# Short Signatures with Short Public Keys From Homomorphic Trapdoor Functions

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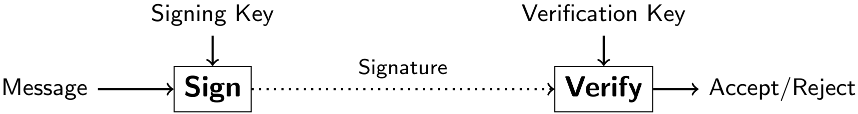
# Stateless Standard-Model Signature Schemes

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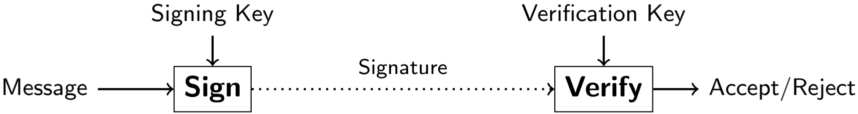


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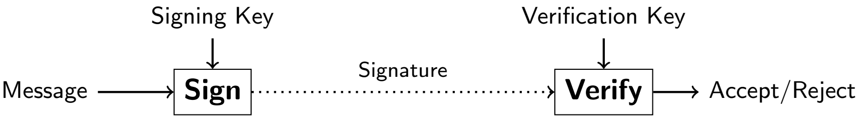


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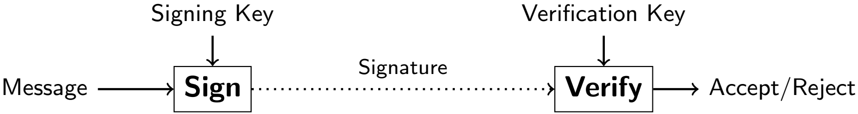


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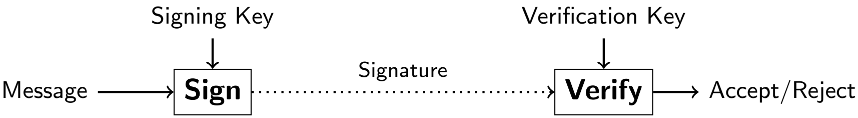


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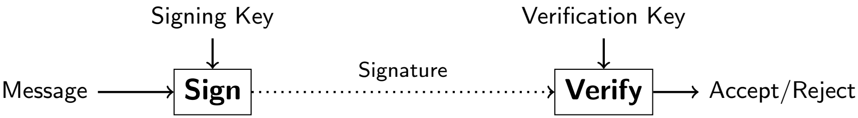


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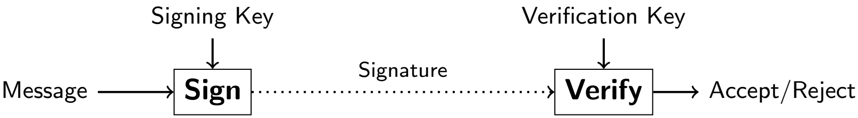


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  - ⋆ Can we do better?

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| Scheme | pk<br>$R_q^{1\times k}$ mat. | sk<br>$R_q^{k\times k}$ mat. | Sig.<br>$R_q^k$ vec. | SIS param<br>$\beta$ |
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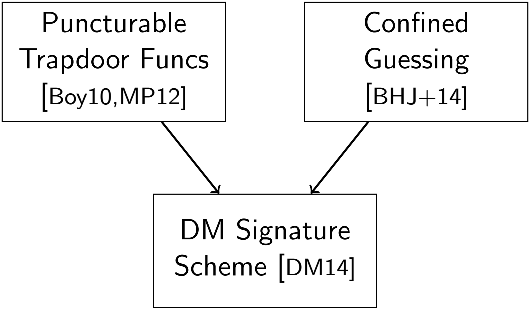
- SIS param can be (large) poly-sized if we set $d = O(\log n / \log\log n)$

# Construction Outline



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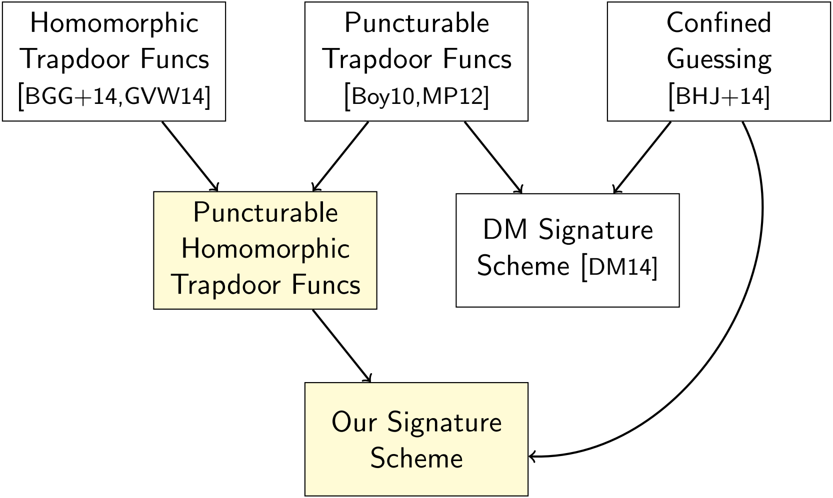

