How Secure is Deterministic Encryption?

Mihir Bellare
Rafael Dowsley
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Deterministic Encryption

Deterministic encryption was introduced by Bellare, Boldyreva and O'neill [BBO07] and offers practical benefits in certain applications such as efficient search on encrypted databases [BBO07] and resilience in the face of low-quality randomness that occurs in many systems [BBN+09,RY10].

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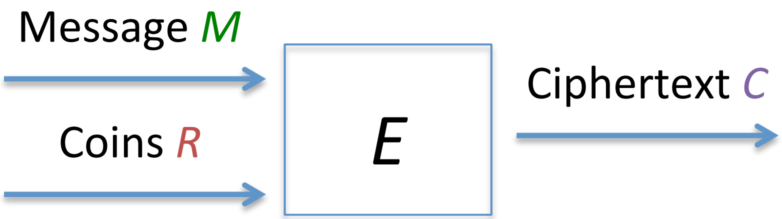

Security Goal: IND-CPA, IND-CCA2

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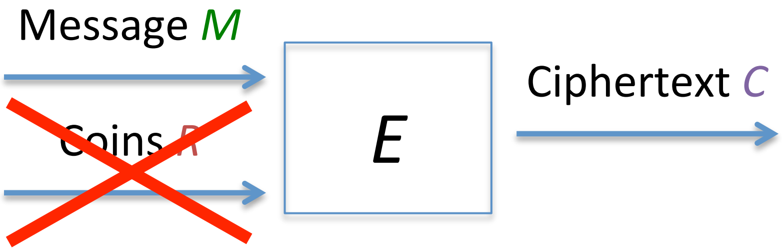

Message M

~~Coins R~~

E

Ciphertext C

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Best Possible!

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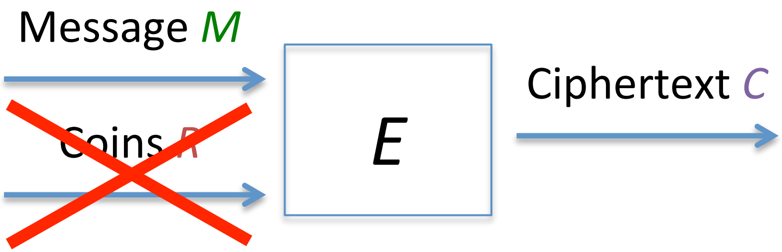

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Best Possible!

Security can be formalized using the PRIV definition [BBO07] or equivalently an IND-style definition [BFOR08], but these definitions are unusual.

Security Definition

Adversary A_M

Challenger

Adversary A_G

M_0,M_1

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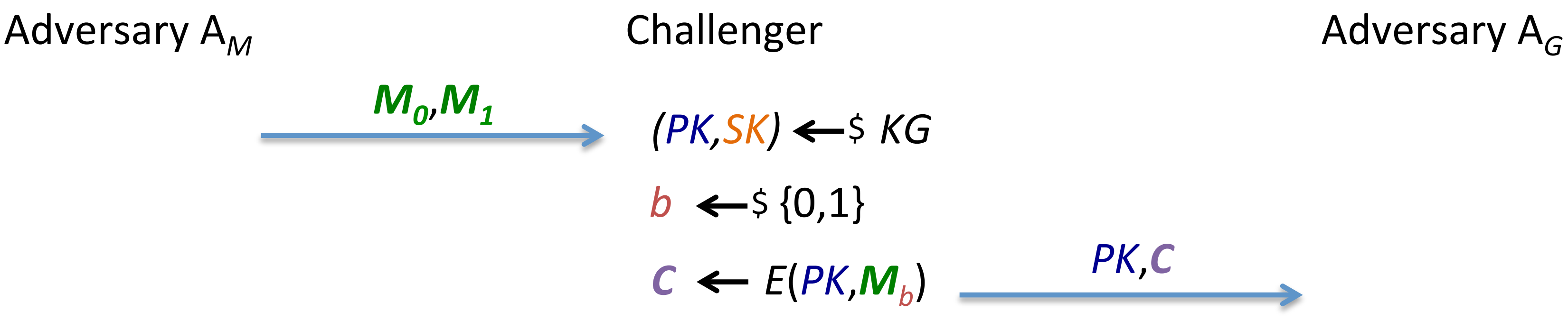

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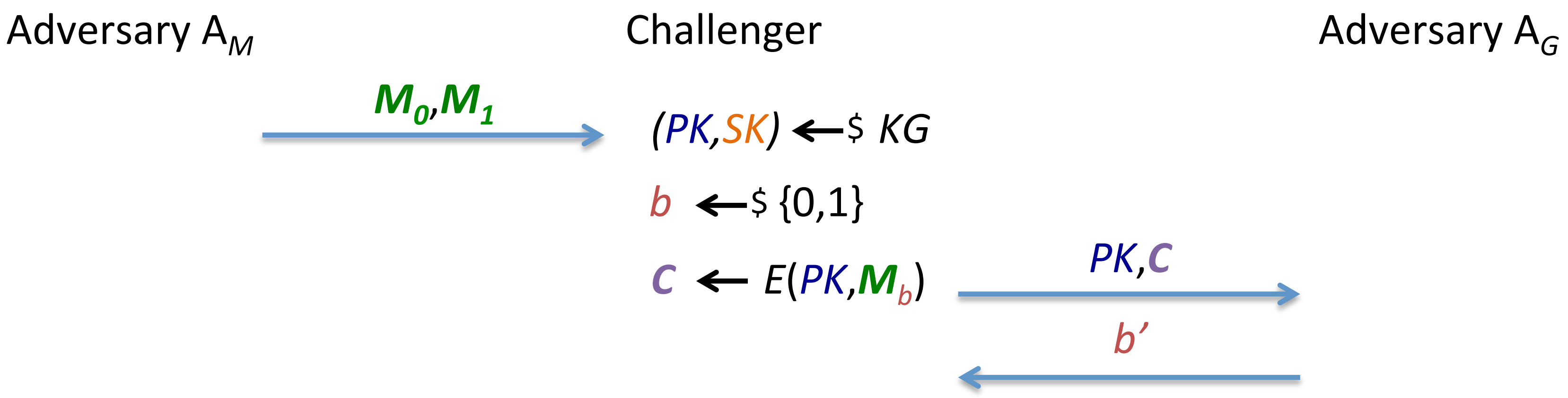

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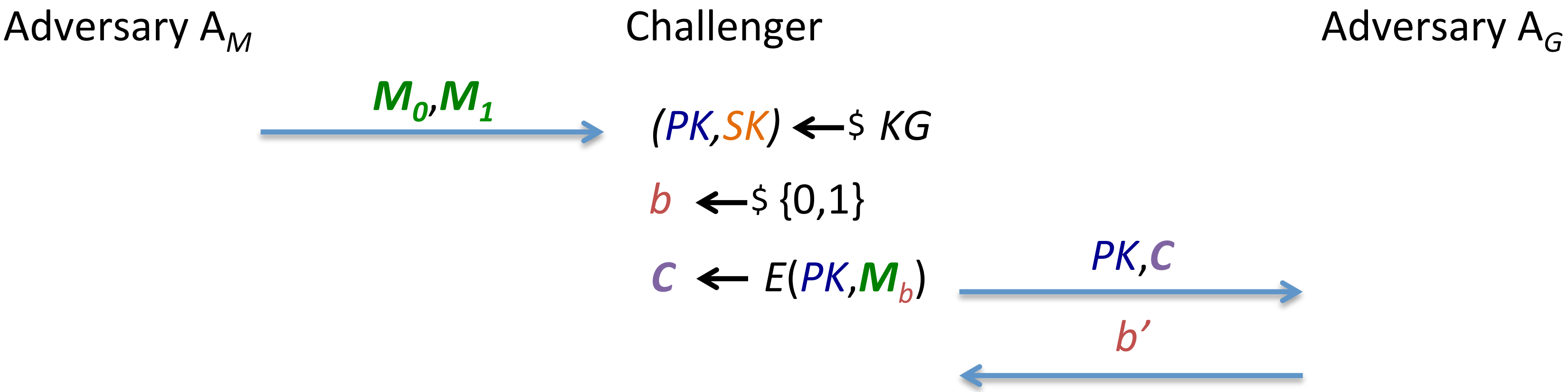

