Compact Ring LWE Cryptoprocessor

CHES 2014

Sujoy Sinha Roy¹, Frederik Vercauteren¹, Nele Mentens¹, Donald Donglong Chen² and Ingrid Verbauwhede¹

> ¹ESAT/COSIC and iMinds, KU Leuven ²Electronic Engineering, City University of Hong Kong

Outline

- Introduction to the ring-LWE problem and an encryption scheme
- Our optimization techniques
- Architecture of the ring-LWE Cryptoprocessor
- Results
- Conclusions

Modern Public Key Schemes

- Modern public-key schemes
 - \succ RSA \leftarrow difficulty of factoring problem
 - ➢ DSA/ECDSA ← difficulty of Discrete Logarithm problem

- Intractable using classical computers
- Threat
 - Quantum computers destroy RSA, DSA and ECDSA

The Ring-LWE Problem

- Defined over a ring $R_q = \mathbb{Z}_q[\mathbf{x}]/\langle f \rangle$
 - A polynomial $a \in R_q$ is chosen uniformly
 - The secret $s \in R_q$ is a fixed polynomial
 - An error polynomial e is sampled from \mathcal{X}_{σ}
 - Compute $t = as + e \in R_q$
 - The ring-LWE distribution on $R_q \times R_q$ consists of tuples (a, t)
- Search ring-LWE problem: given many (\mathbf{a}_i, t_i) samples, find secret s

Ring-LWE : Encryption Scheme

The Encryption Scheme : Key Generation

- Key Generation :
 - Choose two polynomials $r_1, r_2 \leftarrow \chi_{\sigma}$
 - Compute $p = r_1 a \cdot r_2$
 - The secret key is r_2
 - The public key is (a, p)

The Encryption Scheme : Encryption/Decryption

7

• Encryption

- Input message $m \rightarrow$ encoded to polynomial $\bar{m}: \begin{array}{c} 0 \rightarrow 0\\ 1 \rightarrow (q-1)/2 \end{array}$
- Choose $e_1, e_2, e_3 \in R_q$ with coefficients from \mathcal{X}_{σ}
- Ciphertext

$$c_1 = a \cdot e_1 + e_2 \in R_q$$

$$c_2 = p \cdot e_1 + e_3 + \overline{m} \in R_q$$

• Decryption

$$m' = c_1 \cdot r_2 + c_2 \in R_q$$

• Decoding compares the decrypted message coefficients



LWE Cryptosystem : Block Level Diagram

8



Polynomials have 256 or 512 coefficients and each coefficient is 13 or 14 bit Standard deviation is small (less than 5)

Roadmap to Implementation

- Ring-LWE based encryption
 - > Primitives
 - Discrete Gaussian Sampler
 - Polynomial Multiplier
 - ➢ Full cryptosystem
- Discrete Gaussian sampler architecture ---- (Presented in SAC 2013)
 - Knuth-Yao random walk

High Precision Discrete Gaussian Sampling on FPGAs, SAC 2013

Polynomial Multiplication

Polynomial Multiplication Algorithms

- Schoolbook multiplication : complexity n^2
- Karatsuba multiplication : complexity $n^{1.585}$
- FFT based multiplication : **complexity** (*n log n*)

• Number Theoretic Transform (NTT) is a generalization of FFT

 $\triangleright e^{-\frac{2\pi i}{n}} \leftrightarrow n$ -th primitive root of unity in \mathbb{Z}_q

 \succ Involves integer arithmetic modulo q

Polynomial Multiplication : NTT

- Polynomial multiplication : c(x) = a(x)b(x)
 - > 2n point NTT : $c = NTT_{\omega_{2n}}^{-1} \left(NTT_{\omega_{2n}}(a) * NTT_{\omega_{2n}}(b) \right)$