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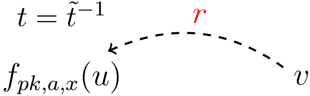


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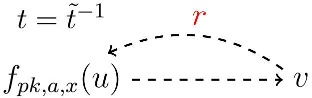


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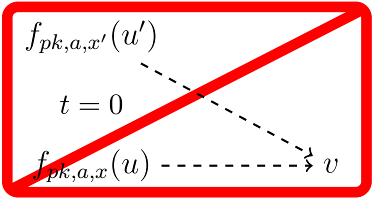


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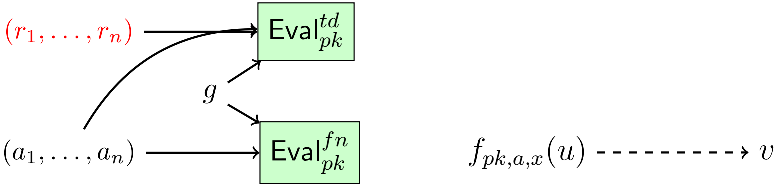


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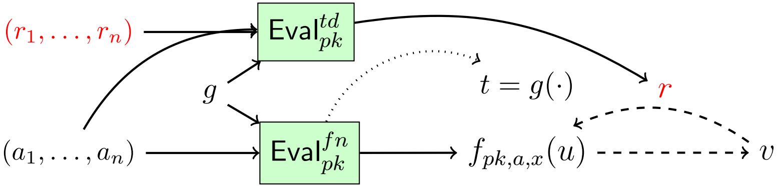


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- Homomorphic Properties
  - Can evaluate funcs $g$ over tags $t_i$ associated with $a_i$
  - Yields new trapdoor function $a$ with tag $t$, trapdoor $r$

# Lattice-Based Construction of PHTDFs

$$f_{(\mathbf{A},\mathbf{B}),-\mathbf{A}\mathbf{R}+t\mathbf{G},\mathbf{x}}(\mathbf{u}) \dashrightarrow \mathbf{v} := [\mathbf{A} \mid -\mathbf{A}\mathbf{R} + t\mathbf{G}]\mathbf{u} + \mathbf{B}\mathbf{x}$$

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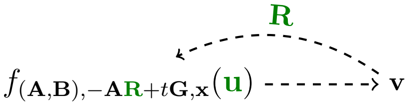


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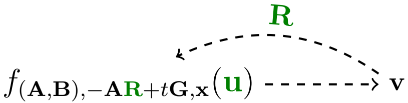


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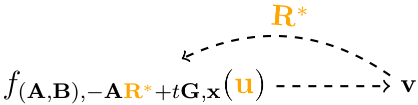


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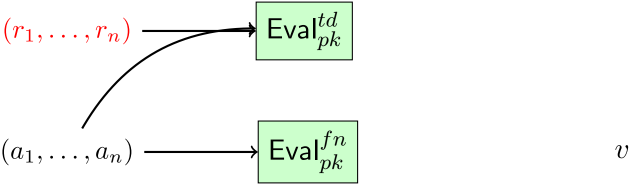
- Larger Trapdoors $\rightarrow$ larger pre-images, larger SIS solutions.

# Signatures from PHTDFs

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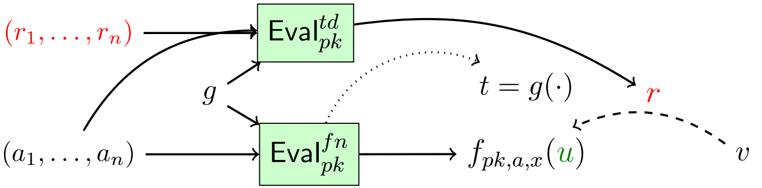


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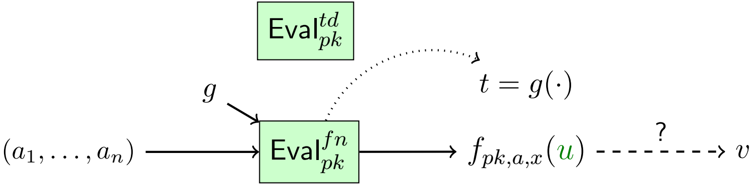


