Two General Attacks on Pomaranch-like Keystream Generators.

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Outline

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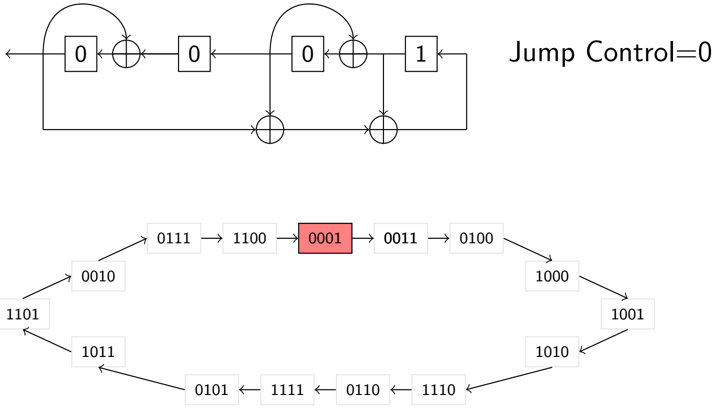
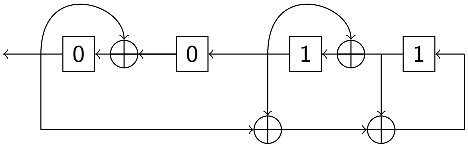
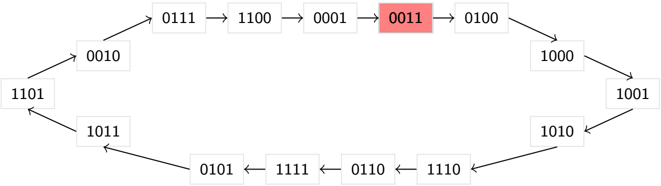
Jump Registers



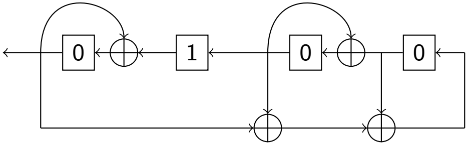
Jump Registers



Jump Control=0



Jump Registers



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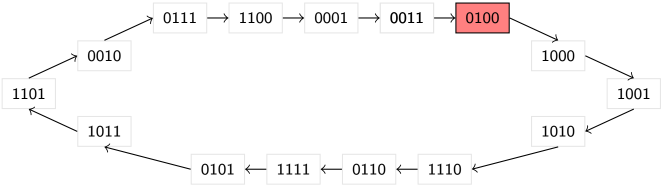

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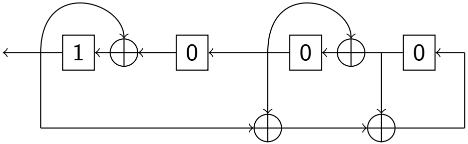

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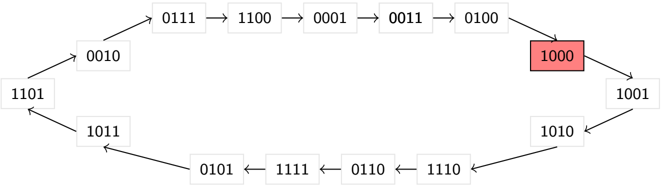

