FSE 2014 @ London, UK
3 March 2014

Improved All-Subkeys Recovery Attacks on FOX, KATAN and SHACAL-2 Block Ciphers

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Sony Corporation

Background

- All-Subkeys Recovery Attacks
  - Proposed at SAC 2012 by Isobe and Shibutani
  - Recover all subkeys instead of user-provided key
  - Applied to CAST-128, KATAN32/48/64, SHACAL-2, Blowfish, FOX

- Function Reduction Technique
  - Proposed at ASIACRYPT 2013 by Isobe and Shinbutani
  - Advanced Technique for All-Subkeys Recovery Attack
  - Applied to *Feistel* Construction
    - Improved generic attacks on several Feistel constructions
    - 8-round attack on CAST-128 (best attack w.r.t. # attacked rounds)
    - Extremely-Low data attacks on 8/10/12-round Camellia-128/192/256

Our Contribution

- Improve "All-Subkeys Recovery Attack"
  - Extend "Function Reduction" to other constructions
    - Exploit structure-dependent properties of Lai-Massey and LFSR-type
  - Utilize "Repetitive All-Subkeys Recovery Attack"
    - Reduce data requirement in each inner ASR attacks
      - Variant of inner loop technique
  - Apply to FOX64/128, KATAN32/48/64, SHACAL-2
    - FOX64/128 : Lai-Massey construction
    - KATAN32/48/64 : Stream-type (LFSR) construction
    - SHACAL-2 : Source-heavy Generalized Feistel construction

Our Results

Best Attacks

Target	# Attacked rounds	Attack Type	Time	Memory	Data	Paper
FOX64	5	Integral	$2^{109.4}$	Not given	2^{9}	[26]
	5	Impossible differential	2^{71}	Not given	2^{90}	[27]
	6	**ASR**	$\mathbf{2^{124}}$	$\mathbf{2^{124}}$	**15**	**Ours**
	7	**ASR**	$\mathbf{2^{124}}$	$\mathbf{2^{124}}$	$\mathbf{2^{30.9}}$	**Ours**
FOX128	5	Integral	$2^{205.6}$	Not given	$2^{116.3}$	[26]
	5	Impossible differential	2^{135}	Not given	28	[27]
	5	ASR	2^{228}	2^{228}	14	[16]
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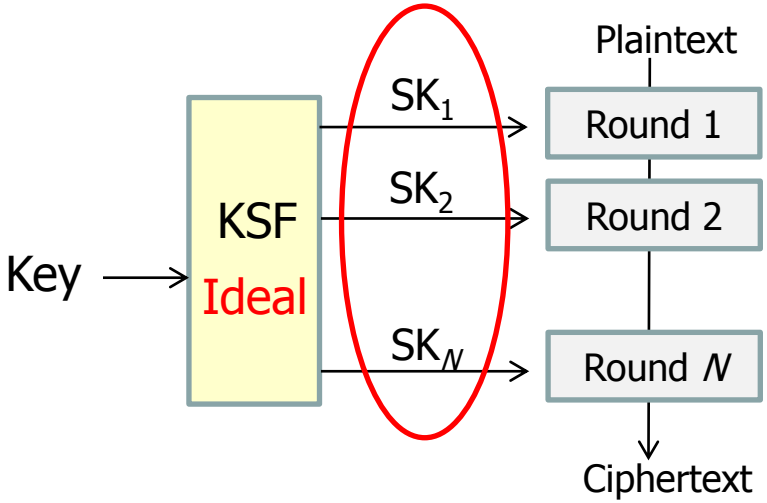
KATAN32	110	ASR	2^{77}	$2^{75.1}$	138	[16]
	114	Differential	2^{77}	Not given	$2^{31.9}$	[2]
	119	**ASR**	$\mathbf{2^{79.1}}$	$\mathbf{2^{79.1}}$	**144**	**Ours**
KATAN48	100	ASR	2^{78}	2^{78}	128	[16]
	105	**ASR**	$\mathbf{2^{79.1}}$	$\mathbf{2^{79.1}}$	**144**	**Ours**
KATAN64	94	ASR	$2^{77.1}$	$2^{79.1}$	116	[16]
	99	**ASR**	$\mathbf{2^{79.1}}$	$\mathbf{2^{79.1}}$	**142**	**Ours**
SHACAL-2	41	ASR	2^{500}	2^{492}	244	[16]
	42	**ASR**	$\mathbf{2^{508}}$	$\mathbf{2^{508}}$	$\mathbf{2^{25}}$	**Ours**

Agenda

1. All-Subkeys Recovery Attacks
2. Function reduction technique
3. Improved Attacks on FOX-64/128
4. Improved Attacks on KATAN-32/48/64
5. Improved Attacks on SHACAL-2
6. Conclusion

All-Subkeys Recovery Attack

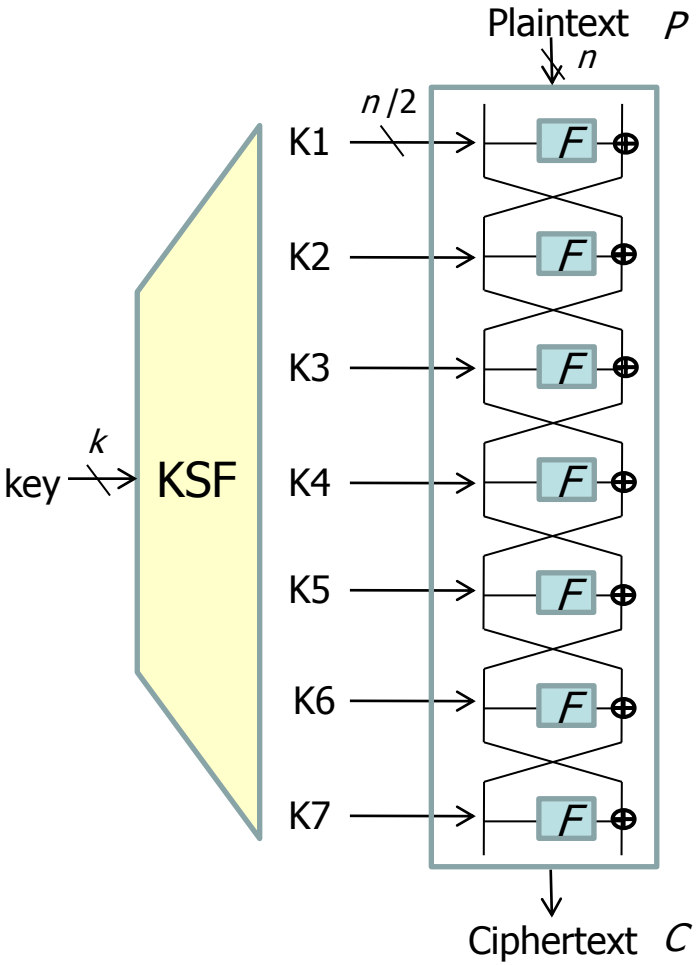
- Find all subkeys instead of user-provided key
  - "Finding all subkeys" ≒ "Finding user-provided key"



- Advantage
  - Analyzing KSF is not necessary
    - Easy to apply to complex KSF!
  - Focus only on data processing part
    - Work on any KSF (even ideal KSF) ⇨ **generic attack!**

How to Recover All Subkeys

- Based on Meet-in-the-Middle Approach
  - Example: 7-round Feistel Cipher w/ k (=2n)-bit key and n-bit block



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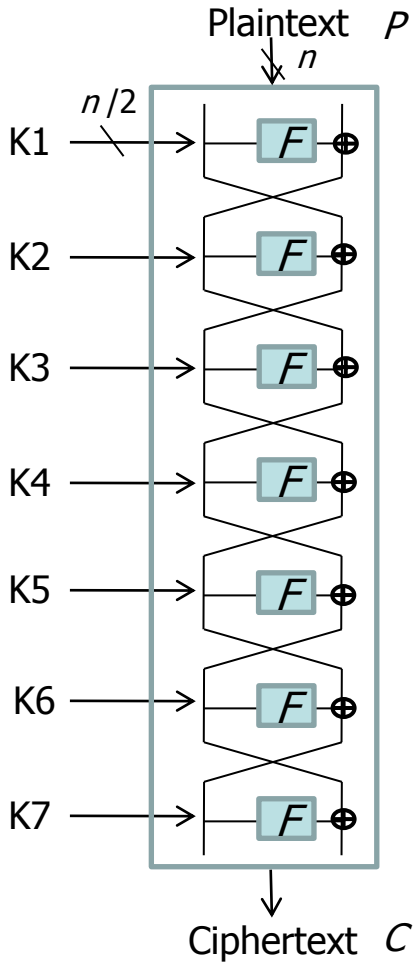

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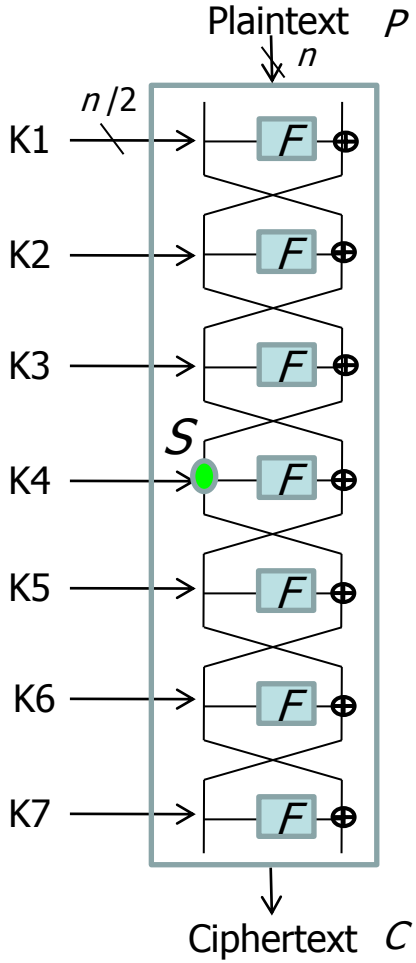

1. Choose s -bit matching state S

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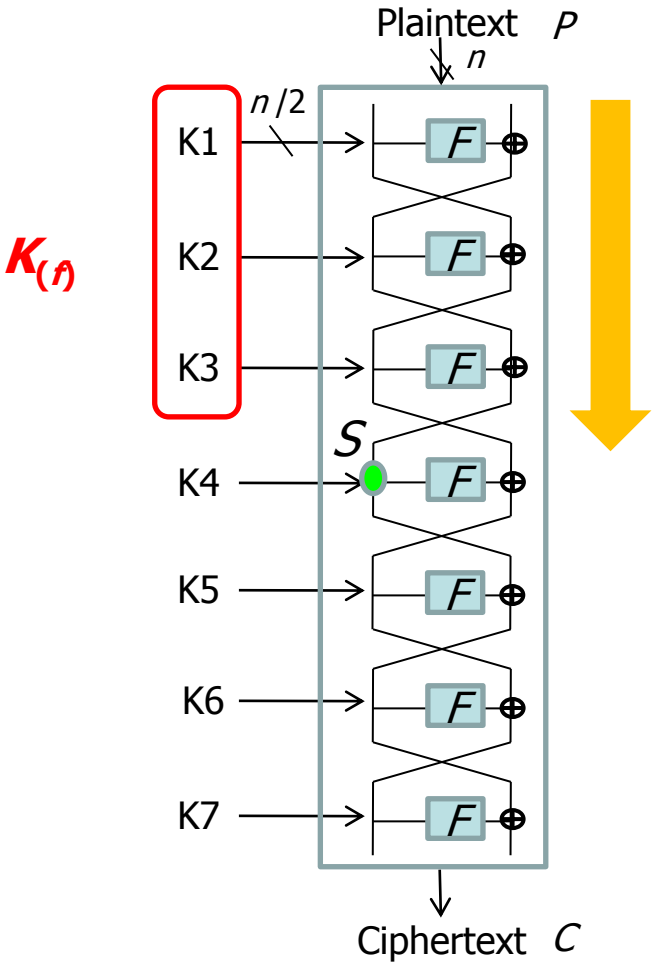

1. Choose s -bit matching state S
2. Compute $S = F_{(f)}(K_{(f)}, P)$ with all $K_{(f)}$ and Make Table of (s, $K_{(f)}$) pairs

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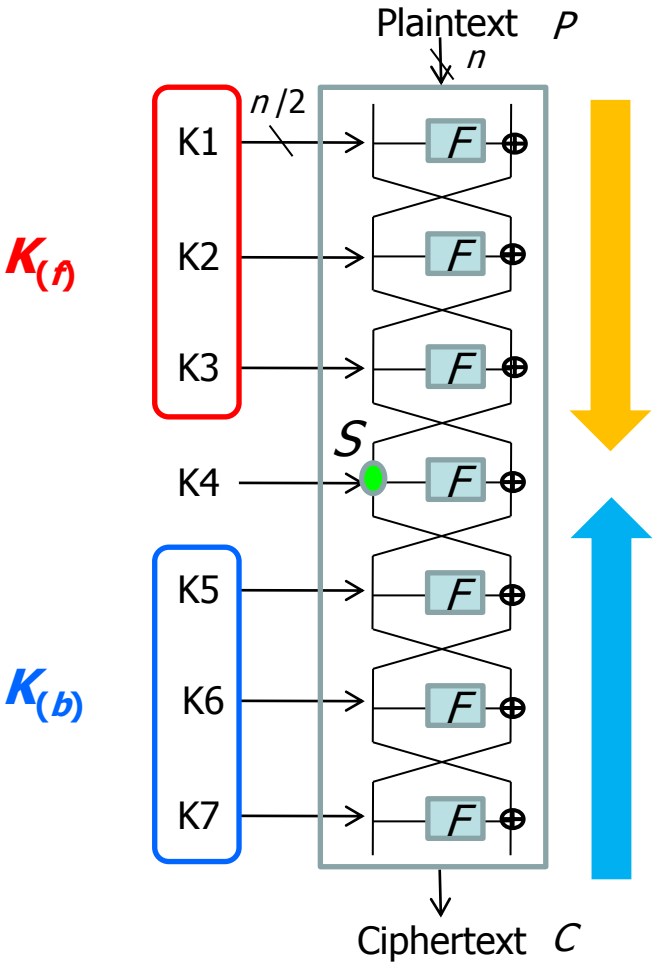

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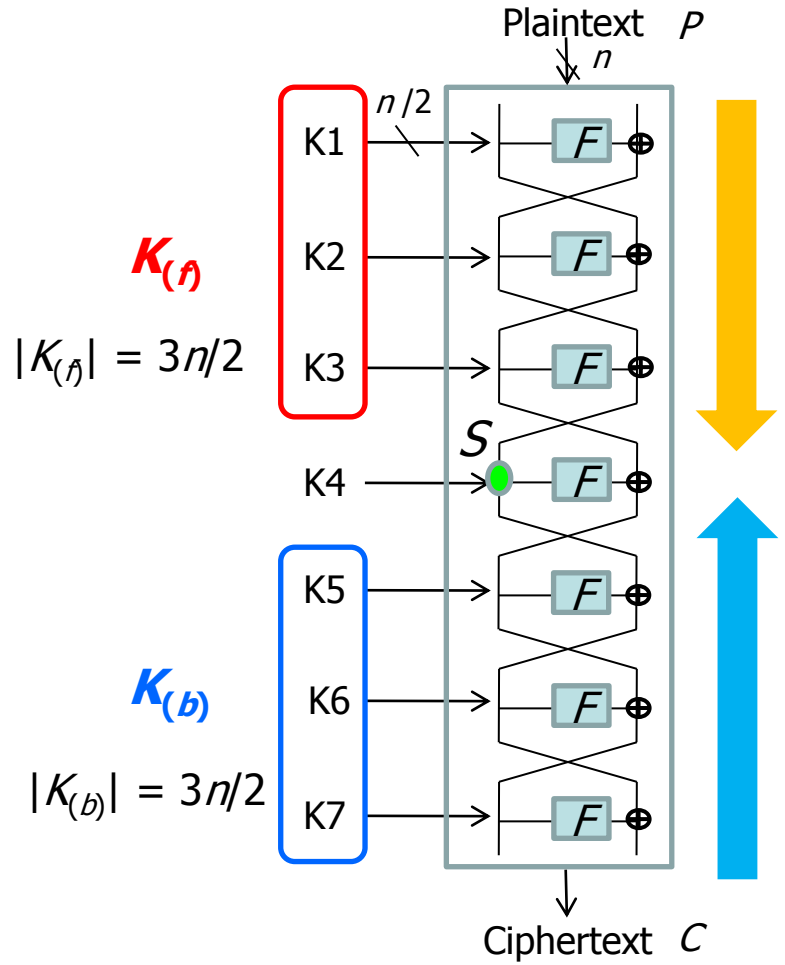

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4. If $S = S'$, regard it as key candidate

surviving key candidates
: $2^{6n/2 - n/2} = 2^{5n/2}$

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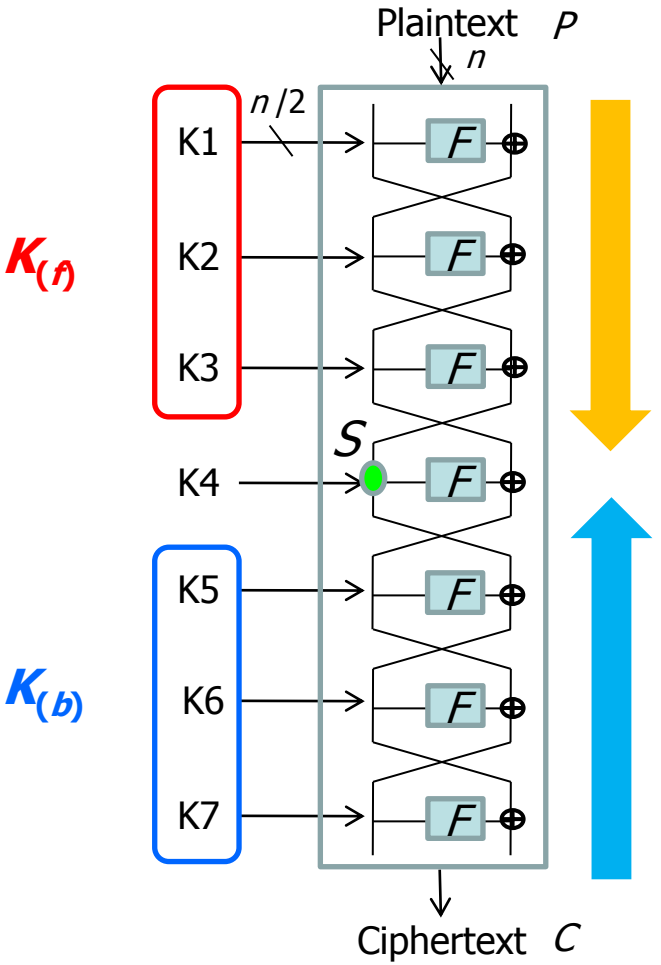