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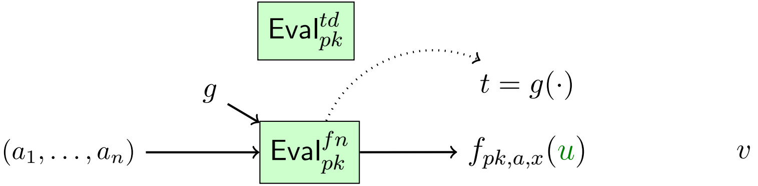


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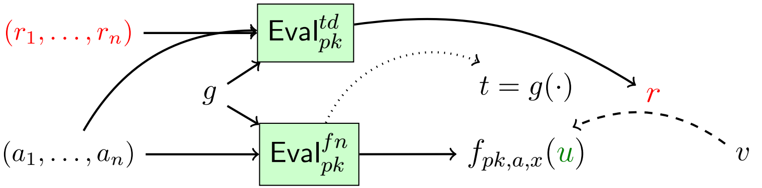


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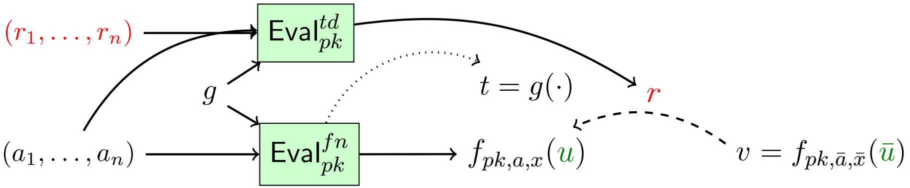


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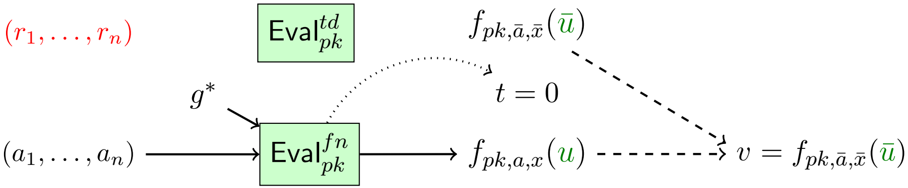


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  2. $\mathcal{A}$ chooses $g^*$ for forgery such that $t = g^*(\cdot) = 0$

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$$t^{(g)} = \underbrace{\underbrace{\underbrace{\underbrace{0}_{t_1^{(g)}}\;1}_{t_2^{(g)}}\;0\;1}_{t_3^{(g)}}\;1\;0\;0\;1}_{t_4^{(g)}}\;\cdots$$

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  - ⋆ Key Point: Embedding of $t^{(g)} - t^{(g')}$ is invertible for $g \neq g'$.

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- Trapdoor $a_0$ in public key has random tag prefix of length $c_{i^*}$.

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| | | | | | | |
|---|---|---|---|---|---|---|
| $a_0$ | $a_1$ | $a_2$ | $\cdots$ | $a_{i^*}$ | $\cdots$ | $a_d$ |
| $\hat{t}_{i^*}$ | $0$ | $0$ | | $1$ | | $0$ |

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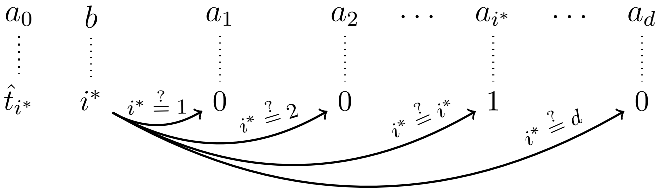


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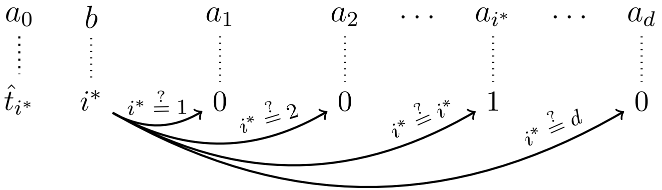


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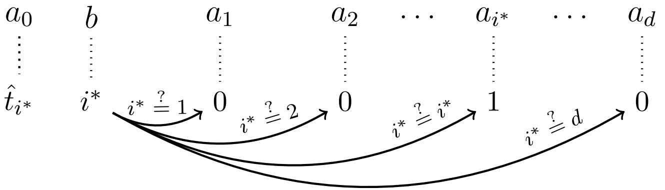


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  - Yields $d^d \operatorname{poly}(n)$ growth instead of $d^d n^{\log d}$ growth

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Hiring? Talk to me!