# High-Performance Scalar Multiplication using 8-Dimensional GLV/GLS Decomposition

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Microsoft Research

YAŞAR UNIVERSITY

# Motivation - I

| | DH | ECDH |
|---|---|---|
| Group | $(\mathbf{F}_{p_1}^{*}, \times)$ | $(E(\mathbf{F}_{p_2}), +)$ |

| Security level (bits) | $\log_2 p_1$ | $\log_2 p_2$ | Ratio DH cost : EC cost |
|---|---|---|---|
| 128 | 3072 | 256 | 10:1 |
| 192 | 7680 | 384 | 32:1 |
| 256 | 15360 | 521 | 64:1 |

Source: NSA – The case for Elliptic Curve Cryptography
http://www.nsa.gov/business/programs/elliptic_curve.shtml

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- E.g. Twisted Edwards curves with extended twisted Edwards coordinates
- See the Explicit-Formulas Database

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### Reduce the **number** of group operations

- Different algorithms to compute the scalar multiplication
- Reduce the number of point additions:
  e.g. use large window sizes
- Reduce the number of point doublings:
  e.g. scalar decomposition

# Reducing the Number of Point Doublings

- *d*-dimensional scalar decomposition
- Decompose a scalar *k* into *d* “mini-scalars” $k_i \approx \sqrt[d]{k}$
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Precompute: $\{\emptyset, P, [\lambda]P, P + [\lambda]P\}$