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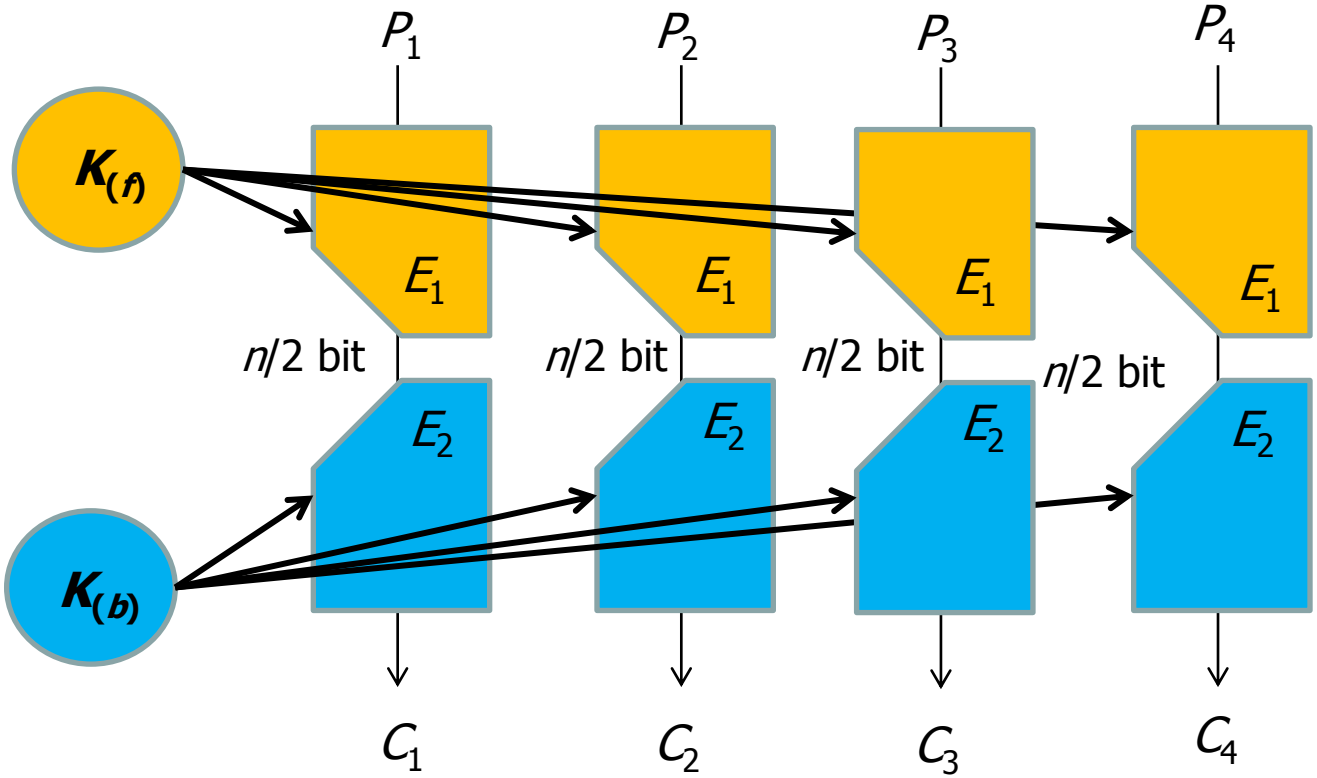
For successful attack
we need to reduce key space

Parallel MitM attack

- Given N (4) plaintext/ciphertexts

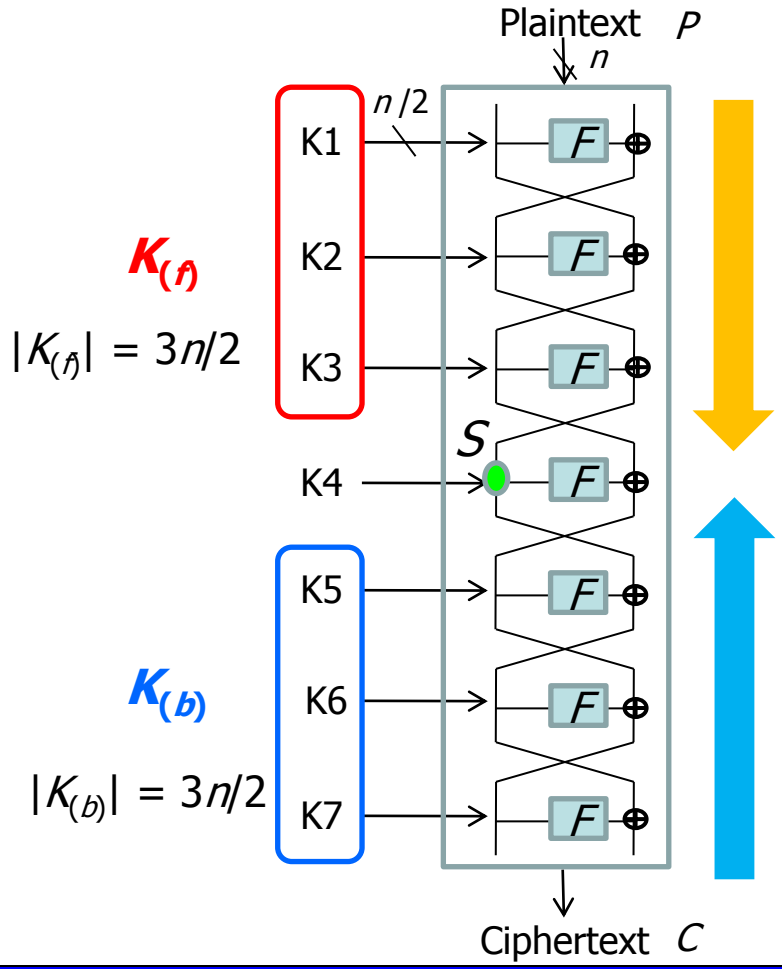


Filter out wrong keys by using N (4) matching state

surviving key candidates : $2^{6n/2 - 4 \cdot n/2} = 2^n (=2^{k/2})$

Evaluation

- Time complexity for filtering wrong keys



-Time complexity :

$$\max\left(2^{|K_{(f)}|}, 2^{|K_{(b)}|}\right) \times N$$

$$= \max\left(2^{3n/2}, 2^{3n/2}\right) \times 4$$

$$= 2^{3n/2+2} = 2^{3n/4+2} < 2^{k}$$

Point for successful attack :

Reduce involved keys $K_{(f)}$ and $K_{(b)}$ in forward and backward computation

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Function Reduction Technique [IS 2013]

- Technique to reduce bits used in $K_{(f)}$ and $K_{(b)}$
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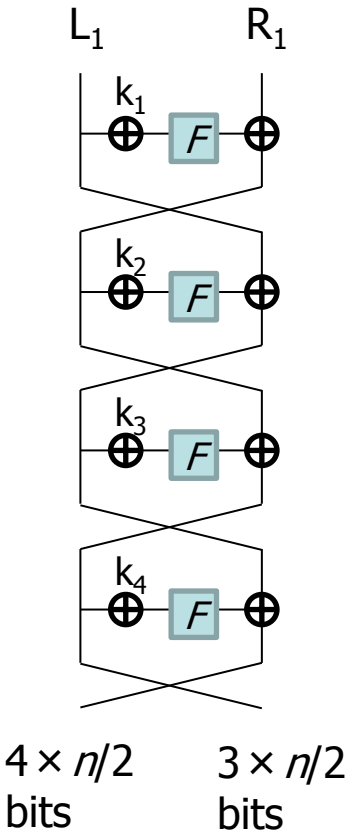

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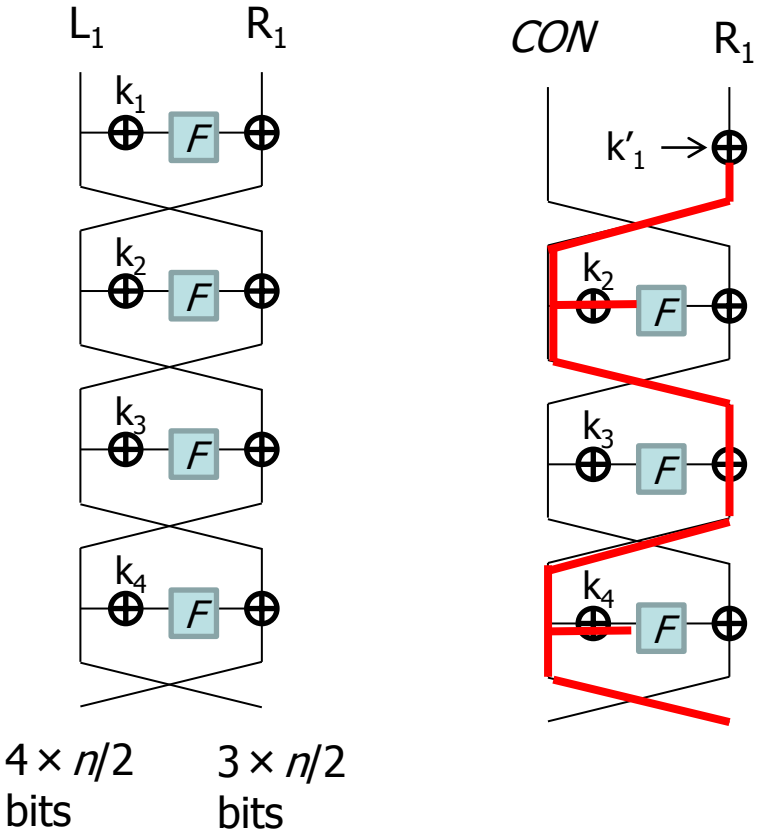

$k'_1 = F(CON \oplus k1)$

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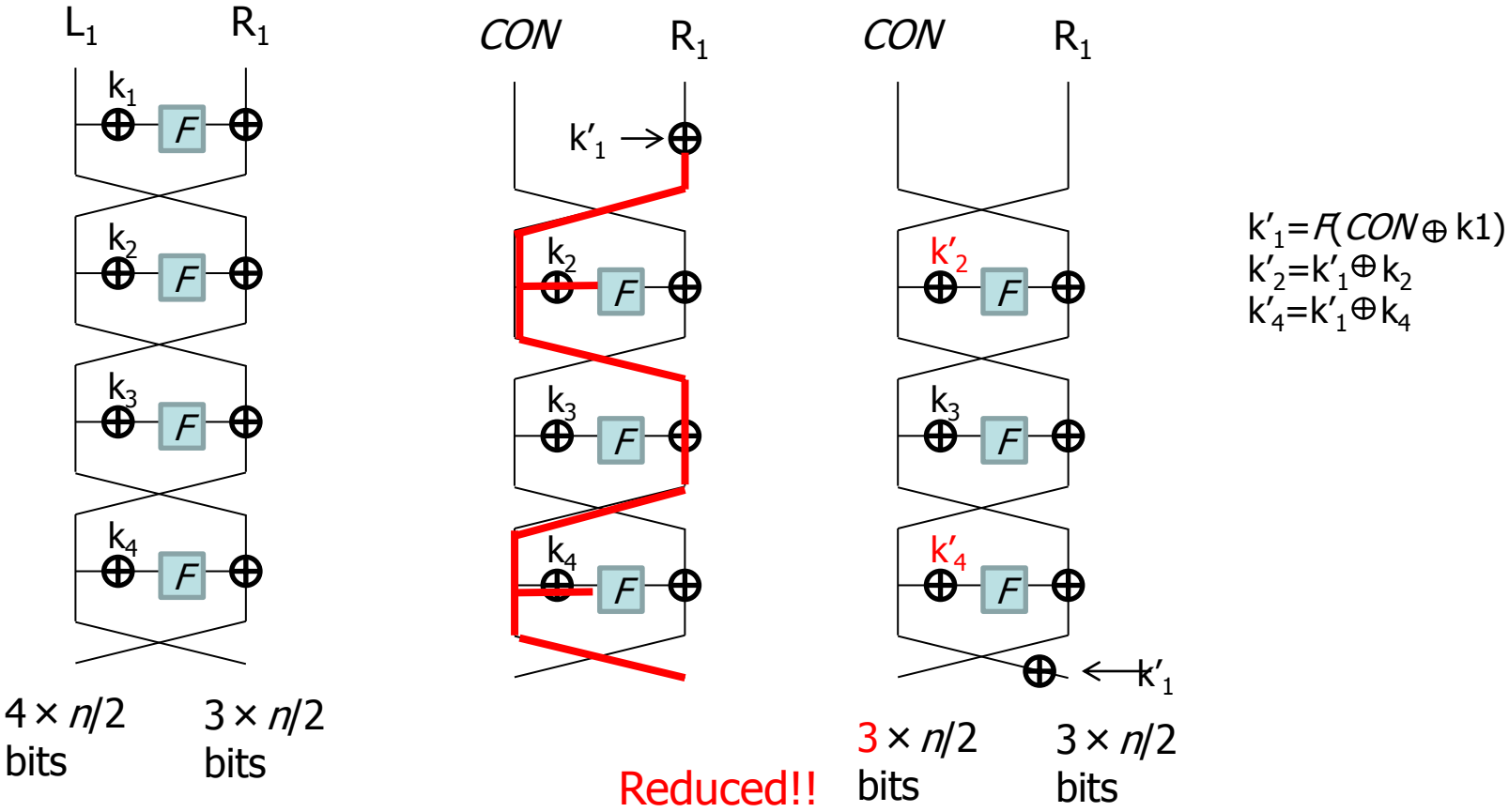

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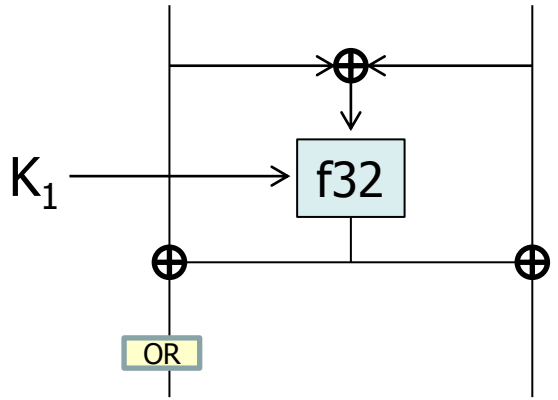
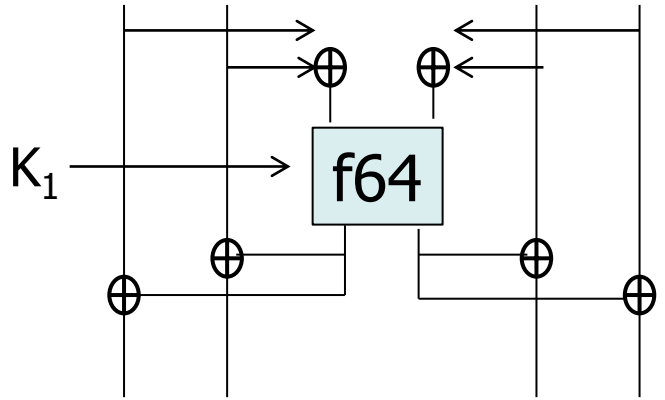
FOX

- Proposed by Junod and Vaudenay @ SAC 2004
- Also known as IDEA NXT (successor of IDEA)
- Based on Lai-Massey scheme
- *Strong* key scheduling function
- Two Variants
  - FOX 64 : 64-bit block and 128-bit key
  - FOX 128 : 128-bit block and 256-bit key

FOX 64

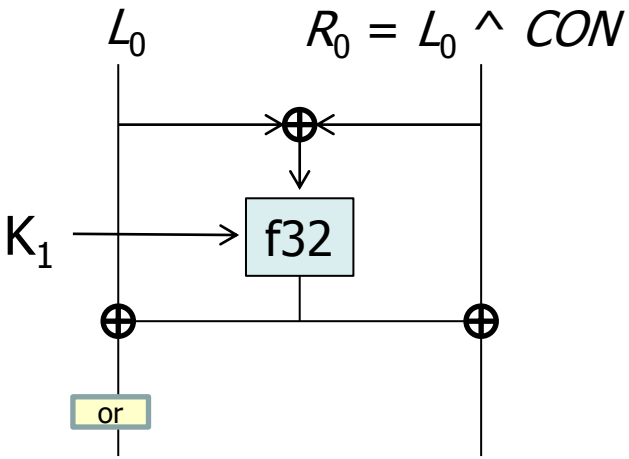


FOX 128



Function Reduction of FOX 64

- Choose plaintext s.t., $R_0 = L_0 \wedge CON$
  - Input to f32 is fixed to constant *CON*



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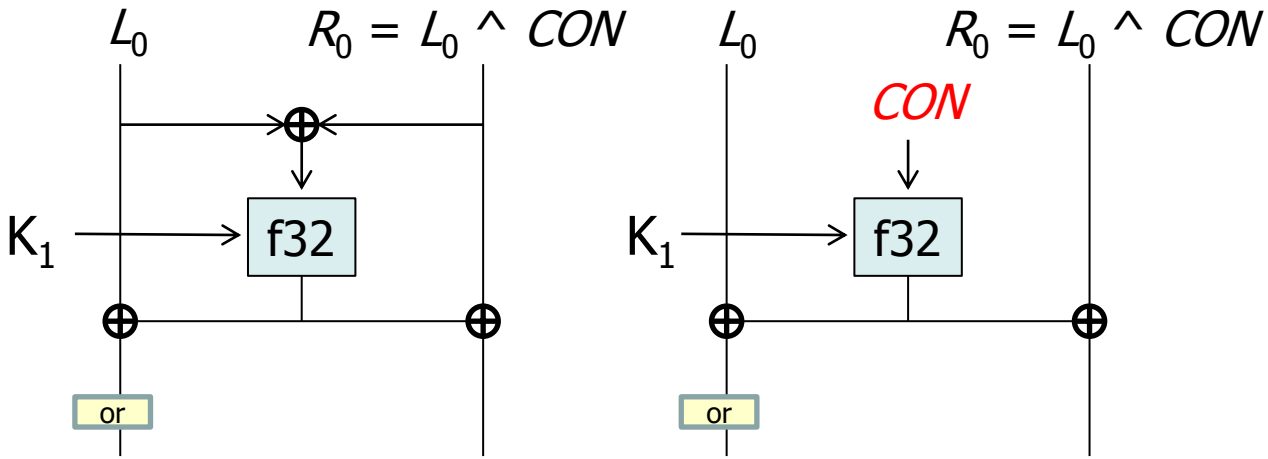

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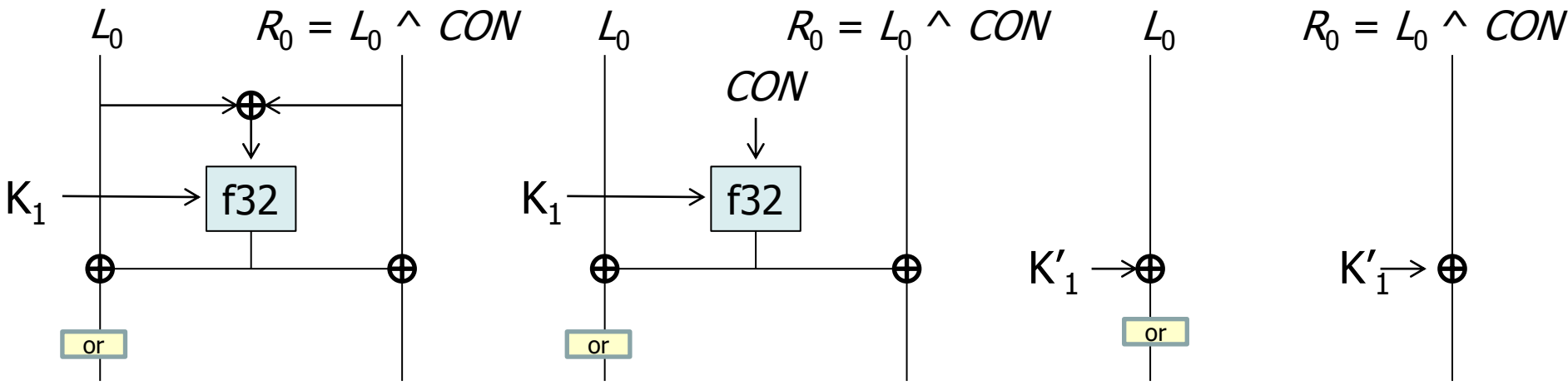