Polynomial Multiplication : NTT

- Polynomial multiplication : c(x) = a(x)b(x)
 - ➤ 2*n* point NTT : $c = NTT_{\omega_{2n}}^{-1} \left(NTT_{\omega_{2n}}(a) * NTT_{\omega_{2n}}(b) \right)$
 - Special optimization : $c(x) = a(x)b(x) \mod f(x)$ where $f = x^n + 1$
 - Negative-wrapped convolution : using n point NTT
 - \triangleright *n* is a power of two and prime $q \equiv 1 \mod 2n$
 - Scaling $a'(x) = a(\omega_{2n} \cdot x)$ and $b'(x) = b(\omega_{2n} \cdot x)$
 - $\succ \quad c' = NTT_{\omega_n}^{-1} \left(NTT_{\omega_n}(a') * NTT_{\omega_n}(b') \right)$
 - Final scaling is required to compute c(x) from $c'(x) = c(\omega_{2n} \cdot x)$

The basic NTT Algorithm

14

Iterative NTT

Input: Polynomial $a(x) \in \mathbb{Z}_q[\mathbf{x}]$ of degree n-1 and n-th primitive root $\omega_n \in \mathbb{Z}_q$ of unity **Output:** Polynomial $A(x) \in \mathbb{Z}_q[\mathbf{x}] = \operatorname{NTT}(a)$ begin 1 $\mathbf{2}$ $A \leftarrow BitReverse(a);$ 3 for m = 2 to n by m = 2m do $\omega_m \leftarrow \omega_n^{n/m}$; $\mathbf{4}$ $\omega \leftarrow 1$: $\mathbf{5}$ for j = 0 to m/2 - 1 do 6 for k = 0 to n - 1 by m do $\mathbf{7}$ 8 $t \leftarrow \omega \cdot A[k+j+m/2];$ $u \leftarrow A[k+j];$ 9 $A[k+j] \leftarrow u+t$; 10 $A[k+j+m/2] \leftarrow u-t;$ 11 12end 13 $\omega \leftarrow \omega \cdot \omega_m$; 14end 15 \mathbf{end} 16 end

The NTT Algorithm

15

Iterative NTT

Input: Polynomial $a(x) \in \mathbb{Z}_q[\mathbf{x}]$ of degree n-1 and n-th primitive root $\omega_n \in \mathbb{Z}_q$ of unity Output: Polynomial $A(x) \in \mathbb{Z}_q[\mathbf{x}] = \operatorname{NTT}(a)$ begin 1 2 $A \leftarrow BitReverse(a);$ 3 for m = 2 to n by m = 2m do $\omega_m \leftarrow \omega_n^{n/m}$; 4 5 $\omega \leftarrow 1$: 6 for j = 0 to m/2 - 1 do 7 for k = 0 to n - 1 by m do 8 $t \leftarrow \omega \cdot A[k+j+m/2];$ 9 $u \leftarrow A[k+j];$ 10 $A[k+j] \leftarrow u+t;$ $A[k+j+m/2] \leftarrow u-t;$ 11 12 end 13 Fixed Computation Cost $\omega \leftarrow \omega \cdot \omega_m$; 14 end 15 end 16 end

NTT Core



Pöppelmann et al. Latincrypt 2012

Our Target : Compact Architecture

Minimize Memory Requirement

Optimization in Area



This increases computation cost !

Computational Optimizations

20

Iterative NTT

1	begin
2	$A \leftarrow BitReverse(a);$
3	for $m = 2$ to n by $m = 2m$ do
4	$\omega_m \leftarrow \omega_N^{N/m}$;
5	for $k = 0$ to $n - 1$ by m do
6	$\omega \leftarrow 1$;
7	for $j = 0$ to $m/2 - 1$ do
8	$t \leftarrow \omega \cdot A[k+j+m/2] ;$
9	$u \leftarrow A[k+j];$
10	$A[k+j] \leftarrow u+t$;
11	$A[k+j+m/2] \leftarrow u-t$;
12	$\omega \leftarrow \omega \cdot \omega_m ;$
13	end
14	\mathbf{end}
15	end
16	end

