On the Security of Tandem-DM

Ewan Fleischmann, Michael Gorski, Stefan Lucks

Bauhaus-University Weimar

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Approaches to building a cryptographic hash function

- From Scratch (MD4, MD5, SHA-0/1, SHA-256/512, RIPEMD, ...)
- From a blockcipher (MMO, DM, MDC-2/4, Tandem-DM, Abreast-DM, ...)
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- 'Secure' (ideal cipher model)
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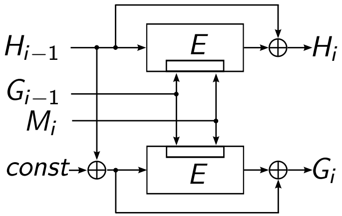
Example: Hirose's FSE'06 proposal



- Rate 1/2, Output size: 2n (i.e. AES-256 256 bits)
- Collision Resistance: $> 2^{124.5}$ for CF
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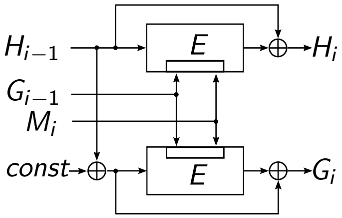

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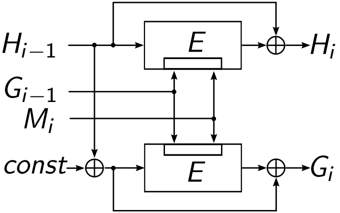

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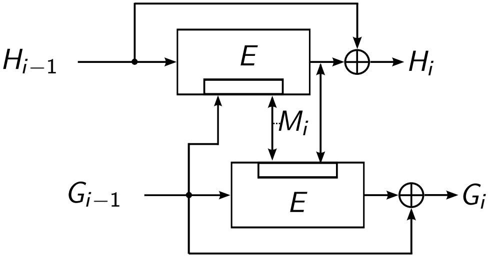
Tandem-DM - a DBL hash function



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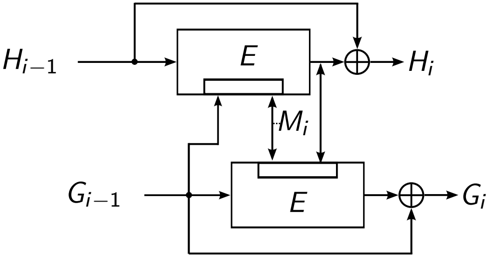

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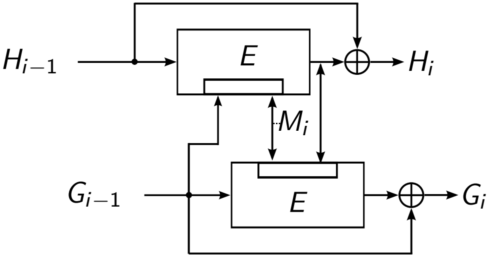

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Security Bound

Theorem (Bound for Collision Resistance)

Let F be the Tandem-DM compression function and n, q be natural numbers with $q < 2^n$. Let $N' = 2^n - q$ and let α be any positive number with $eq/N' \leq \alpha$ and $\tau = \alpha N'/q$ (and e^x being the exponential function). Then

$$\mathbf{Adv}_F^{\mathrm{COLL}}(q) \leq q 2^n e^{q\tau(1-\ln\tau)/N'} + 4q\alpha/N' + 6q/(N')^2 + 2q/(N')^3.$$

Security Bound

Corollary/Conjecture

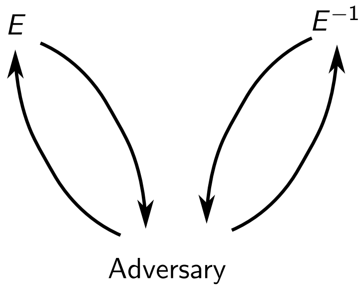
For the compression function Tandem-DM, instantiated with AES-256, any adversary asking less than $2^{120.4}$ (backward or forward) oracle queries cannot find a collision with probability greater than $1/2$. In this case, $\alpha = 24.0$.