$K'_1 = f32(CON, K_1)$

depend only on key bits

Function Reduction of FOX 64

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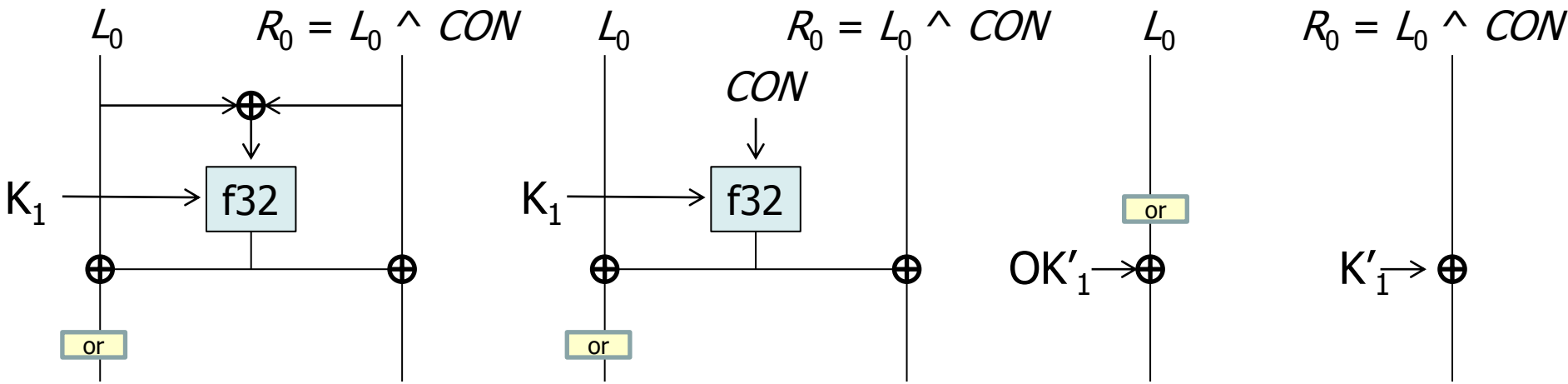
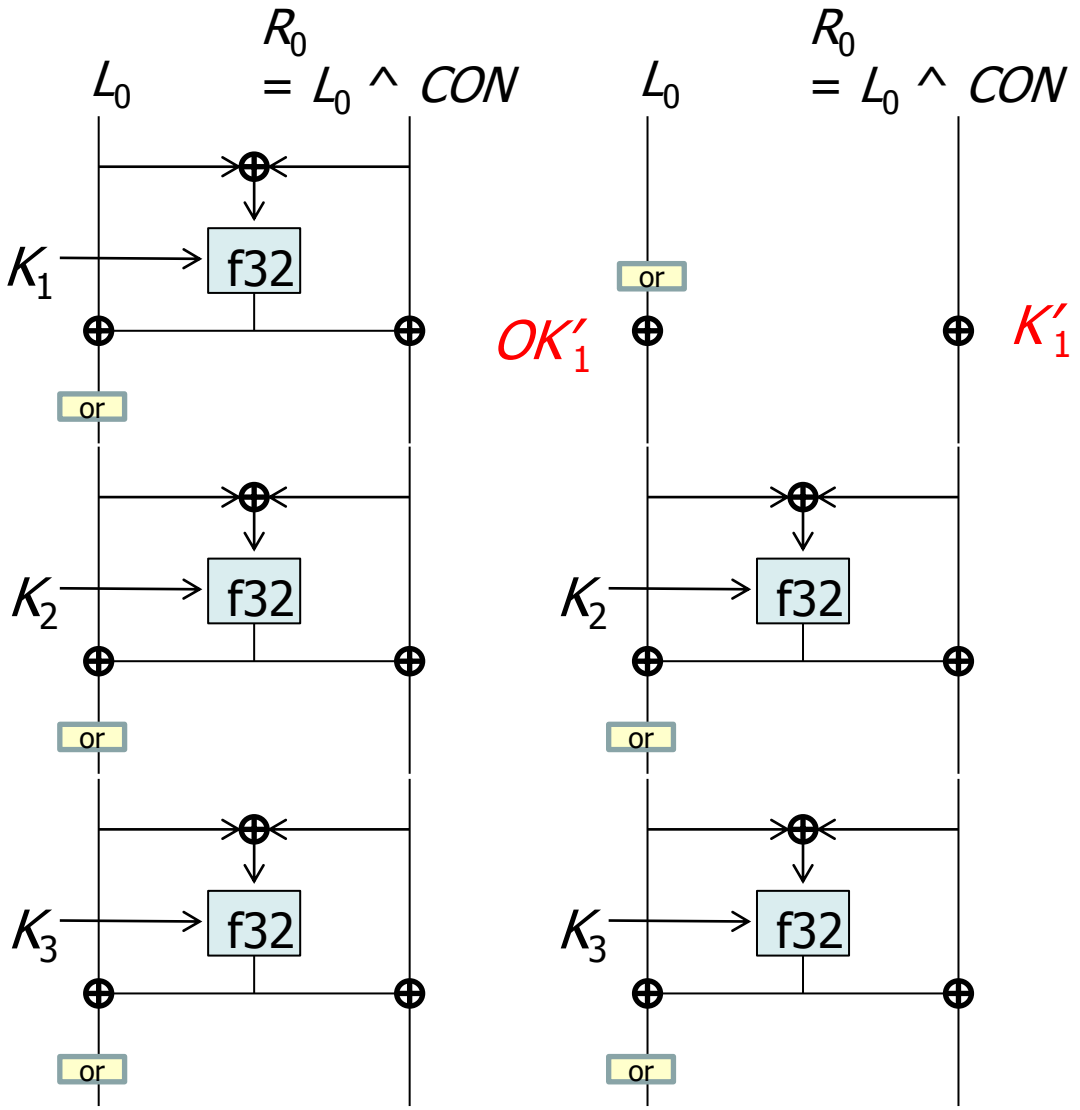


$K'_1 = f32(CON, K_1)$
$OK'_1 = or\ (f32(CON, K_1))$

3-round Function Reduction of FOX64



$K'_1 = \text{f32}(CON, K_1)$

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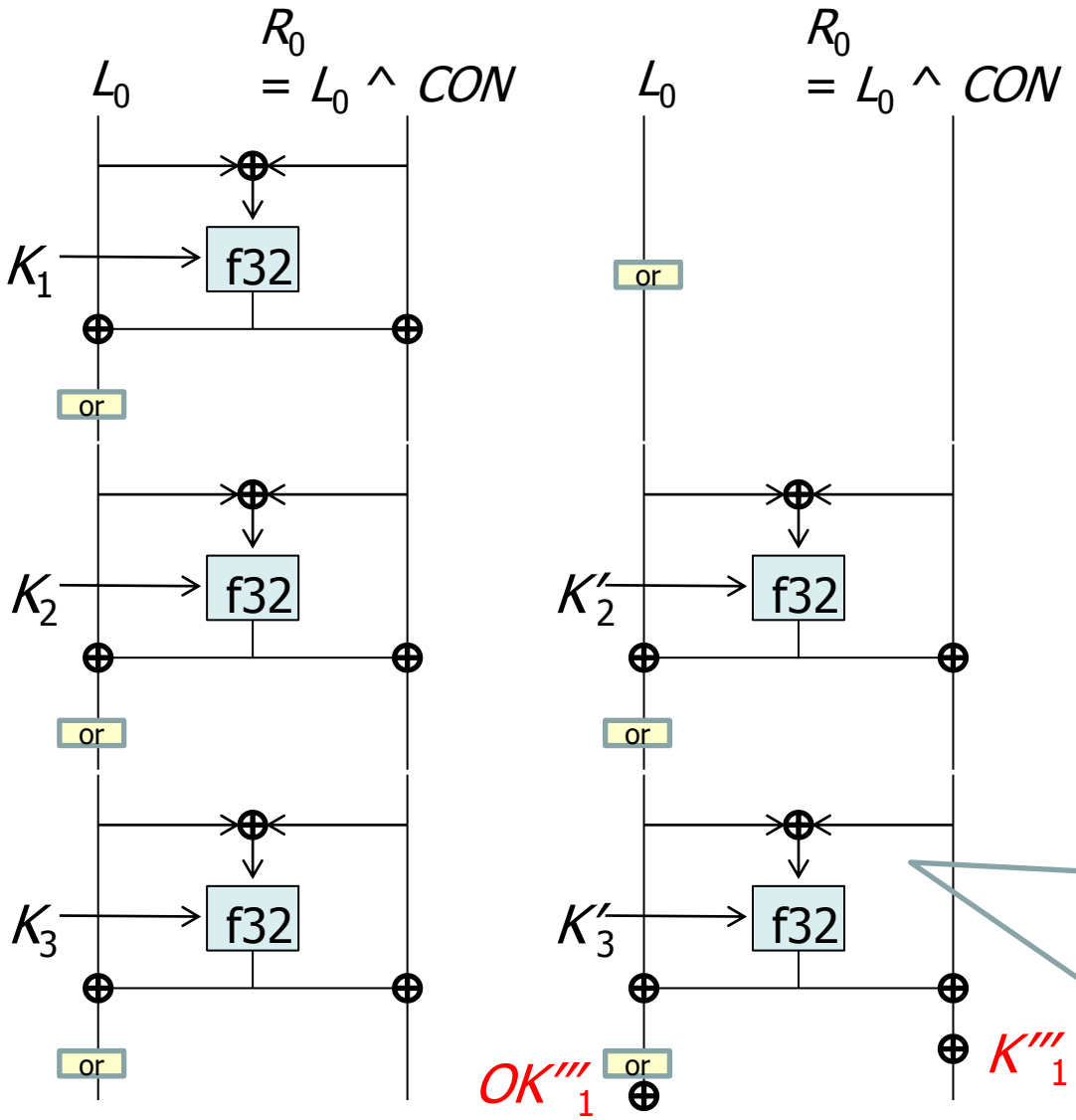

$K'_1 = \text{f32}(CON, K_1)$
$OK'_1 = \text{or}(\text{f32}(CON, K_1))$
$LK'_2 = LK_2 \wedge OK'_1 \wedge K'_1$
$K''_1 = K'_1 \wedge LK_2$
$OK''_1 = \text{or}(OK'_1 \wedge LK_2)$
$K'_2 = LK_2 \,||\, RK_2$
$LK'_3 = LK_3 \wedge OK''_1 \wedge K''_1$
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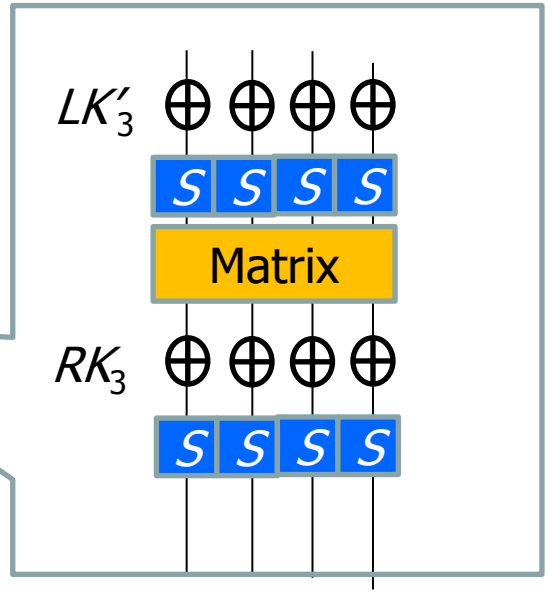

6-round Attack on FOX 64

- 3-round Function Reduction in forward direction

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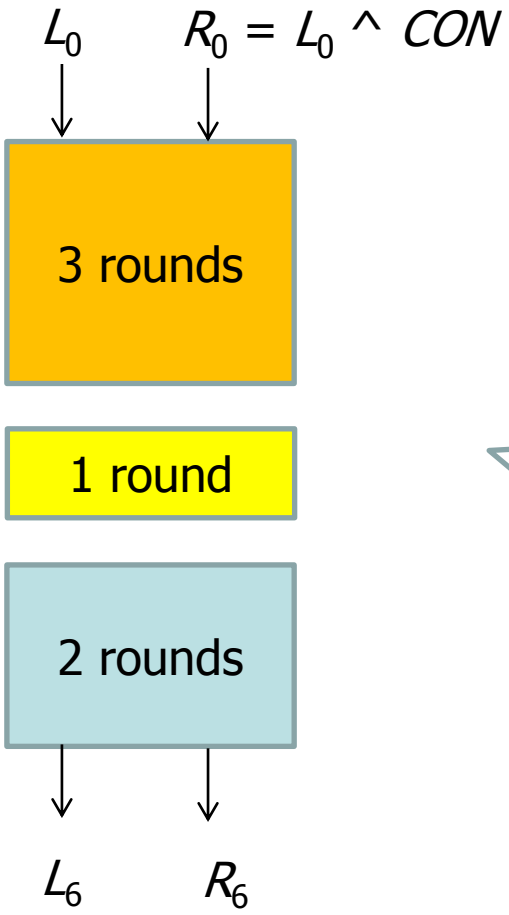

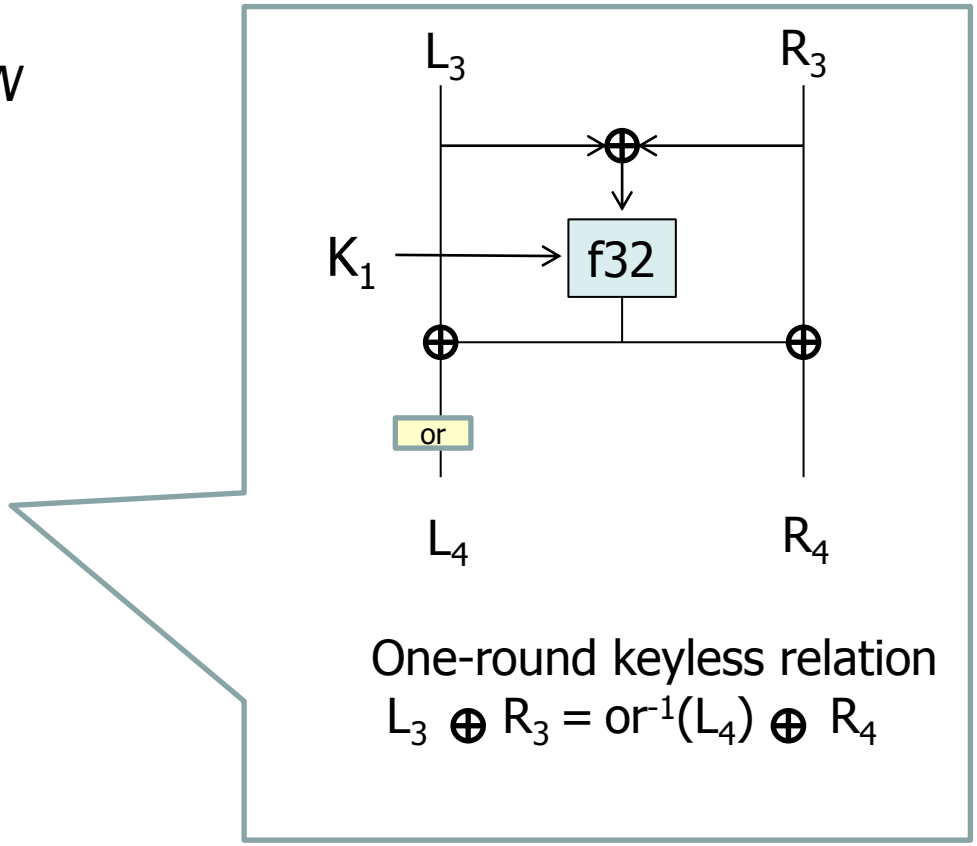

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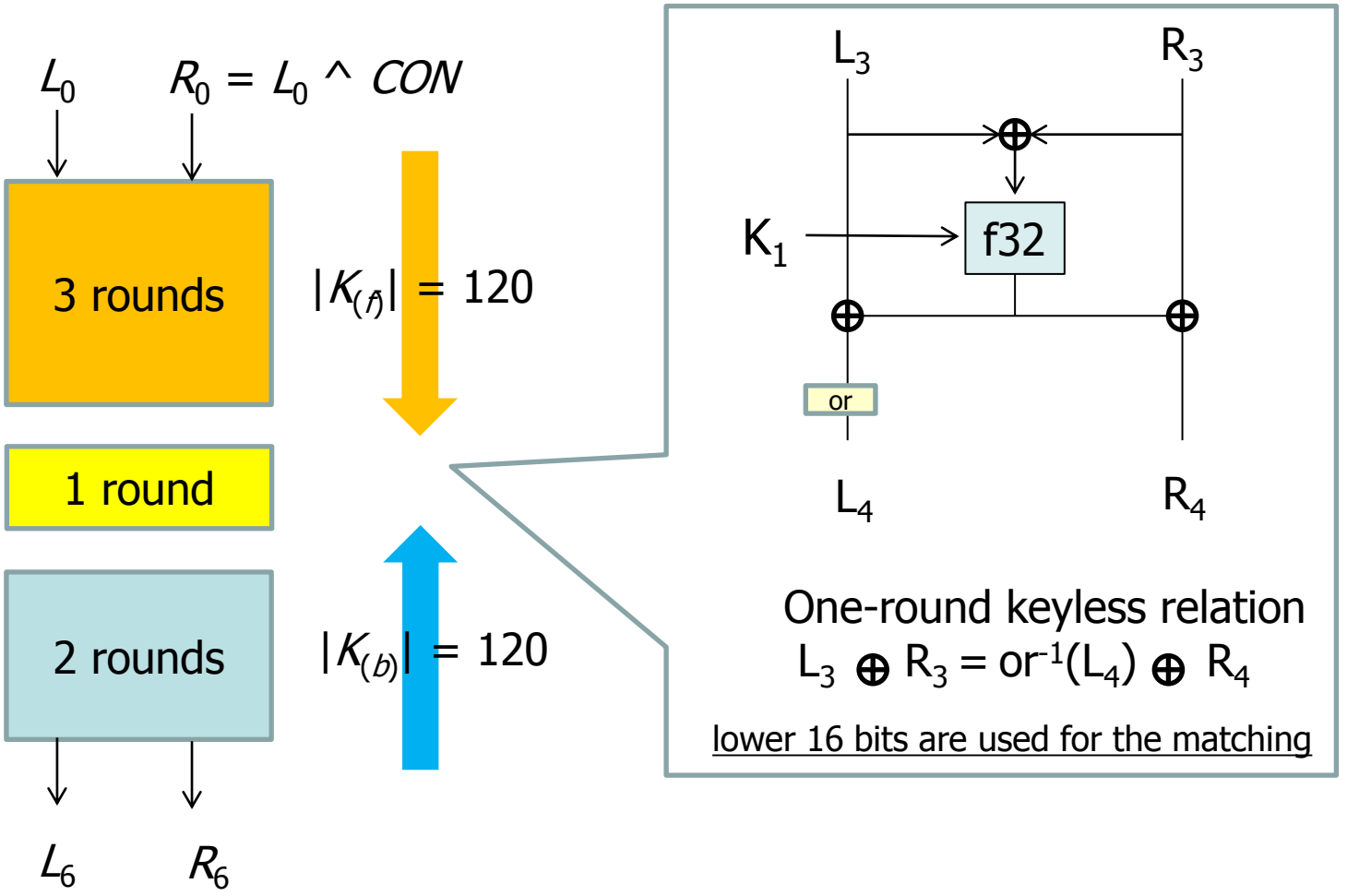

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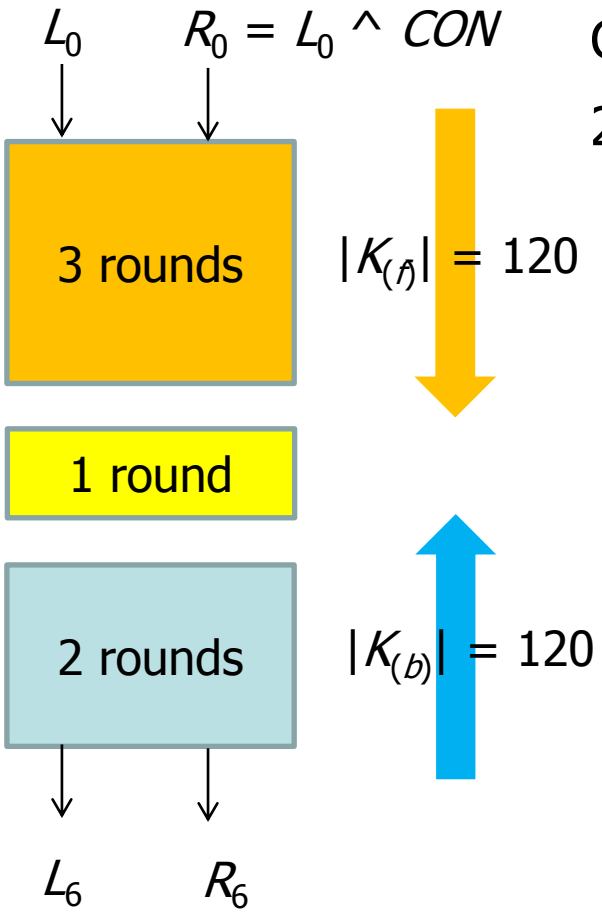