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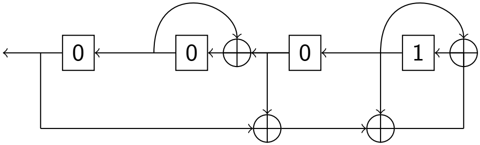
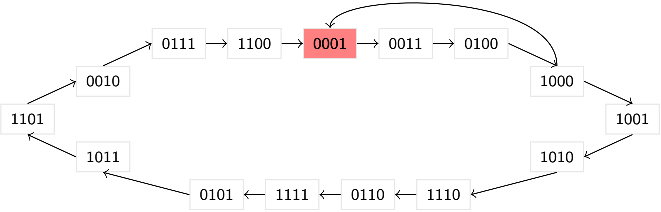
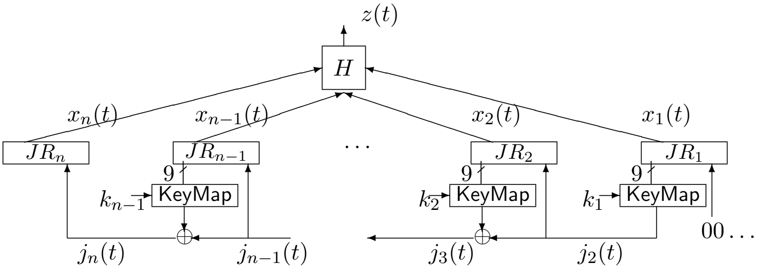


Jump Control=1

Jump Index=12



Description of Pomaranch

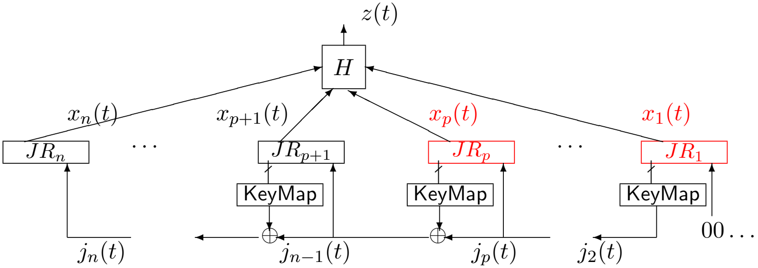


- n jump registers $JR_1, \dots, JR_n$ of length L.
- Jump sequence $j_2, \dots, j_n$ used to clock jump registers.
- KeyMap, key-dependent function, $f : \mathbb{F}_2^9 \mapsto \mathbb{F}_2$.
- Filter function H.

Different Designs

- 5 different proposals for Pomaranch ciphers.
- 3 different design ideas:
  1. Same jump register, linear filter function.
  2. Different jump registers, linear filter function.
  3. Different jump registers, non-linear filter function.

Period of Jump Registers



- Period of JR_1 is denoted T_1, $T_1 = 2^L - 1$.
- Period for register JR_p is $T_p = T_1^p \approx 2^{pL}$.

Hence

$$x_i(t) = x_i(t + T_1^i),\ 1 \leq i \leq p.$$

- Useful for p such that $T_1^p < 2^{|K|}$, $|K|$ =Key size.

Period of Jump Registers

Take samples at time t and $t + T_1^p$:

$$z(t)+z(t+T_1^p) = H\big(x_1(t),\ldots,x_n(t)\big)+H\big(x_1(t+T_1^p),\ldots,x_n(t+T_1^p)\big).$$

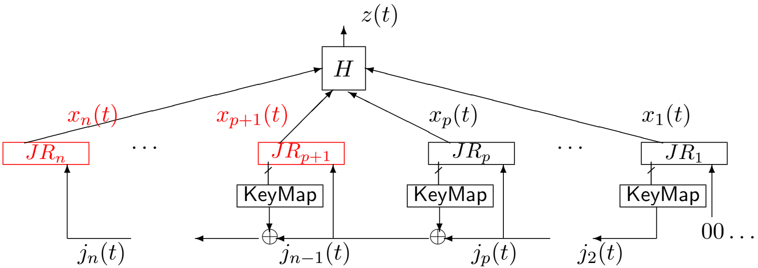
- Linear H:

$$z(t) + z(t + T_1^p) = \sum_{i=p+1}^{n} x_i(t) + x_i(t + T_1^p).$$

- Non-linear H: $H(t)$ and $H(t + T_1^p)$ have p inputs $x_1, \ldots, x_p$ in common, $0 \leq p \leq n$.

