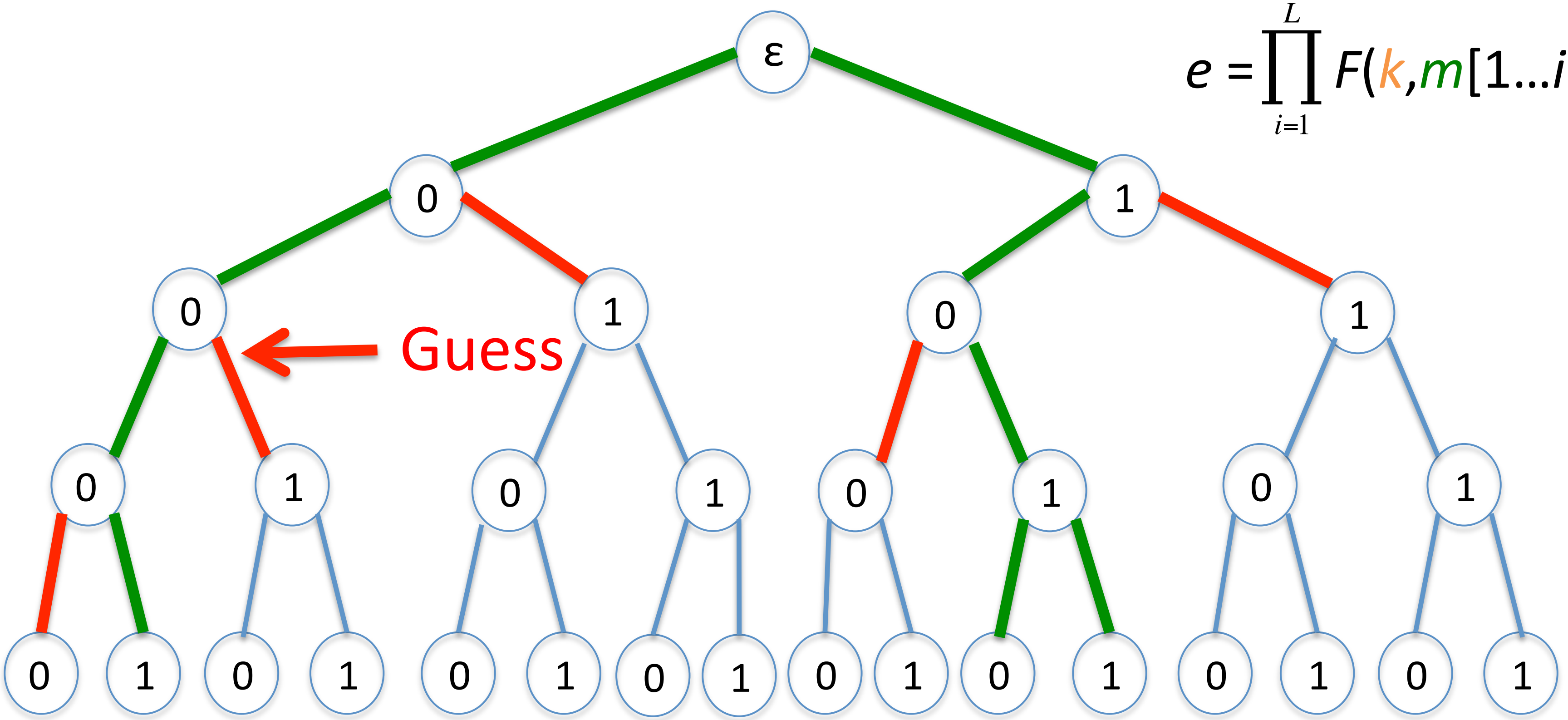

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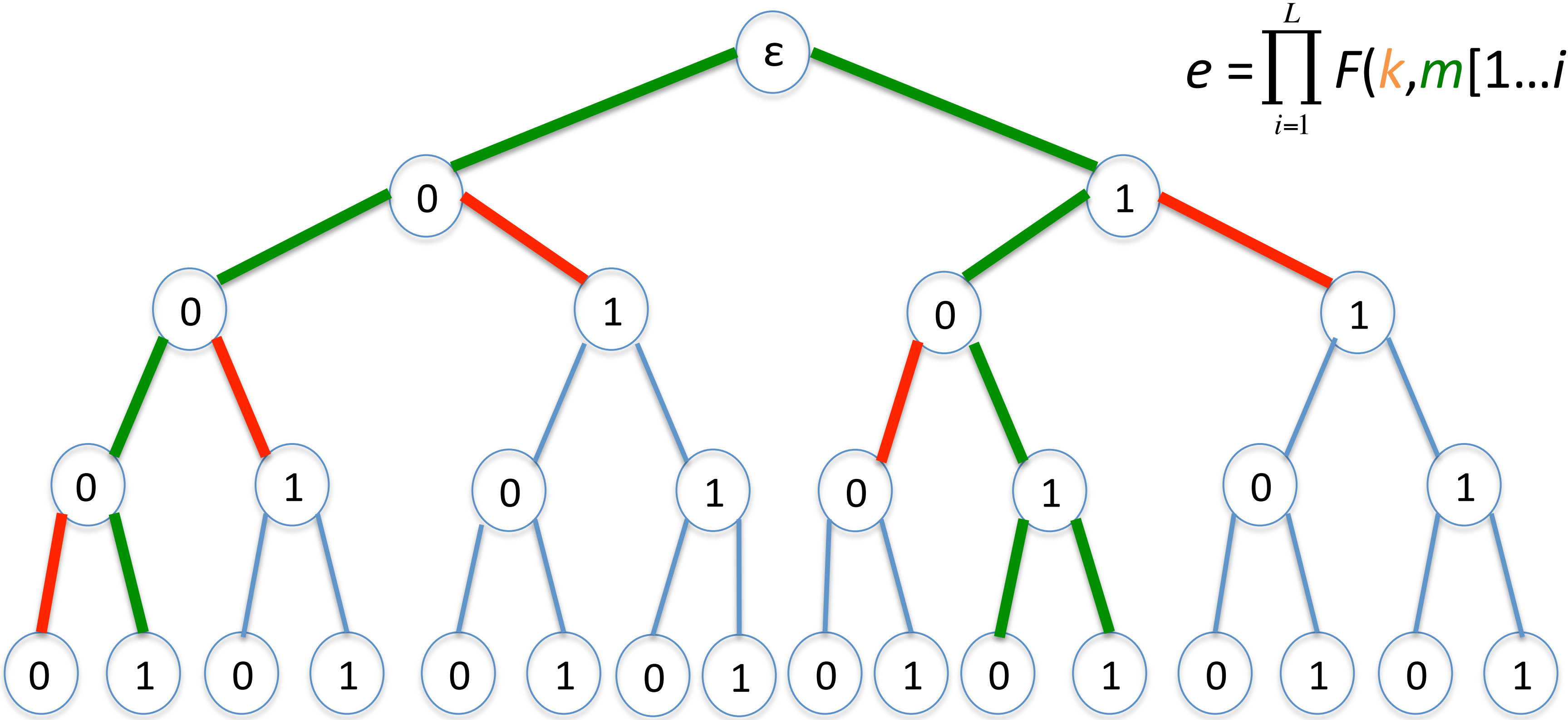

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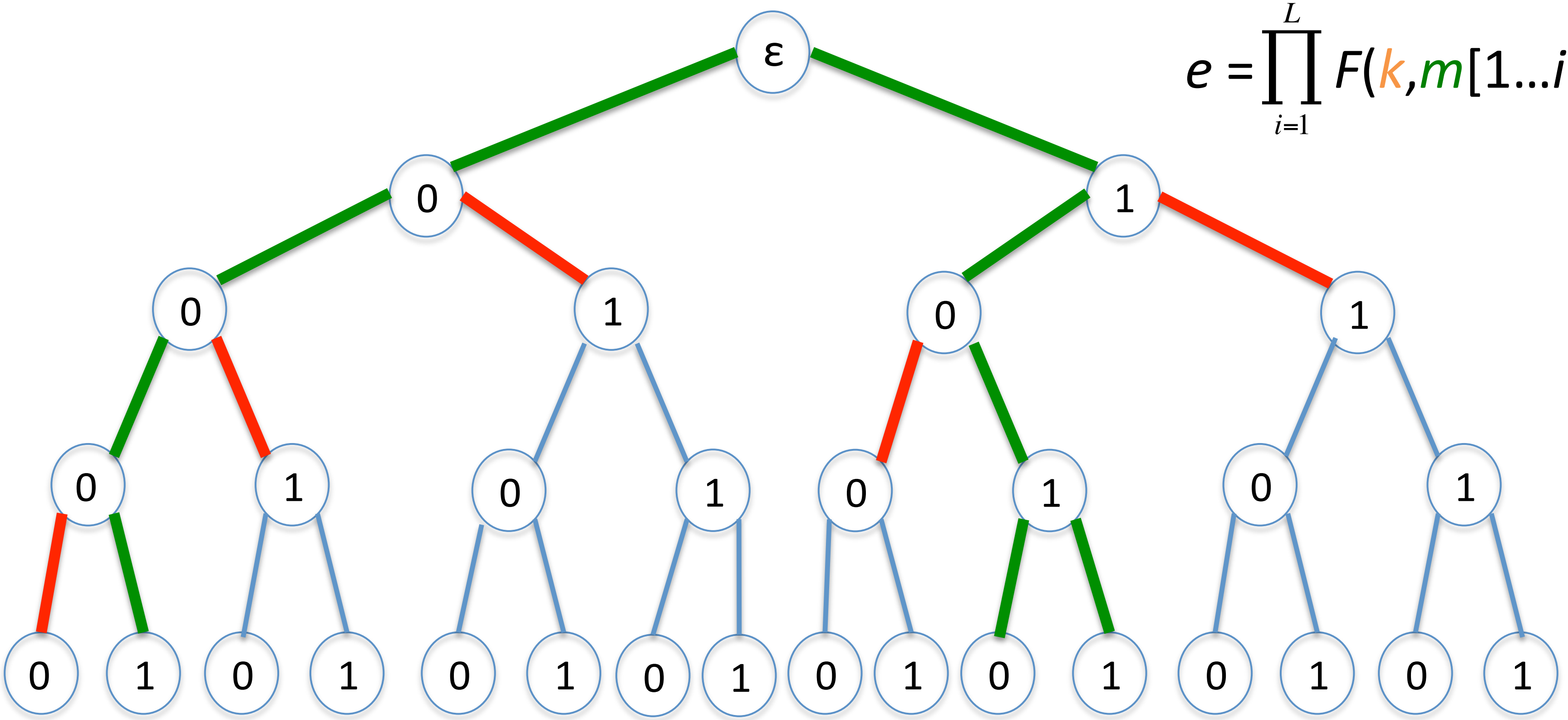

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We do not try to guess the exit branch.

