RUHR-UNIVERSITÄT BOCHUM

Lattice-based Proxy Re-encryption

PKC 2014, 26.03.14

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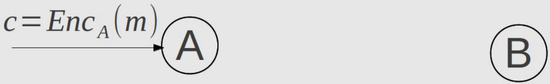
für IT-Sicherheit

Outline

The informal definition of a Proxy Re-Encyption



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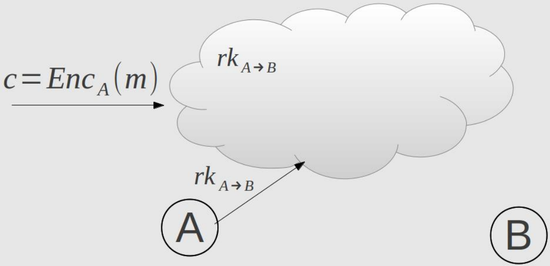

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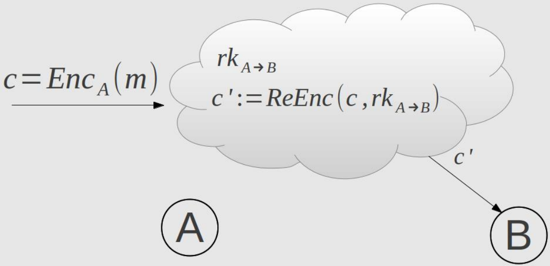

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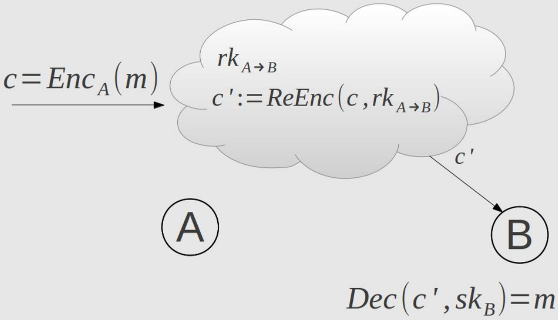

The formal Definition

Definition 1 (Proxy Re-Encryption)

A *unidirectional* Proxy Re-Encryption (PRE) is a tuple of algorithms:

- $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^n)$
- $c_{\mathsf{pk}} \leftarrow \mathsf{Enc}(\mathsf{pk}, m)$
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- $c' \leftarrow \mathsf{ReEnc}(\mathsf{rk}_{\mathsf{pk} \to \mathsf{pk}'}, c_{\mathsf{pk}})$

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$b \leftarrow \{0,1\}$ $c^* = \mathsf{Enc}(\mathsf{pk}^*, m_b)$	$\xrightarrow{c^*}$	
If $b = b'$ output 1 else output 0	$\xleftarrow{b'}$	$b' \in \{0,1\}$

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- Collusion 'safe'
- Key optimal
- Non-transitive
- Proxy invisibility

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PRE overview

	Unidirectional	Non-interactive	Collusion-safe	Assumption	Security Model
[BBS98]	✗	✗	✗	DDH	IND-CPA

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This work	✓	✓	✓	LWE	IND-CCA1

Main result

Theorem 2

Our unidirectional Proxy Re-Encryption scheme is IND-CCA1-secure assuming the hardness of decision-LWE.

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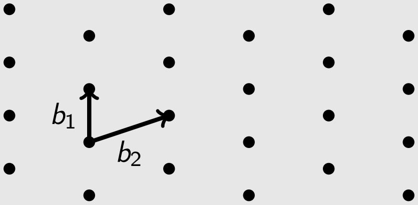
Lattice definition

- Lattice Λ of dimension m is a discrete additive subgroup of $\mathbb{Z}^m$.



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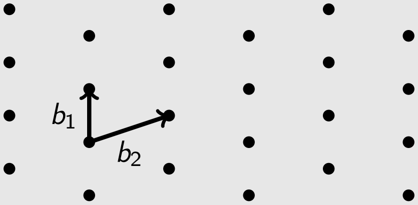
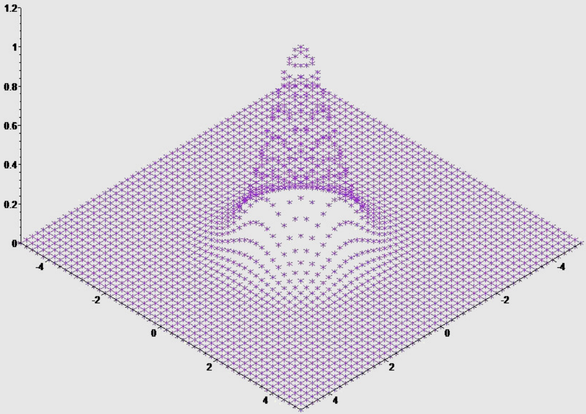


- Basis $\mathbf{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_k\} : \Lambda(\mathbf{B}) = \{\mathbf{Bz} : \mathbf{z} \in \mathbb{Z}^k\}$.