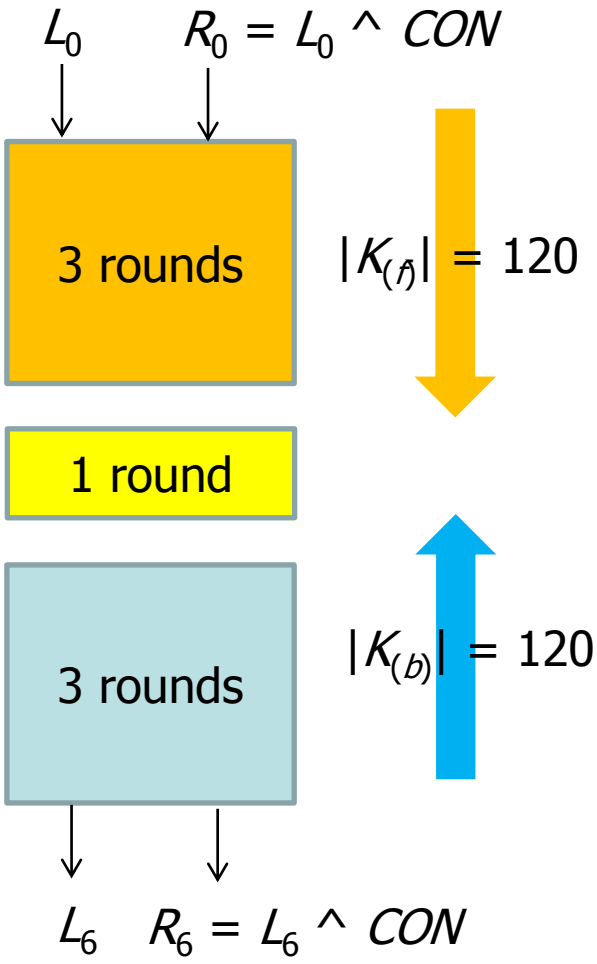
Given 15 data (15 parallel ASR attacks),

2^{240} candidates are reduced to $2^0(2^{240-16\cdot15}) = 1$

Time : $2^{120} \times 15 = 2^{124}$
Data : 15
Memory : $2^{120} \times 14 = 2^{124}$

7-round Attack on FOX64

- 3-round Function Reduction in both directions



- Require 15 plaintext/ciphertext pairs s.t.
  - Plaintext = $L_0 || L_0 \wedge CON$ (32 bit condition)
  - Ciphertext = $L_6 || L_6 \wedge CON$ (32 bit condition)
- To find these data, it requires more than 2^{32} plaintexts satisfying $L_0 || L_0 \wedge CON$ (32 bit condition).
- However, degree of freedom of plaintext is only 32 bit L_0

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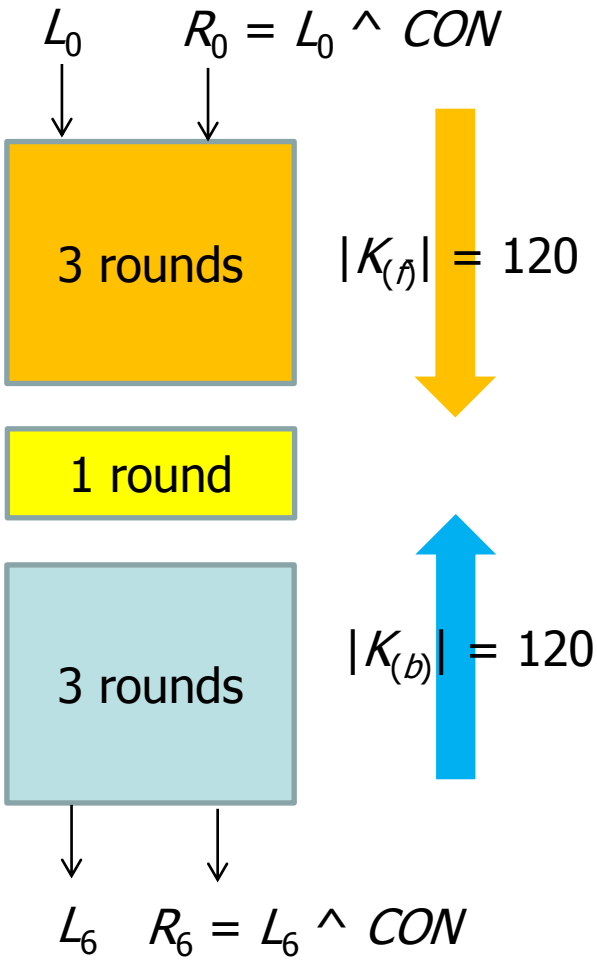

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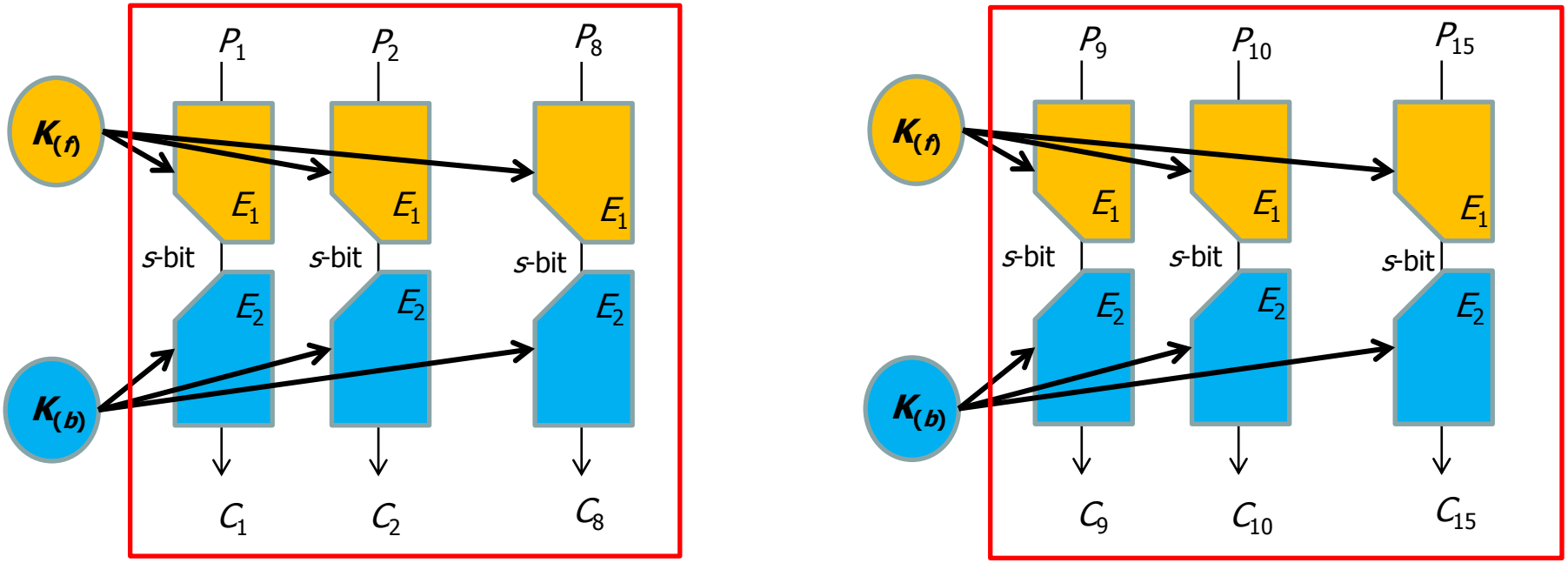
- However, degree of freedom of plaintext is only 32 bit L_0

Repetitive ASR approach

Repetitive All-Subkeys Recovery Attack

- Divide Parallel MitM into M parts
  - Enable mounting each ASR with less data
  - Example : $M = 2$ (8 and 7)



8 desired data can be obtained from 2^{31} ciphertexts

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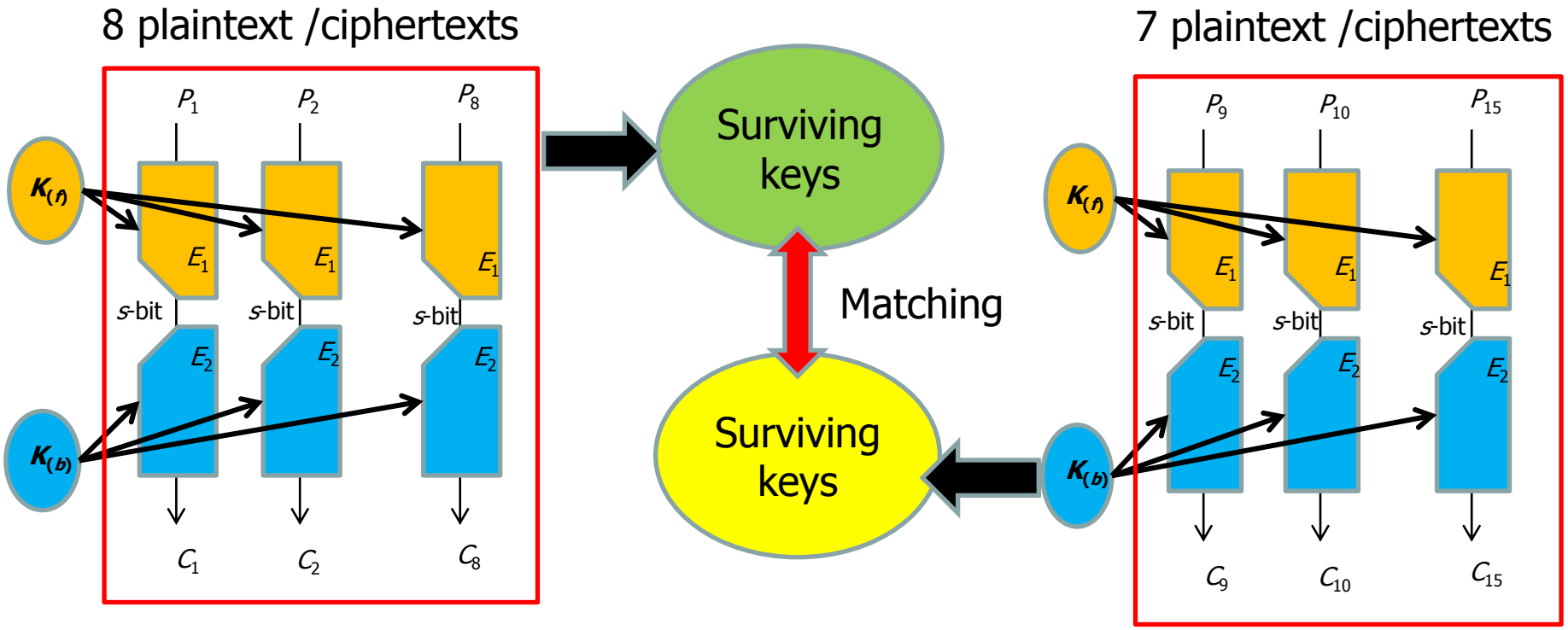

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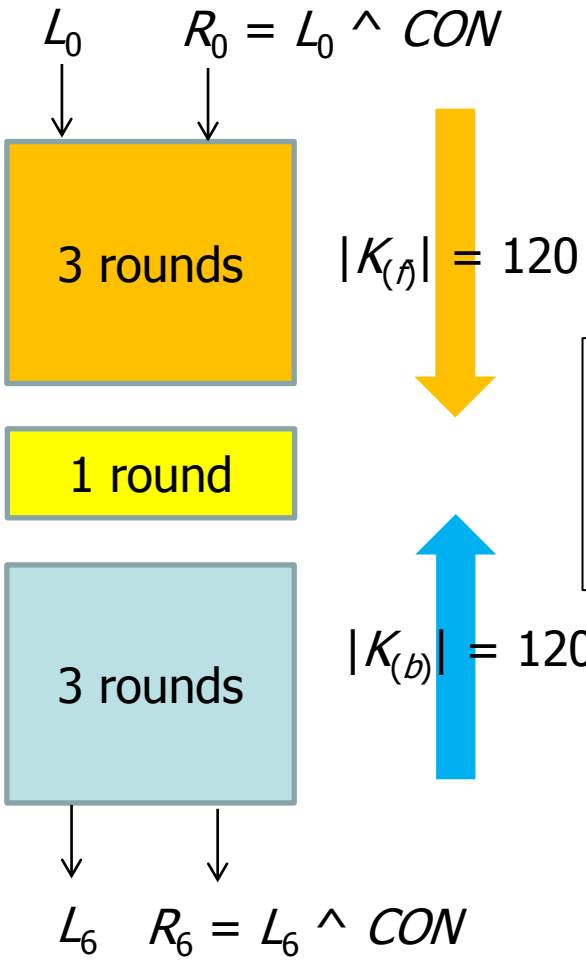

Time : $2^{120} \times 8 + 2^{120} \times 7 = 2^{124}$
Data : 2^{31}
Memory : $2^{120} \times 14 = 2^{123}$

Results

- Update best single-key attacks on FOX64 and FOX128 w.r.t. number of attacked rounds

Best Attacks

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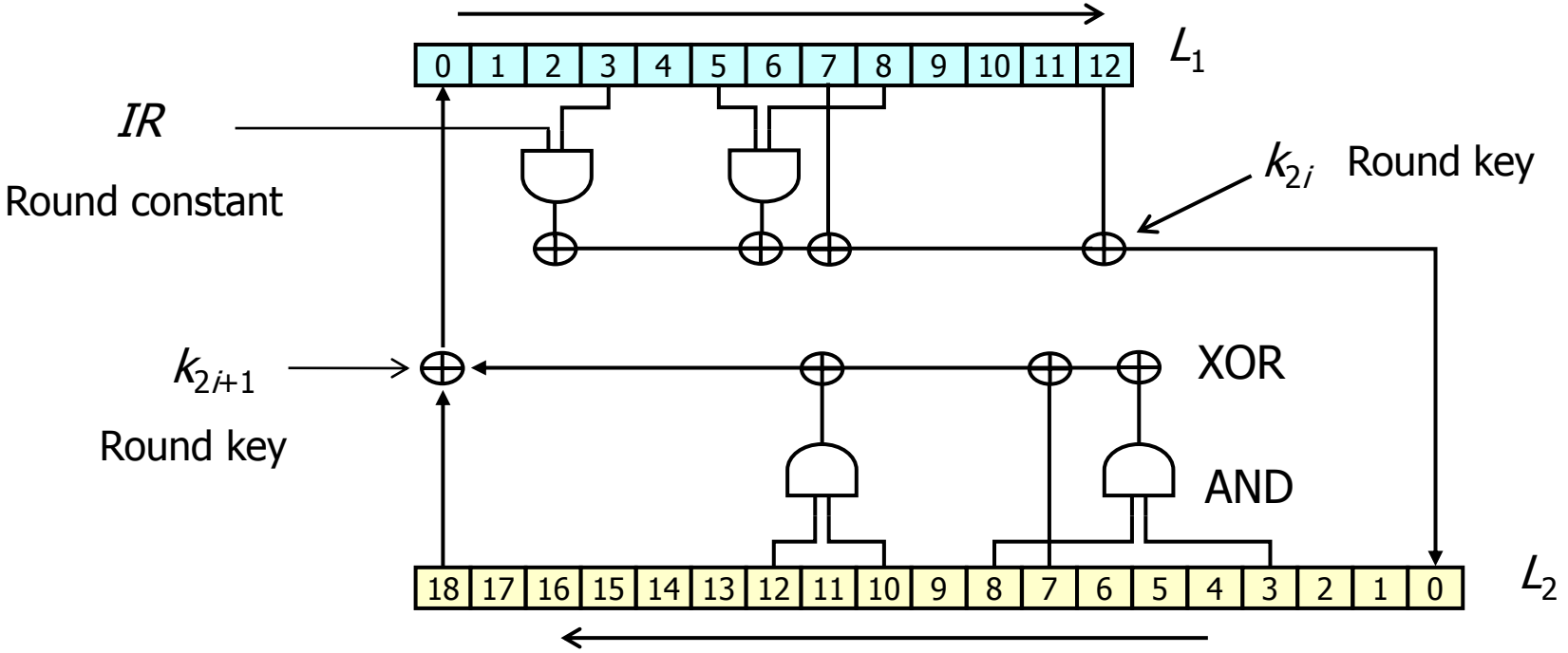
4. Improved Attacks on KATAN-32/48/64
5. Improved Attacks on SHACAL-2
6. Conclusion

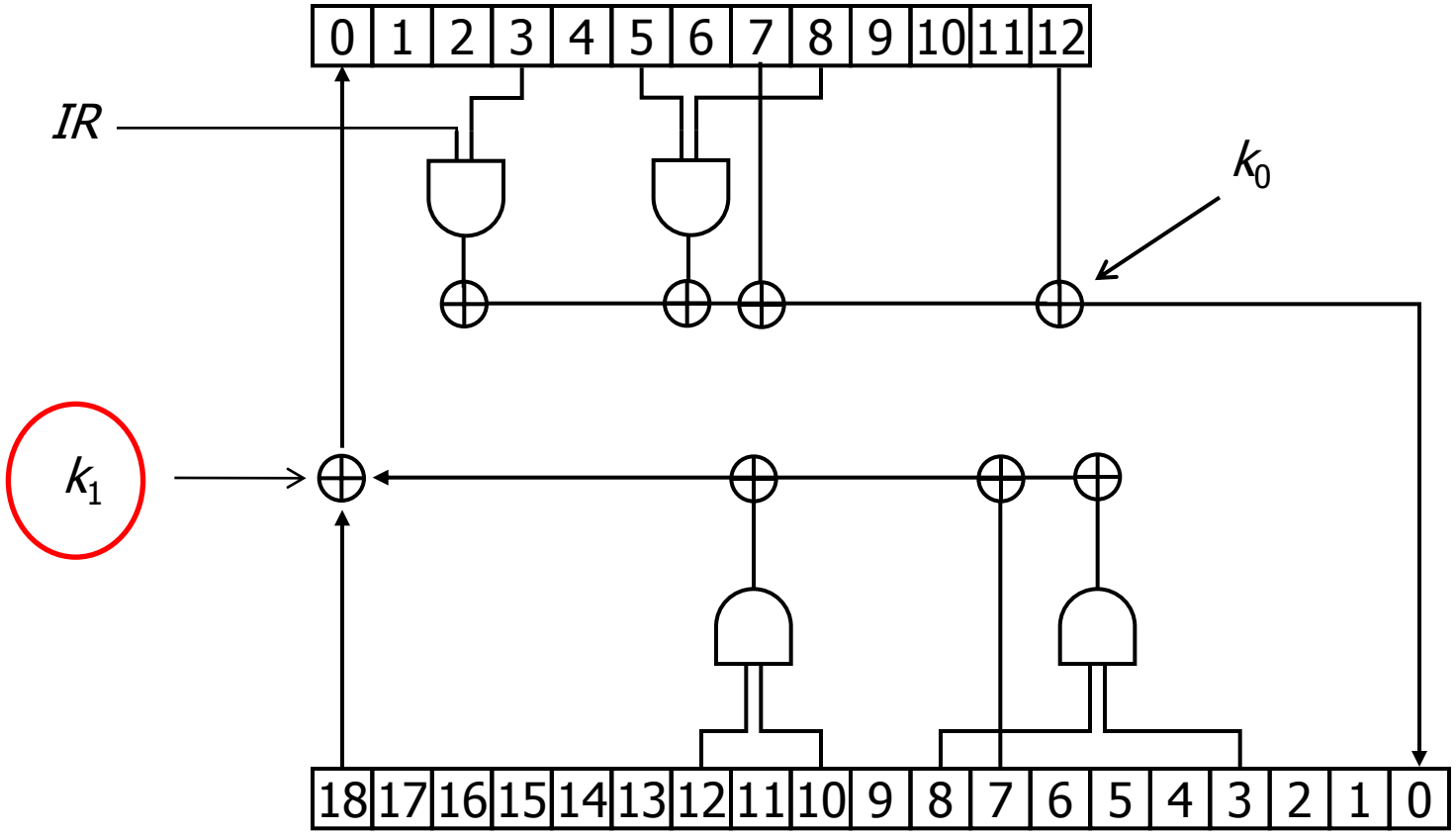
KATAN Family

- Ultra lightweight block cipher (CHES 2010)
- block size : 32/48/64 bits, key size : 80 bits
- Based on Stream cipher Trivium
  - 254-round LFSR-type construction