Naïve Approach

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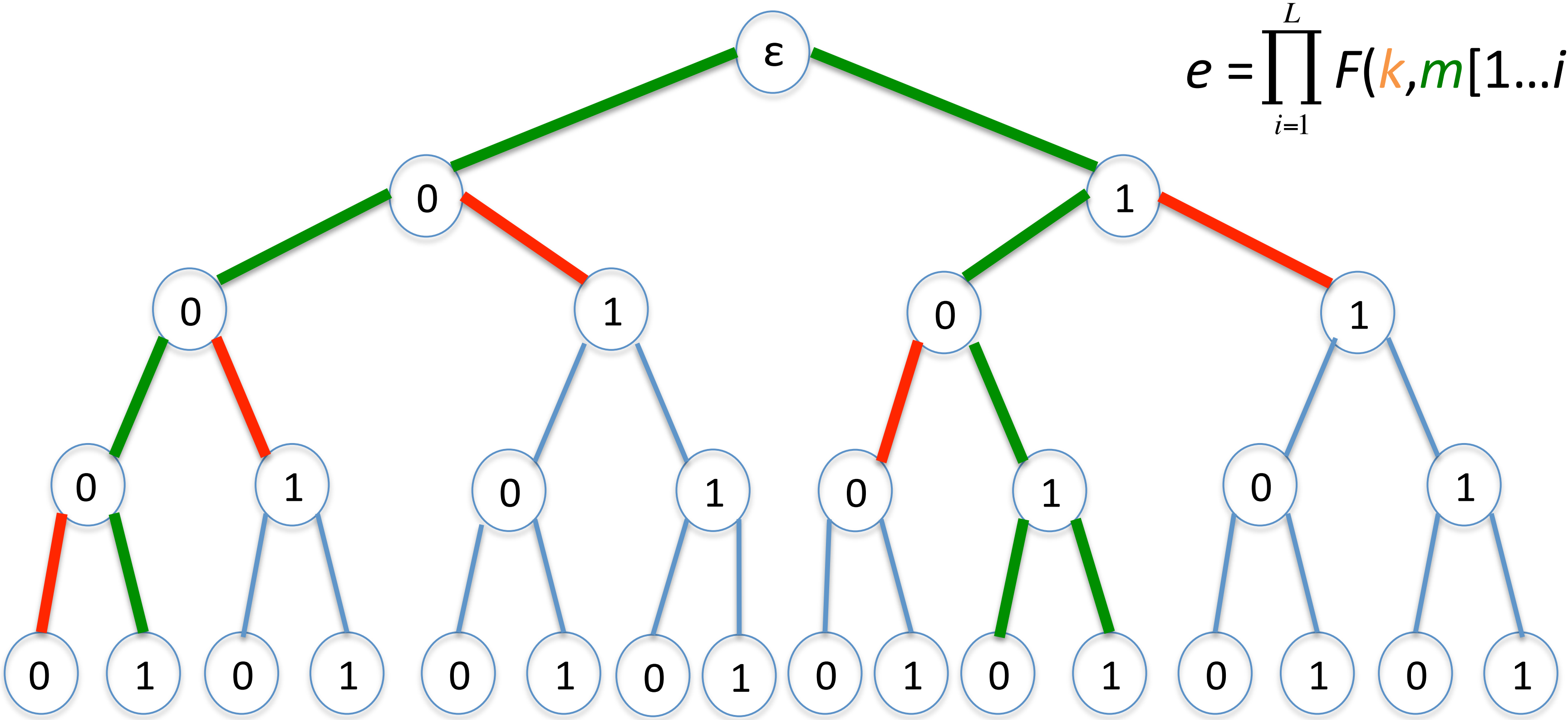

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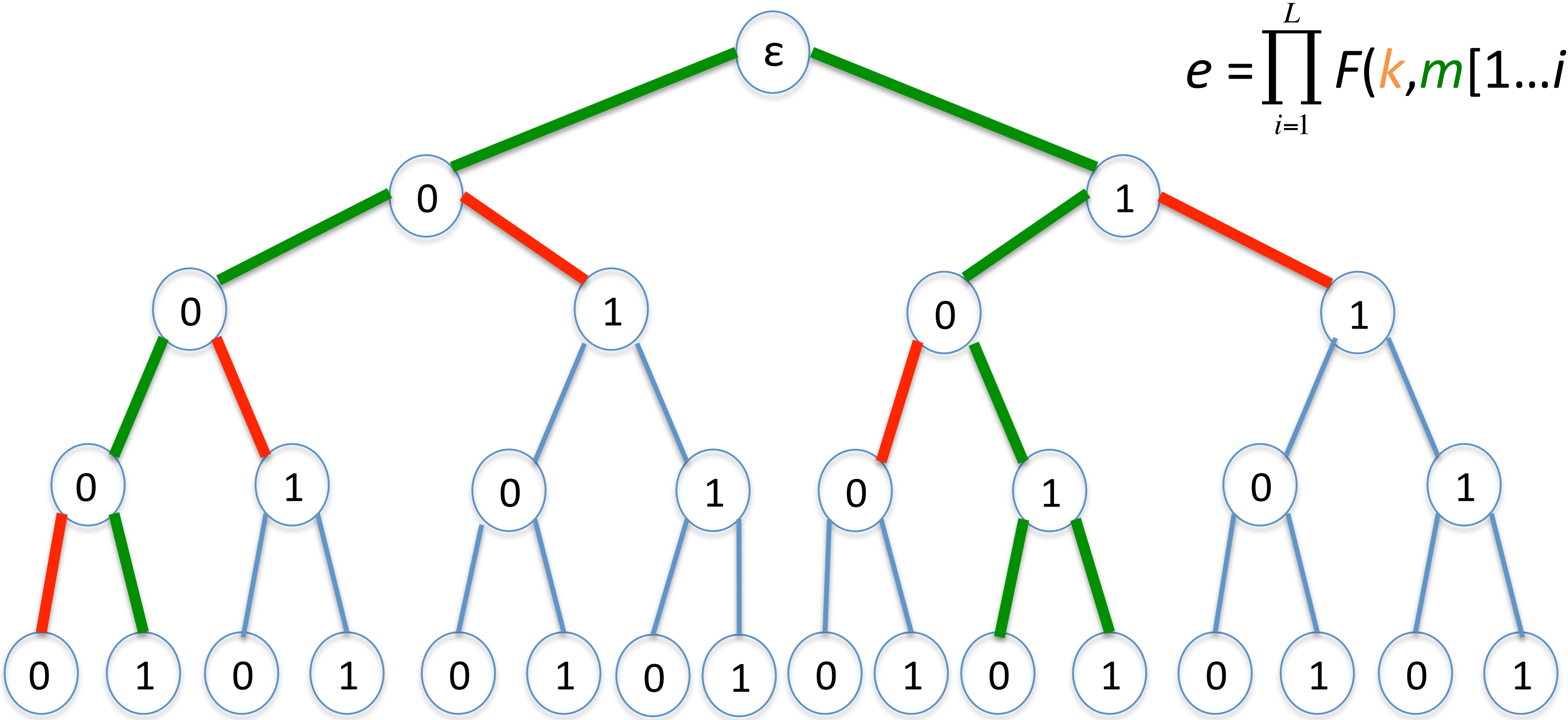

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With overwhelming probability all exit branches are good. Very bad parameters.

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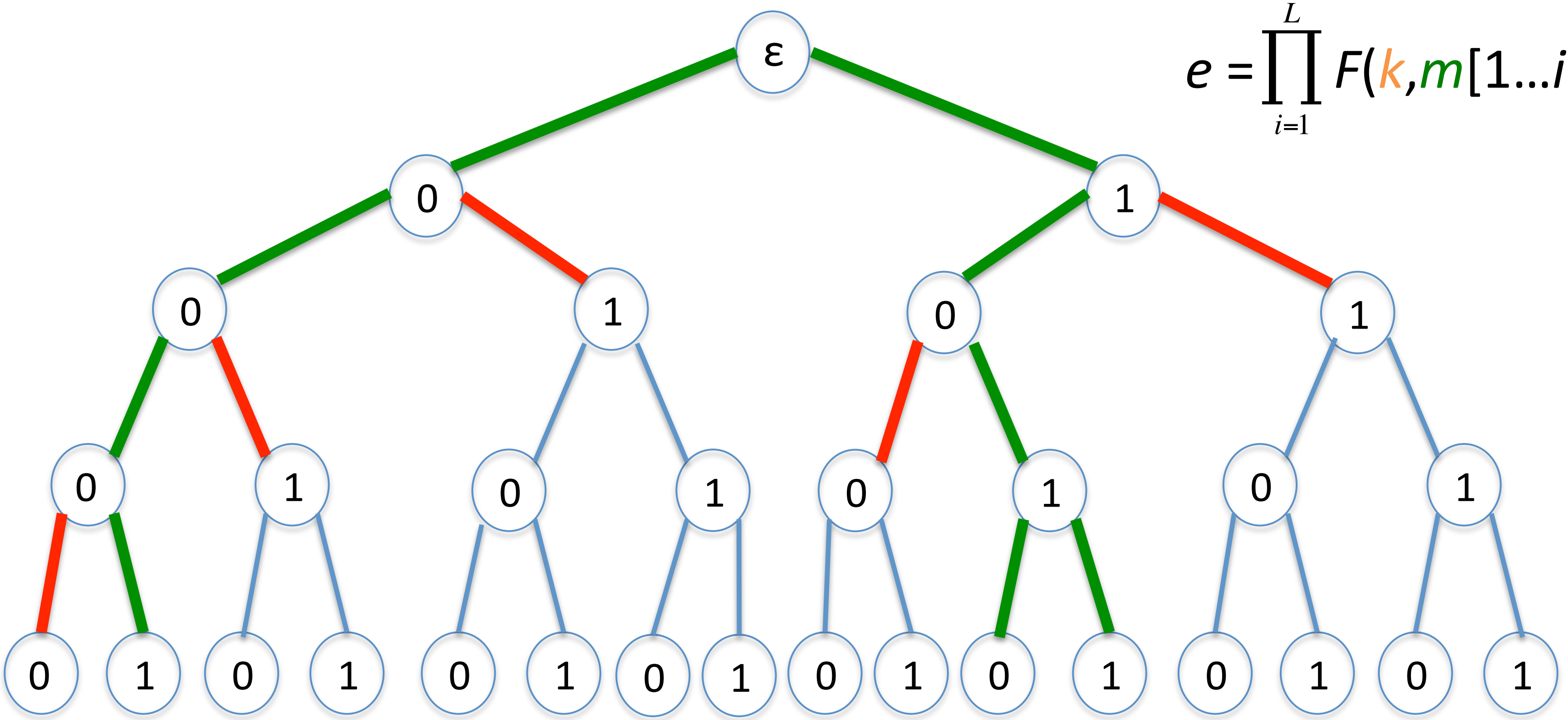

Only require the numbers to be non-α-smooth with constant probability.

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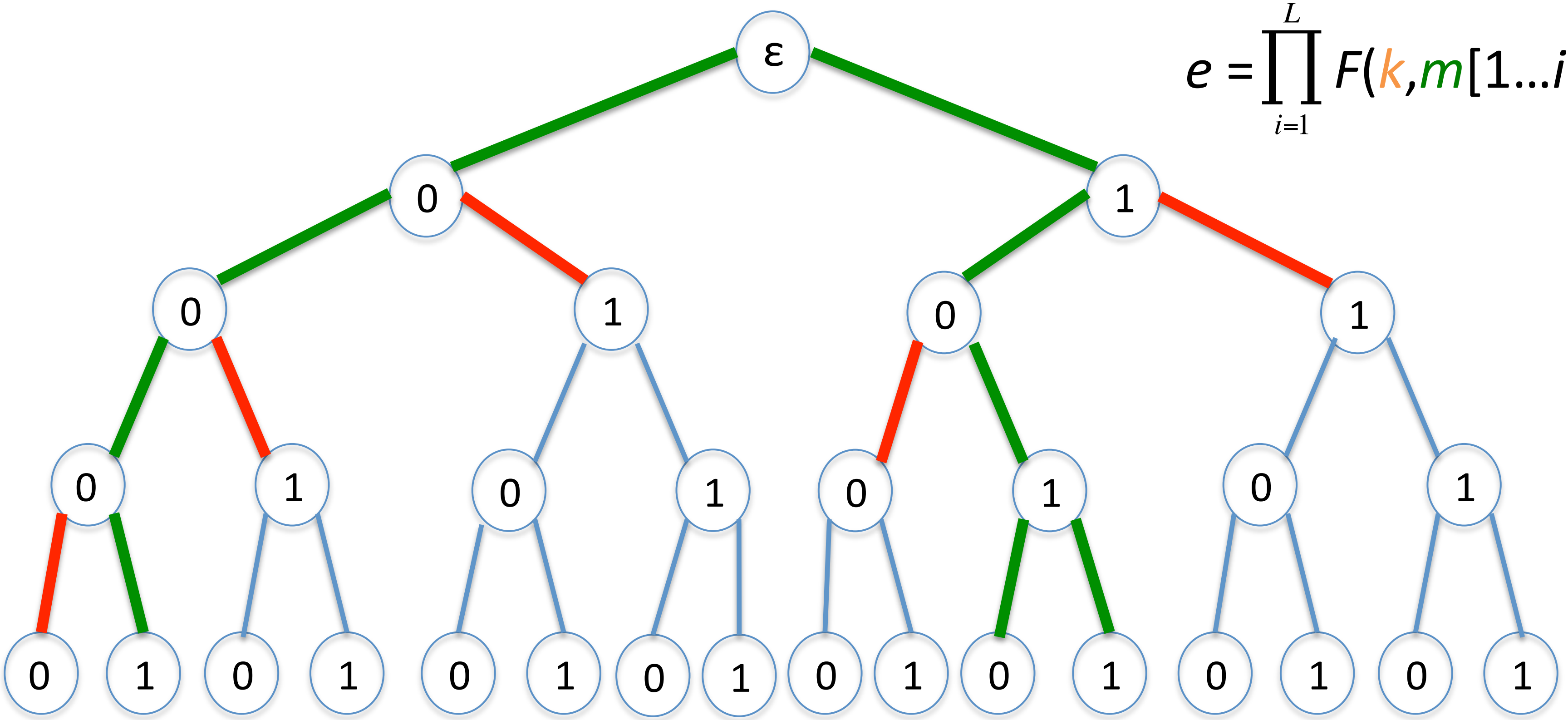

For every possible valid forgery, there should be a non-α-smooth number in its root to message path that is not in the sub-tree of queried messages.