Example: $d = 2$

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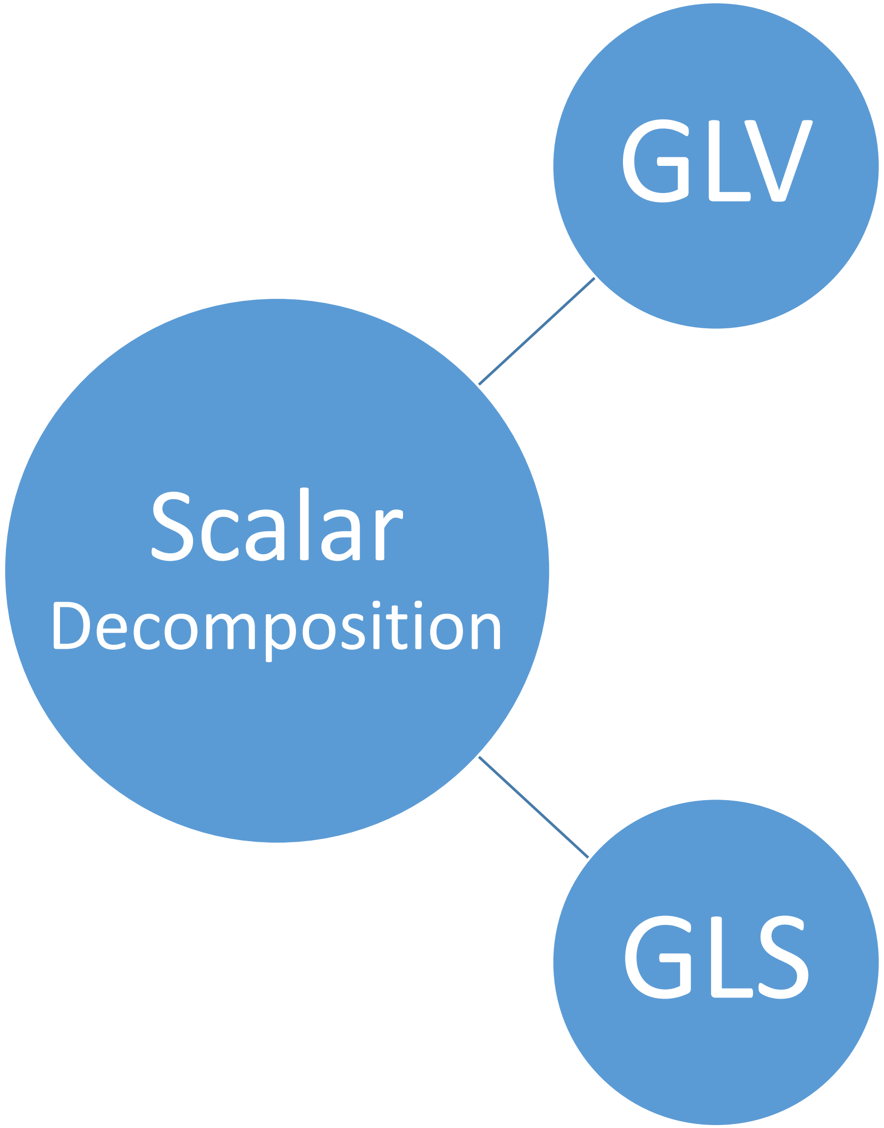
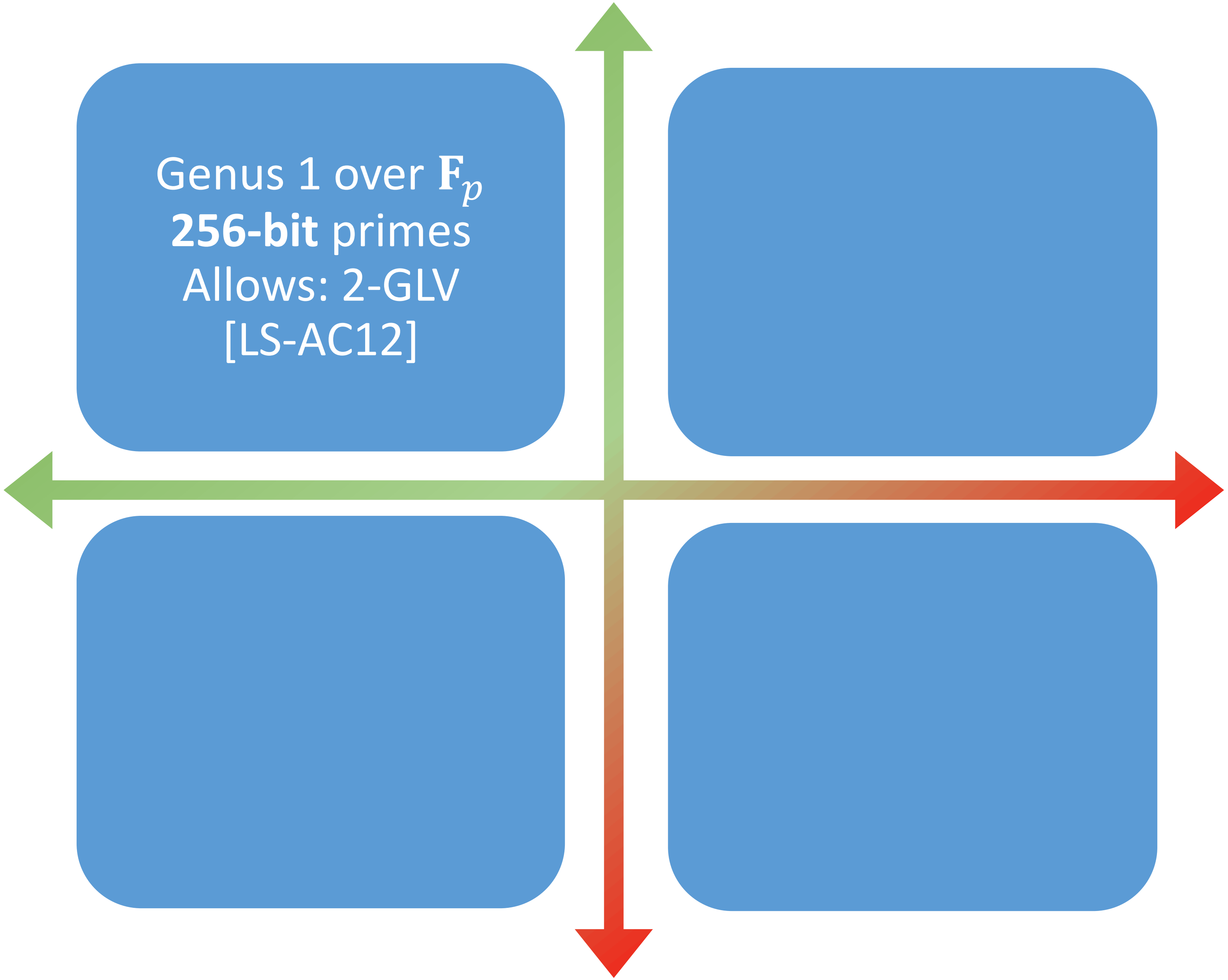



- Gallant, Lambert, Vanstone [GLV-C01]
- Use non-trivial endomorphism
- Larger endomorphism ring means larger dimensional scalar decomposition