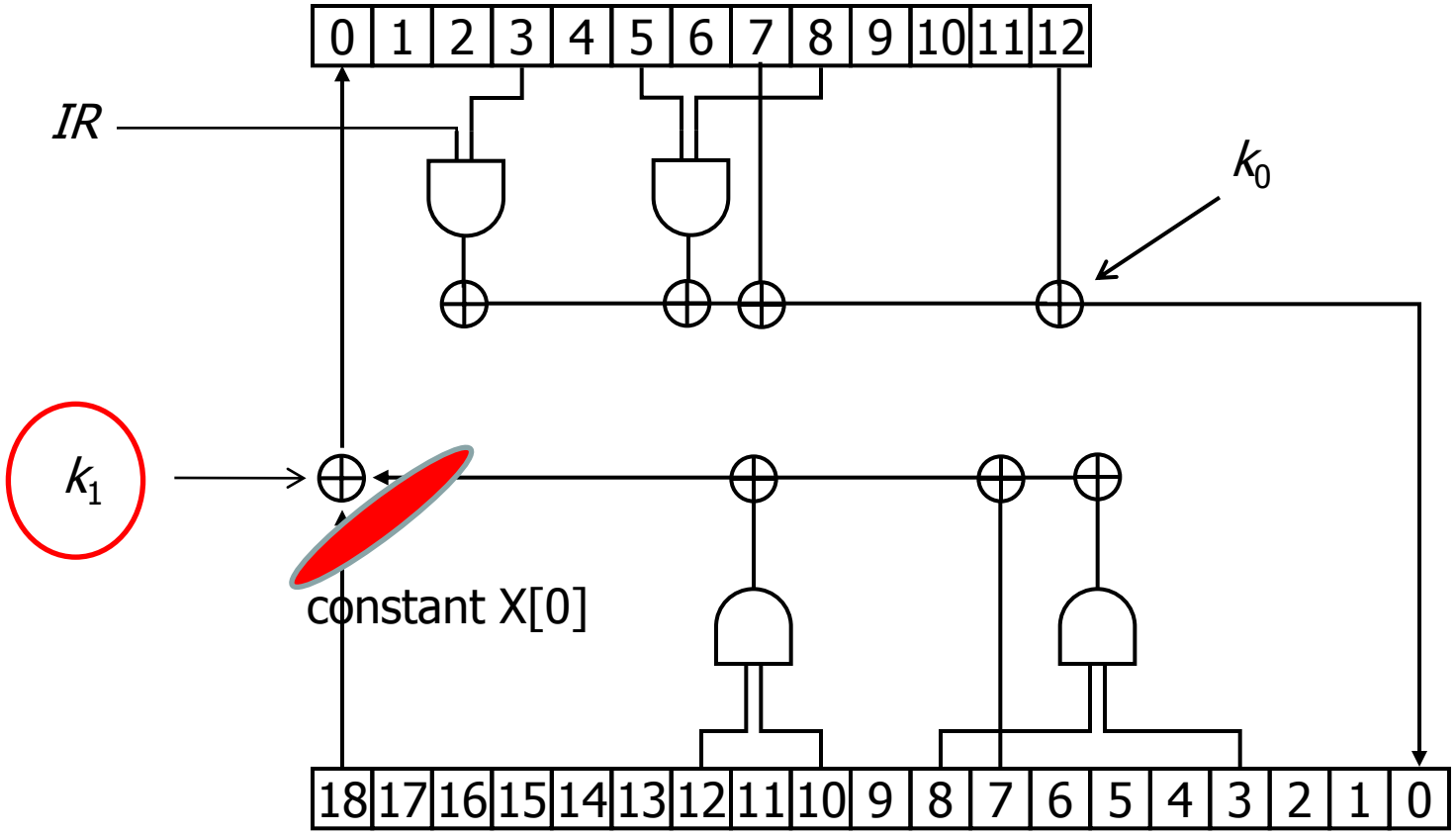
Function Reduction of KATAN32

$i = 0$

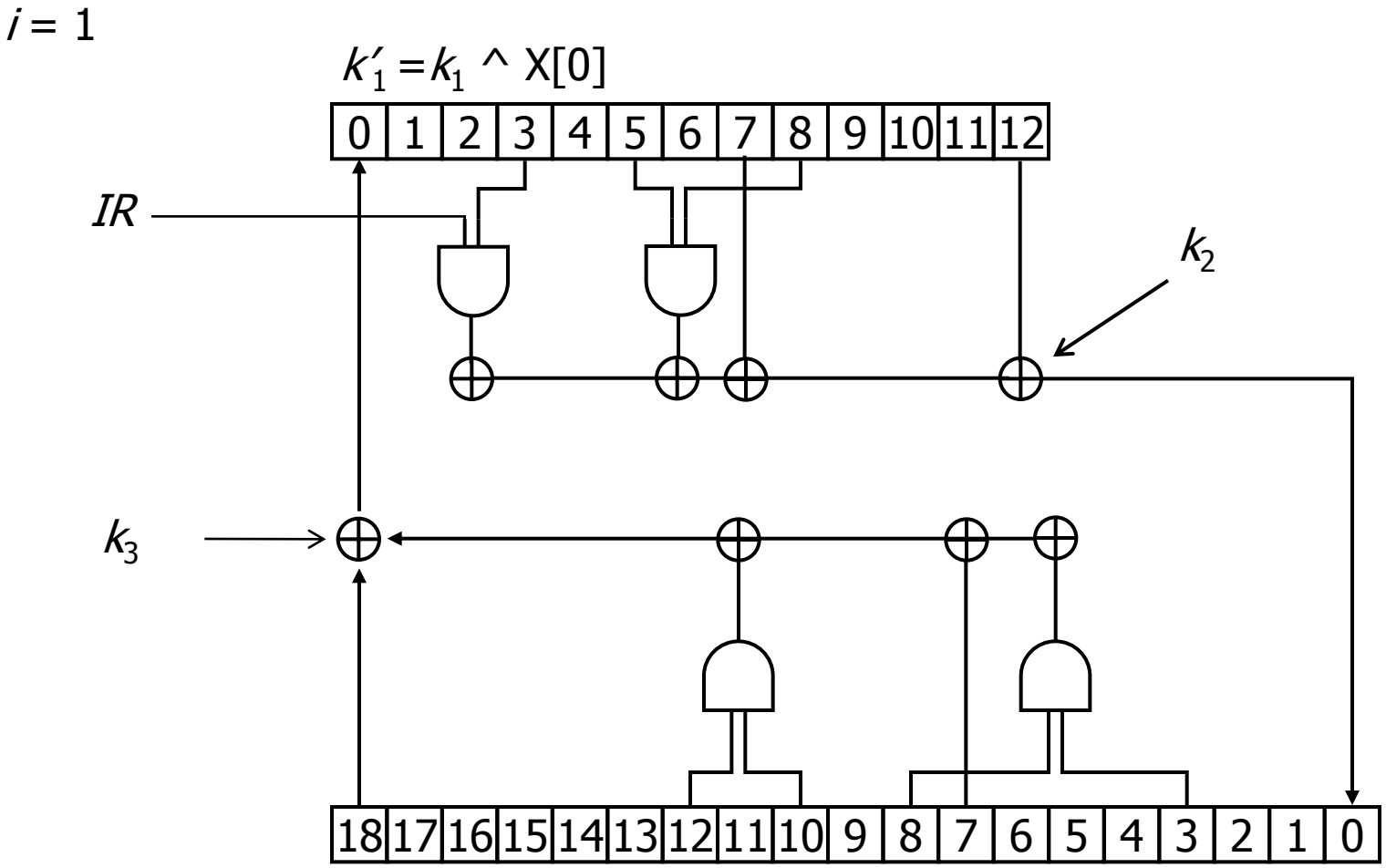


Function Reduction of KATAN32

$i = 0$



Function Reduction of KATAN32



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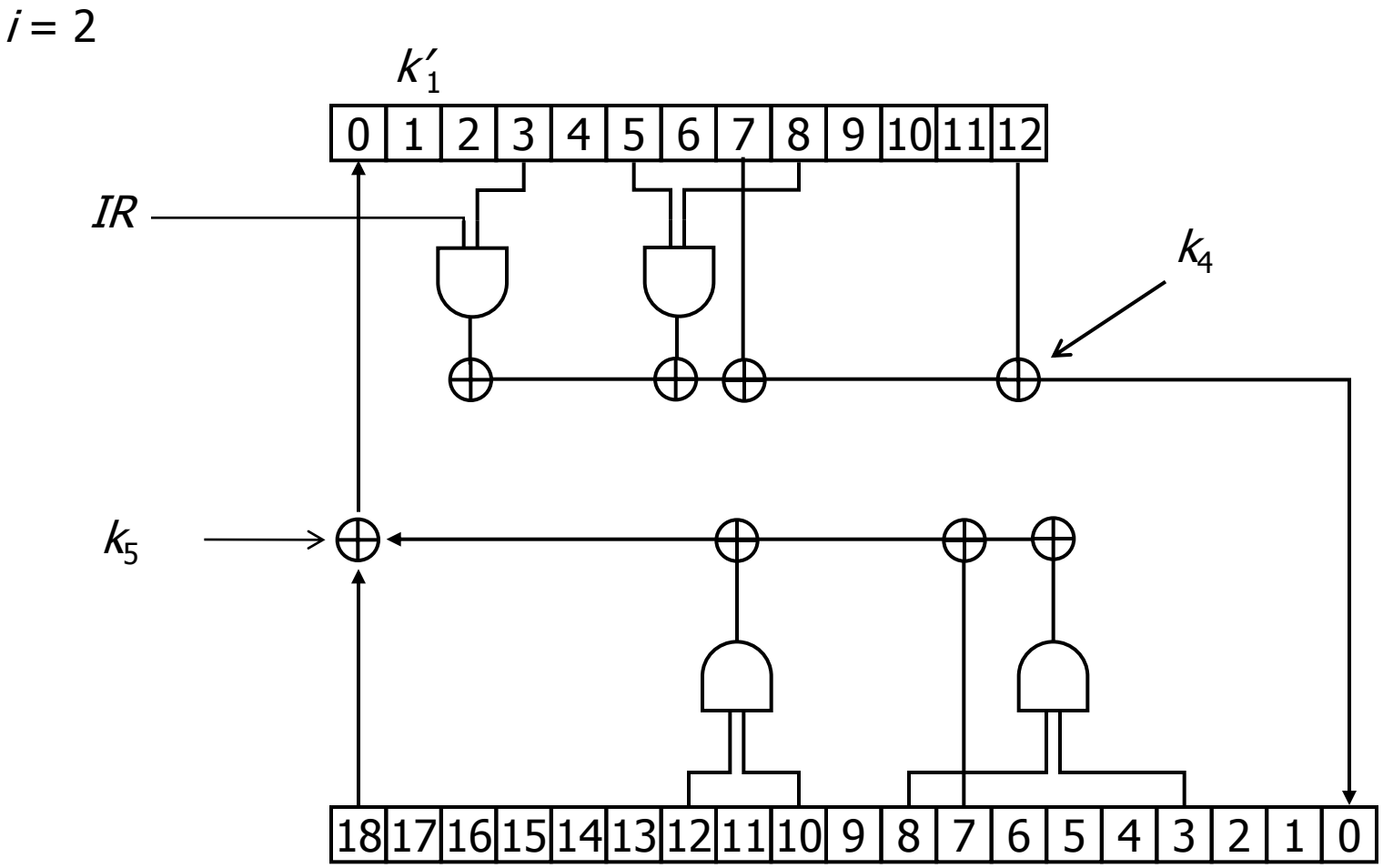
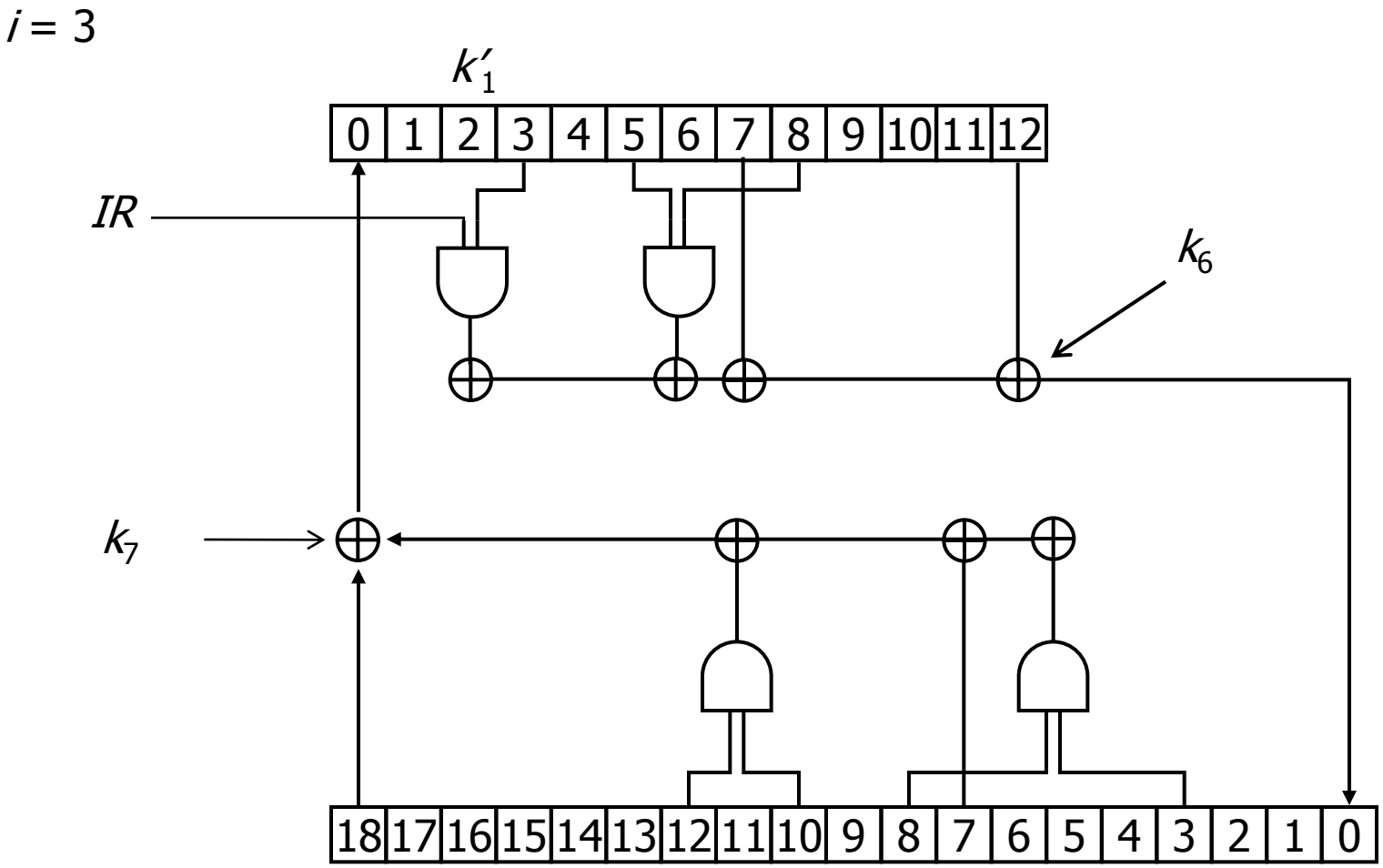
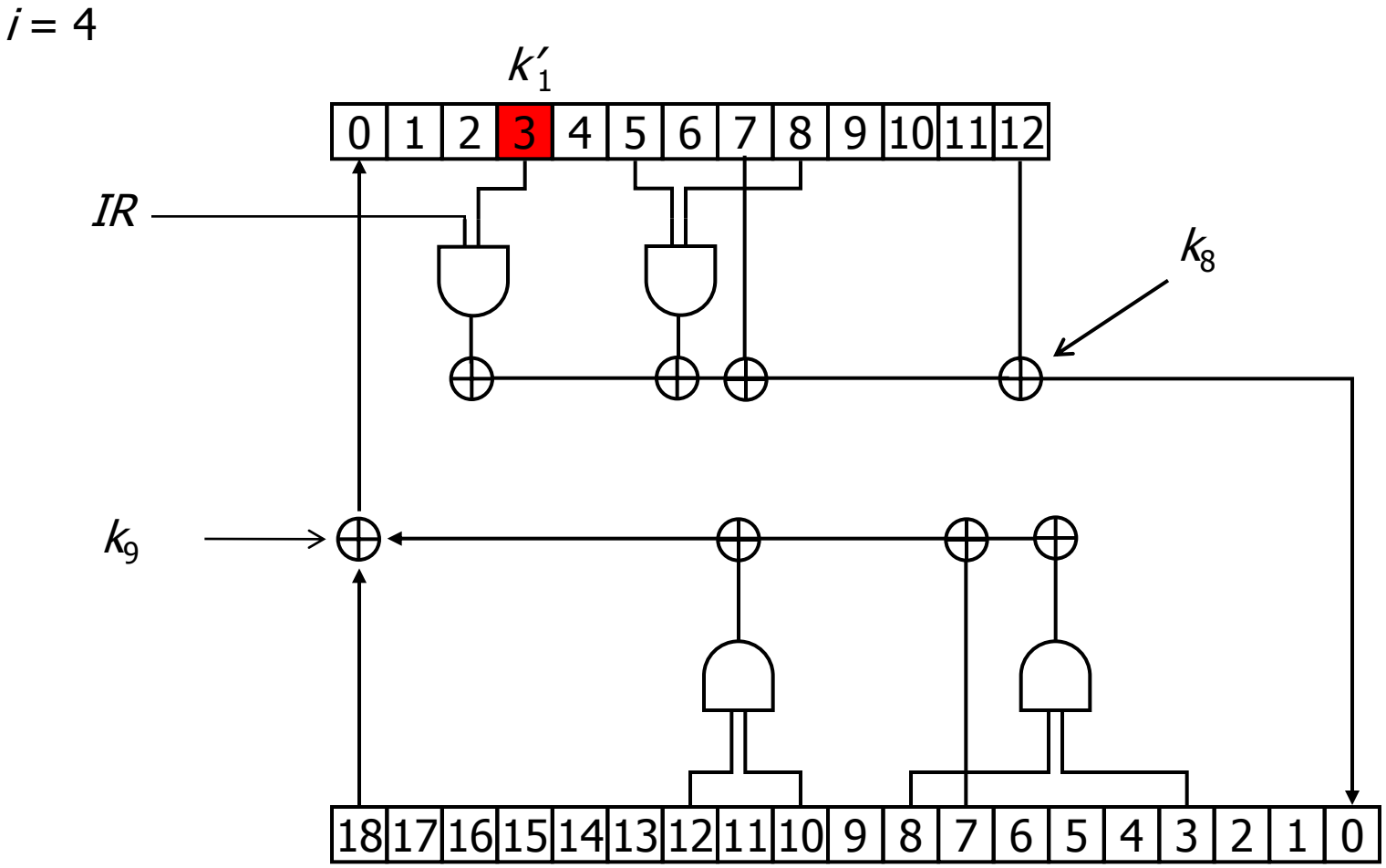