$$\Pr\left(z(t) + z(t + T_i^p) = 0\right) = \frac{1}{2}(1 - \delta),\ -1 \leq \delta \leq 1.$$

Linear Approximation of $JR_{p+1}, \ldots, JR_n$



For $JR_l,\ p+1 \le l \le n$, search for a set $\mathcal{A}$ of size w such that

$$\Pr\Big(\sum_{i\in\mathcal{A}} x_l(t+i) = 0\Big) = \frac{1}{2}(1-\varepsilon), \quad -1 \le \varepsilon \le 1,$$

Design Idea 1: Linear H, same registers

- Linear approximation of one register, (applies to all registers);
- Samples at time t and $t + T_1^p$.

$$\sum_{i \in \mathcal{A}} z(t+i) + z(t+i+T_1^p) =$$

$$= \sum_{j=p+1}^{n} \sum_{i \in \mathcal{A}} \left(x_j(t+i) + x_j(t+i+T_1^p)\right).$$

- Bias of $\sum_{i \in \mathcal{A}} x_i(t+i)$ is ε, we have $2(n-p)$ such relations

$$\varepsilon_{tot} = \varepsilon^{2(n-p)}$$

- $1/\varepsilon_{tot}^2$ samples needed to distinguish the cipher from a truly random source.

Design Idea 1: Linear H, same registers

Theorem

The computational complexity and the number N of keystream bits needed to reliably distinguish the Pomaranch family of stream ciphers using a linear filter function and n jump registers of the same type is upper bounded by

$$N \leq T_1^p + \frac{1}{\varepsilon^{4(n-p)}}, \quad p > 0,$$

where ε is the bias of the best linear approximation of the jump register.

- Pomaranch v1 (128 bit): Keystream and Complexity$=2^{71}$.
- Pomaranch v2 (80 bit): Keystream and Complexity$=2^{56}$.
  (128 bit): Keystream and Complexity$=2^{77}$.

Design Idea 2: Linear H, Different Registers

- Samples at time t and $t + T_1^p$.
- Linear approximation for all registers jointly.
- Bias for the approximation of register i is ε_i, total bias is given by

$$\varepsilon_{tot} = \prod_{i=p+1}^{n} \varepsilon_i^2.$$

Design Idea 2: Linear H, Different Registers

Theorem

Assuming there is a linear relation that is biased in all registers. The computational complexity and the number N of keystream bits needed to reliably distinguish the Pomaranch family of stream ciphers using a linear filter function and n jump registers of different types is upper bounded by

$$N \leq T_1^p + \frac{1}{\prod_{i=p+1}^{n} \varepsilon_i^4}, \quad p > 0,$$

where ε_i is the bias of jump register JR_i.

- Pomaranch v3 (128 bit): Keystream and Complexity$=2^{126}$. (Without frame length restriction)

Design Idea 3: Nonlinear H, Different Registers

- Consider the case, (can easily be extended), when the filter function H can be written on the form

$$H(x_1, \ldots, x_n) = G(x_1, \ldots, x_{n-1}) + x_n.$$

- $G(t)$ and $G(t + T_1^p)$ have p inputs $x_1, \ldots, x_p$ in common,

$$\Pr\left(z(t) + z(t + T_i^p) = 0\right) = \frac{1}{2}(1 - \delta),\ -1 \leq \delta \leq 1.$$

- Find linear approximation for JR_n.

Design Idea 3: Nonlinear H, Different Registers

$$H(x_1, \ldots, x_n) = G(x_1, \ldots, x_{n-1}) + x_n. \tag{1}$$

Theorem

The computational complexity and the number N of keystream bits needed to reliably distinguish the Pomaranch family of stream ciphers using a filter function of the form (1) is upper bounded by

$$N \leq T_1^p + \frac{1}{(\varepsilon^2 \delta^w)^2}.$$

where ε is the a biased approximation of weight w of register JR_n and δ is the bias of
$G\big(x_1(t), \ldots, x_{n-1}(t)\big) + G\big(x_1(t + T_1^p), \ldots, x_{n-1}(t + T_i^p)\big)$.