Gaussians on Lattices

$$v \leftarrow D_{\Lambda,s} \Leftrightarrow v \propto \rho_s(\mathbf{x}) = \exp(-\frac{\pi\|\mathbf{x}\|^2}{s^2})$$



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- Public $\left[\mathbf{A}\right] \in \mathbb{Z}_q^{n\times m}$, $q = \mathrm{poly}(n), m \approx n \log q$

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$\mathbf{u} := f_{\mathbf{A}}(\mathbf{x}) = \mathbf{A}\mathbf{x} \mod q \in \mathbb{Z}_q^n$	$g_{\mathbf{A}}(\mathbf{s}, \mathbf{e}) = \mathbf{s}^t\mathbf{A} + \mathbf{e}^t \mod q \in \mathbb{Z}_q^m$

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$f_{\mathbf{A}}^{-1}$: sample $\mathbf{x}' \leftarrow D_{\Lambda_{\mathbf{u}},s}$ s.t. $\mathbf{A}\mathbf{x}' = \mathbf{u}$	$g_{\mathbf{A}}^{-1}$: find the unique $\mathbf{s}$ (or $\mathbf{e}$)

G-trapdoor [PM12]

- For a uniform $\mathbf{A}_0 \in \mathbb{Z}_q^{n \times \bar{m}}$ and a short $\mathbf{R} \leftarrow \mathbb{Z}^{\bar{nk} \times nk}$ define

$$\mathbf{A} = [\mathbf{A}_0 \mid \mathbf{G}] \begin{bmatrix} \mathbf{I} & -\mathbf{R} \\ & \mathbf{I} \end{bmatrix} = [\mathbf{A}_0 \mid \mathbf{G} - \mathbf{A}_0\mathbf{R}]$$

for some $\mathbf{G}$ with easy $f_{\mathbf{G}}^{-1}$ and $g_{\mathbf{G}}^{-1}$.

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- $[\mathbf{A}_0 \mid \mathbf{A}_0\mathbf{R}]$ is uniform by the leftover hash lemma, so is $\mathbf{A}$.
- $\mathbf{A} \cdot \begin{bmatrix} \mathbf{R} \\ \mathbf{I} \end{bmatrix} = \mathbf{G}$

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Outline

Extended G-trapdoor

- Idea: generate multiple $\mathbf{R}$-transformations

$$\mathbf{A} = [\mathbf{A}_0 \mid \underbrace{\mathbf{G} - \mathbf{A}_0\mathbf{R}_1}_{\text{trapdoor for } f_{\mathbf{A}}} \mid \overbrace{\mathbf{G} - \mathbf{A}_0\mathbf{R}_2}^{\text{trapdoor for } g_{\mathbf{A}}}]$$

- $\mathbf{R}_1$ allows to sample short vectors (i.e. generate rk)
- $\mathbf{R}_2$ allows to invert $\mathbf{s}^t\mathbf{A} + \mathbf{e}^t$ (i.e. decrypt)

Encryption

- $\mathsf{pk} = [\mathbf{A}_0 \mid \mathbf{G} - \mathbf{A}_0\mathbf{R}_1 \mid \mathbf{G} - \mathbf{A}_0\mathbf{R}_2] \in \mathbb{Z}_q^{n \times m}$, $\mathsf{sk} := [\mathbf{R}_1 \mid \mathbf{R}_2]$

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- $\text{Enc}(mes, \text{pk})$:

$$\mathbf{c}_1 = \mathbf{s}^t \cdot \text{pk} + \mathbf{e}_1^t \mod q,$$
$$\mathbf{c}_2 = \mathbf{s}^t \cdot \mathbf{A}_{aux} + \mathbf{e}_2^t + \texttt{enc}(mes) \mod q,$$

for $\mathbf{s} \xleftarrow{\$} \mathbb{Z}_q^n$, $\mathbf{e}_1, \mathbf{e}_2 \leftarrow D_s$, $\mathbf{A}_{aux} \xleftarrow{\$} \mathbb{Z}_q^{n \times nk}$ and $\texttt{enc}(mes) := mes \cdot \lfloor \frac{q}{2} \rfloor$.