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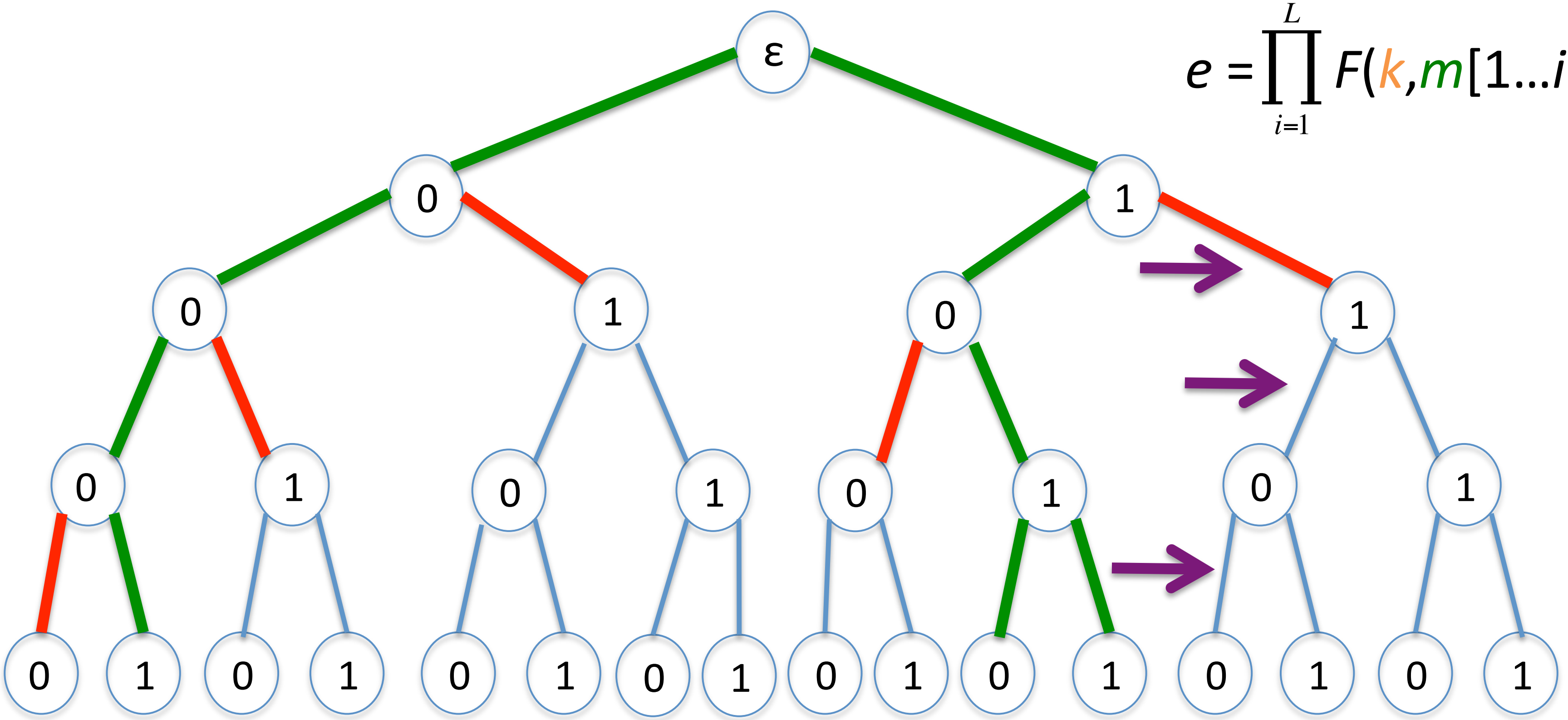

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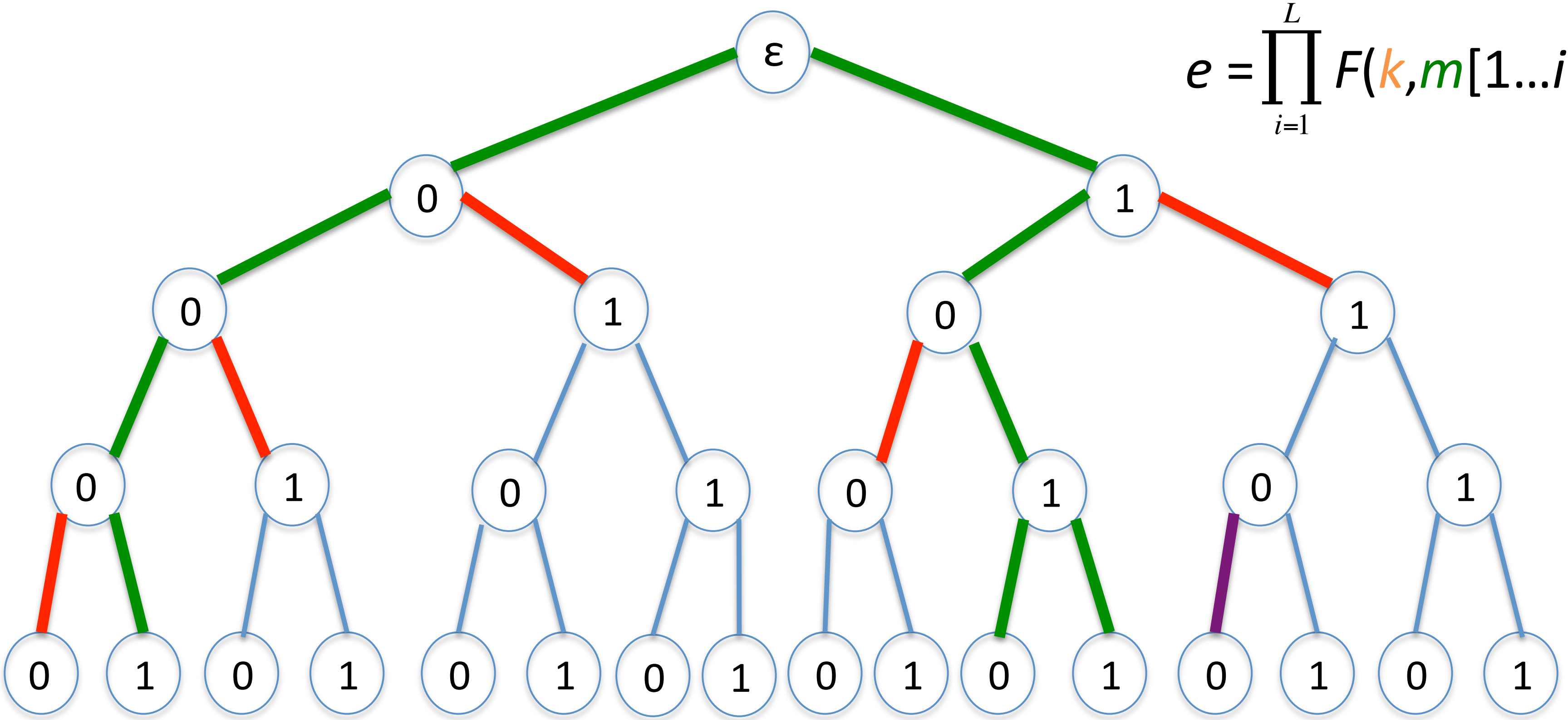

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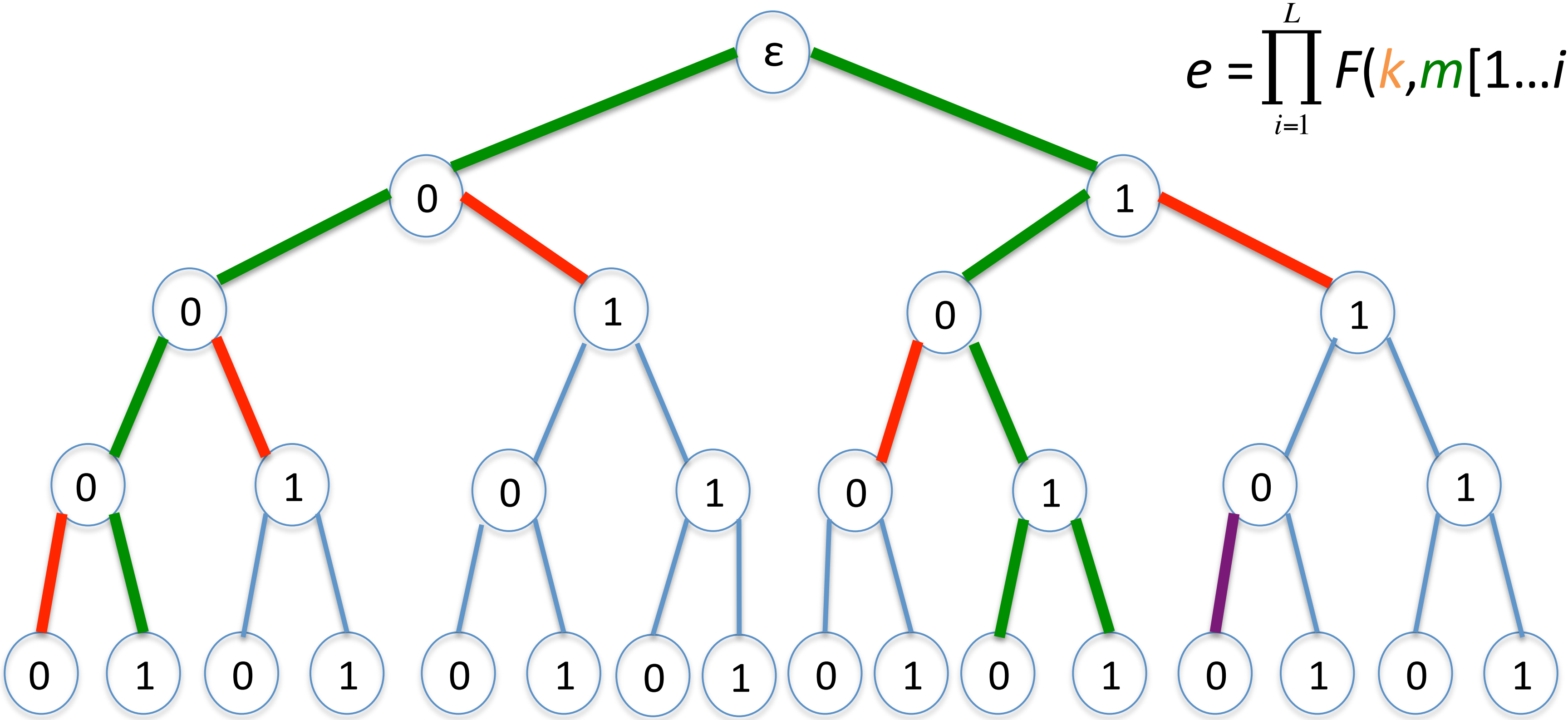

Adapt analysis technique from Gennaro et al. [GHR99] to analyze the probability that a non-smooth number divides the product of some random numbers.