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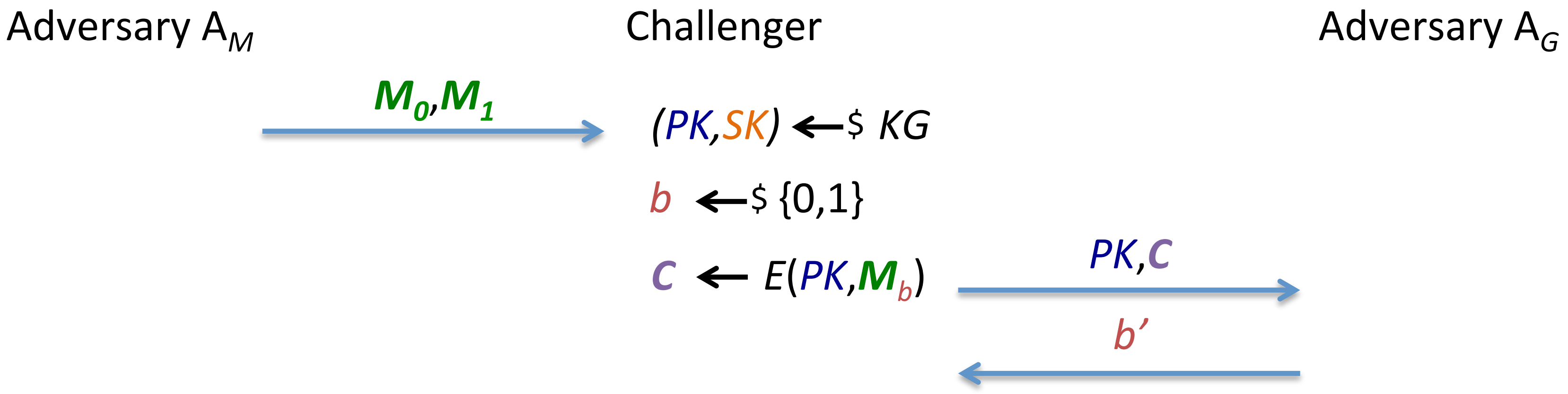

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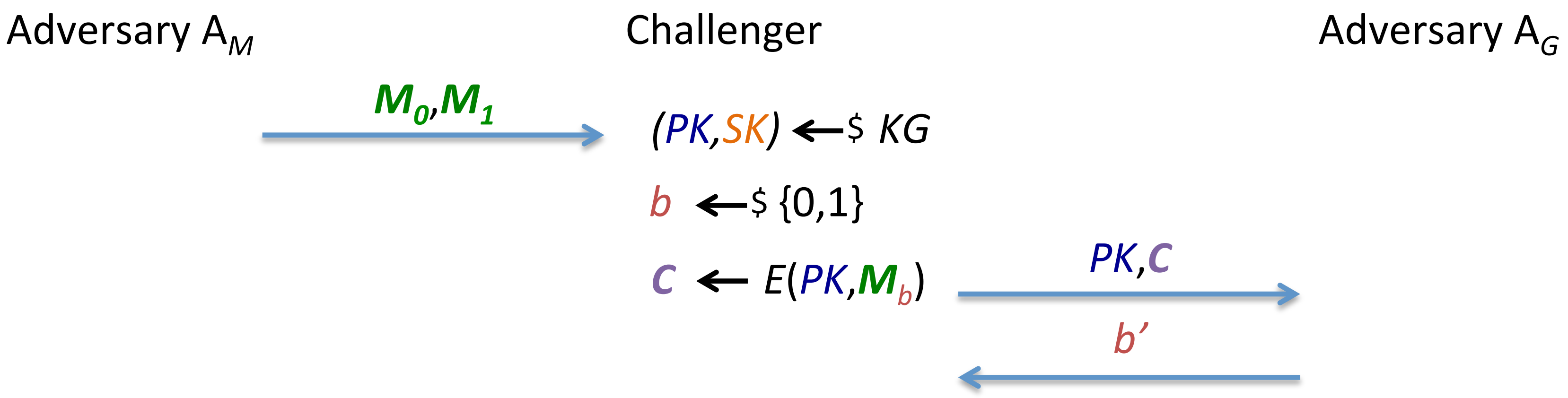

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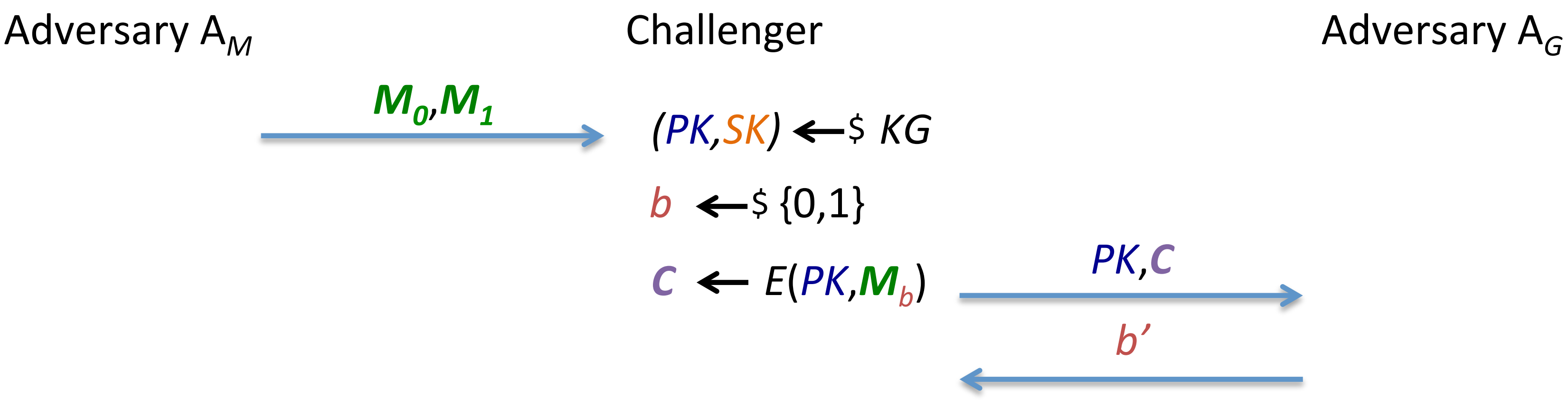

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Questions

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In the case of randomized PKE the answer to these questions is YES, but for deterministic encryption the situation is different and our results will show some subtle points about security definitions for deterministic PKE.