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- $\mathsf{Dec}(\mathbf{c} = (\mathbf{c}_1, \mathbf{c}_2, \mathbf{A}_{aux}), sk)$: recover $\mathbf{s}$ using $\mathbf{R}_2$:

$$\mathbf{c}_1 \begin{bmatrix} \mathbf{R}_2 \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix} = \mathbf{s}^t[\mathbf{G}] + \widetilde{\mathbf{e}}^t \mod q.$$

Re-Encrytion key generation

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- Extend to matrices in column-wise way:

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$$\cdot[\mathbf{X}_0]$$

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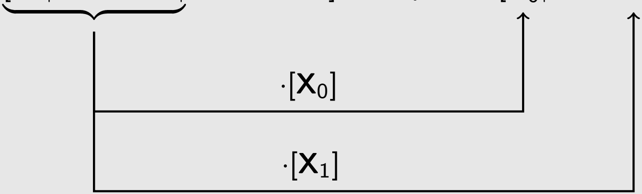

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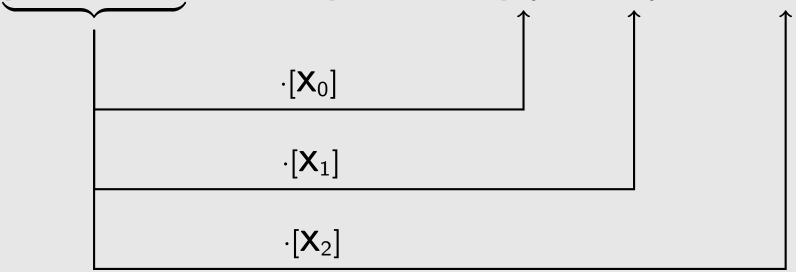

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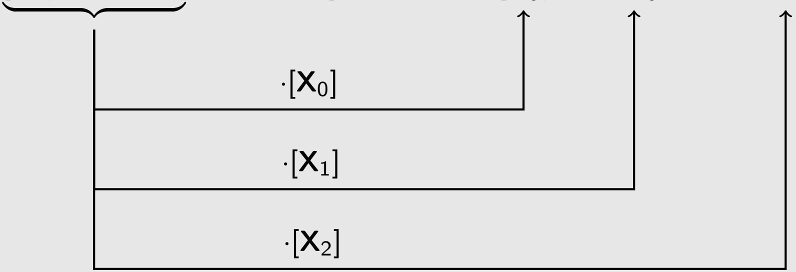

$$\mathrm{rk}_{\mathrm{pk}\to\mathrm{pk}'} = \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 & \mathbf{X}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \in \mathbb{Z}^{m\times m}, \text{ where all } \mathbf{X} \text{ are gaussian.}$$

Re-Encryption

So for $\mathbf{c}_1 = \mathbf{s}^t[\mathbf{A}_0 \mid \mathbf{G} - \mathbf{A}_0\mathbf{R}_1 \mid \mathbf{G} - \mathbf{A}_0\mathbf{R}_2] + \mathbf{e}^t \mod q$

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So for $\mathbf{c}_1 = \mathbf{s}^t[\mathbf{A}_0 \mid \mathbf{G} - \mathbf{A}_0\mathbf{R}_1 \mid \mathbf{G} - \mathbf{A}_0\mathbf{R}_2] + \mathbf{e}^t \mod q$

- $\mathbf{c}_1' = \mathsf{ReEnc}(\mathbf{c}_{pk}, rk_{\mathsf{pk}\rightarrow\mathsf{pk}'}) = \mathbf{c}_{pk} \cdot \mathsf{rk}_{\mathsf{pk}\rightarrow\mathsf{pk}'}$

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- $\mathbf{c}_1' = \mathrm{ReEnc}(\mathbf{c}_{pk}, rk_{\mathrm{pk}\to\mathrm{pk}'}) = \mathbf{c}_{pk} \cdot \mathrm{rk}_{\mathrm{pk}\to\mathrm{pk}'}$
- $\mathbf{c}_1' = \mathbf{s}^t[\mathbf{A}_0' \mid \mathbf{G} - \mathbf{A}_0'\mathbf{R}_1' \mid \mathbf{G} - \mathbf{A}_0'\mathbf{R}_2'] + \widetilde{\mathbf{e}}^t \mod q,$

where $\widetilde{\mathbf{e}}^t = (\mathbf{e}_0, \mathbf{e}_1)^t \cdot \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 & \mathbf{X}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$ is as small as

$$\approx \sqrt{3} \cdot \|\mathbf{e}_0\mathbf{X}_2 + \mathbf{e}_1\|.$$

Summary

Proxy re-encryption scheme that

- is based on hard problems on lattices
- is unidirectional
- does not require a trusted party to generate re-encryption keys
- uses the 'Extended $\mathbf{G}$-trapdoor'.

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Many thanks for your attention!

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