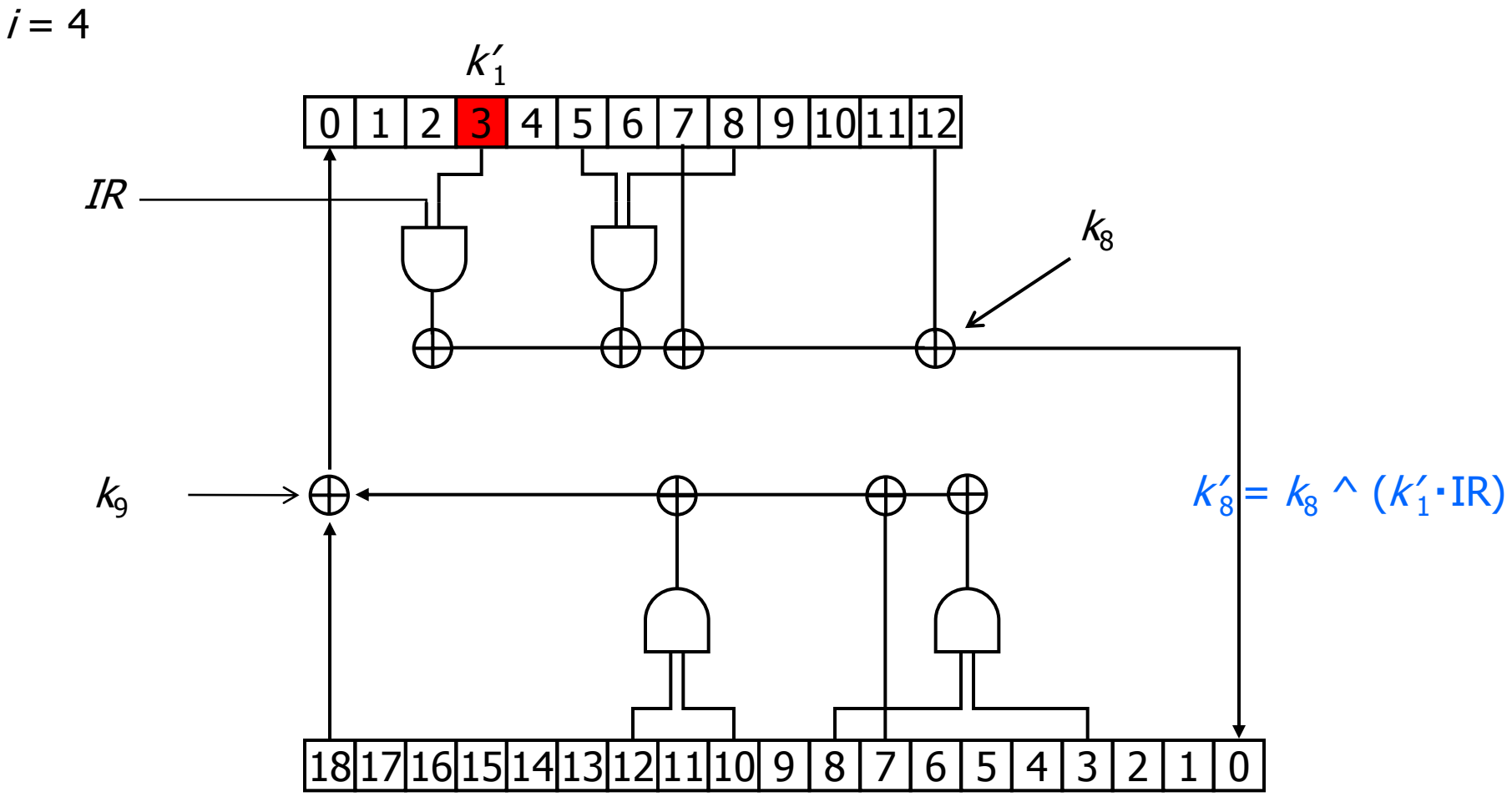
Function Reduction of KATAN32



Function Reduction of KATAN32

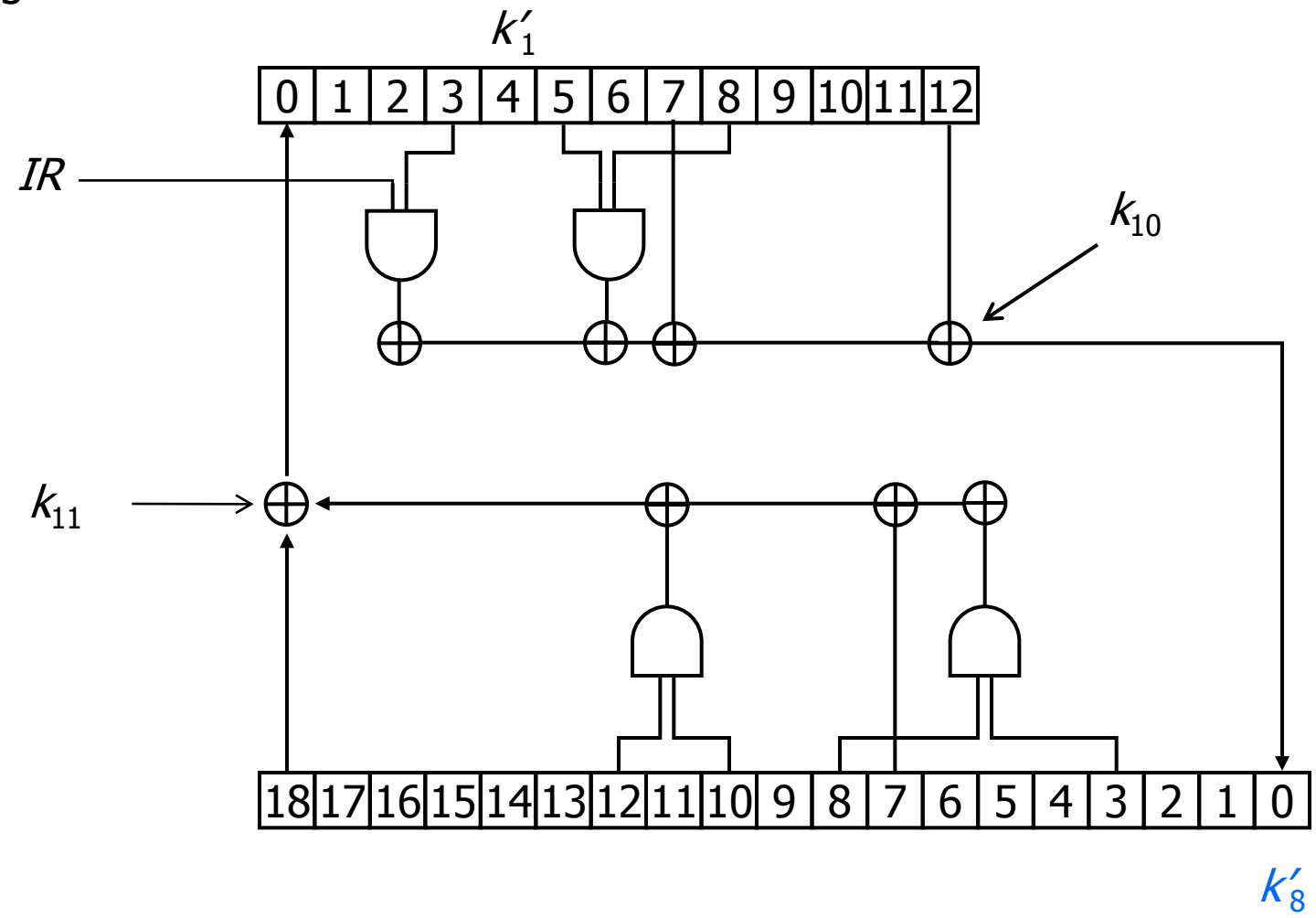


Function Reduction of KATAN32

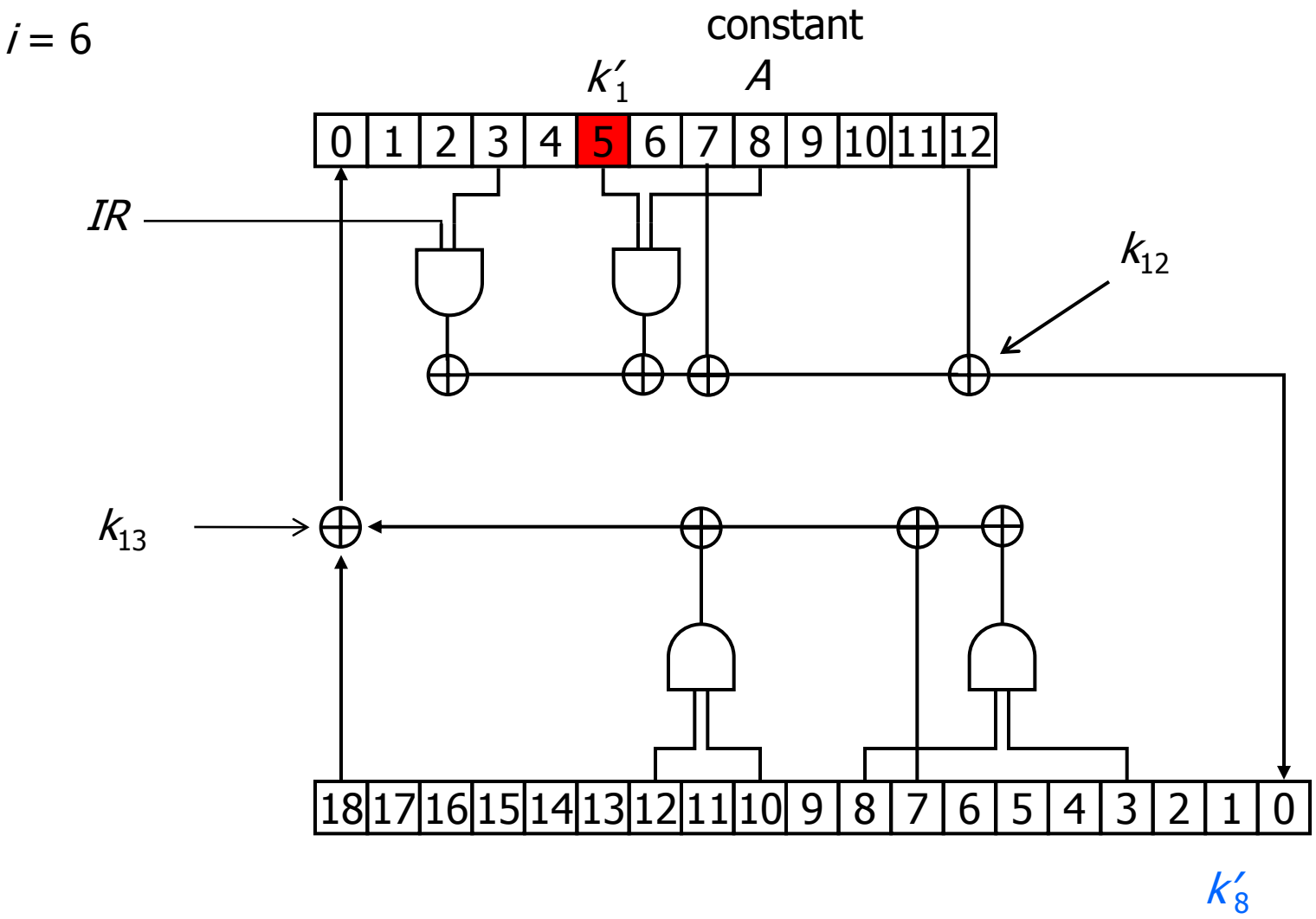


Function Reduction of KATAN32

$i = 5$



Function Reduction of KATAN32



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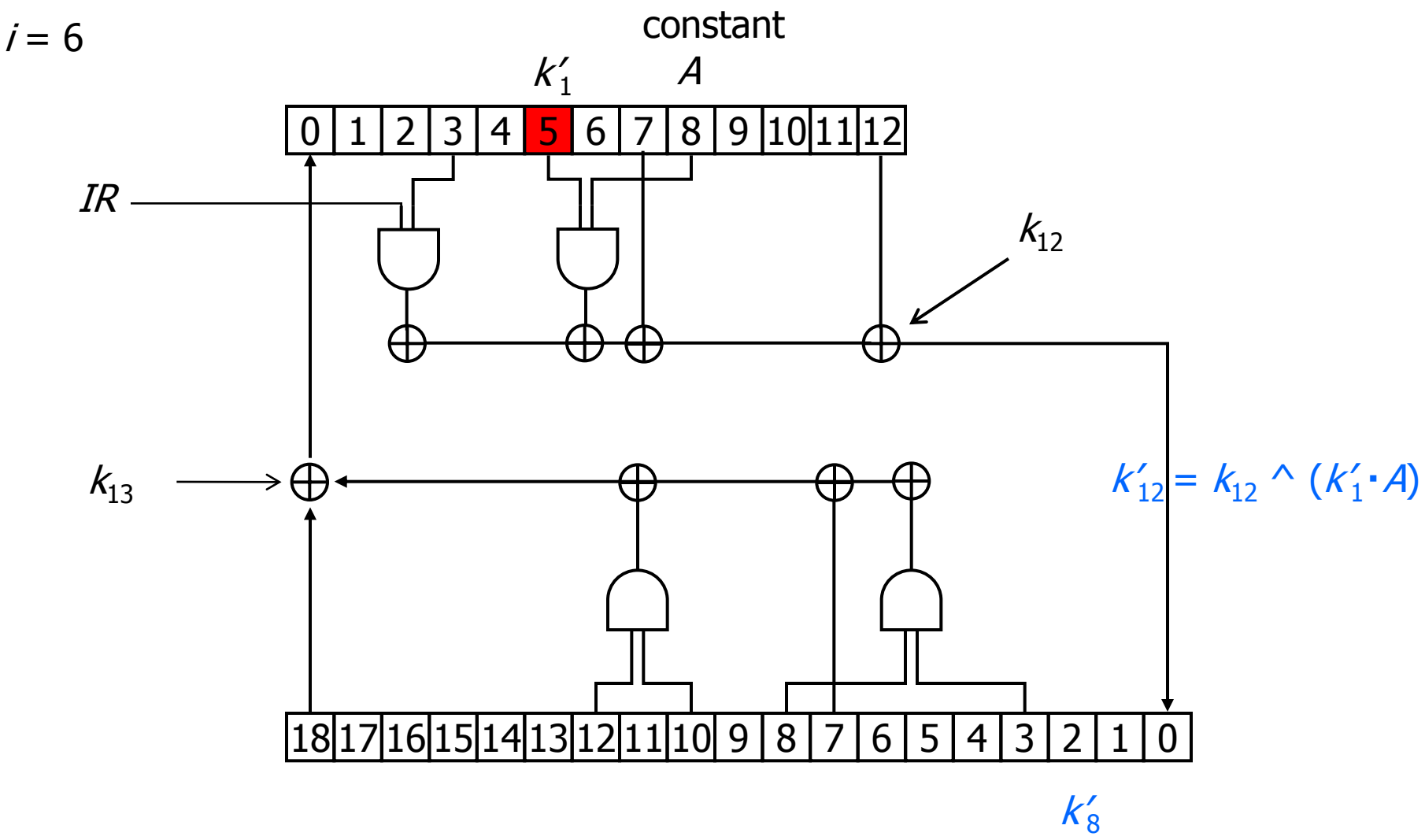
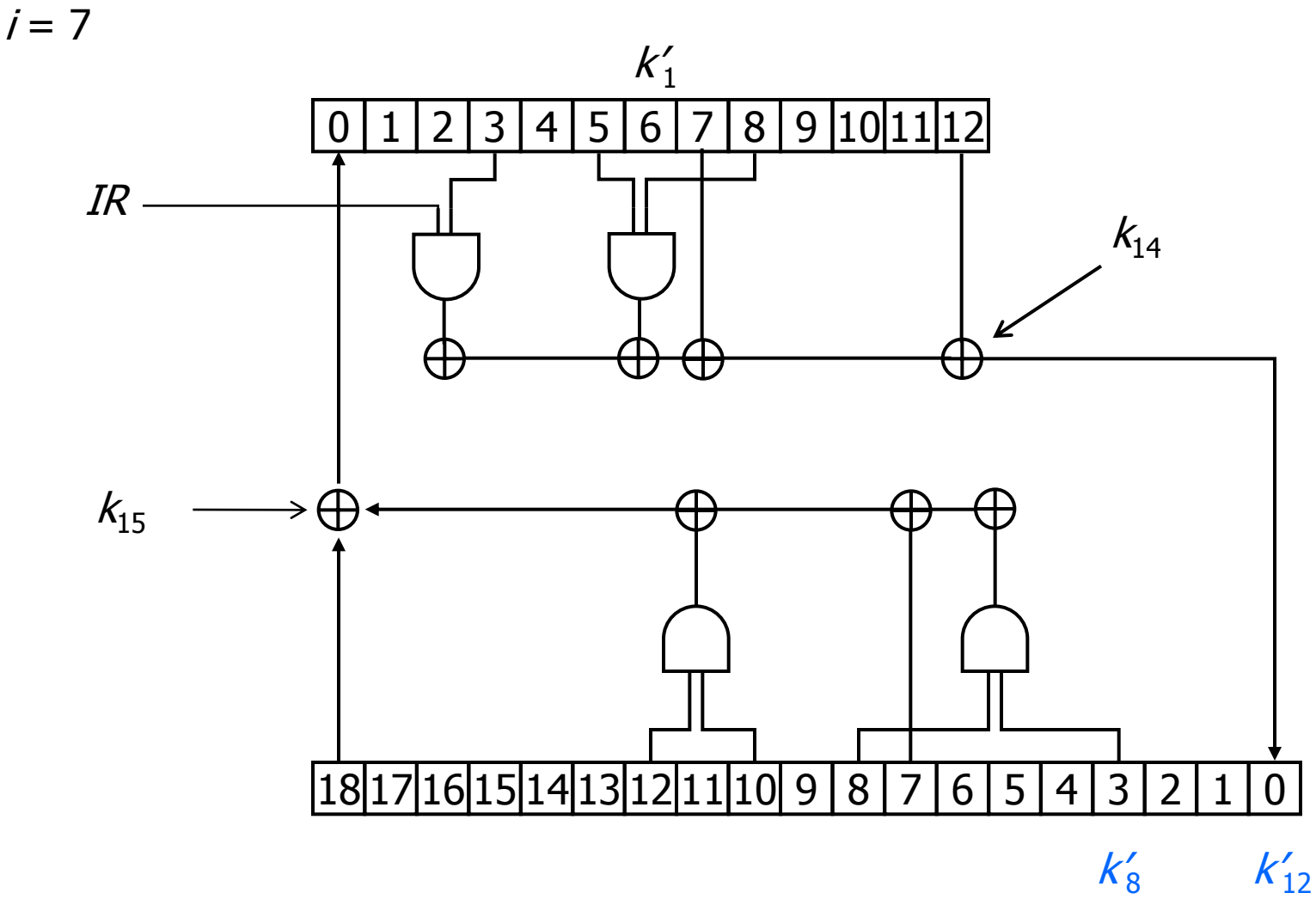
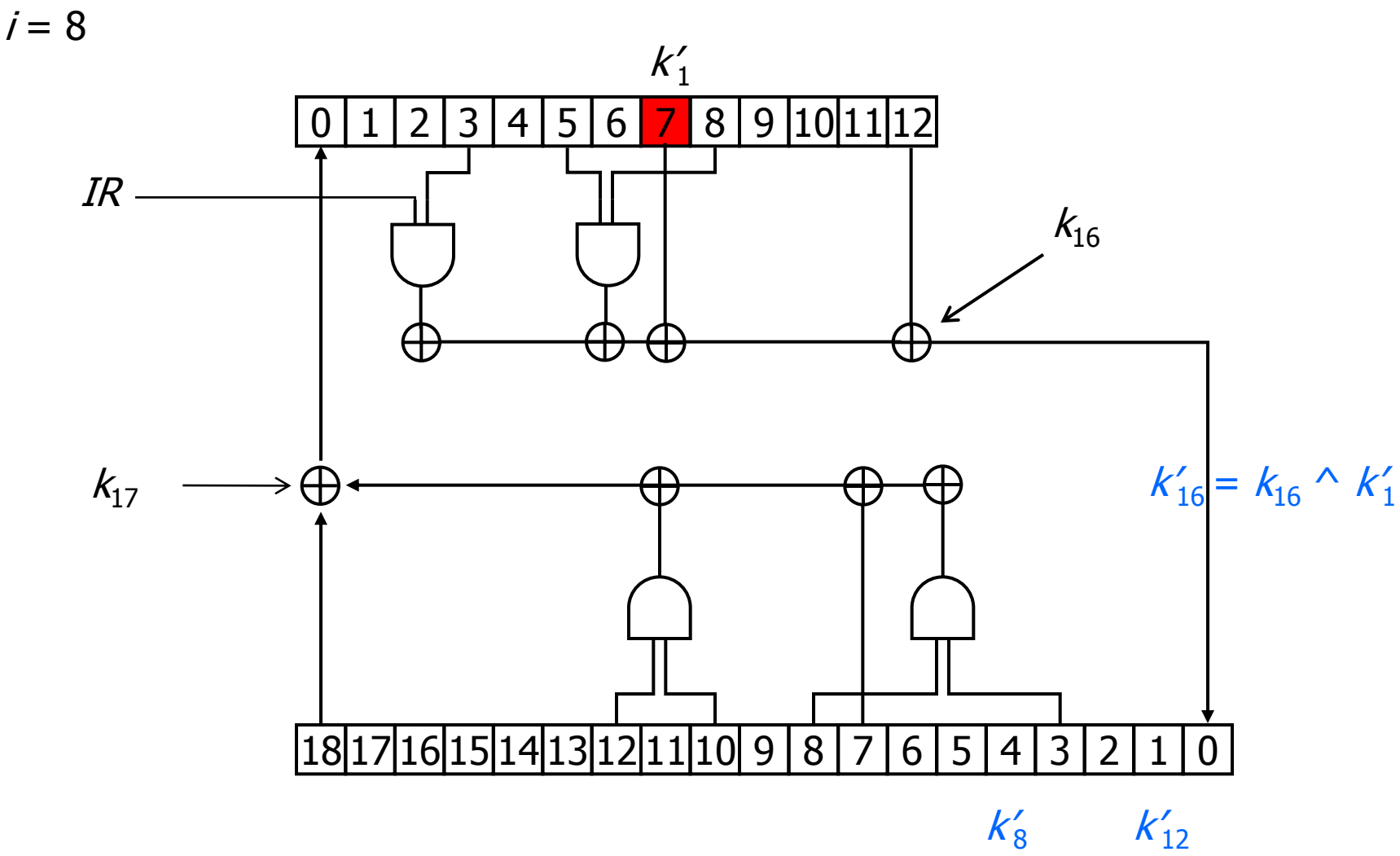