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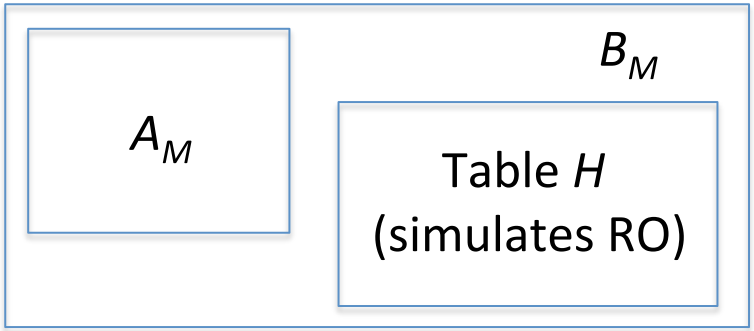

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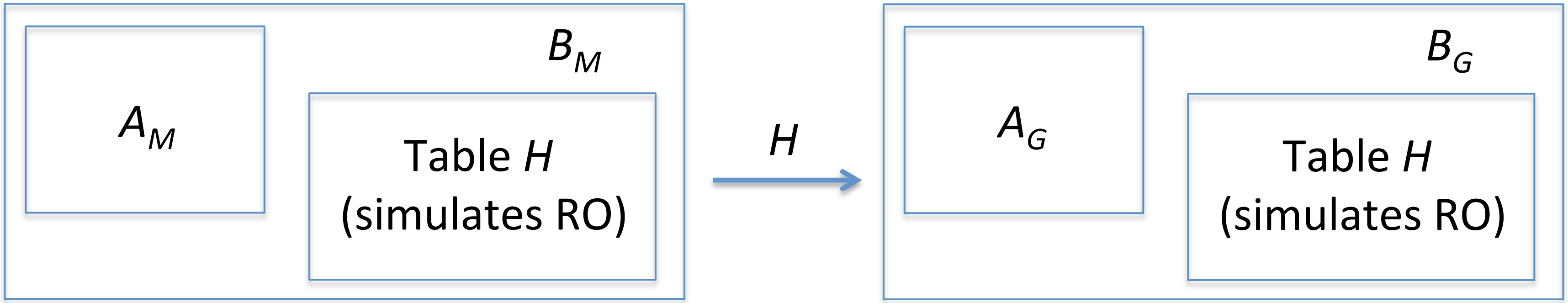

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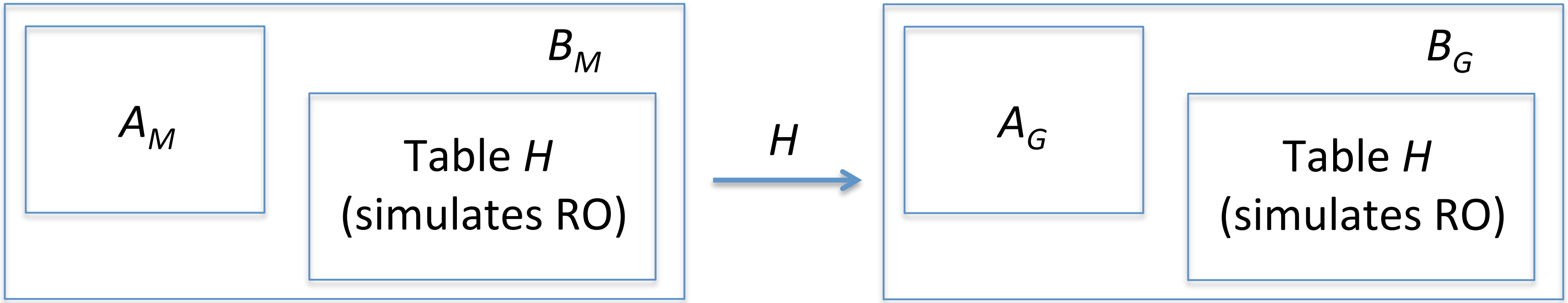

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A_M

Table H
(simulates RO)

H

B_G

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The claim and the simulation argument hardly seem specific to randomized PKE.

The Case of Deterministic PKE

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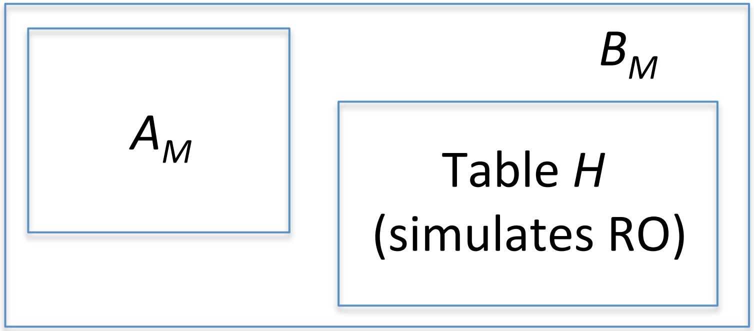

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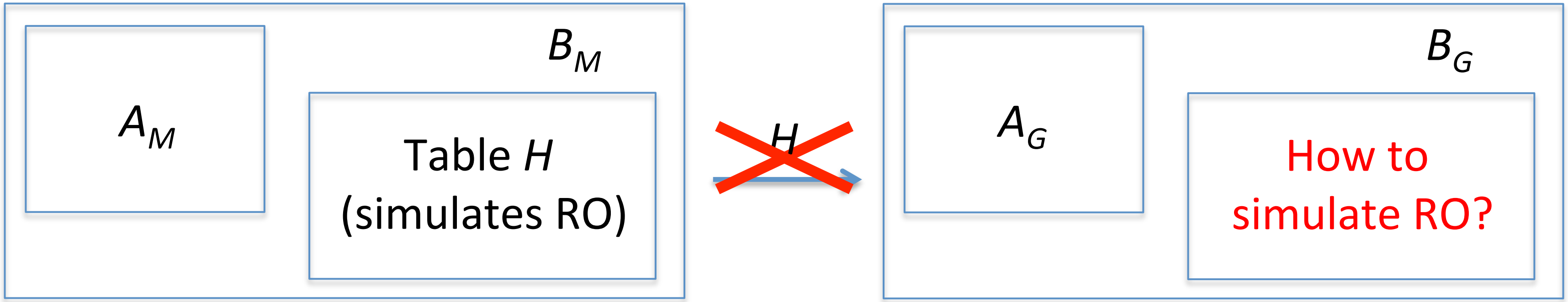

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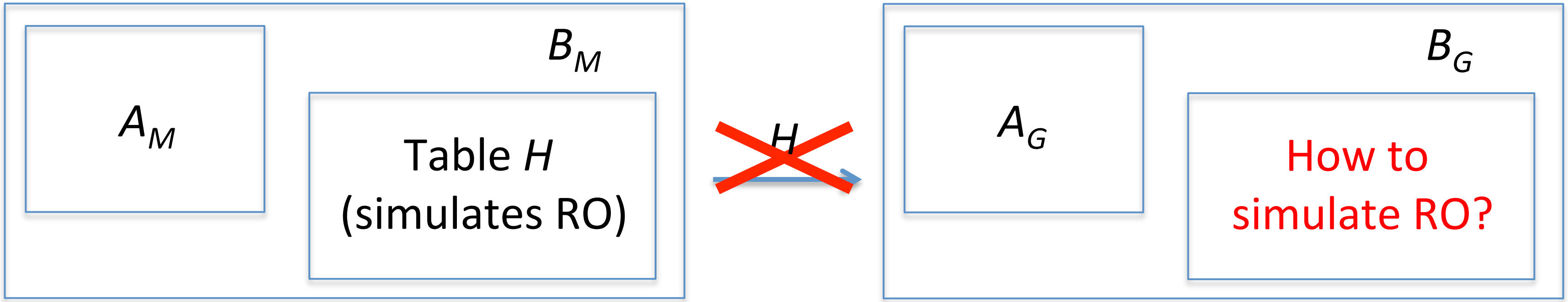

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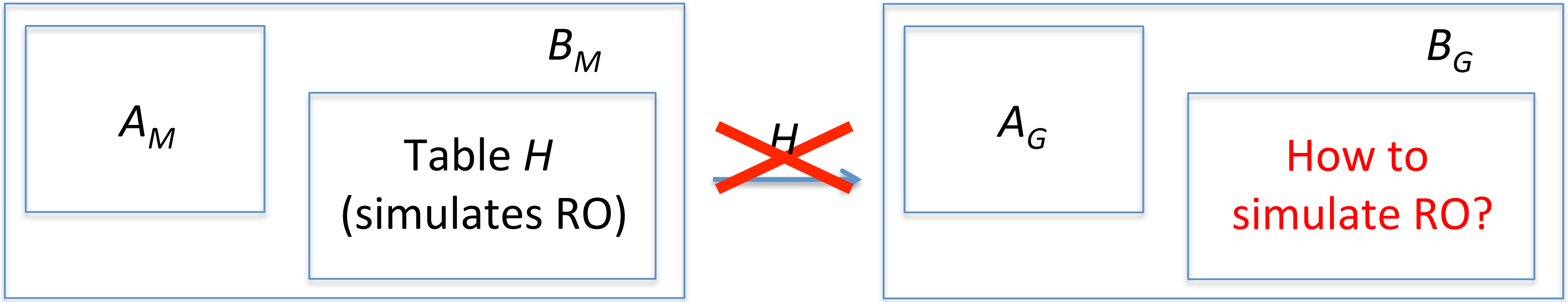

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It is not completely clear if the security implication always holds for deterministic PKE: whether we could prove it or not depended on details of the security definition.