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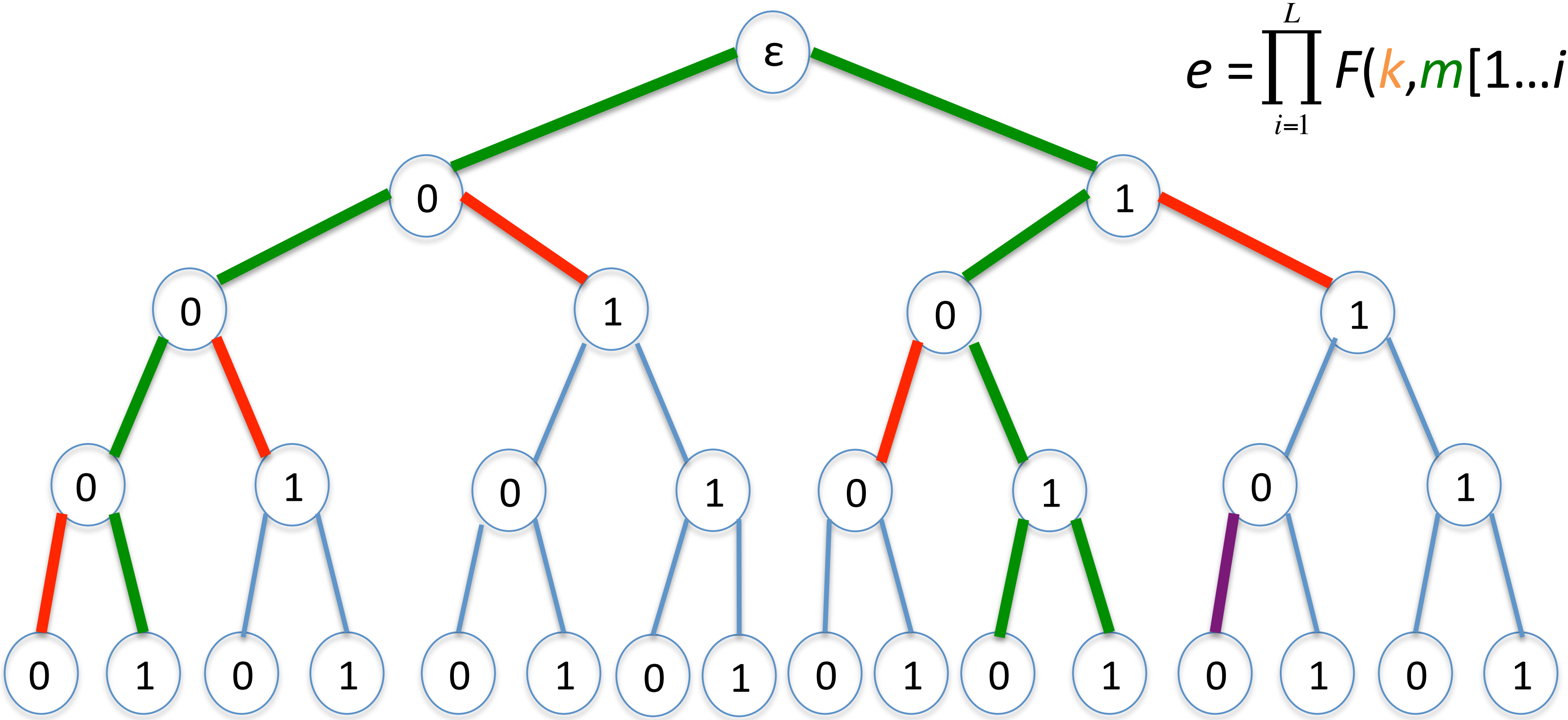

Adapt analysis technique from Gennaro et al. [GHR99] to analyze the probability that a non-smooth number divides the product of some random numbers.

If the number associated with the branch in **purple** does not divide the product of the ones in **green**, then it is possible to solve the strong RSA problem.

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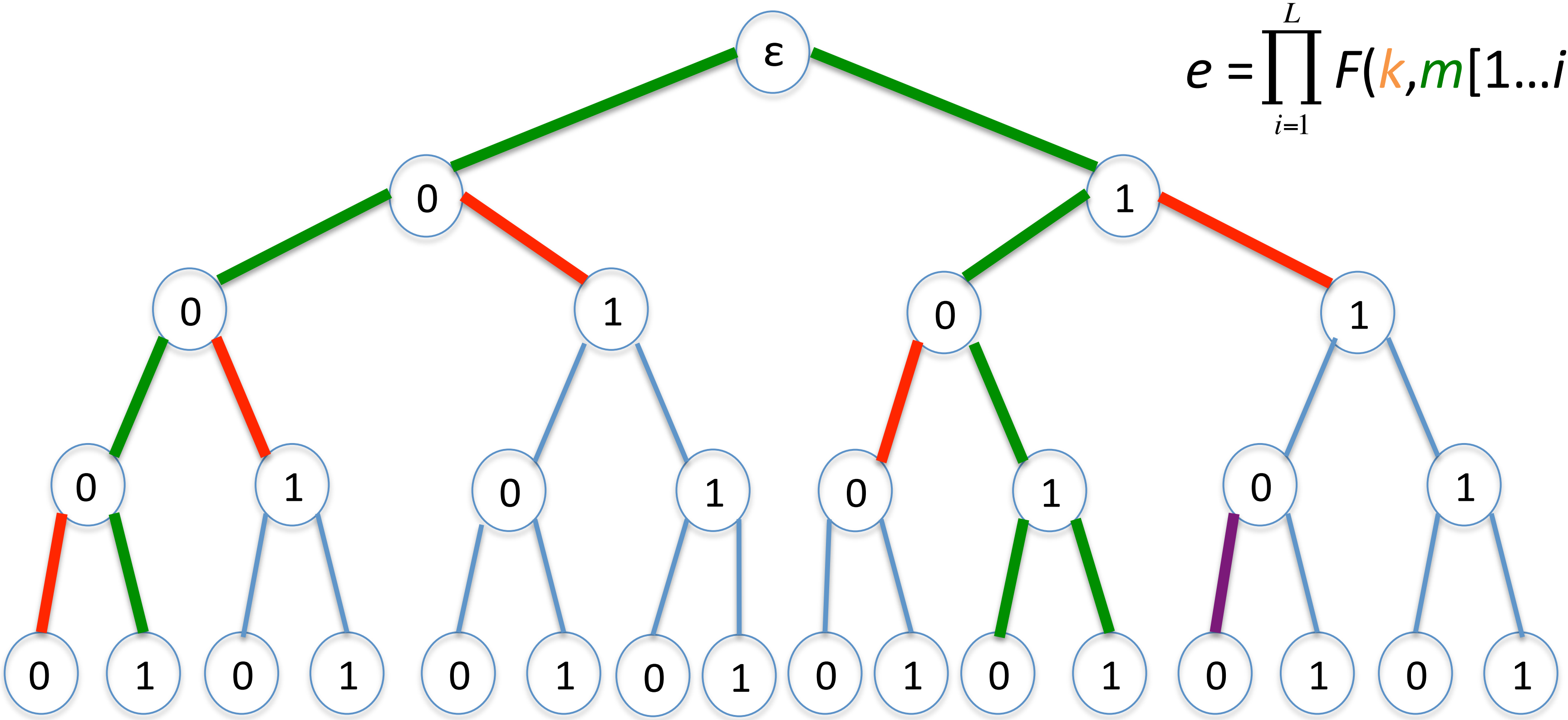

Given an strong RSA instance (n, y) and the signature query, compute g backwards:

$$g = y^{\prod \text{green numbers}} \bmod n$$

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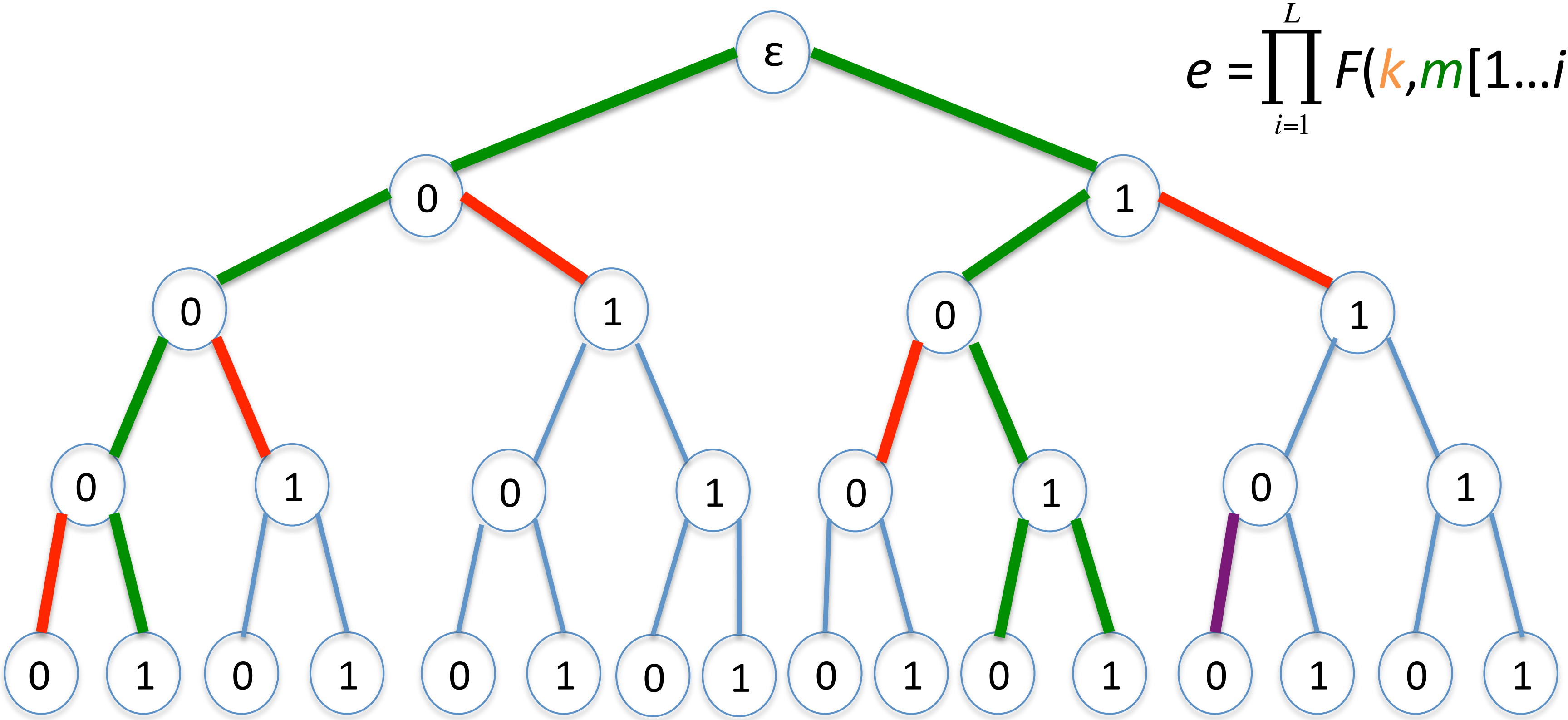

Given a forged signature (m^*, σ^*), it is possible to solve the strong RSA problem if

$$\gcd(e^*, \prod \textit{green numbers}) > 1$$

$$e^* = \prod F(k, m^*[1 \ldots i])$$

Smoothness Analysis

$$Sign(sk, m) = g^{1/e} \bmod n$$

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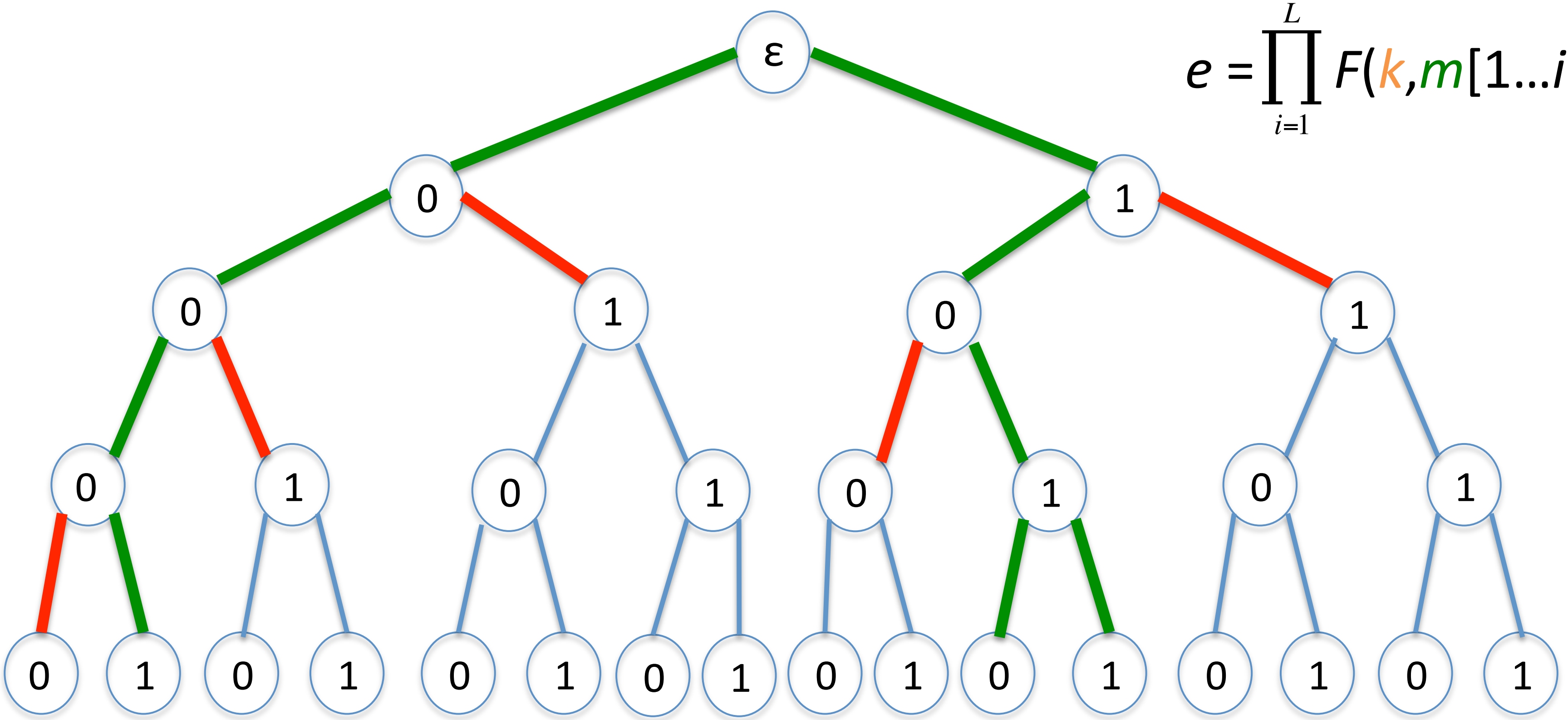