- Galbraith, Lin, Scott [GLS-JoC11]
- Independent of the endomorphism
- Works for E($\mathbf{F}_{p^m}$) with $m > 1$

# Overview of GLV/GLS techniques – 128-bit security



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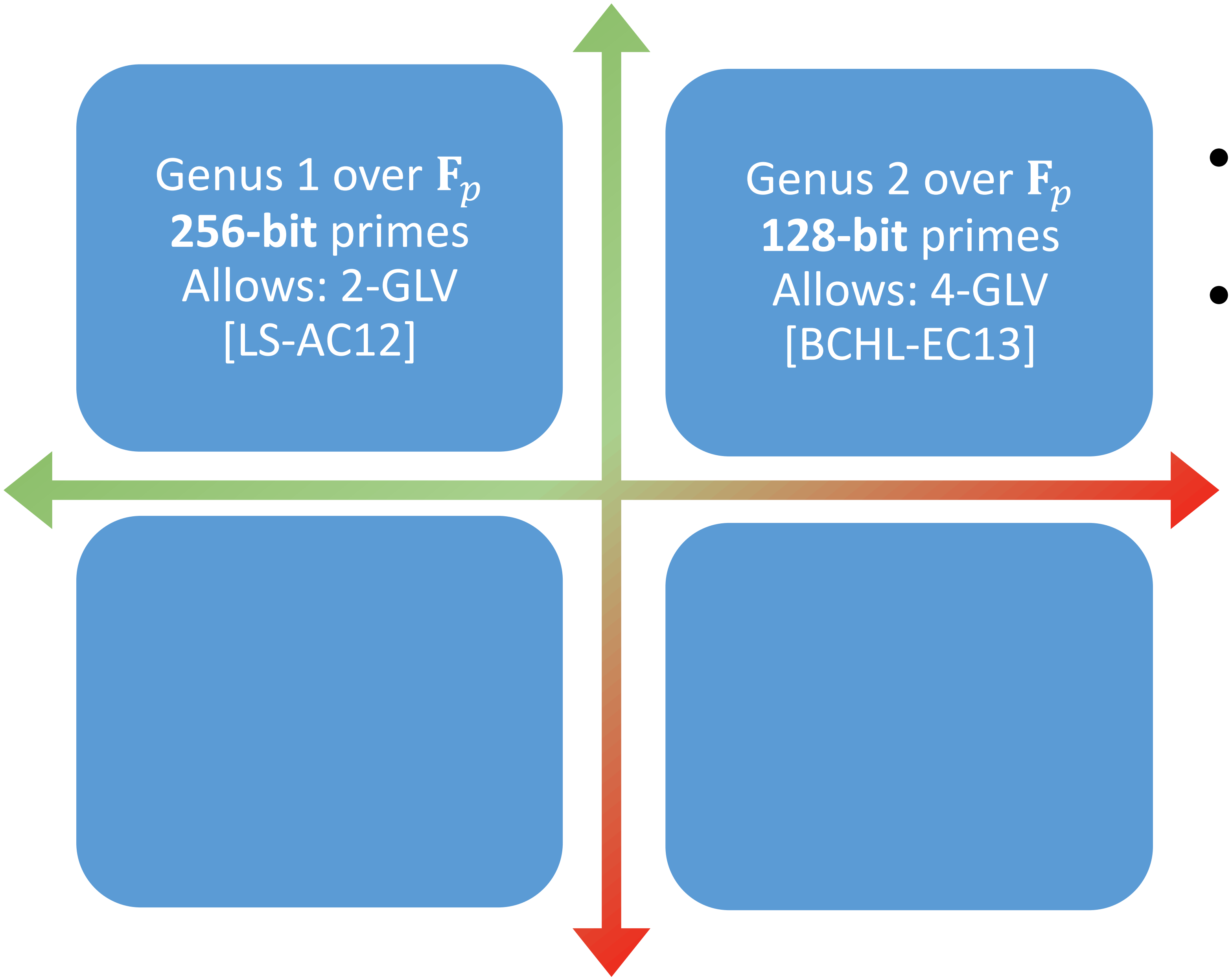


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- But genus > 3 is considered insecure