Iterative NTT

1 begin $\mathbf{2}$ $A \leftarrow BitReverse(a);$ $\mathbf{3}$ for m = 2 to n by m = 2m do $\omega_m \leftarrow \omega_N^{N/m}$; $\mathbf{4}$ $\mathbf{5}$ $\omega \leftarrow 1$; for j = 0 to m/2 - 1 do 6 for k = 0 to n - 1 by m do $t \leftarrow \omega \cdot A[k + j + m/2]$; $\mathbf{7}$ 8 9 $u \leftarrow A[k+j];$ $A[k+j] \leftarrow u+t$; 10 $A[k+j+m/2] \leftarrow u-t;$ 11 12end 13 $\omega \leftarrow \overline{\omega \cdot \omega_m}$; 14 \mathbf{end} 15end 16 end

Pöppelmann et al. Latincrypt 2012

Our

Aysu et al. HOST 2013

Reduction in fixed computation overhead

- 21
- Forward negative wrapped NTT requires **pre-scaling** of the input polynomials

$$\bar{a} = (a_0, \psi a_1, ..., \psi^{n-1} a_{n-1})$$
 $\bar{b} = (b_0, \psi b_1, ..., \psi^{n-1} b_{n-1})$ where $\psi = \omega_{2n}$

• Our implementation is **free** from this pre-scaling

Reduction in the pre-scaling overhead

22

Iterative NTT

Input: Polynomial $a(x) \in \mathbb{Z}_q[\mathbf{x}]$ of degree n-1 and n-th primitive root $\omega_n \in \mathbb{Z}_q$ of unity **Output:** Polynomial $A(x) \in \mathbb{Z}_q[\mathbf{x}] = \text{NTT}(a)$ 1 begin $\mathbf{2}$ $A \leftarrow BitReverse(a);$ $\mathbf{3}$ for m = 2 to n by m = 2m do $\omega_m \leftarrow \omega_n^{n/m}$; 4 $\omega \leftarrow 1$; $\mathbf{5}$ for j = 0 to m/2 - 1 do 6 for k = 0 to n - 1 by m do $\mathbf{7}$ 8 $t \leftarrow \omega \cdot A[k+j+m/2];$ $u \leftarrow A[k+j];$ 9 10 $A[k+j] \leftarrow u+t;$ $A[k+j+m/2] \leftarrow u-t;$ 11 12end 13 $\omega \leftarrow \omega \cdot \omega_m$; 14 end 15end 16 end

Iterative NTT

Input: Polynomial $a(x) \in \mathbb{Z}_q[\mathbf{x}]$ of degree n-1 and n-th primitive root $\omega_n \in \mathbb{Z}_q$ of unity **Output:** Polynomial $A(x) \in \mathbb{Z}_q[\mathbf{x}] = \operatorname{NTT}(a)$ begin 1 $A \leftarrow BitReverse(a);$ $\mathbf{2}$ 3 for m = 2 to n by m = 2m do $\omega_m \leftarrow \omega_n^{n/m}$; $\mathbf{4}$ $\mathbf{5}$ $\omega \leftarrow \omega_{2m}$; for j = 0 to m/2 - 1 do 6 for k = 0 to n - 1 by m do 7 $t \leftarrow \omega \cdot A[k+j+m/2];$ 8 9 $u \leftarrow A[k+j];$ 10 $A[k+j] \leftarrow u+t;$ $A[k+j+m/2] \leftarrow u-t$; 11 12end 13 $\omega \leftarrow \omega \cdot \omega_m$; \mathbf{end} 14 15 \mathbf{end} **16** end