Theorem (Bound for Preimage Resistance)

Let $F := F^{TDM}$ be the Tandem-DM compression function. For every $N' = 2^n - q$ and $q > 1$

$$\mathbf{Adv}_F^{Inv}(q) \leq 2q/(N')^2.$$

Model for the proof (1)



Query History $\mathcal{Q}$: $\{\}$

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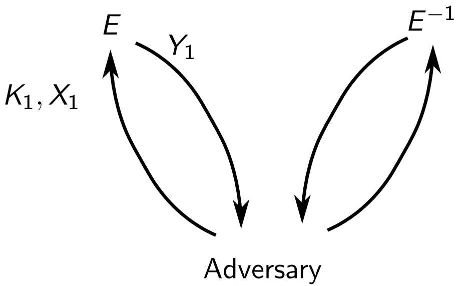

<u>Query History $\mathcal{Q}$:</u> $\{(X_1, K_1, Y_1)\}$

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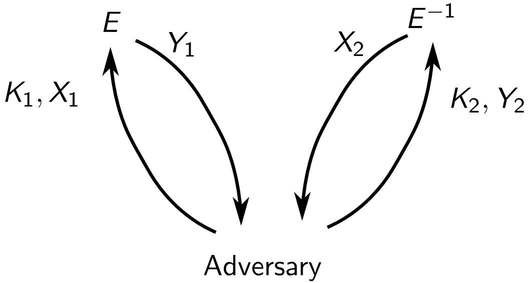

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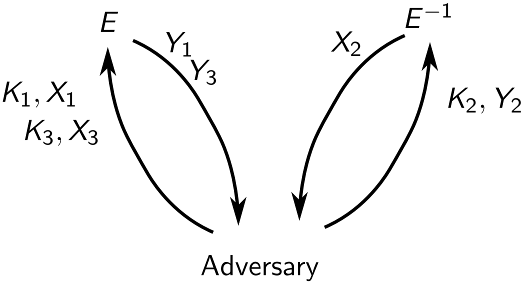

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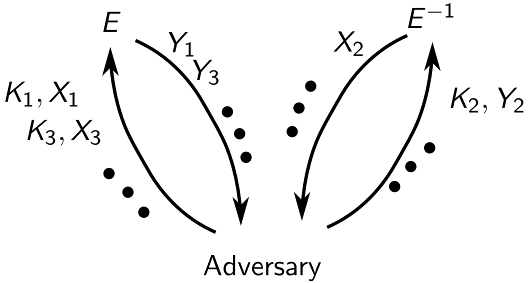

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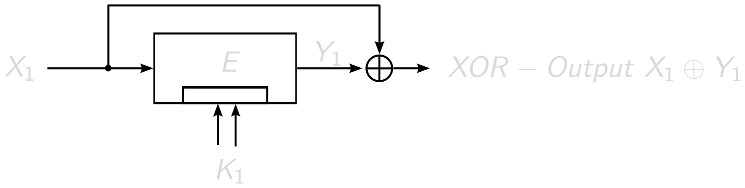

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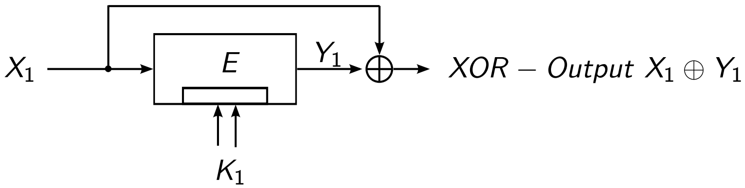

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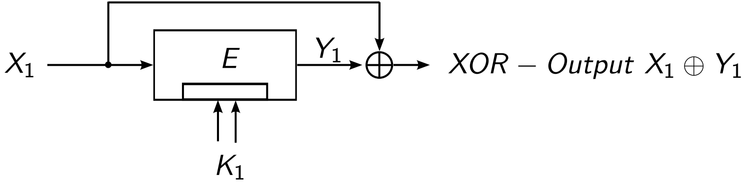