Non-Uniform Adversaries

Idea: use a q-wise independent hash function h to simulate the random oracle. Hardwire h into the circuits of B_m and B_G.

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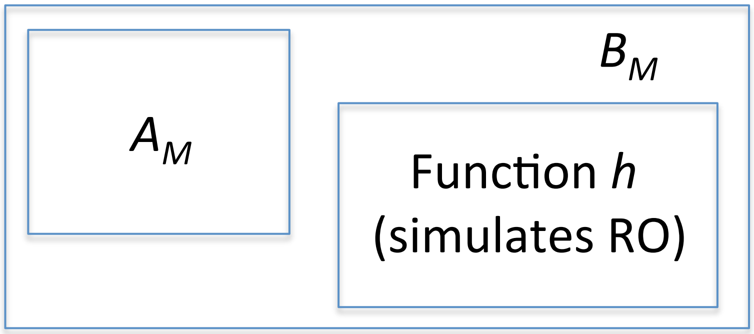

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Function h
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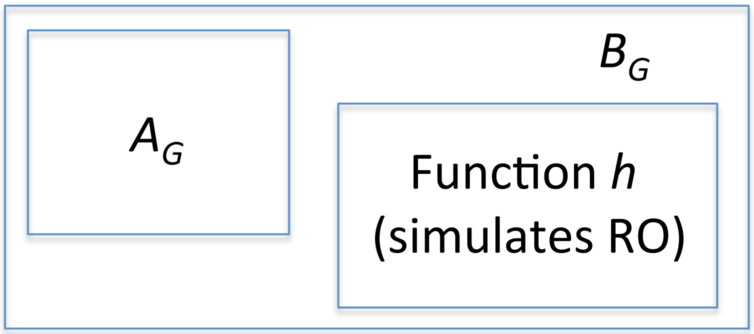

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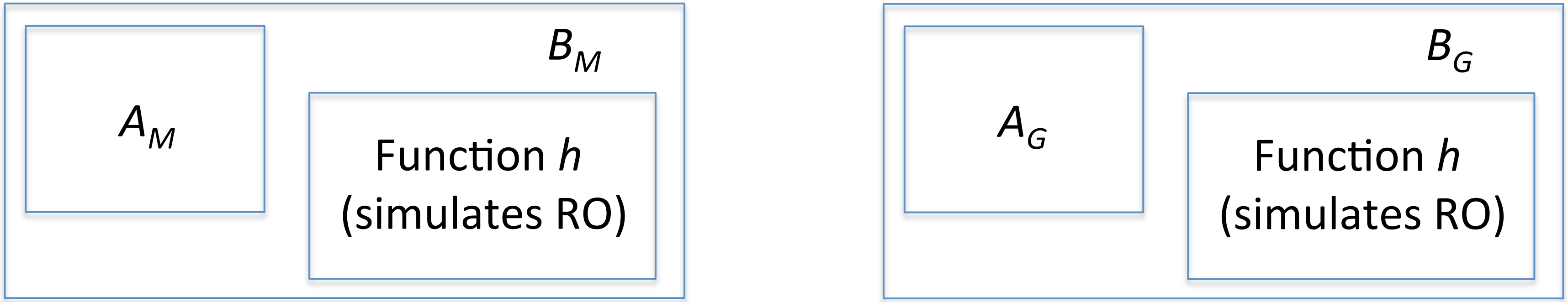

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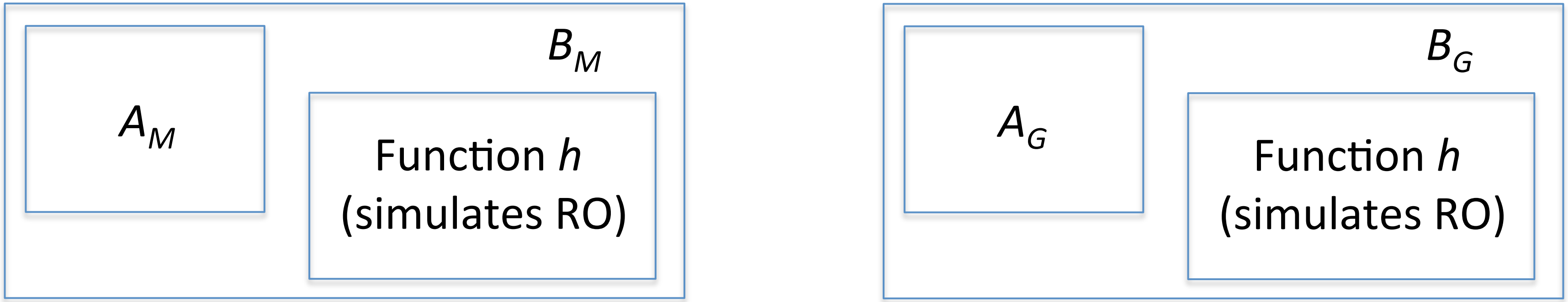

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Note that q-wise independence is a non-adaptive condition, while the RO queries are adaptive. But it is possible to handle this in the analysis.

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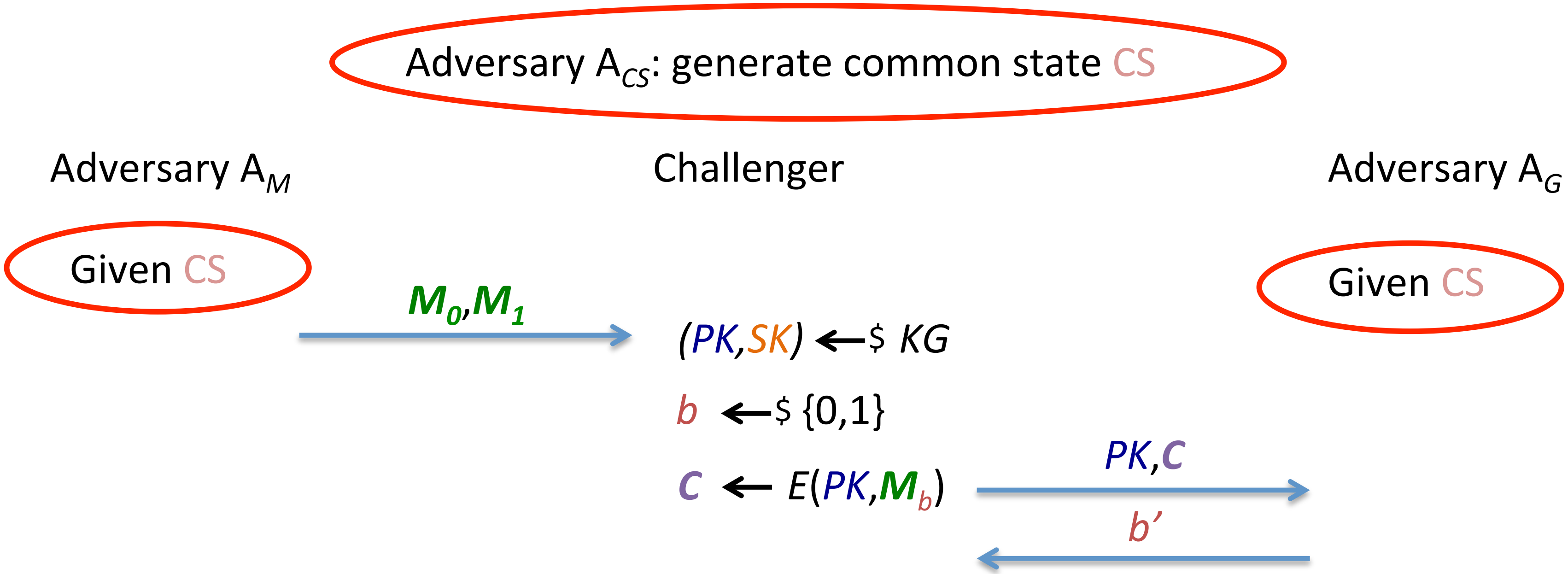
Intuitively it is hard to imagine how a standard model scheme can be insecure in the RO model if the messages have high min-entropy conditioned on the RO.