$\#E(\mathbf{F}_{p_1}) \approx \#\mathrm{Jac}_C(\mathbf{F}_{p_2})$ with

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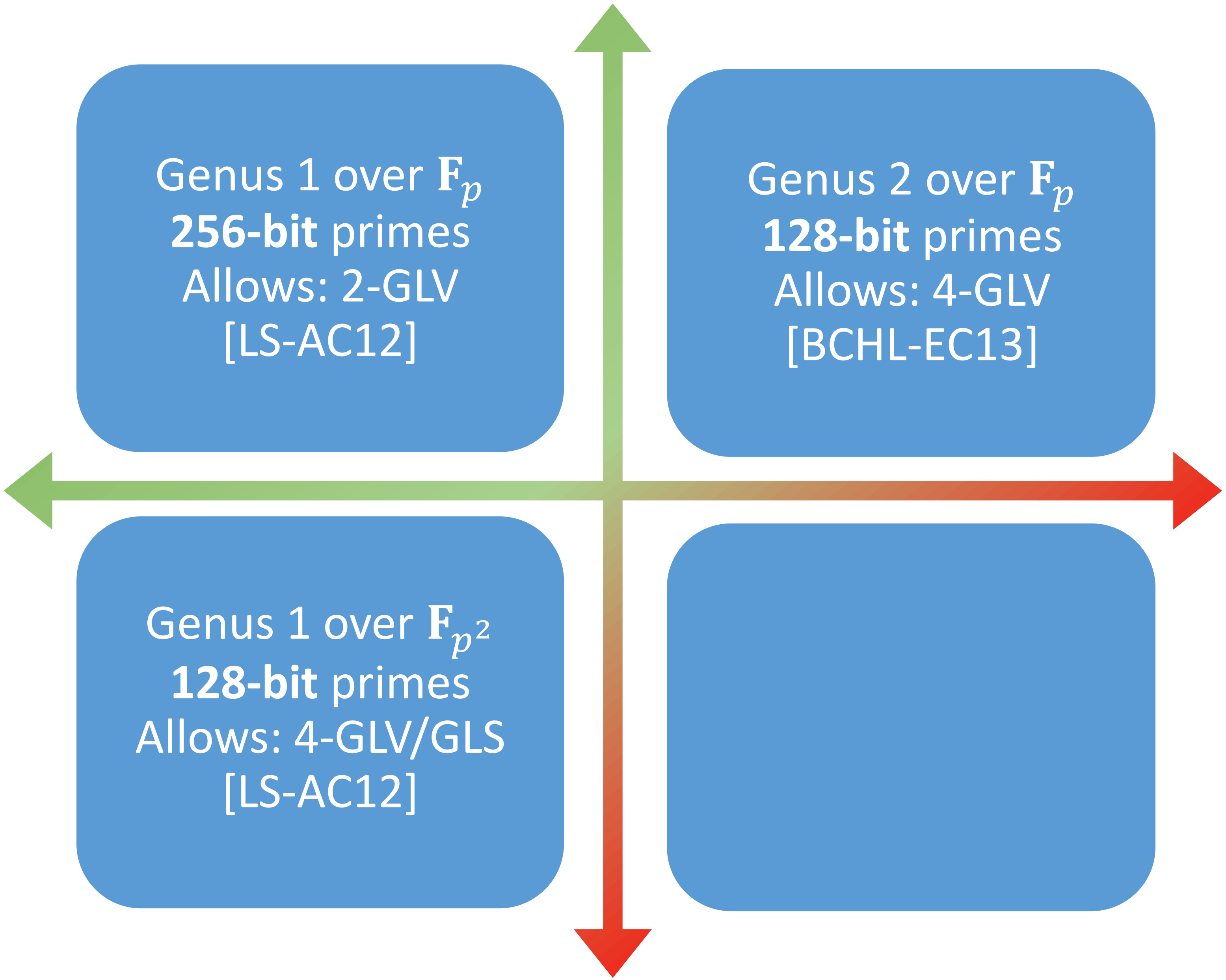


$\#E\left(\mathbf{F}_{p_1}\right) \approx \#E\left(\mathbf{F}_{p_2^2}\right)$ with

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- Extension degree of *m*>1 means we can use GLS
- When *m* is too large subexponential attacks

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Genus 1 over $\mathbf{F}_p$
**256-bit** primes
Allows: 2-GLV
[LS-AC12]

Genus 2 over $\mathbf{F}_p$
**128-bit** primes
Allows: 4-GLV
[BCHL-EC13]

Genus 1 over $\mathbf{F}_{p^2}$
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Allows: 4-GLV/GLS
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Fits in a single word on high-end servers

Ideal candidate for embedded platforms

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What about security?