Function Reduction of KATAN32



Function Reduction of KATAN32



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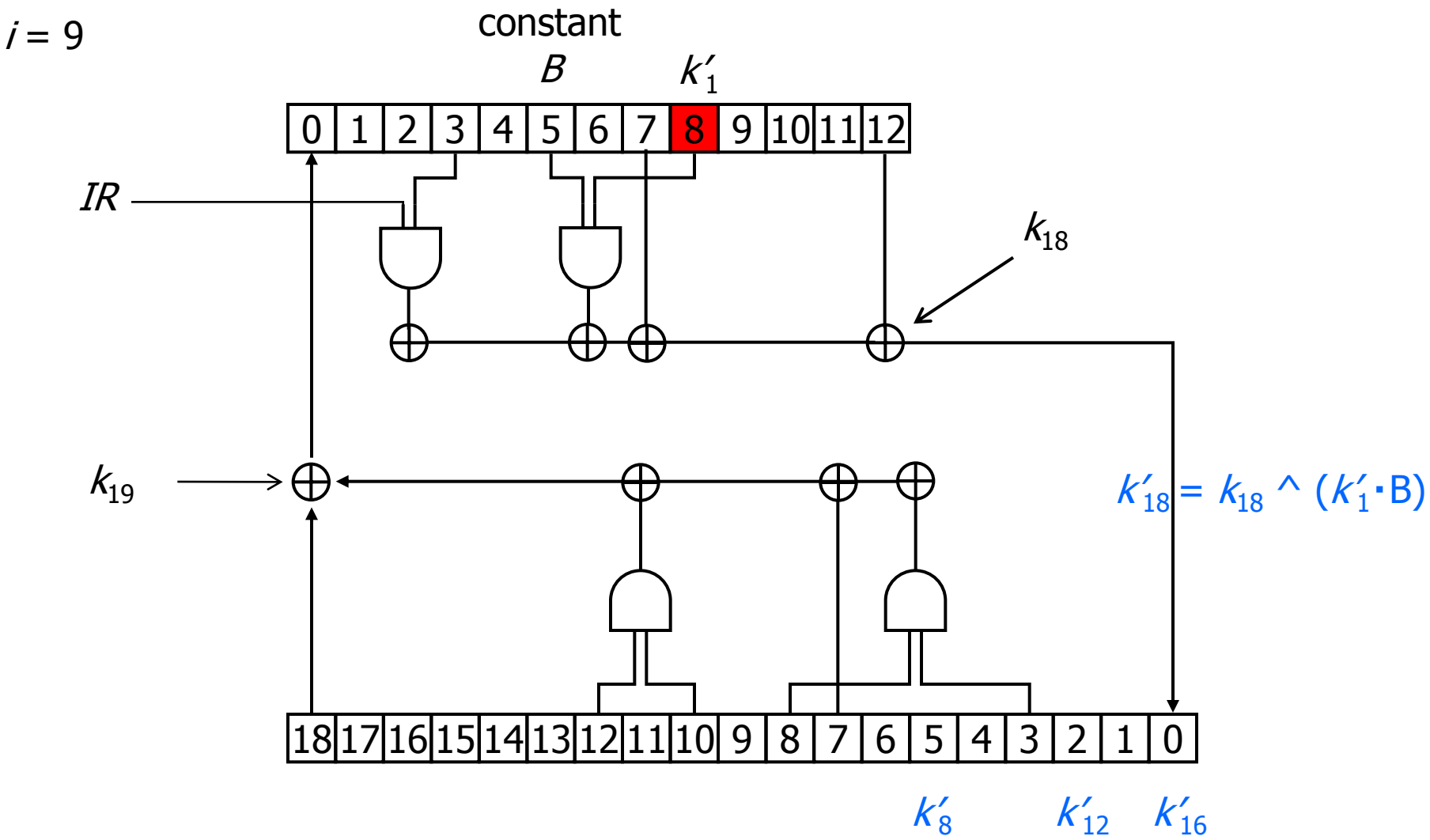
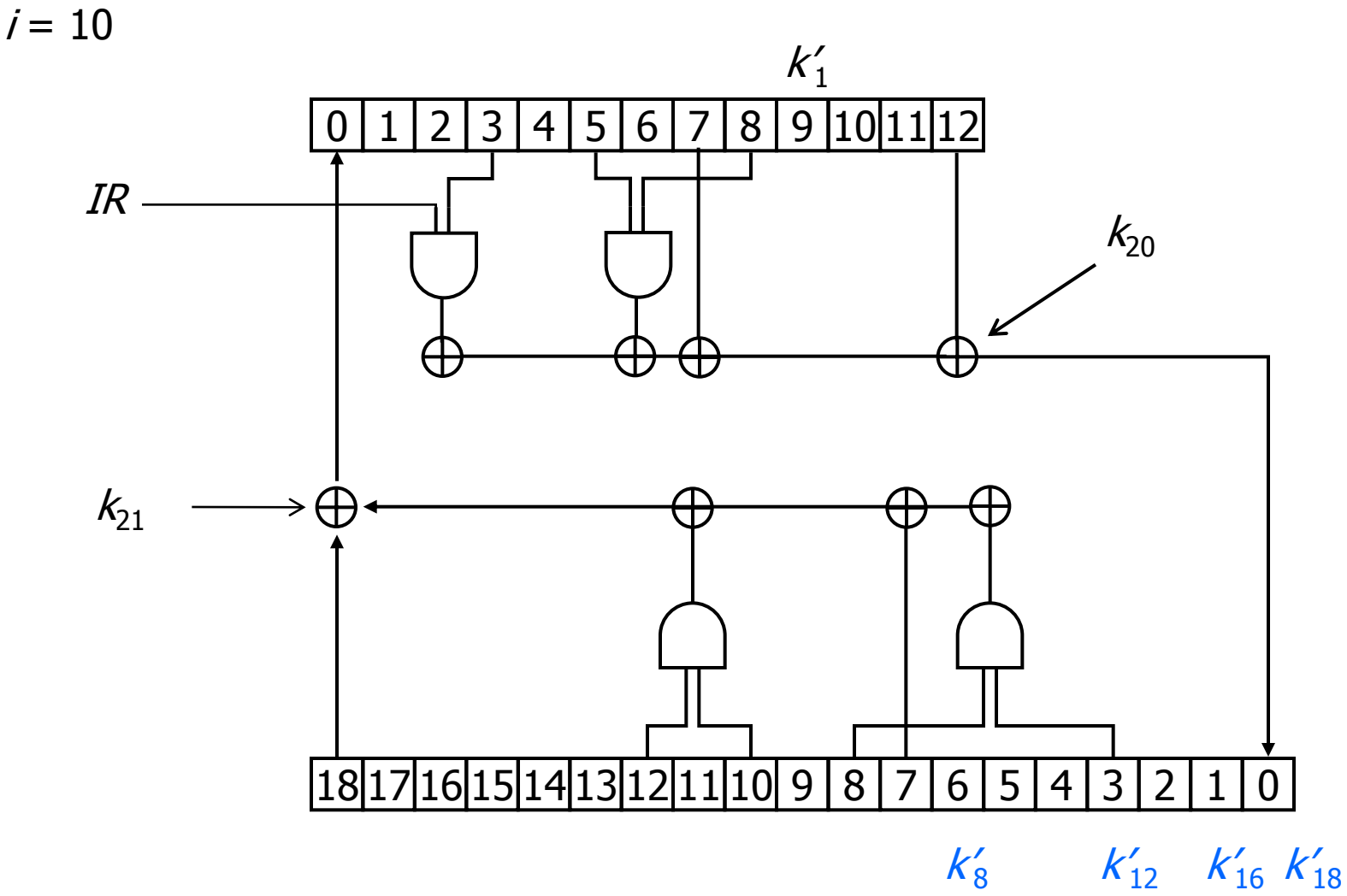
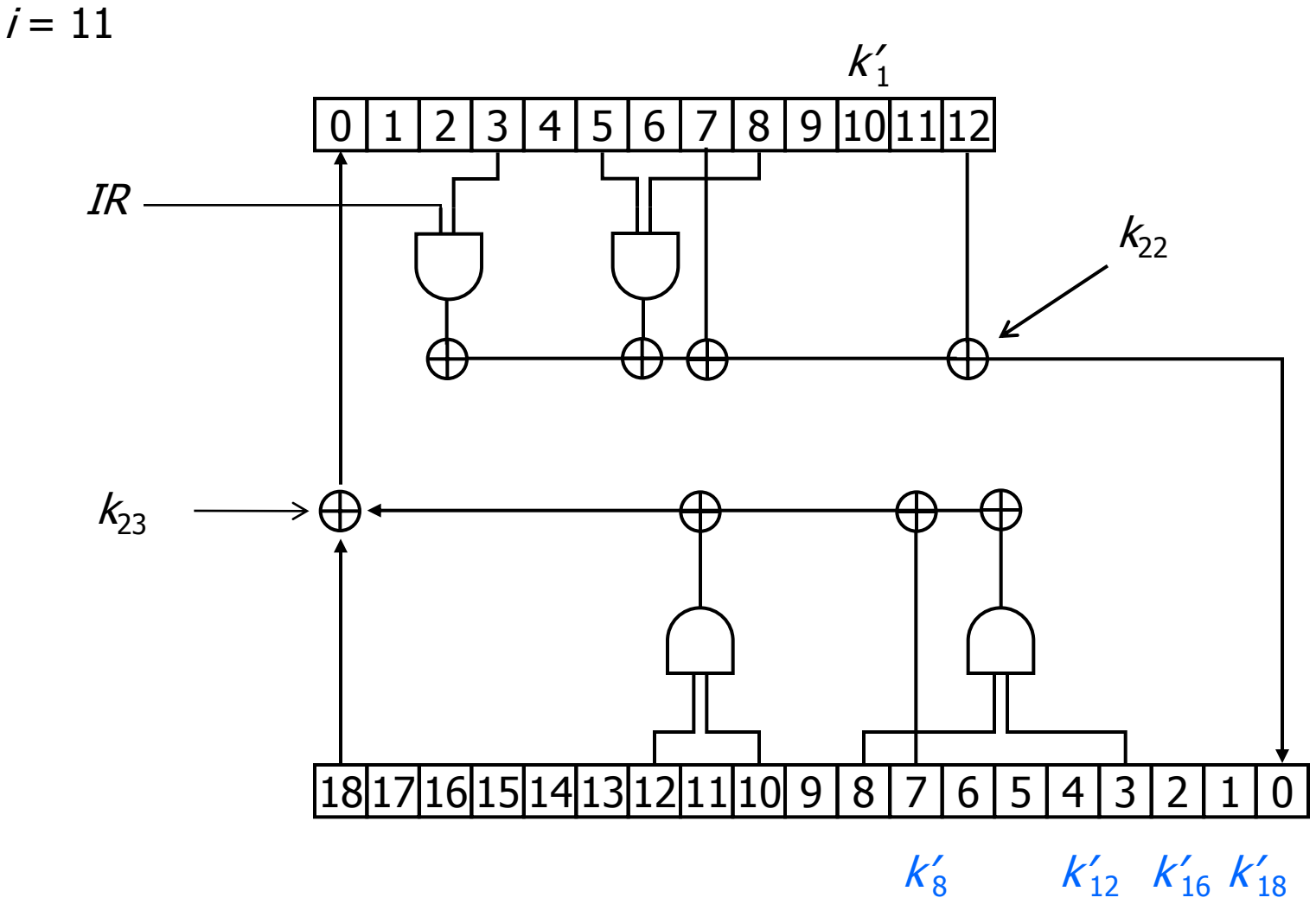