Three Stage Adversaries



Adversary A_{CS}: generate common state CS

Adversary A_M

Challenger

Adversary A_G

Given CS

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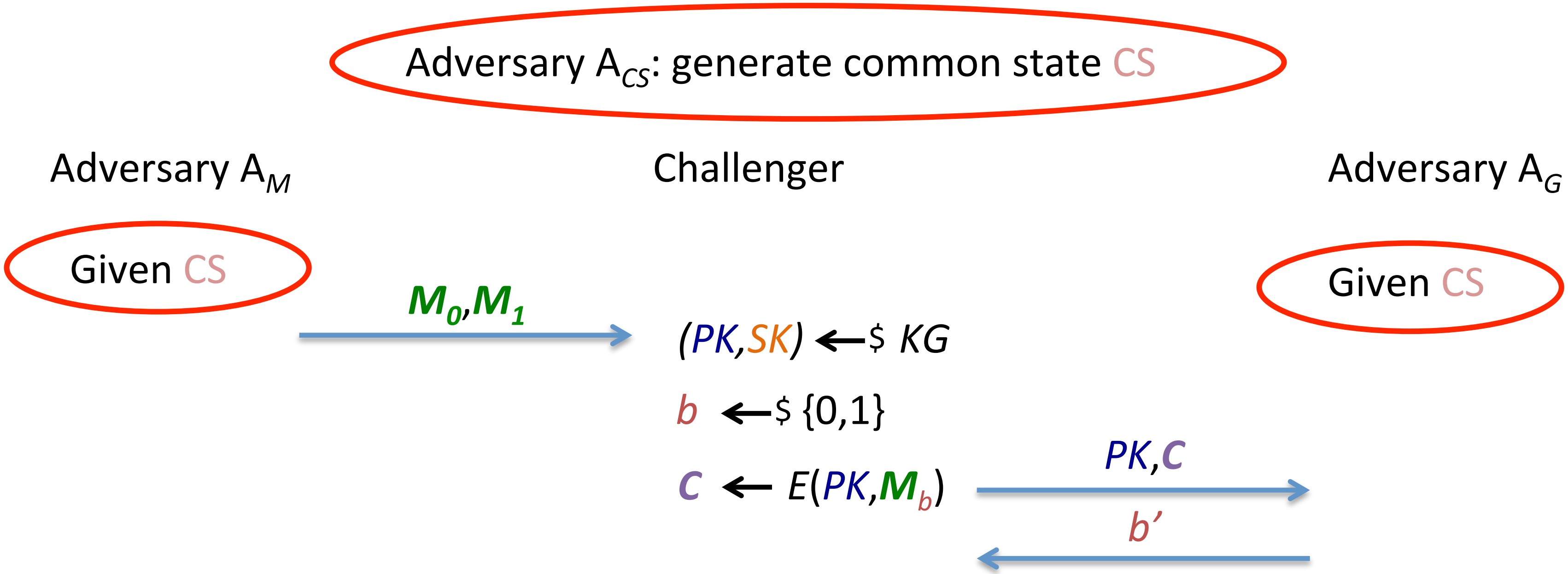

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Takeaway: Use the definition with three stage adversaries.

Is it possible to achieve security against selective opening attacks?

SOA-M

Challenger$_D$ Adversary A

$M \leftarrow\$ \; D$; $(PK,SK) \leftarrow\$ \; KG$

$R \leftarrow\$ \; \{0,1\}^{rn}$; $C \leftarrow E(PK, M;R)$ $\xrightarrow{PK,C}$

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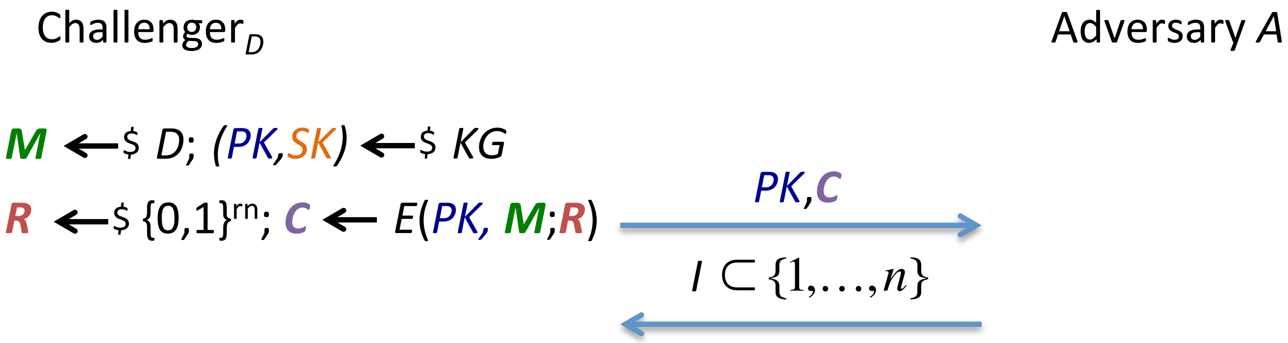

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$$PK, C \longrightarrow$$

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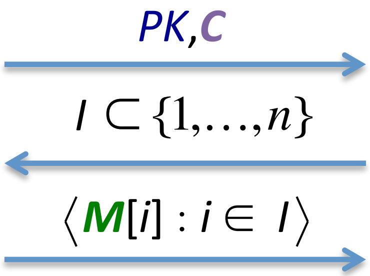

PK,C

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$\langle M[i] : i \in I \rangle$

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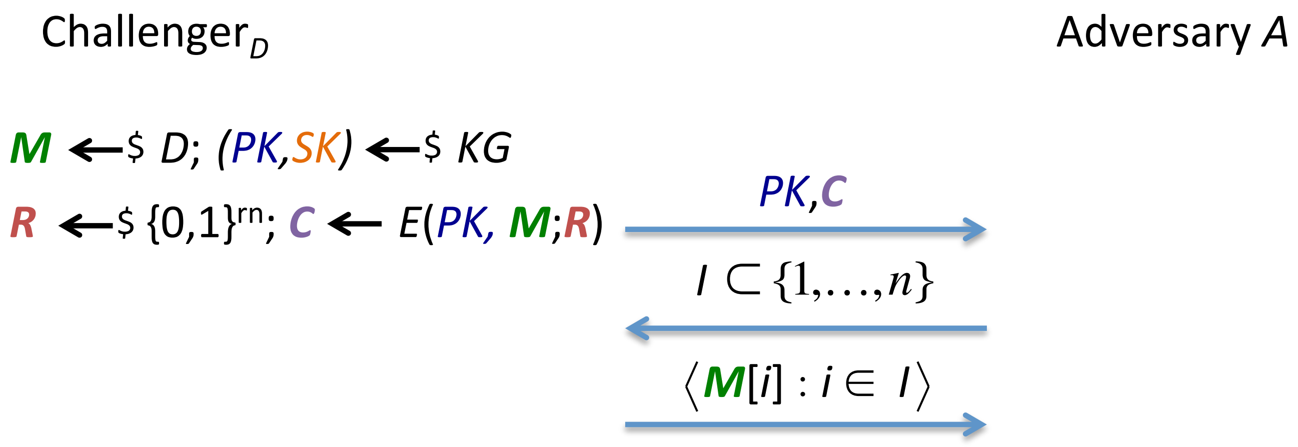