Start analysis with the exit branches. If one of them is smooth, analyze its children and repeat this process recursively.

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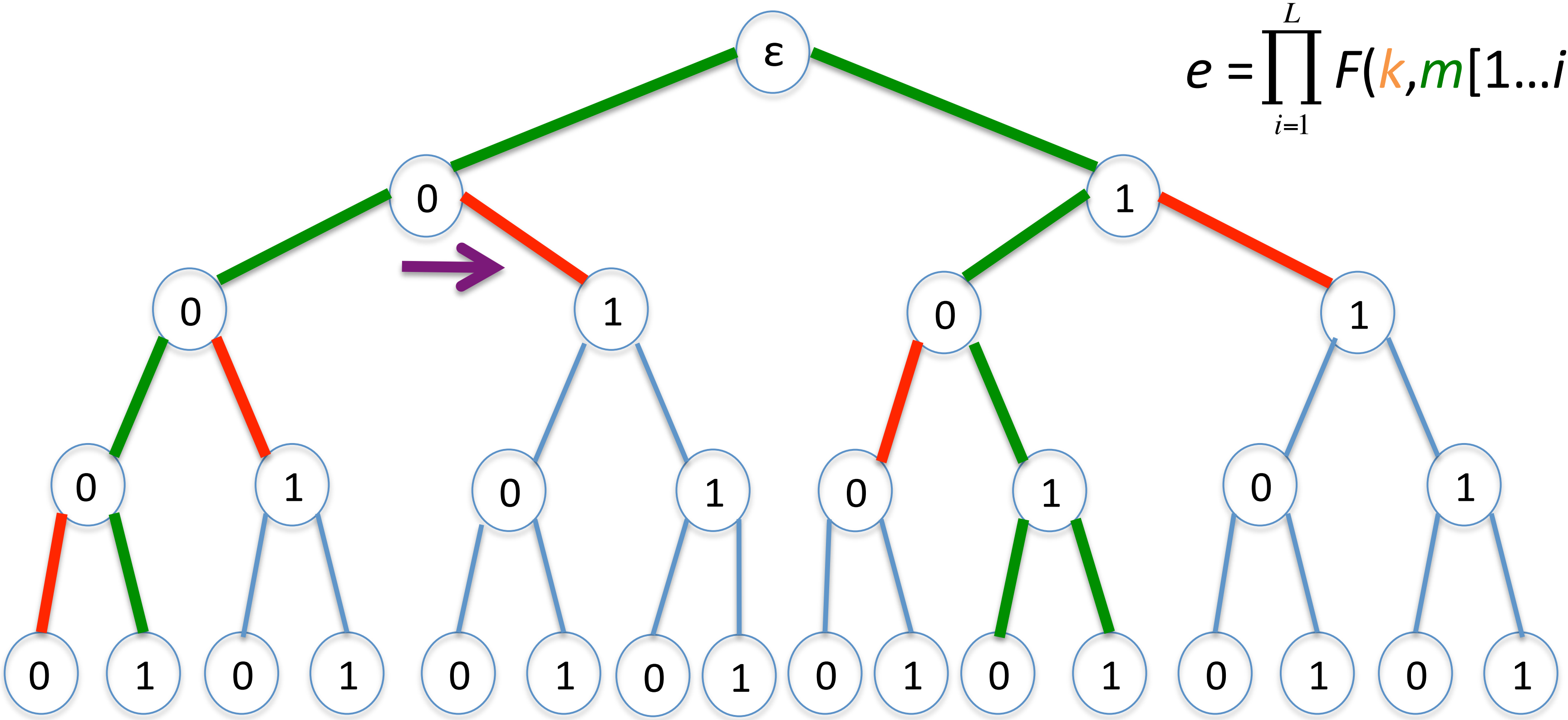

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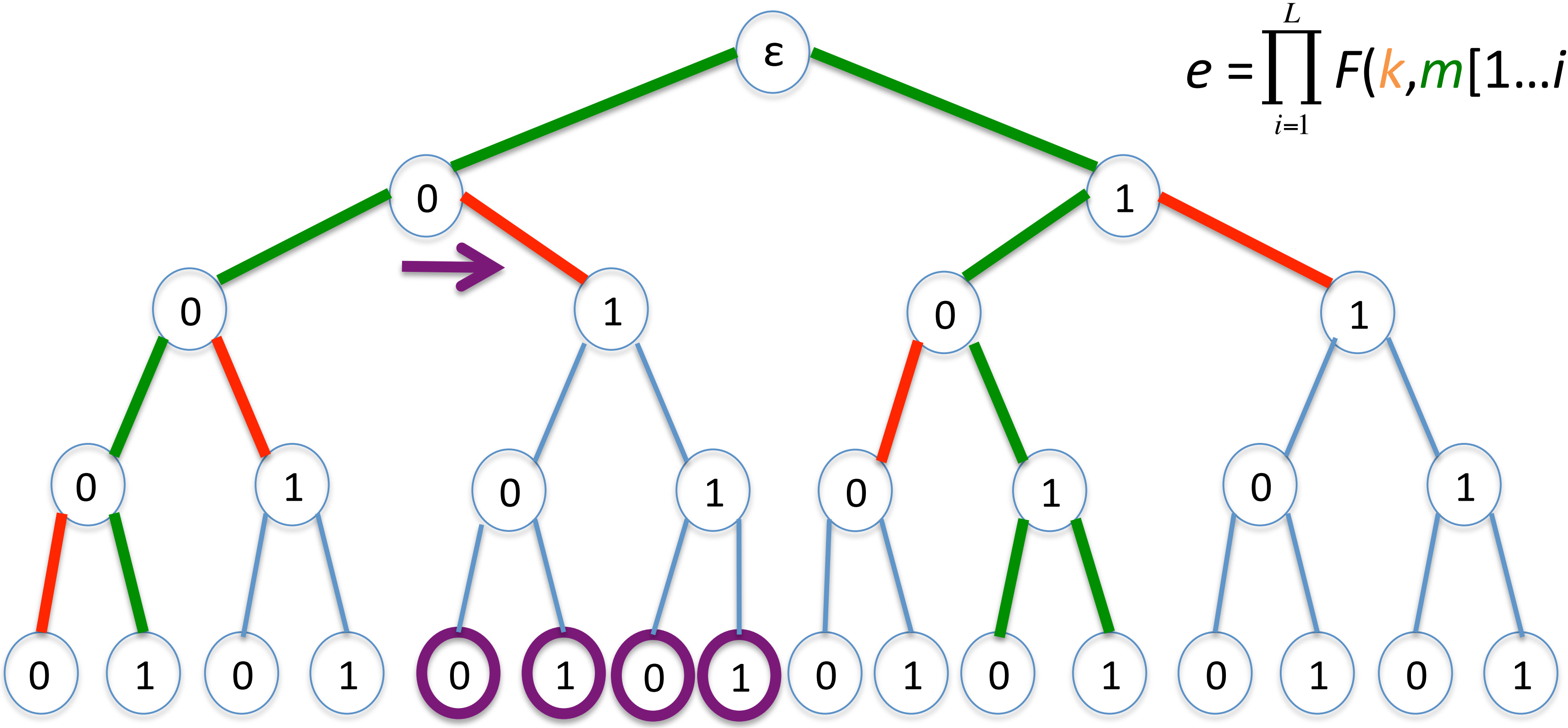

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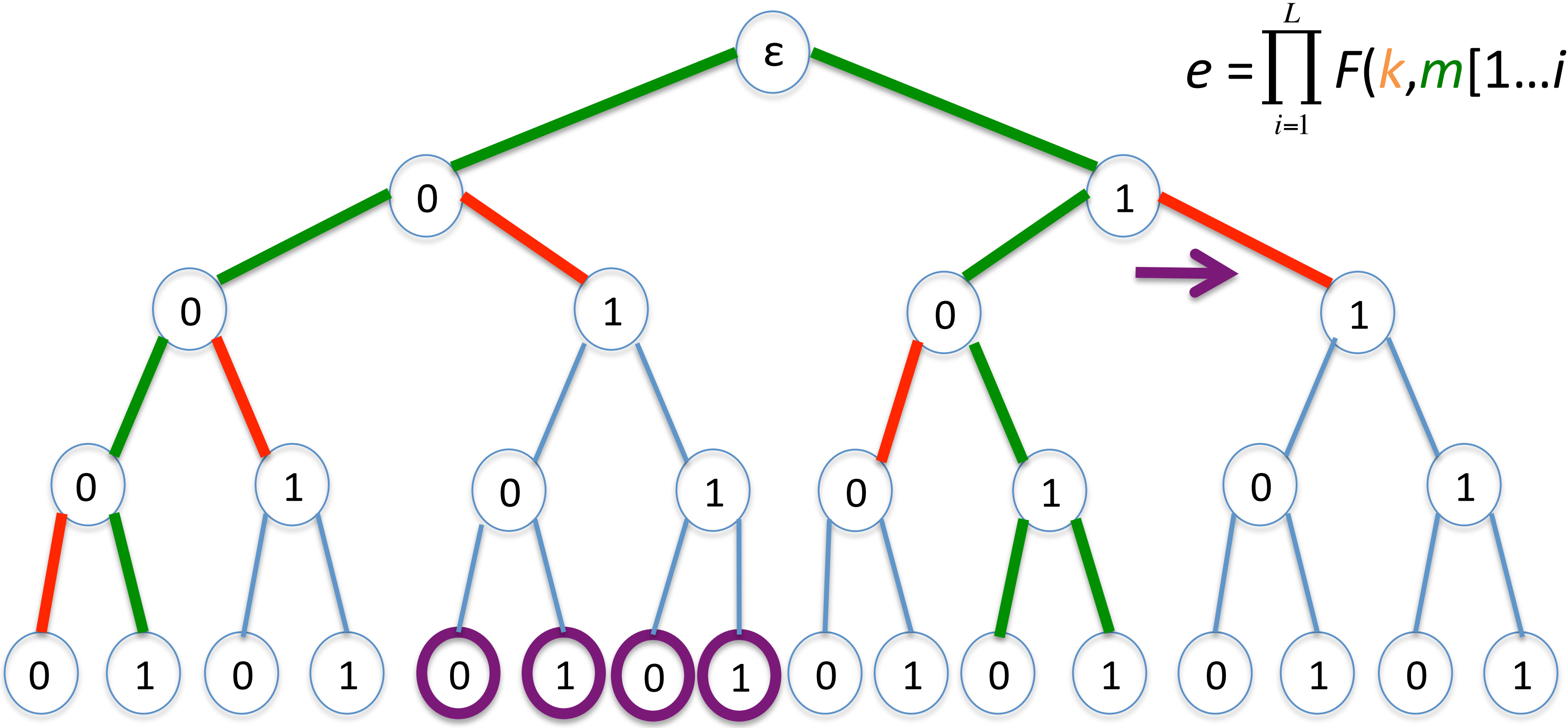