Reduction in pre-scaling overhead

Optimization Step 3 : Memory Access Scheme

Memory Access : Motivation 1

25

Iterative NTT

```
1 begin
 \mathbf{2}
            A \leftarrow BitReverse(a);
 3
            for m = 2 to n by m = 2m do
                  \omega_m \leftarrow \omega_N^{N/m} ;
  \mathbf{4}
  \mathbf{5}
                  \omega \leftarrow \omega_{2m}
                  for j = 0 to m/2 - 1 do
  6
                        for k = 0 to n - 1 by m do
  \mathbf{7}
  8
                              t \leftarrow \omega \cdot A[k+j+m/2];
                              u \leftarrow A[k+j];
  9
                              A[k+j] \leftarrow u+t \ ;
10
                               A[k+j+m/2] \leftarrow u-t;
11
12
                        \mathbf{end}
13
                        \omega \leftarrow \omega \cdot \omega_m;
14
                  end
15
            end
16 end
```

• Two coefficients are accessed from RAM

• One arithmetic operation is performed



Arithmetic blocks remains idle

Memory Access : Motivation 2

- Xilinx BRAM Slice of 18Kb
 - ➢ Width of words : 36 bits
 - > Depth of RAM : 512
- Polynomials in LWE
 - ➤ LWE256 : q= 7681 (13 bit)
 - ➤ LWE512 : q= 12289 (14 bit)
- Two coefficients can be stored in one word



Our memory efficient NTT ...

Optimization : Memory Access



In this scheme

- Two coefficients are in one word
- Two pairs are processed together

Optimization : Memory Access

29

Iterative NTT : Memory Efficient Version

Input: Polynomial $a(x) \in \mathbb{Z}_q[\mathbf{x}]$ of degree n-1 and n-th primitive root $\omega_n \in \mathbb{Z}_q$ of unity **Output:** Polynomial $A(x) \in \mathbb{Z}_q[\mathbf{x}] = \operatorname{NTT}(a)$ 1 begin $A \leftarrow BitReverse(a);$ /* Coefficients are stored in the memory as proper pairs */ $\mathbf{2}$ for m = 2 to n/2 by m = 2m do 3 $\mathbf{4}$ $\omega_m \leftarrow m$ -th primitiveroot(1); $\omega \leftarrow squareroot(\omega_m)$ or 1 /* Depending on forward or backward NTT */; $\mathbf{5}$ for j = 0 to m/2 - 1 do 6 $\mathbf{7}$ for k = 0 to n/2 - 1 by m do 8 $(t_1, u_1) \leftarrow (A[k+j+m/2], A[k+j]) /* \text{ From MEMORY}[k+j] */;$ 9 $(t_2, u_2) \leftarrow (A[k+m+j+m/2], A[k+m+j]) /* MEMORY[k+j+m/2]$ 10 $t_1 \leftarrow \omega \cdot t_1$: 11 $t_2 \leftarrow \omega \cdot t_2$: $(A[k+j+m/2], A[k+j]) \leftarrow (u_1 - t_1, u_1 + t_1);$ 12 $(A[k+m+j+m/2], A[k+m+j]) \leftarrow (u_2 - t_2, u_2 + t_2);$ 13 $MEMORY[k+j] \leftarrow (A[k+j+m], A[k+j]);$ 14 $MEMORY[k + j + m/2] \leftarrow (A[k + j + 3m/2], A[k + j + m/2]);$ 15end 1617 $\omega \leftarrow \omega \cdot \omega_n$; No idle cycles 18end 19end **BRAM efficiency**

Optimization : Ring-LWE Encryption Scheme

Pöppelmann, SAC 2013

Encryption

$$\begin{split} \tilde{e_1} &\leftarrow NTT(e_1) \\ c_1 &\leftarrow INTT(\tilde{a} * \tilde{e_1}) + \tilde{e_2} \\ c_2 &\leftarrow INTT(\tilde{p} * \tilde{e_1}) + e_3 + \bar{m} \end{split}$$