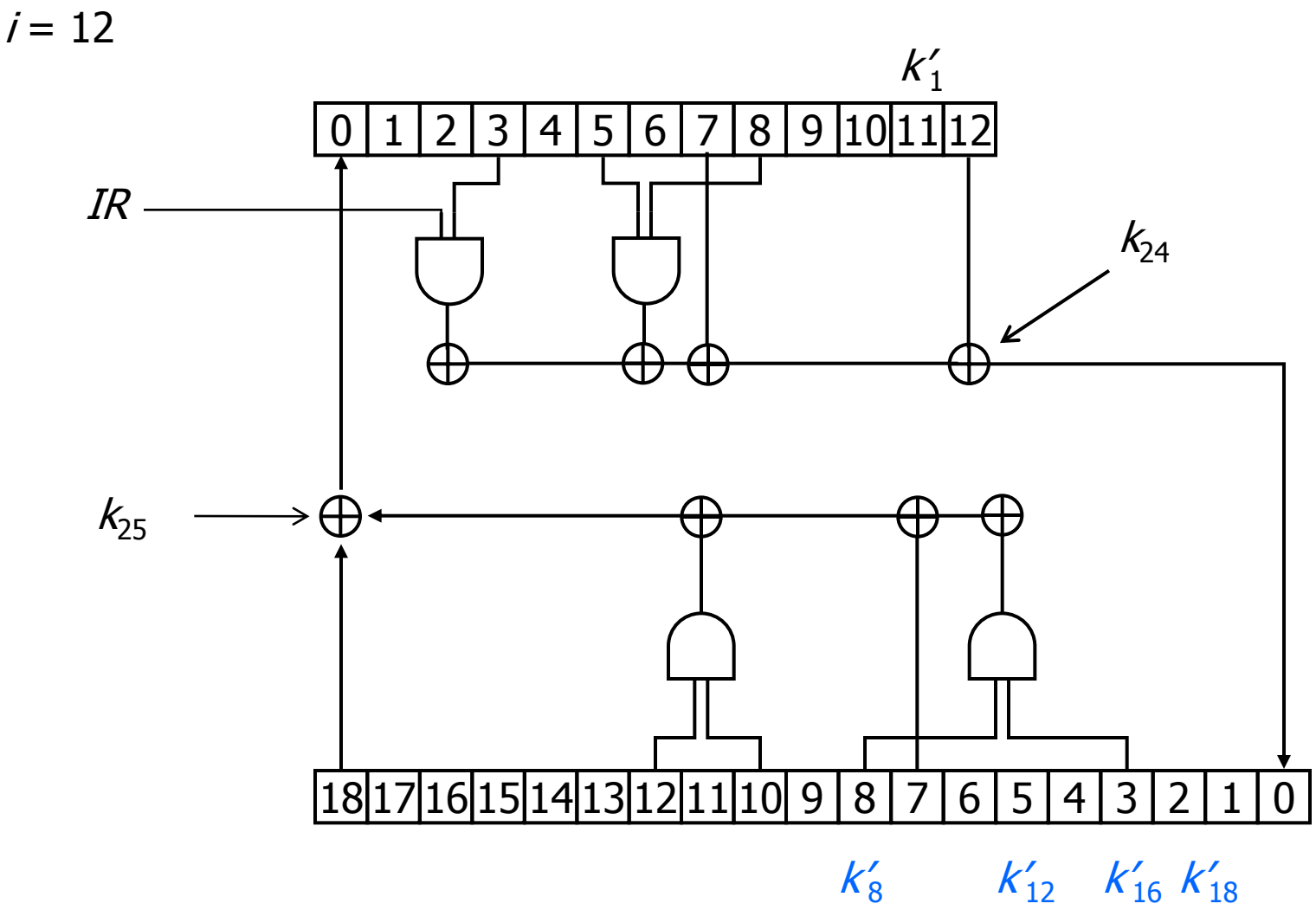
Function Reduction of KATAN32



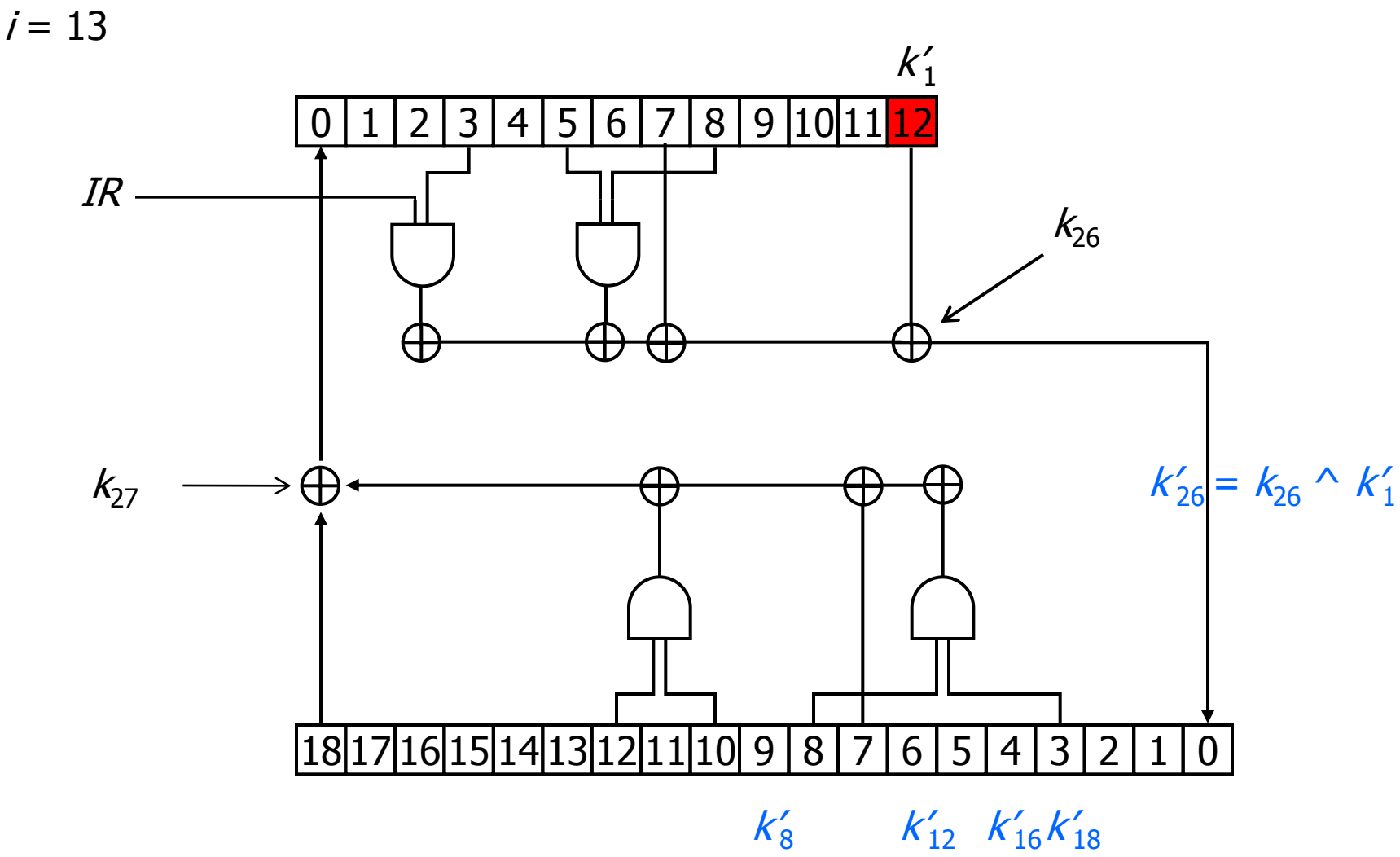
Function Reduction of KATAN32



Function Reduction of KATAN32

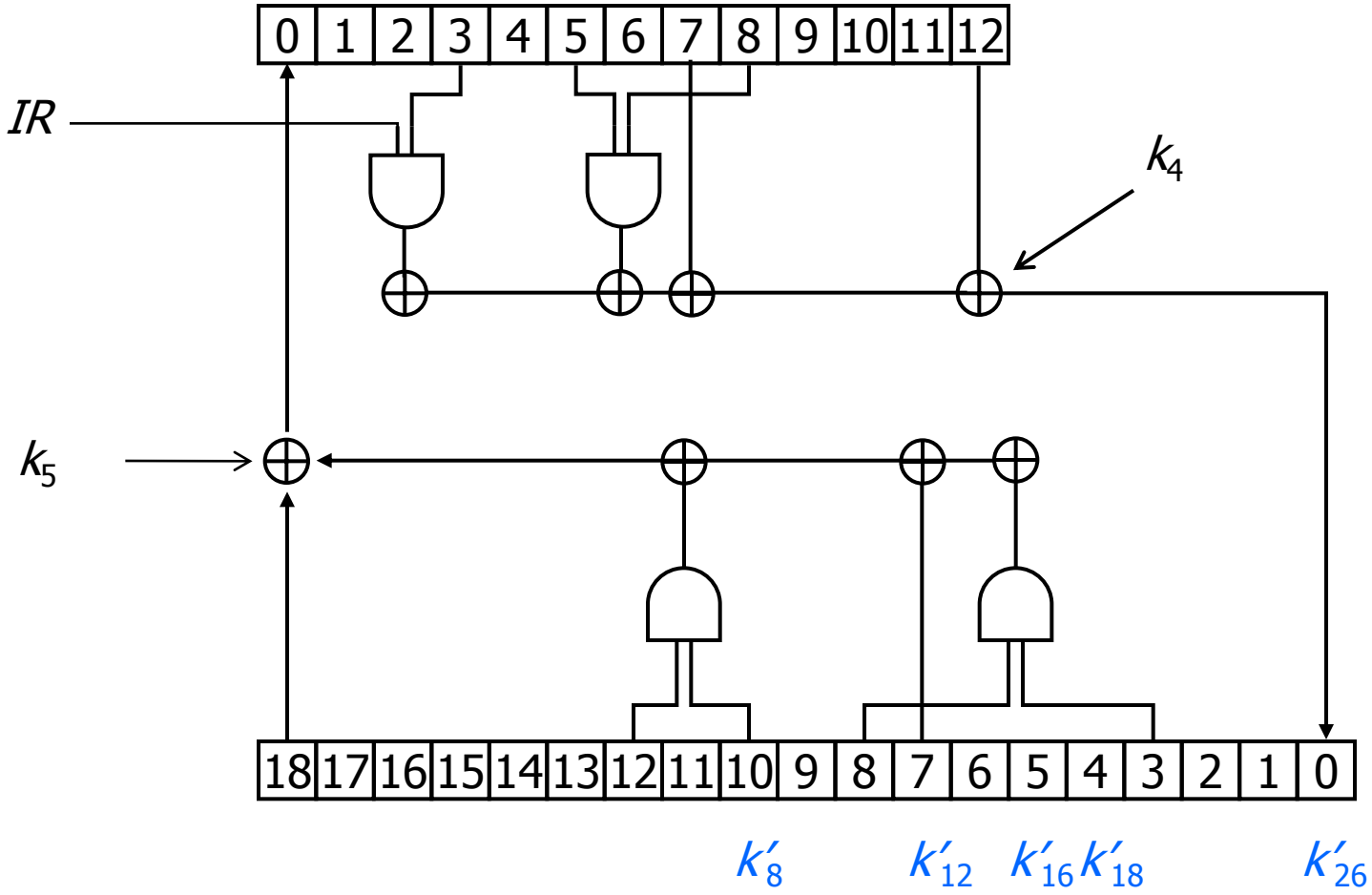


Function Reduction of KATAN32



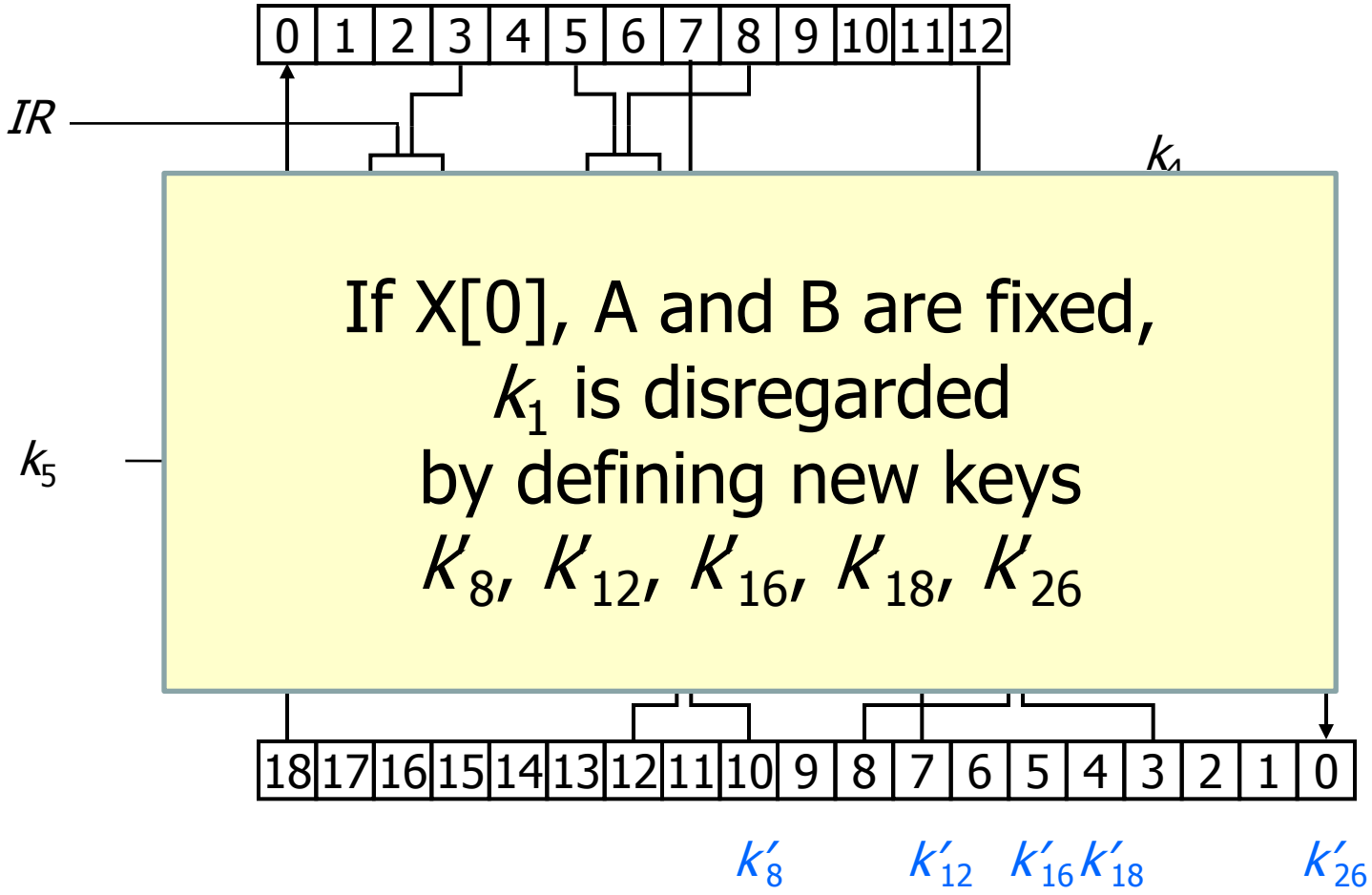
Function Reduction of KATAN32

$i = 13$



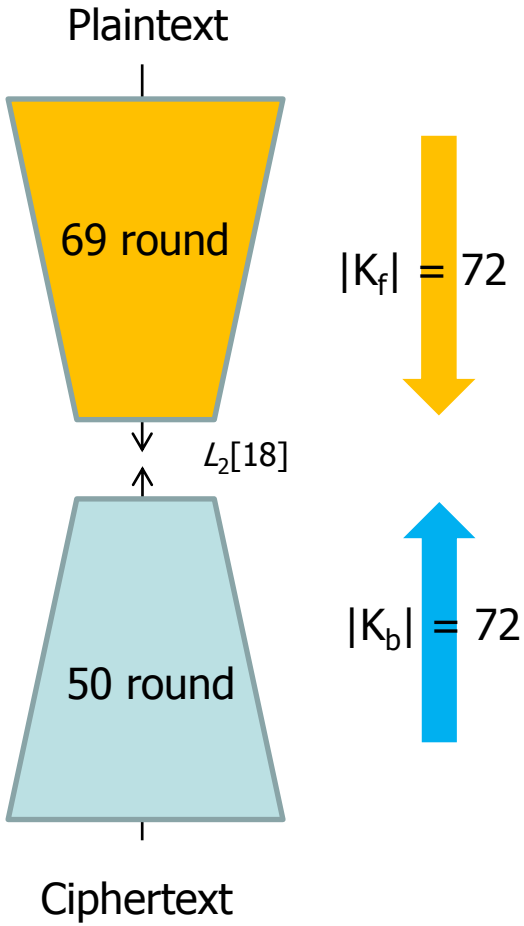
Function Reduction of KATAN32

$i = 13$



119-round Attack on KATAN32

- 8-bit function reduction in forward direction
  - by controlling 23 bits of plaintexts



Given 144 data,

2^{144} candidates are reduced to $2^0(2^{240-1\cdot144}) = 1$

Time : $2^{72} \times 144 = 2^{79.1}$
Data : 144
Memory : $2^{72} \times 144 = 2^{79.1}$

Results

- Update best single-key attacks on KATAN 32/48/64 w.r.t. number of attacked rounds

Best Attacks

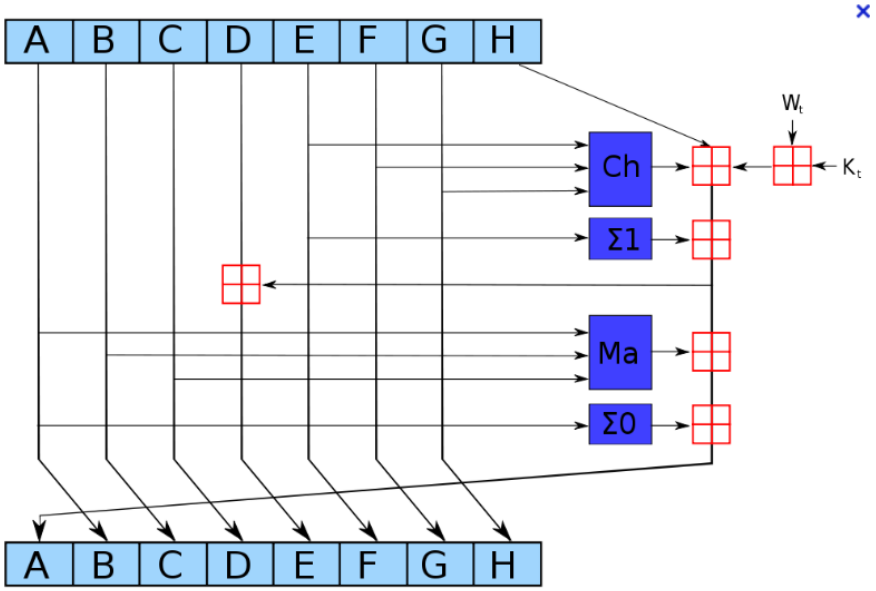
Target	#Attack round	Attack Type	Time	Memory	Data	Paper
KATAN32	110	ASR	2^{77}	$2^{75.1}$	138	[16]
	114	Differential	2^{77}	Not given	$2^{31.9}$	[2]
	119	**ASR**	**$2^{79.1}$**	**$2^{79.1}$**	**144**	**Ours**
KATAN48	100	ASR	2^{78}	2^{78}	128	[16]
	105	**ASR**	**$2^{79.1}$**	**$2^{79.1}$**	**144**	**Ours**
KATAN64	94	ASR	$2^{77.1}$	$2^{79.1}$	116	[16]
	99	**ASR**	**$2^{79.1}$**	**$2^{79.1}$**	**142**	**Ours**

Agenda

1. All-Subkeys Recovery Attacks
2. Function reduction technique
3. Improved Attacks on FOX-64/128
4. Improved Attacks on KATAN-32/48/64
5. Improved Attacks on SHACAL-2
6. Conclusion

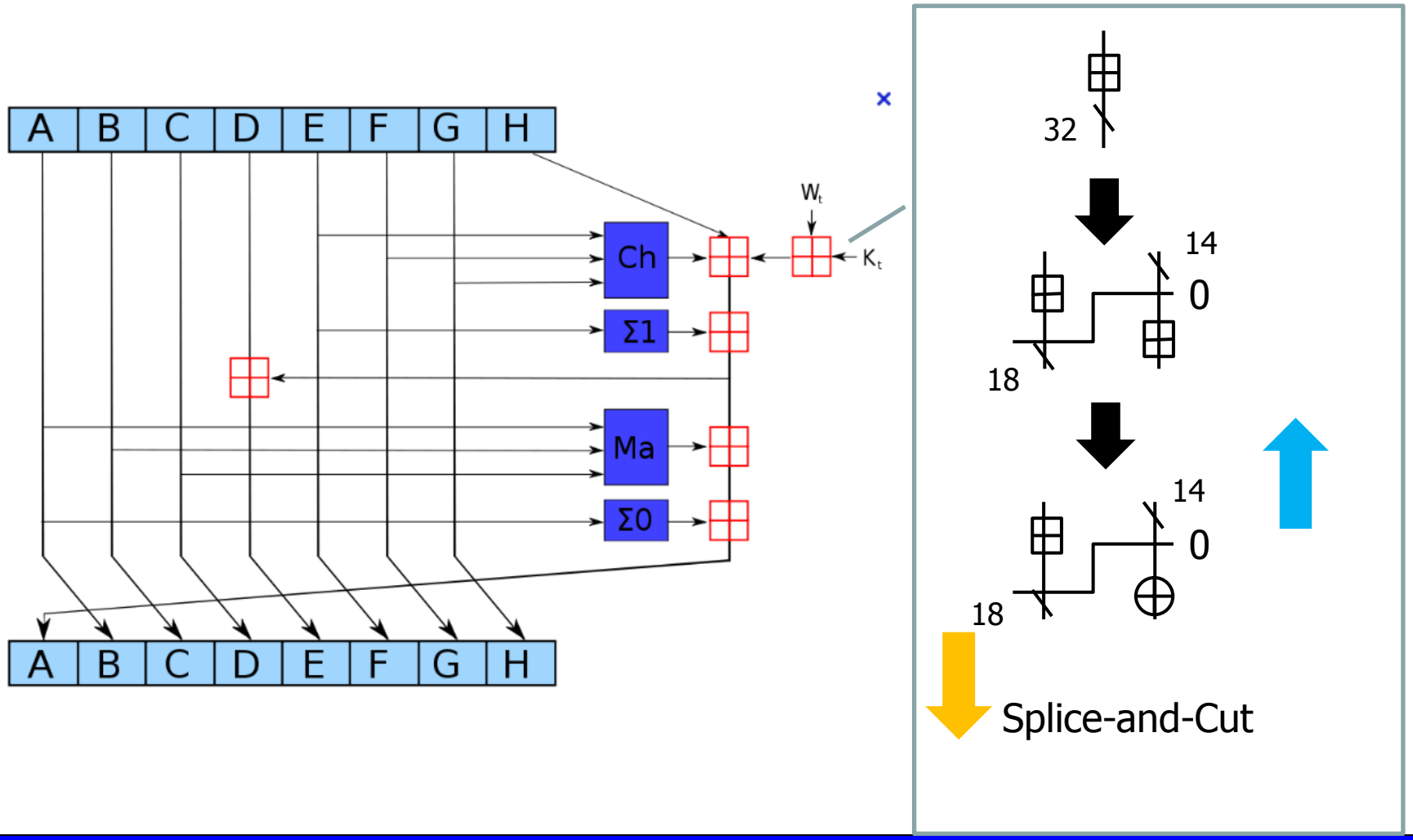
SHACAL-2

- Selected by NESSIE portfolio
- block size : 256 bits, key size : <= 512 bits
- Based on SHA-256 compression function
  - 64-round GFN like construction
- Best Attack : 41 round ASR attack



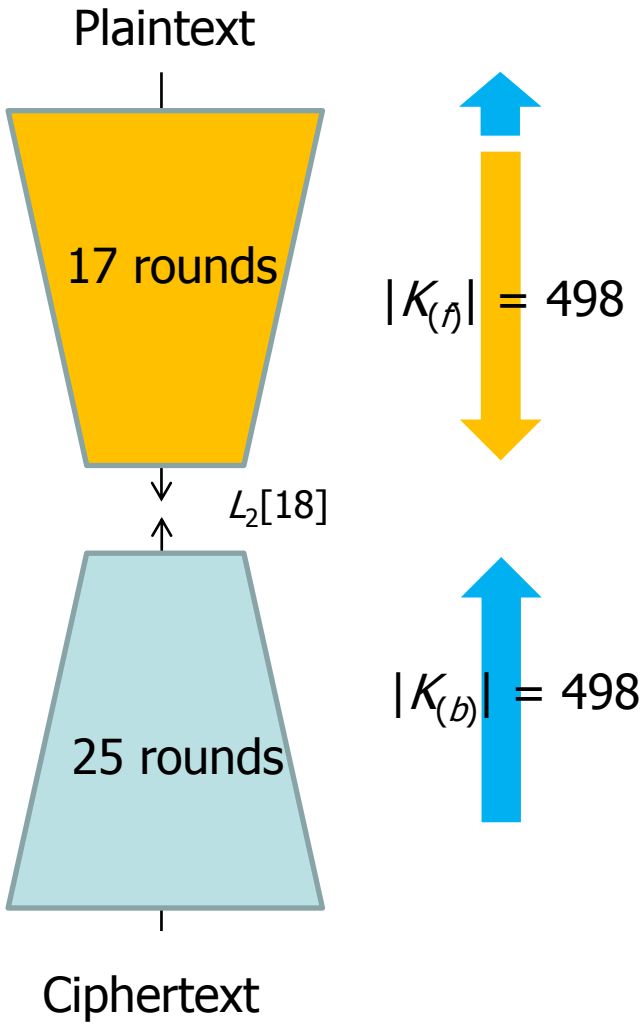
Key Linearization of SHACAL-2

- Key Linearization w/ splice-and-cut



42-round Attack on SHACAL-2

- Splice-and-Cut approach



Given 996 data,

2^{996} candidates are reduced to $2^0(2^{240-1\cdot144})$
$= 1$

Time : $2^{498} \times 996 = 2^{508}$
Data : 2^{25}
Memory : $2^{498} \times 996 = 2^{508}$

Conclusion

- Improved "All-Subkeys Recovery Attack"
  - Extended "Function Reduction" to other constructions
    - Exploit structure-dependent properties of Lai-Massey and LFSR-type
  - Utilized "Repetitive All-Subkeys Recovery Attack"
  - Updated best single-key attacks of FOX64/128, KATAN32/48/64, SHACAL-2
    - 7-round attacks on FOX64/128
    - 119/110/99-round attacks on KATAN32/48/64
    - 42-round attack on SHACAL-2

Thank you for your attention!