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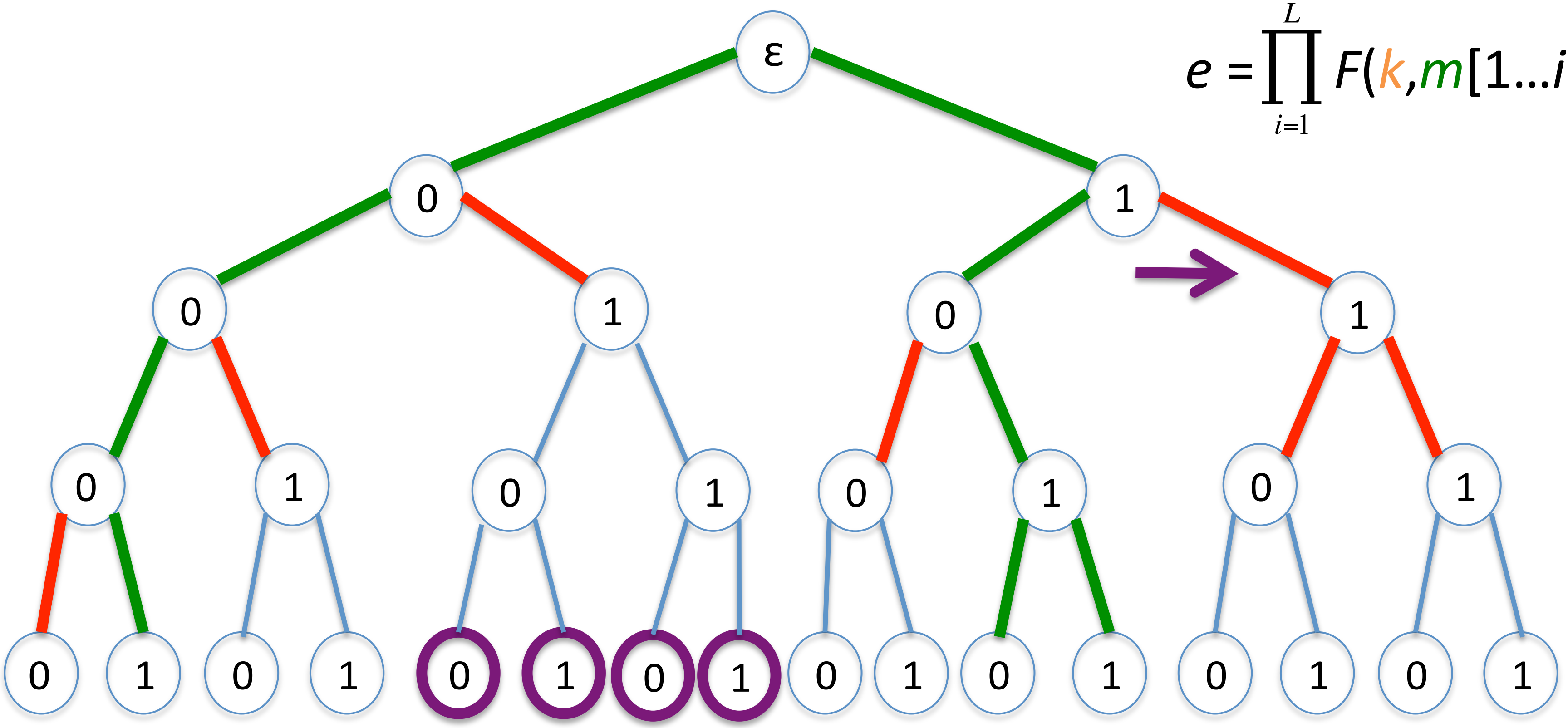

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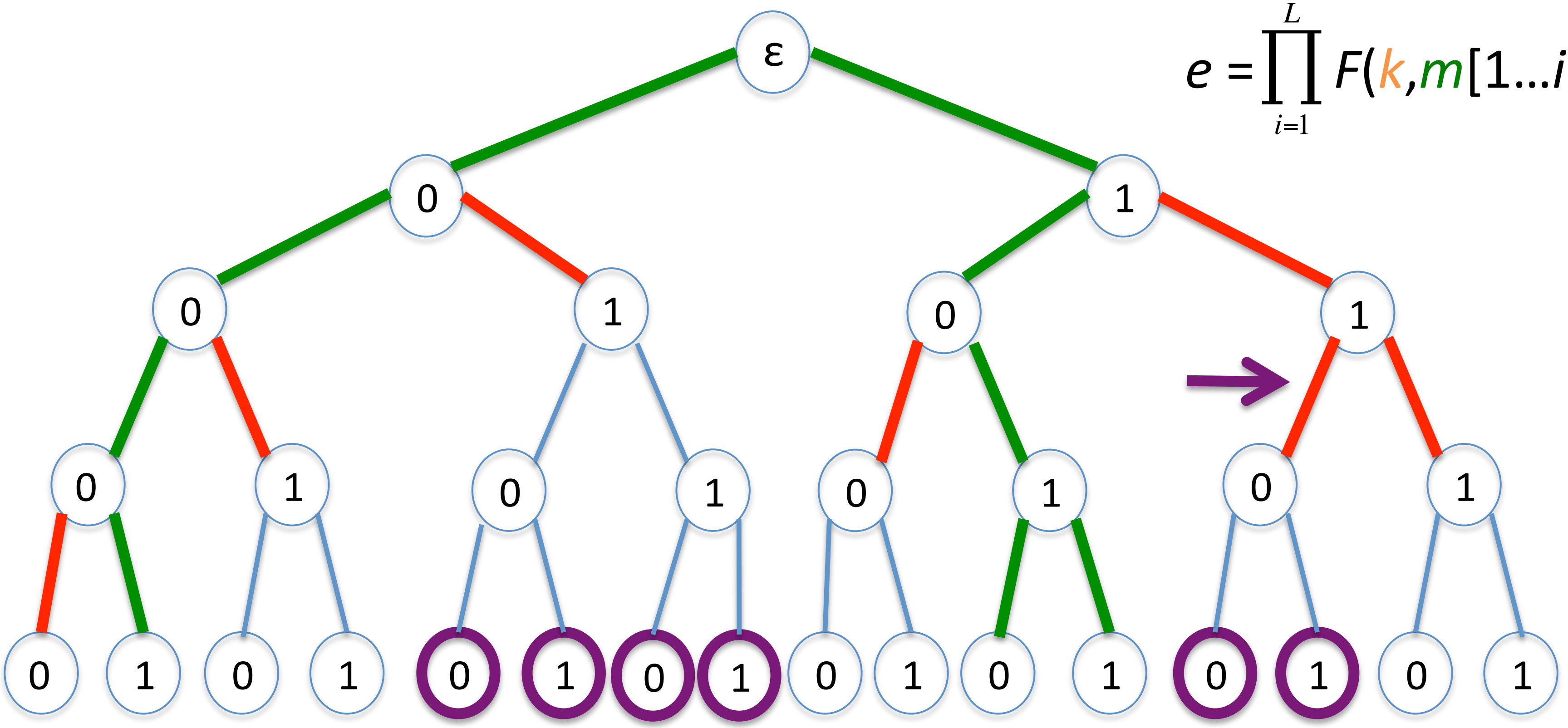

Start analysis with the exit branches. If one of them is smooth, analyze its children and repeat this process recursively.

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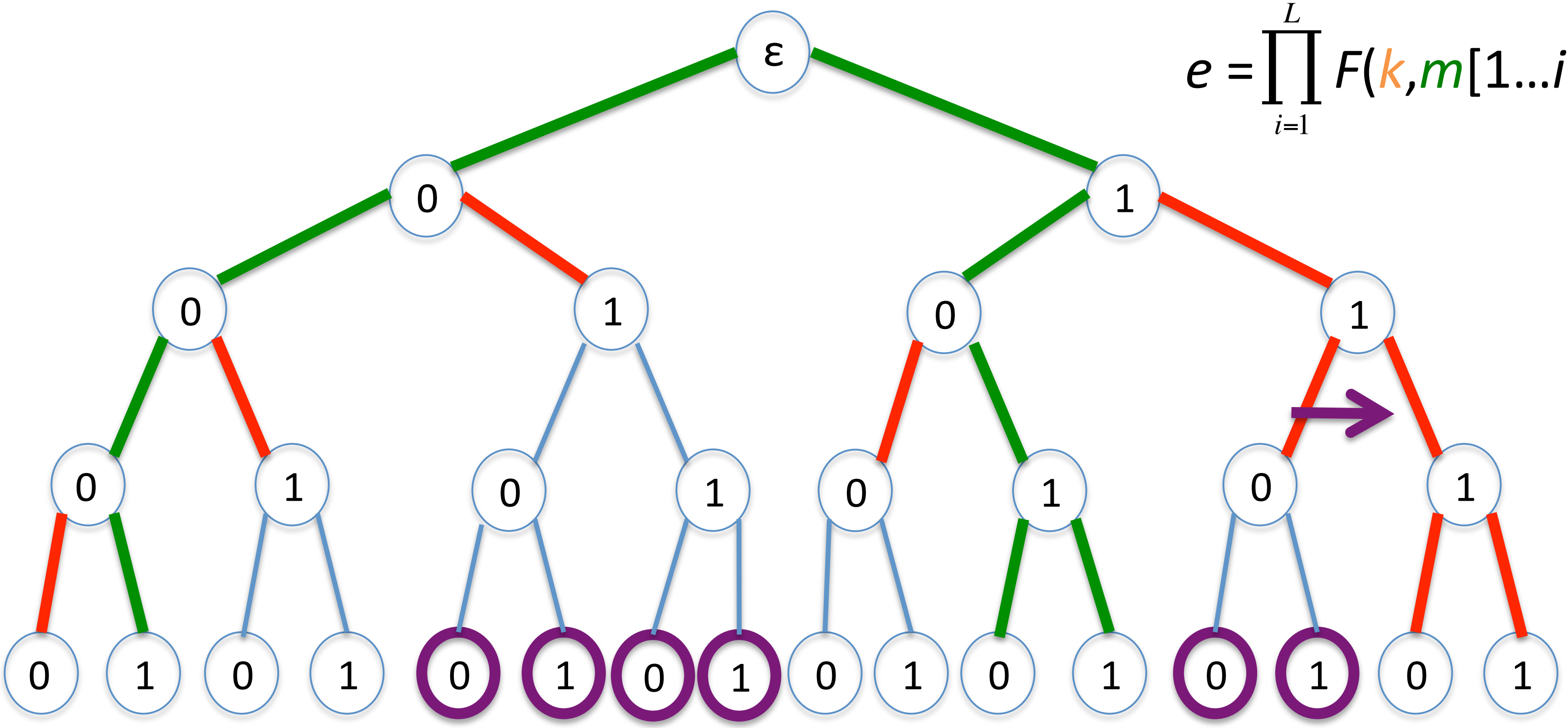