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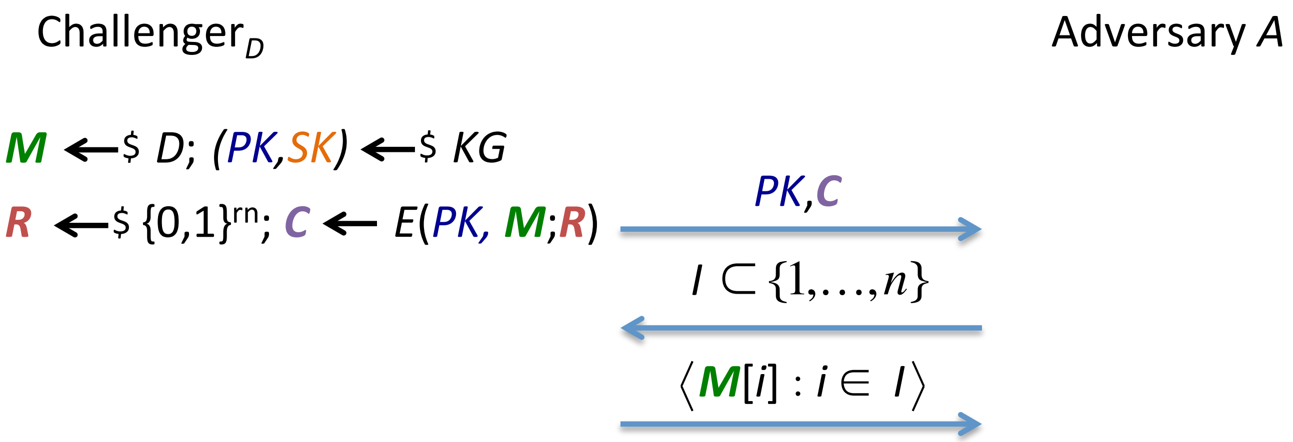

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IND-CPA security implies SOA-M security [BY09].

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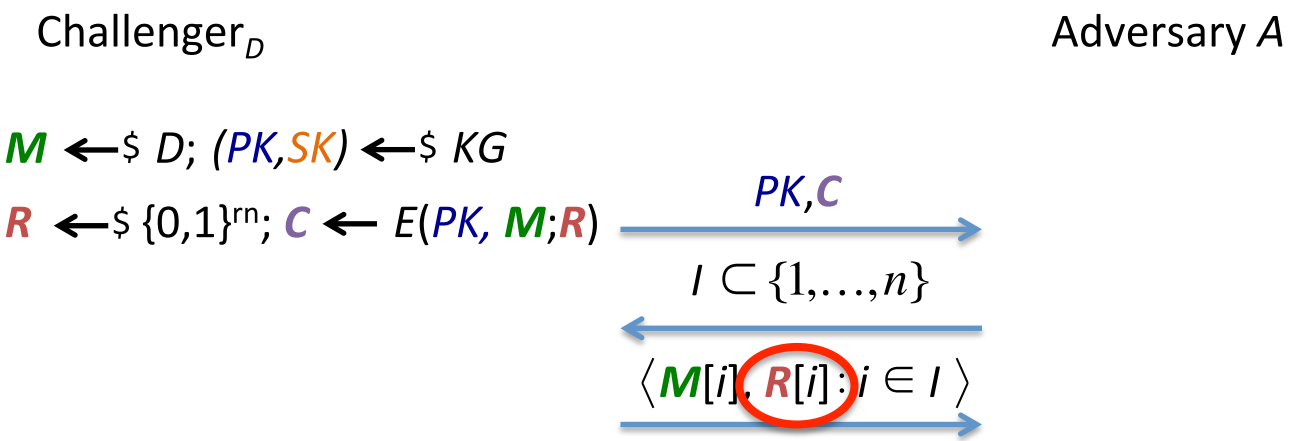

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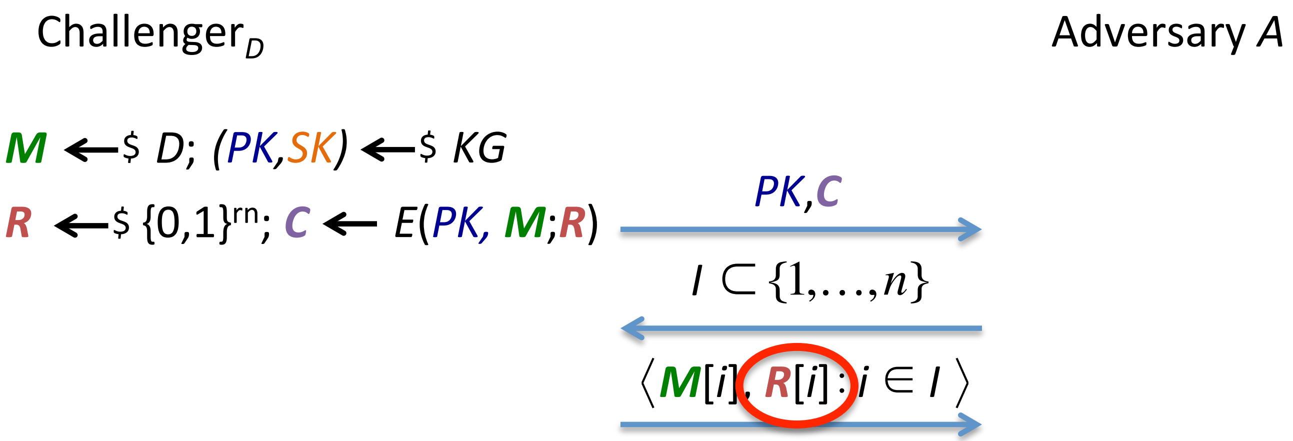

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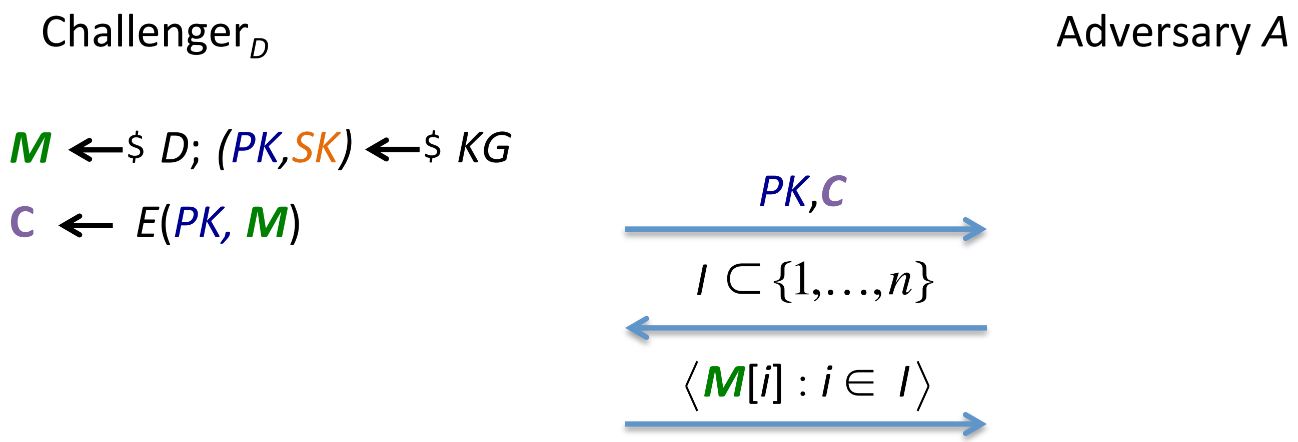
IND-CPA security implies SOA-M security [BY09].