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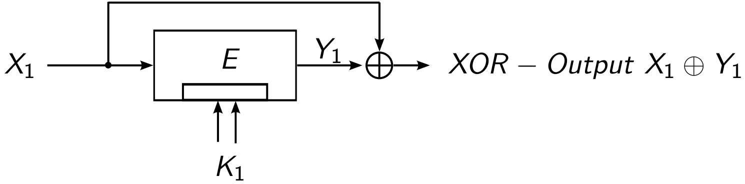

1. Adversary A wins if queries can be assembled to hash two distinct colliding words
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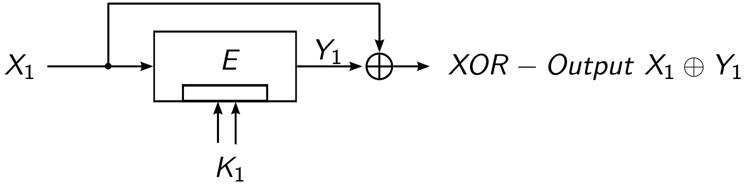

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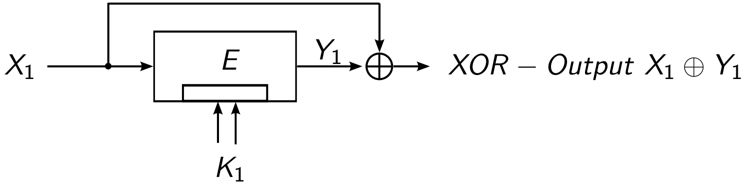

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Main Idea

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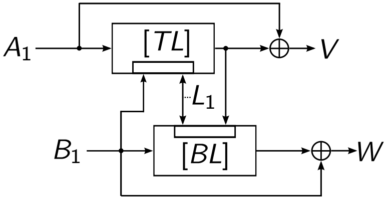
Upper bound the probability of the adversary making a query that can be used as the *final* query to complete a collision.



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1. We examine the adversarys queries one at a time as they come in
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Proof Overview

1. Exhibit predicates $\textsc{Lucky}(\mathcal{Q})$, $\textsc{Win1}(\mathcal{Q})$, $\textsc{Win2}(\mathcal{Q})$ and $\textsc{Win3}(\mathcal{Q})$ such that
2. $\textsc{Coll}^{TDM}(\mathcal{Q}) \Rightarrow \textsc{Lucky}(\mathcal{Q}) \vee \textsc{Win1}(\mathcal{Q}) \vee \textsc{Win2}(\mathcal{Q}) \vee \textsc{Win3}(\mathcal{Q})$
3. Upper bound separately the probabilities $\Pr[\textsc{Lucky}(\mathcal{Q})]$, $\Pr[\textsc{Win1}(\mathcal{Q})]$, $\Pr[\textsc{Win2}(\mathcal{Q})]$ and $\Pr[\textsc{Win3}(\mathcal{Q})]$
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Example Case: $Win1(\mathcal{Q}) = \neg\textsc{Lucky}(\mathcal{Q}) \wedge Fit1(\mathcal{Q})$

Fit$1(\mathcal{Q})$**:** The last query is used only once in position TL. Note that this is equal to the case where the last query is used only once in position TR.

$\textsc{Fit}1a(\mathcal{Q})$	*all queries used in the collision are pairwise different,*
$\textsc{Fit}1b(\mathcal{Q})$	$\mathrm{BL} = \mathrm{TR}$ *and* BR *is different to* TL, BL, TR,
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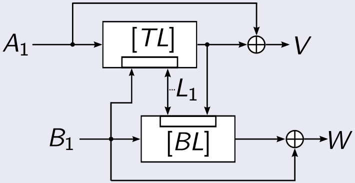

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