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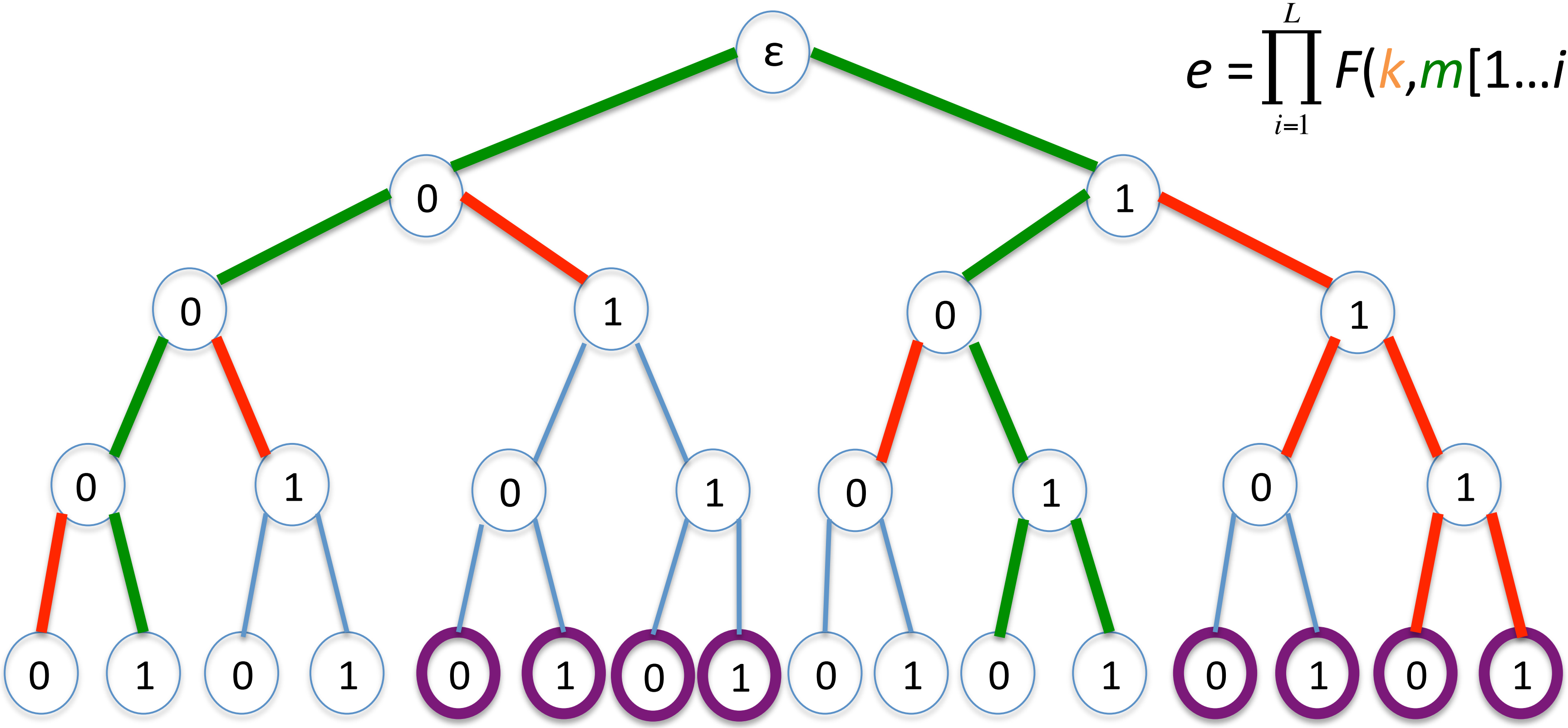

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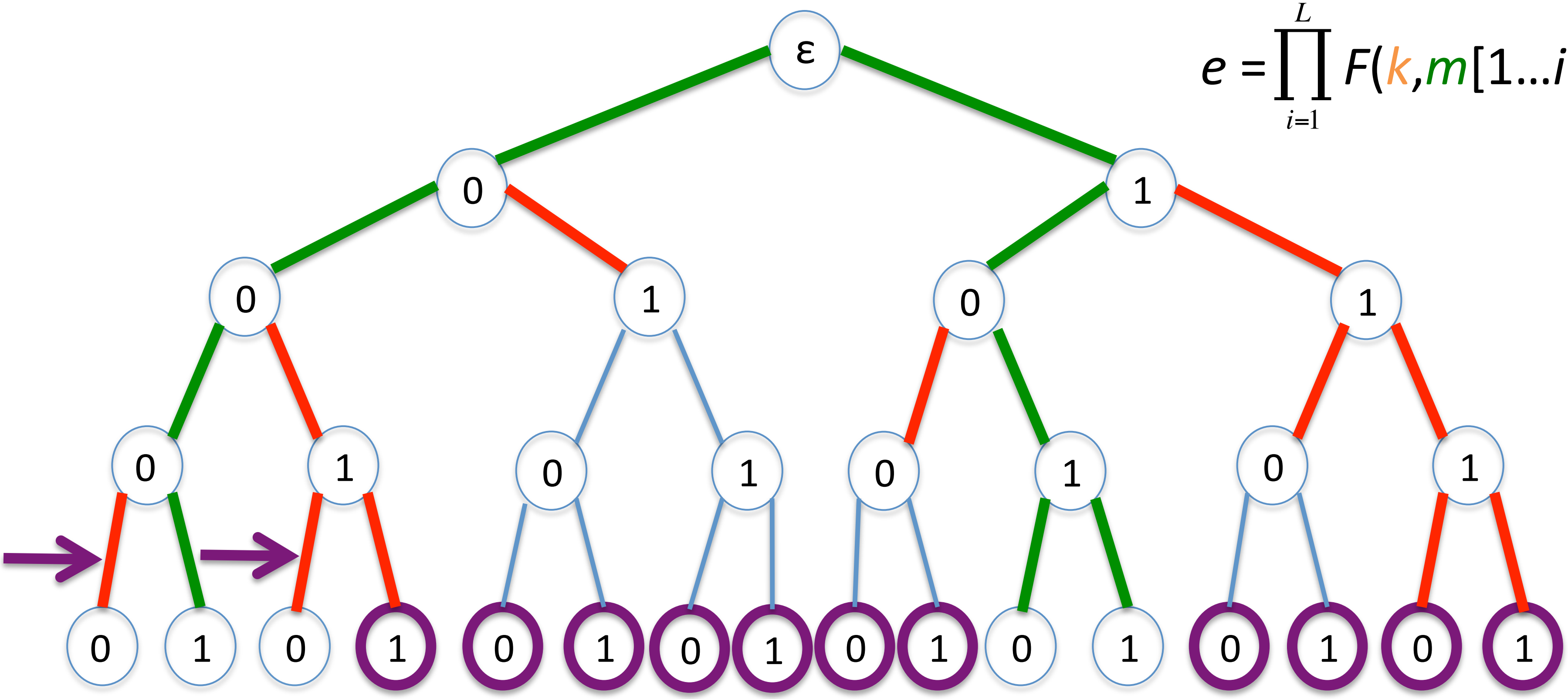

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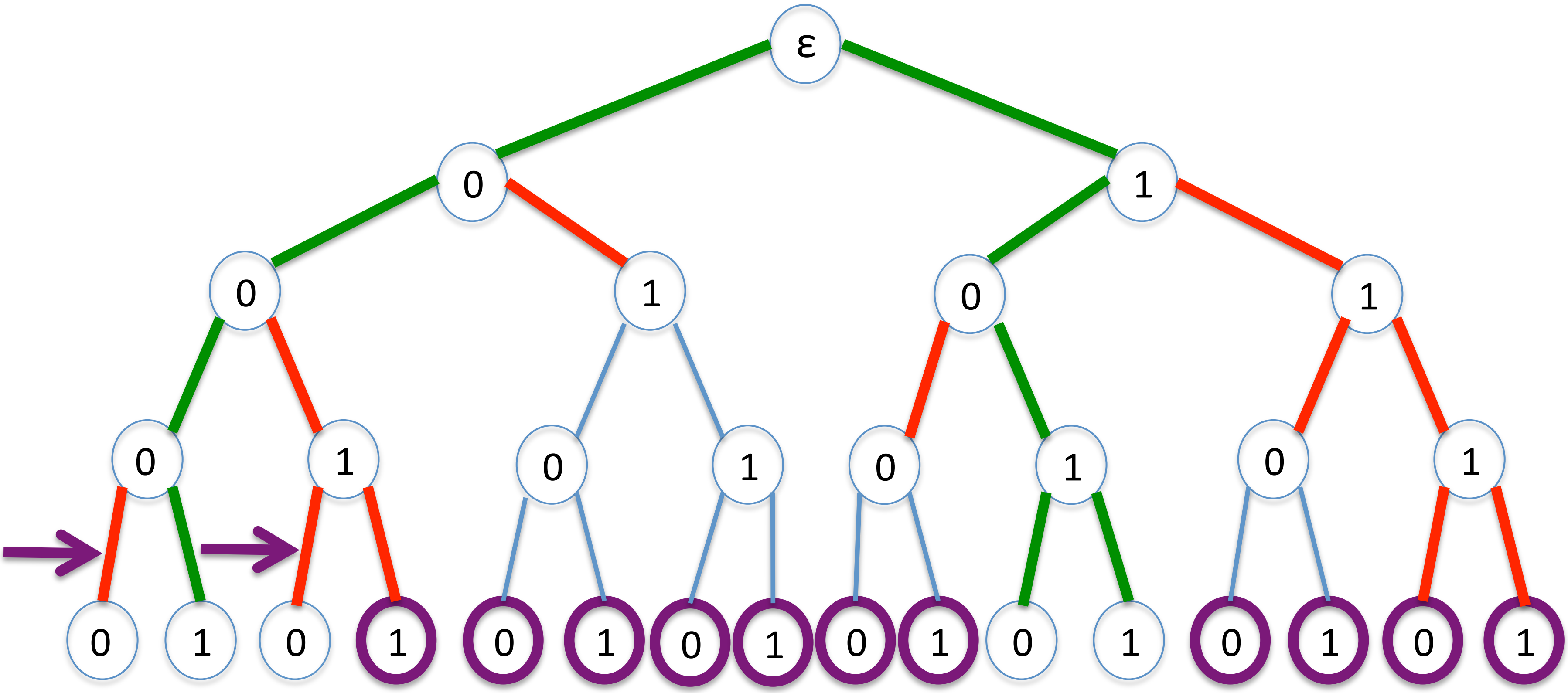
Problem: How to deal with exit branches in the last level/non-finished recursions.

Smoothness Analysis



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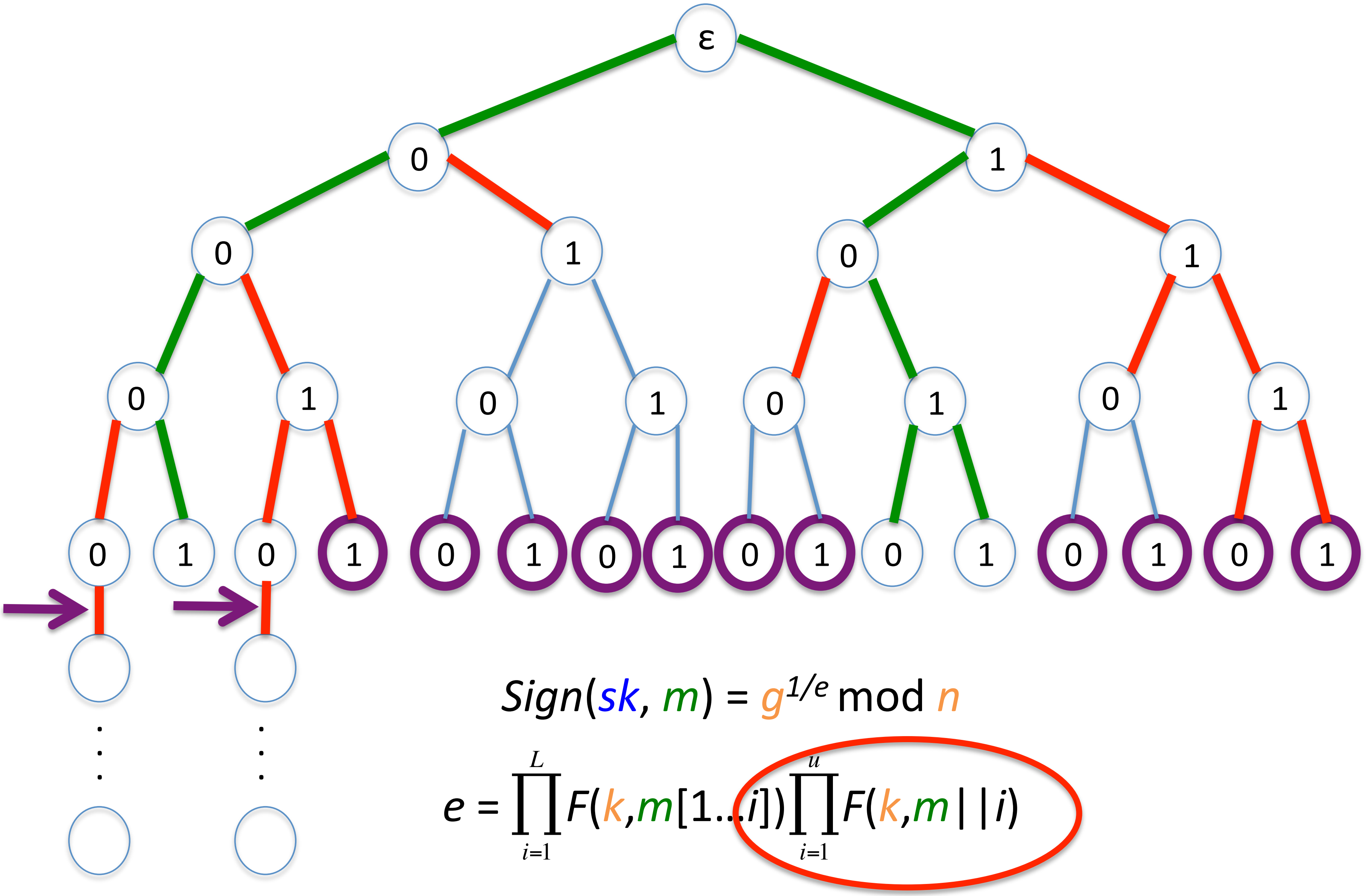