How to efficiently use 8-GLV/GLS?

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# Fast Primes – NIST-like and Montgomery Friendly

[BCHL-EC13] uses Mersenne prime: $2^{127} - 1$, we try: $2^{61} - 1$
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Interleaved radix-$2^b$ Montgomery multiplication

$$C \equiv A \cdot B \cdot 2^{-bn} \bmod p, \mu = -p^{-1} \bmod 2^b, A = \sum_{i=0}^{n-1} a_i 2^{bi}$$

for $i = 0$ to $n - 1$ do

$$C = C + a_i \cdot B$$
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$= 2^{32}(2^{29}-0)-1$

Interleaved radix-$2^b$ Montgomery multiplication

Example: $2^b\left(2^{\tilde{b}}-c\right)-1$

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## Generic Attack: Pollard rho

- [Pollard-MoC78]
- $\sqrt{(\pi r)/(2\#\mathrm{Aut})}$, where $\#\mathrm{Aut} \geq 2$
- For the GLV/GLS curves $\#\mathrm{Aut} = 10$

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- [P. Gaudry. J. Symb. Comp. 09], [K. Nagao. ANTS 10]

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**Advantage**
Smaller scalars

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- $\psi$: $\mathrm{Jac}(C) \rightarrow \mathrm{Jac}(C), \; \psi(D) = [\lambda]D$, for $0 < \lambda < r$
- Decompose the scalar using [Park,Jeong,Lim-EC02]

Lookup table: $L[i] = \sum_{\ell=0}^{7}\left(\left\lfloor \frac{i}{2^\ell} \right\rfloor \bmod 2\right) \cdot D_\ell$ for $0 \leq i < 2^8$

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**Constant** time
$32\mathrm{D} + 7\psi$
$(22 + 2 \times 32)\mathrm{A}$

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Two lookup tables:

$$T_1[i] = \sum_{\ell=0}^{3}\left(\left\lfloor \frac{i}{2^\ell} \right\rfloor \bmod 2\right) \cdot D_\ell$$

$$T_2[i] = \sum_{\ell=0}^{3}\left(\left\lfloor \frac{i}{2^\ell} \right\rfloor \bmod 2\right) \cdot D_{\ell+4}$$

for $0 \le i < 2^4$

**Non-constant** time
$32\mathrm{D} + 16\psi$
$(13 + 2 \times 32)\mathrm{A}$

# Performance Results – x86

**Platform**: Intel Core i7-3520M Ivy Bridge (2893.484 MHz), hyperthreading turned off and over-clocking (“turbo boost”) disabled

We didn’t aim for record-performance.
There is room for improvement!

| Reference | $(g, \mathrm{K})$ | CT | Bit sec | $10^3$ cycles |
|---|---|---|---|---|
| NIST-p224 | $(1, \mathbf{F}_p)$ | | 112 | 302 |
| [B-PKC06] curve25519 | $(1, \mathbf{F}_p)$ | | 126 | 182 |
| [FPH-ep13] 4-GLV/GLS | $(1, \mathbf{F}_{p^2})$ | | 125 | 92 |
| [BCHL-EC13] Kummer | $(2, \mathbf{F}_p)$ | | 125 | 117 |
| [BCHL-EC13] 4-GLV | $(2, \mathbf{F}_p)$ | | 125 | 156 |
| $2^{61} - 1$, Kummer | $(2, \mathbf{F}_{p^2})$ | | 103 | 108 |
| $2^{61} - 1$, 8-GLV/GLS | $(2, \mathbf{F}_{p^2})$ | | 105 | 100 (88) |

- 3x faster than NIST
- Slightly less secure

# Performance Results – x86

**Platform**: Intel Core i7-3520M Ivy Bridge (2893.484 MHz), hyperthreading turned off and over-clocking ("turbo boost") disabled

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| [BCHL-EC13] 4-GLV | $(2, \mathbf{F}_p)$ | | 125 | 156 |
| $2^{61} - 1$, Kummer | $(2, \mathbf{F}_{p^2})$ | | 103 | 108 |
| $2^{61} - 1$, 8-GLV/GLS | $(2, \mathbf{F}_{p^2})$ | | 105 | 100 (88) |