While IND-CPA (or even IND-CCA2) does not imply SOA-C security [BDWY12], it is possible to achieve SOA-C security [BHY09].

SOA for Deterministic PKE



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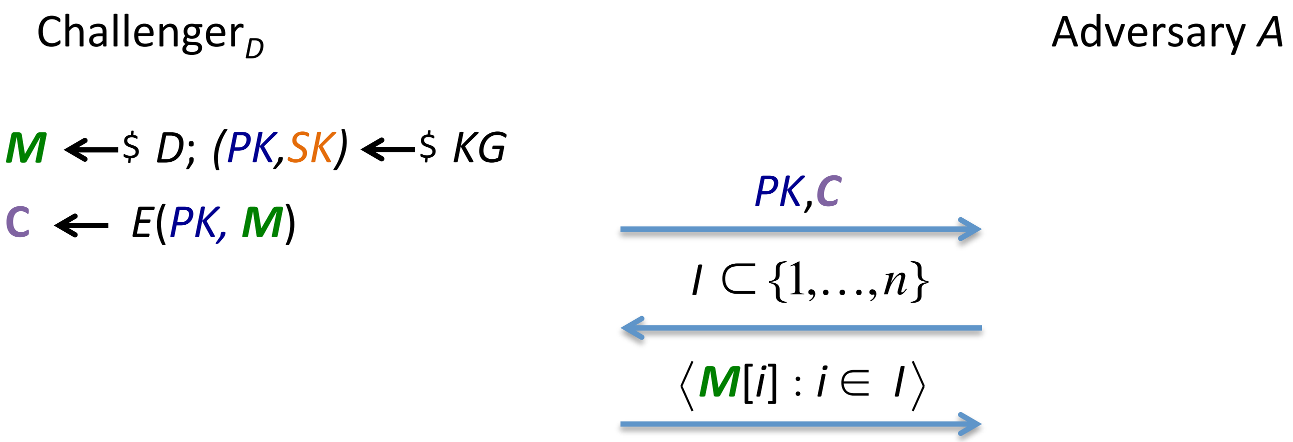

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Since the difficulty of achieving SOA security for randomized PKE lies in exposure of the coins, one might get the impression that SOA-security would be trivial to achieve for deterministic PKEs.

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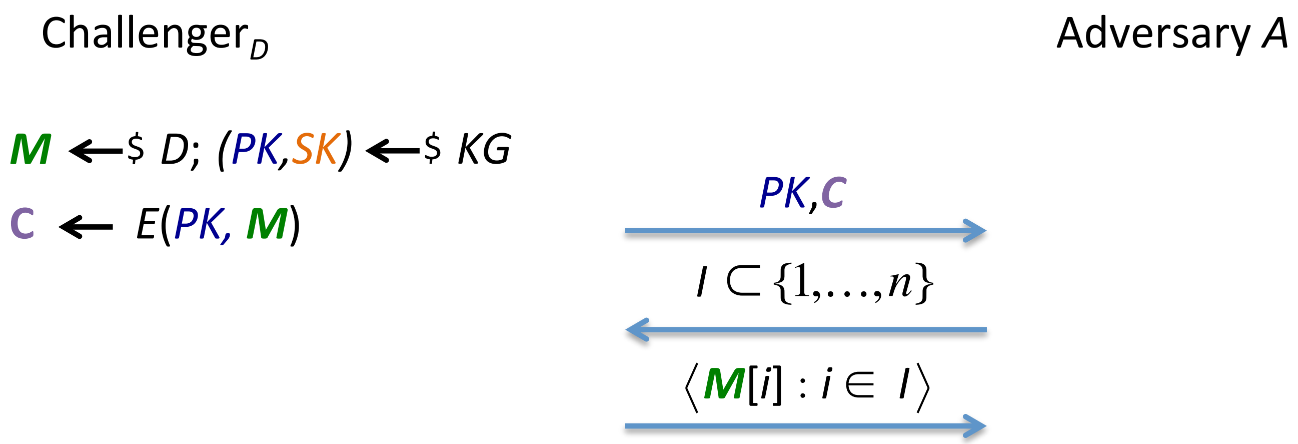

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$$\xleftarrow{I \subset \{1,\ldots,n\}}$$

$$\xrightarrow{\langle M[i] : i \in I \rangle}$$

Security means A cannot figure out anything about $\langle M[i] : i \notin I \rangle$.

Since the difficulty of achieving SOA security for randomized PKE lies in exposure of the coins, one might get the impression that SOA-security would be trivial to achieve for deterministic PKEs.

Contrary is true: unachievable.

SOA Security Definition

Formalized using a simulation-based definition.

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The adversary in our result actually uses uniform, independent messages.

Result

In the case of randomized PKE, Bellare, Dowsley, Waters and Yilek [BDWY12] showed that any scheme satisfying a certain binding property is not SOA-secure.

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For deterministic PKE, we show that every scheme admits a verification algorithm that tests the extent to which the encryption induced by a public key (even dishonestly-chosen ones) is an injective function. If it is far from injective, it gets detected, otherwise we have some sort of binding.

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Adapt technique of Bellare et al. to show that no deterministic PKE is SOA-secure.

IND-Style Definition

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Problems even for randomized PKE: very limited set of message distributions or non-polynomial time games.

Does single-user security implies multi-user security?