Decryption

$$\begin{split} \tilde{c_1} &\leftarrow NTT(c_1) \\ m' &\leftarrow INTT(\tilde{c_1} * \tilde{r_2}) + c_2) \end{split}$$

$$\#NTT = 5$$

Our Scheme

Encryption

$$\tilde{e_1} \leftarrow NTT(e_1); \quad \tilde{e_2} \leftarrow NTT(e_2) (\tilde{c_1}, \tilde{c_2}) \leftarrow \left(\tilde{a} * \tilde{e_1} + \tilde{e_2}; \ \tilde{p} * \tilde{e_1} + NTT(e_3 + \bar{m}) \right)$$

Decryption

$$m' = INTT(\tilde{c_1} * \tilde{r_2} + \tilde{c_2})$$

#NTT = 4

The NTT Core

The NTT Core



Memory Efficient NTT

33

Iterative NTT : Memory Efficient Version

Input: Polynomial $a(x) \in \mathbb{Z}_q[\mathbf{x}]$ of degree n-1 and n-th primitive root $\omega_n \in \mathbb{Z}_q$ of unity **Output:** Polynomial $A(x) \in \mathbb{Z}_q[\mathbf{x}] = \operatorname{NTT}(a)$ begin 1 $A \leftarrow BitReverse(a);$ /* Coefficients are stored in the memory as proper pairs */ $\mathbf{2}$ for m = 2 to n/2 by m = 2m do 3 $\mathbf{4}$ $\omega_m \leftarrow m$ -th primitiveroot(1); $\omega \leftarrow squareroot(\omega_m)$ or 1 /* Depending on forward or backward NTT */; $\mathbf{5}$ for j = 0 to m/2 - 1 do 6 $\mathbf{7}$ for k = 0 to n/2 - 1 by m do 8 $(t_1, u_1) \leftarrow (A[k+j+m/2], A[k+j]) /* \text{ From MEMORY}[k+j] */;$ 9 $(t_2, u_2) \leftarrow (A[k+m+j+m/2], A[k+m+j]) /* MEMORY[k+j+m/2]$ 10 $t_1 \leftarrow \omega \cdot t_1$: 11 $t_2 \leftarrow \omega \cdot t_2$: $(A[k+j+m/2], A[k+j]) \leftarrow (u_1 - t_1, u_1 + t_1);$ 12 $(A[k+m+j+m/2], A[k+m+j]) \leftarrow (u_2 - t_2, u_2 + t_2);$ 13 $MEMORY[k+j] \leftarrow (A[k+j+m], A[k+j]);$ 14 $MEMORY[k + j + m/2] \leftarrow (A[k + j + 3m/2], A[k + j + m/2]);$ 15end 1617 $\omega \leftarrow \omega \cdot \omega_n$; 18end 19end







35_













The NTT Core : Pipeline



The Ring-LWE Cryptoprocessor



Results

Our Ring-LWE Cryptoprocessor : Results on Virtex 6

Parameters	Device	m LUTs/FFs/	Freq	$\operatorname{Cycles}/\operatorname{Time}(\mu s)$	
		DSPs/BRAM18	(MHz)	Encryption	Decryption
(256, 7681, 11.32)	V6LX75T	1349/860/1/2	313	6.3k/20.1	2.8k/9.1
(512, 12289, 12.18)		1536/953/1/3	278	13.3k/47.9	5.8k/21

Comparison with previous Ring-LWE Implementation

	Parameters	Device	m LUTs/FFs/	Freq	Cycles/7	$\Gamma \mathrm{ime}(\mu s)$
			DSPs/BRAM18	(MHz)	Encryption	Decryption
1	(256, 7681, 11.32)	V6LX75T	1349/860/1/2	313	6.3k/20.1	2.8k/9.1
	(512, 12289, 12.18)		1536/953/1/3	278	13.3k/47.9	5.8k/21
2	(256, 7681, 11.32)	V6LX75T	4549/3624/1/12	262	6.8k/26.2	4.4k/16.8
	(512, 12289, 12.18)	V6LX75T	5595/4760/1/14	251	13.7k/54.8	8.8k/35.4