- Pomaranch v3 (80 bit): Keystream and Complexity$=2^{71}$.

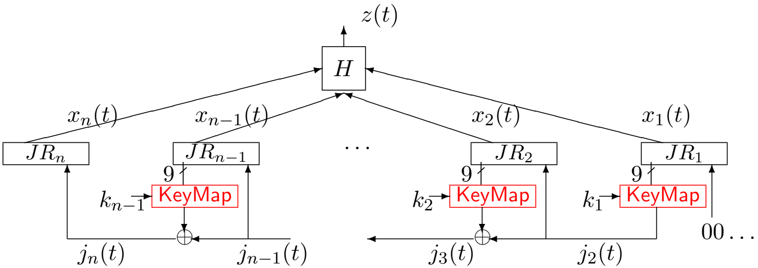
Attack II: Square Root Resynchronization Attack

- Divide the internal state into two parts,

$$State = (State_K,\ State_{K+IV}).$$

  $State_K$ only holds the key.
  $State_{K+IV}$ depends on both key and IV.
- If key size $|K| > |State_{K+IV}|/2$, attack succeeds with complexity below exhaustive key search.
- One attack scenario:
  - Fixed key.
  - One long keystream sequence from one IV.
  - Intercept ciphertexts from many IVs, knowing l plaintext bits of every ciphertext.
  - Goal is to recover more of the plaintext for one message.

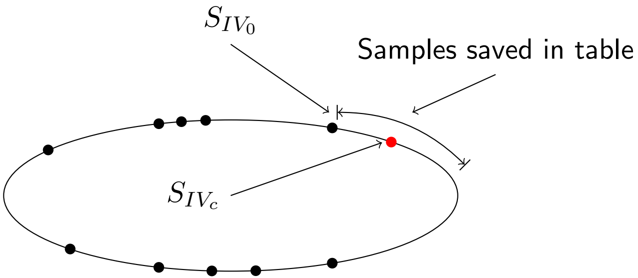
Square Root Resynchronization Attack on Pomaranch



- KeyMap, is key dependent but independent of IV.

Square Root Resynchronization Attack on Pomaranch

- Samples taken as, $S(t) = \big(z(t), z(t+1), \ldots, z(t+nL-1)\big)$.
- Fixed key defines state graph of size $(2^L - 1)^n \approx 2^{nL}$.
- Store large amount of samples from IV_0 in table.
- Find IV_c, such that $S_{IV_c}(t_c) = S_{IV_0}(t_0)$.



Attack Complexities

- Assume we have $2^{\beta nL}$ samples from IV_0.
- We need samples from $2^{(1-\beta)nL}$ different IVs to find collision.
- Time:
  - Sort table $\beta nL 2^{\beta nL}$.
  - Search in table $\beta nL 2^{(1-\beta)nL}$
- Memory: $nL2^{\beta nL}$

Results

		Attack I Keystream/Compl.	Attack II Memory/IVs/Compl.
Pomaranch v1	128 bit	2^{71} / 2^{71}	2^{67} / 2^{63} / 2^{63}
Pomaranch v2	80 bit	2^{56} / 2^{56}	2^{45} / 2^{42} / 2^{42}
	128 bit	2^{77} / 2^{77}	2^{67} / 2^{63} / 2^{63}
Pomaranch v3	80 bit	2^{71} / 2^{71}	2^{58} / 2^{54} / 2^{54}
	128 bit	2^{126} / $2^{126}*$	2^{71} / 2^{98} / 2^{104}

* Without frame length restriction.

Conclusions

- Presented the best distinguisher so far on all version and variants of Pomaranch in terms of computational complexity.
- Presented a general resynchronization attack that works for all ciphers where $|K| > |State_{K+IV}|/2$.
- First attack presented on Pomaranch Version 3.