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#Recursions

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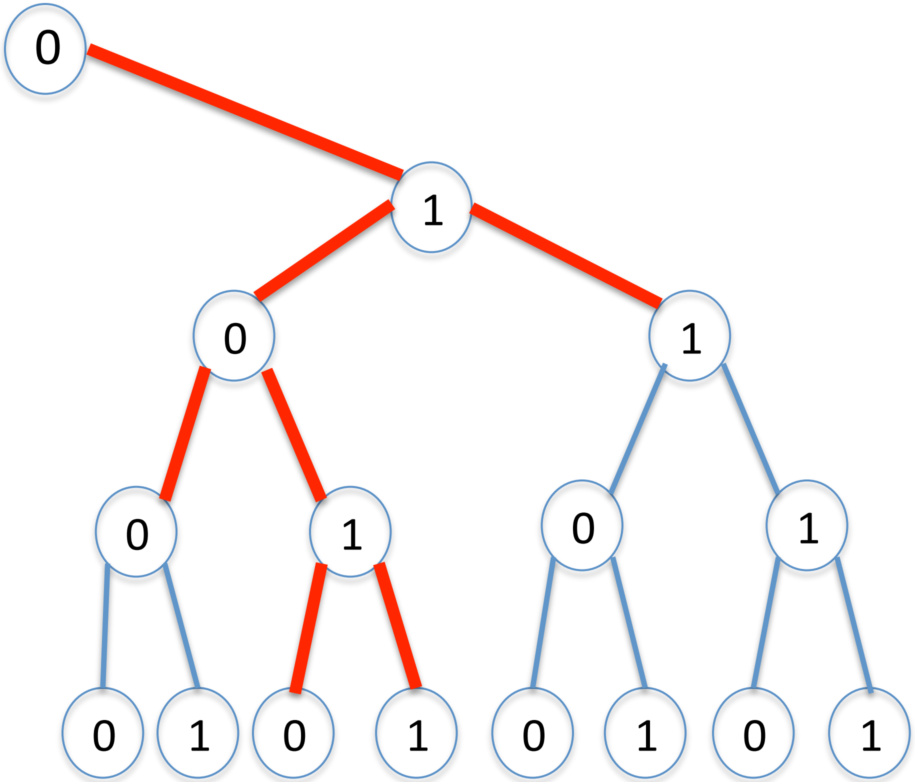

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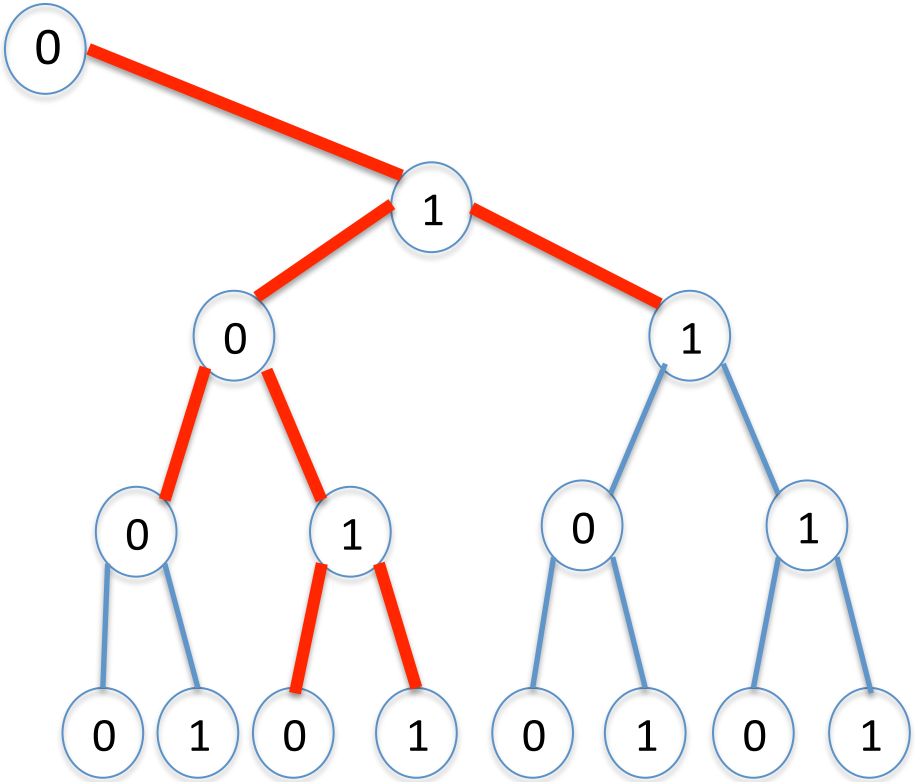

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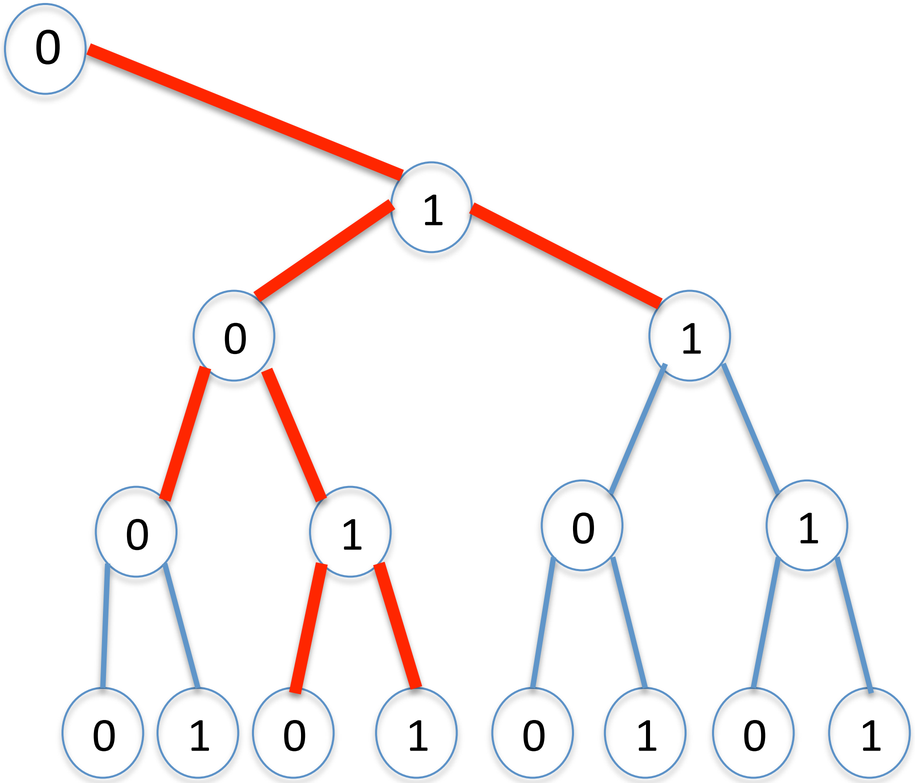

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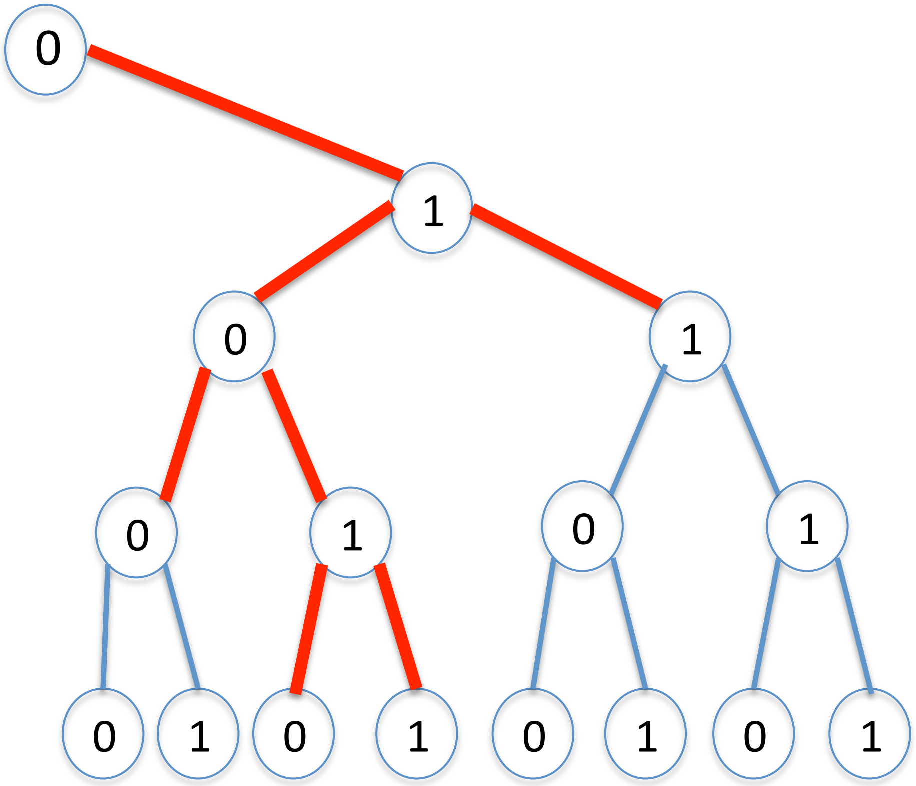

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Upper bound at each level: 2L.

Each level can have at most double the calls as the previous one.

If the previous level had at most L calls, then the current one has at most 2L. Otherwise the previous level had between L and 2L calls and we apply the Chernoff bound to these Bernoulli random variables.

Smoothness Analysis

In the context of elliptic-curve signatures Coron, Handschuh and Naccache [CHN99] avoided point counting by working over larger underlying fields.

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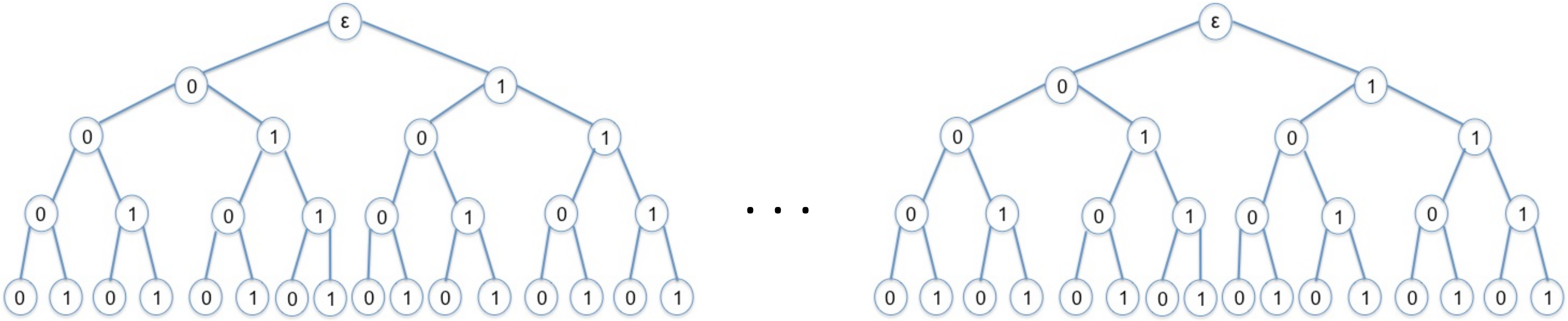

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- ✧ Our scheme does not need to generate primes while signing.
- ✧ Public-key and signature size competitive with other schemes.
- ✧ Weakness: verification performance.
- ✧ Could be a stepping-stone to more practical signatures in the standard model.

Thank You!