- **1** RLWE, Our Implementation
- 2 RLWE, Pöppelmann et al. SAC 2013

Comparison with ECC (ECIES)

	Parameters	Device	m LUTs/FFs/	Freq	Cycles/7	$\Gamma \mathrm{ime}(\mu s)$
			DSPs/BRAM18	(MHz)	Encryption	Decryption
1	(256, 7681, 11.32)	V6LX75T	1349/860/1/2	313	6.3k/20.1	2.8k/9.1
2	Binary-233	V5LX85T	18097/-/5644/0	156	$\approx 3.8 \text{k}/24.6$	$\approx 1.9 k/12.3$

- **1 RLWE, Our Implementation**
- 2 ECC, Rebeiro et al. CHES 2012

Conclusions and Future Work

Conclusions

- Hardware implementation of an instruction-set ring-LWE processor
 - Optimizations in the NTT
 - Architecture level accelerations
 - Best area-time performance

Future Work

- Lattice based signature scheme and fully-homomorphic encryption
 - Discrete Gaussian sampling
 - Larger polynomial multiplier

Thank You

Backup Slides

Comparison with NTRU

Implementation	Parameters	Device	m LUTs/FFs/	Freq	Cycles/7	$\Gamma \mathrm{ime}(\mu s)$
Algorithm			DSPs/BRAM18	(MHz)	Encryption	Decryption
RLWE	(256, 7681, 11.32)	V6LX75T	1349/860/1/2	313	6.3k/20.1	2.8k/9.1
	(512, 12289, 12.18)		1536/953/1/3	278	13.3k/47.9	5.8k/21
NTRU (ICM-09)	NTRU-251	XCV1600E	27292/5160/14352/0	62.3	-/1.54	-/1.41
Kamal et al.						

Comparison with McEliece

	Implementation	Security	Device	LUTs/FFs/	Freq	Cycles/	$\Gamma \mathrm{ime}(\mu s)$
	Algorithm			DSPs/BRAM18	(MHz)	Encryption	Decryption
1	RLWE	≈ 128	Virtex 6	1349/860/1/2	313	6.3k/20.1	2.8k/9.1
2	McEliece	≈ 128	Virtex 6	≈ 20 k/-/-/488	254	1.2k/4.7	233k/920

1 RLWE, Our Implementation

2 McEliece Cryptosystem by Ghosh et al. IEEE TC, 2014

All Comparisons

Implementation	Parameters	Device	m LUTs/FFs/	Freq	Cycles/7	$\Gamma \mathrm{ime}(\mu s)$
Algorithm			DSPs/BRAM18	(MHz)	Encryption	Decryption
RLWE	(256, 7681, 11.32)	V6LX75T	1349/860/1/2	313	6.3k/20.1	2.8k/9.1
	(512, 12289, 12.18)		1536/953/1/3	278	13.3k/47.9	5.8k/21
RLWE $(SAC-13)$	(256, 7681, 11.32)	V6LX75T	4549/3624/1/12	262	6.8k/26.2	4.4k/16.8
Pöppelmann et al.	(512, 12289, 12.18)	V6LX75T	5595/4760/1/14	251	13.7k/54.8	8.8k/35.4
RLWE (ISCAS-14)	(256, 4096, 8.35)	S6LX9	317/238/95/1	144	136k/946	-
Pöppelmann et al.			112/87/32/1	189	-	66k/351
ECC (CHES-12)	Binary-233	V5LX85T	18097/-/5644/0	156	$\approx 3.8 \text{k}/24.6$	$\approx 1.9 k/12.3$
Rebeiro et al.						
NTRU (ICM-09)	NTRU-251	XCV1600E	27292/5160/14352/0	62.3	-/1.54	-/1.41
Kamal et al.						