mIND Security

Adversary A_{CS}: generate common state CS

Adversary A_M

Challenger

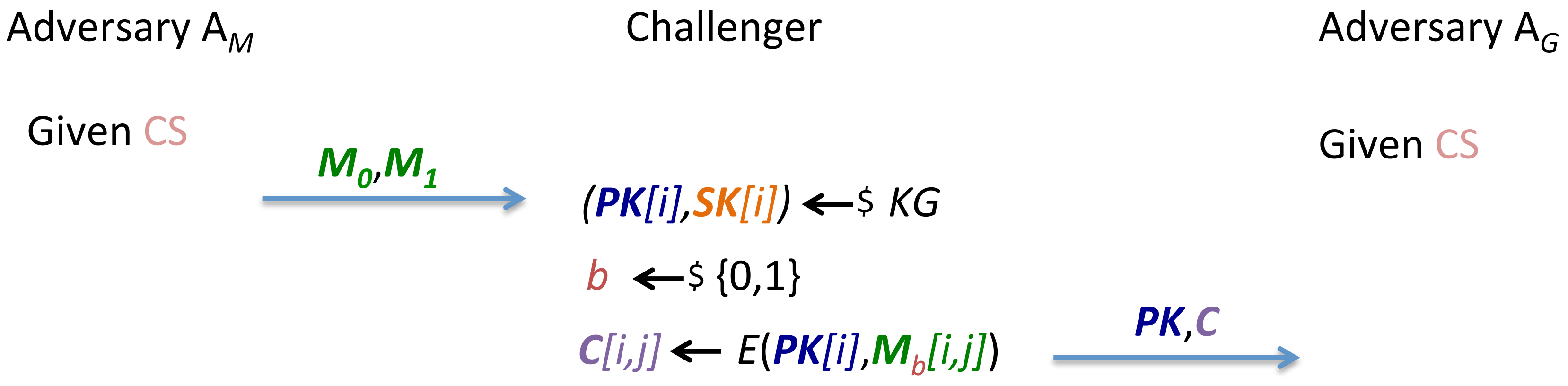
Adversary A_G

Given CS

Given CS

mIND Security

Adversary A_{CS}: generate common state CS



Adversary A_M

Given CS

M_0, M_1

Challenger

$(PK[i], SK[i]) \leftarrow\$ \; KG$

$b \leftarrow\$ \; \{0,1\}$

$C[i,j] \leftarrow E(PK[i], M_b[i,j])$

PK, C

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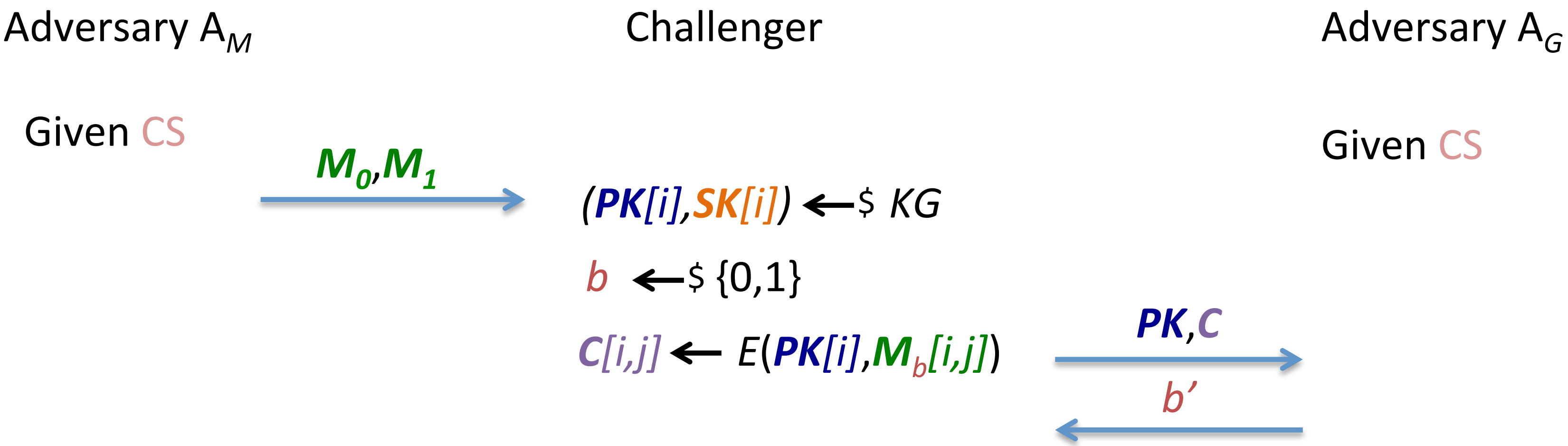

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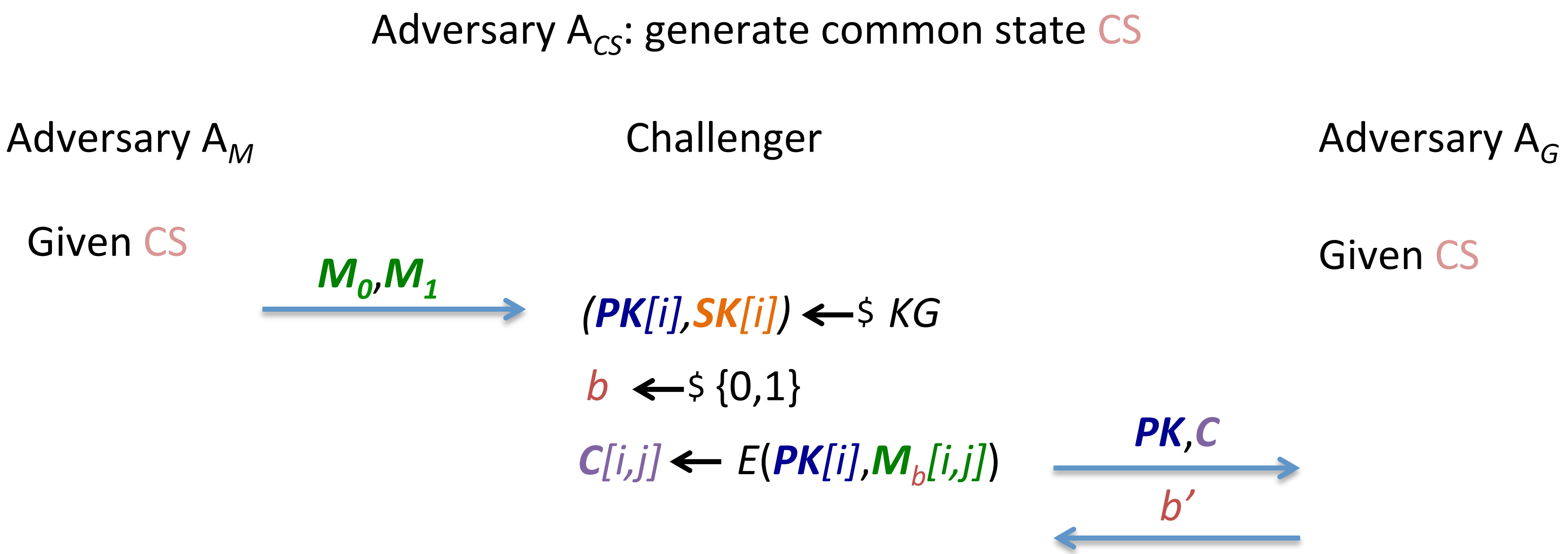

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Adversary A_G

Given CS

Adversary (A_{CS},A_M,A_G) wins if $b'=b$. Security requires $2\Pr[b'=b]-1$ to be negligible.

There are three essential restrictions without which security is not achievable:

1) A_{CS} and A_M do not get the public keys.

2) All messages in M_0,M_1 must have high min-entropy and there are no repeated messages in the same row of the matrix.

3) A_M cannot pass state to A_G.

Result

For randomized PKE, single-user security implies multi-user security [BBM00,BPS00]. What about deterministic PKE?

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Theorem: Assume there exists an IND-secure deterministic PKE scheme. Then there exists a deterministic PKE scheme that is IND-secure, but not mIND-secure.

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Theorem: Assume there exists an IND-secure deterministic PKE scheme. Then there exists a deterministic PKE scheme that is IND-secure, but not mIND-secure.

Insecure even for two users.

Proof Idea

Case 1: All deterministic PKE schemes are insecure for 2 users.

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Trivial to establish the theorem.

Case 2: There exists a deterministic PKE scheme that is secure for 2 users.

Let *DE* be a scheme which is secure for 2 users. Then we construct a modified scheme *DE’* which is secure for a single user, but not for 2 users.

Proof Idea

```
$DE'.PG(1^{\lambda})$:
    $\pi \xleftarrow{\$} DE.PG(1^{\lambda})$
    $(PK^*, SK^*) \xleftarrow{\$} DE.KG(\pi)$
    Return $\pi^* = (\pi, PK^*)$
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$DE'.E(\pi^*, PK, M)$:
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$DE'.D(\pi^*, SK, C')$:
    $M \leftarrow DE.D(\pi, SK, C)$
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PK^* can be viewed as a key of a dummy second user of the old scheme.

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Then the fact that DE' is IND-secure follows from the security against 2 users of the original scheme.

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$DE'.D(\pi^*, SK, C')$:
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Return $M$
```

The fact that *DE′* is not secure for 2 users follows from the fact that the second part of the ciphertexts can be used to check whether the messages encrypted to different users are the same or not.

Summary

- Consider using the definition with three stage adversaries. For the one with two stage adversaries it is not clear whether security in the standard model implies security in the random oracle model.

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Summary

✧ Consider using the definition with three stage adversaries. For the one with two stage adversaries it is not clear whether security in the standard model implies security in the random oracle model.

✧ It is not possible to obtain deterministic PKE which are secure against selective opening attacks (at least for simulation-based definitions).

✧ Single-user security does not imply multi-user security for deterministic PKE.

Thank You!