- Similar performance
- 105 vs 126 bits security

# Performance Results – ARM

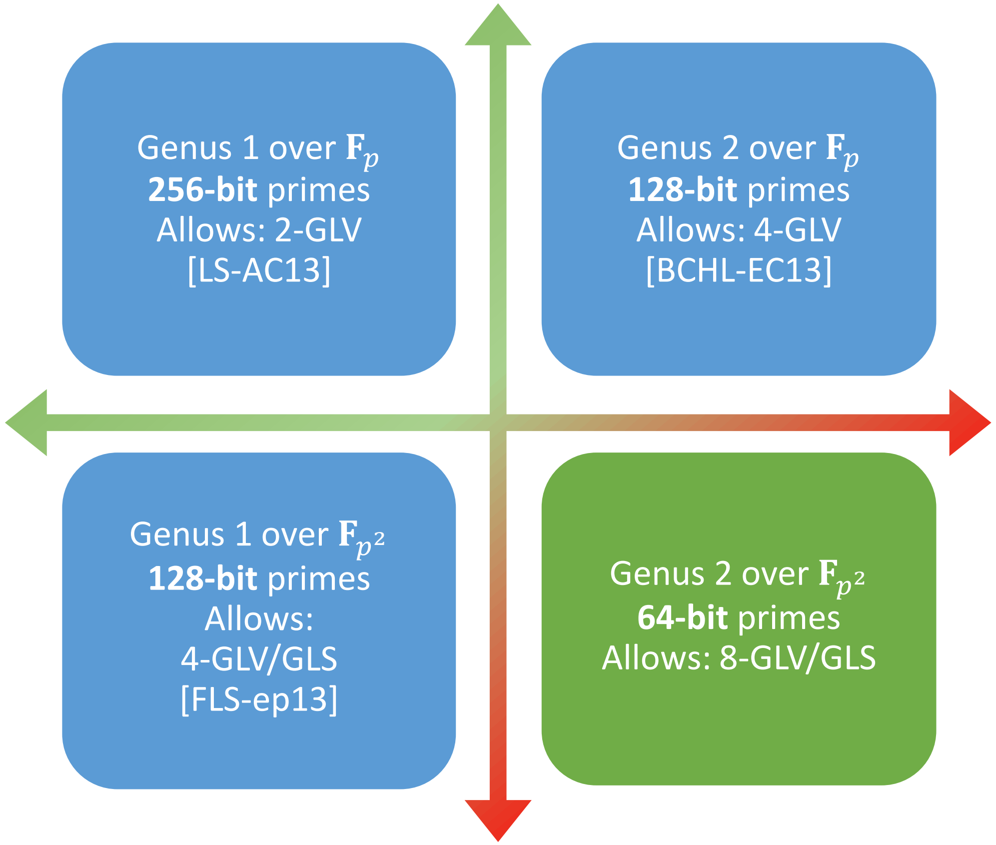
**Platform**: BeagleBoard-xM (1 GHz Cortex-A8 ARM core)
Use the Montgomery-friendly arithmetic

We didn't aim for record-performance.
There is room for improvement!

- Order of magnitude faster than NIST-p224
- Our performance in the same ballpark
- Lower security level

| | Platform | $(g, \mathrm{K})$ | CT | Bit sec | $10^3$ cycles |
|---|---|---|---|---|---|
| [MTS-SASP11] NIST-p224 | Cortex-A8 | $(1, \mathbf{F}_p)$ | | 112 | 7805 |
| [BS-CHES12] curve25519 | Cortex-A8 w NEON | $(1, \mathbf{F}_p)$ | | 126 | 527 |
| [FPH-ep13] 4-GLV/GLS | Cortex-A9 | $(1, \mathbf{F}_{p^2})$ | | 125 | 417 |
| [H-ep12] twisted Edwards | Cortex-A9 | $(1, \mathbf{F}_p)$ | | 125 | 616 |
| $2^{61}-1$, Kummer | Cortex-A8 | $(2, \mathbf{F}_{p^2})$ | | 103 | 767 |
| $2^{61}-1$, 8-GLV/GLS | Cortex-A8 | $(2, \mathbf{F}_{p^2})$ | | 105 | 617 (576) |

# Conclusions




- ✓ Genus 2 over $\mathbf{F}_{p^2}$ allows to work with 64-bit primes
- ✓ Interesting for both high-end 64-bit servers and embedded 32-bit devices
- ✓ Precomputing the lookup table for 8-GLV/GLS is more involved than for 2- and 4-GLV/GLS
- ✓ Although faster attacks exist, still provides sufficient security

See our full paper:
Cryptology ePrint Archive: **Report 2013/146**
The x86 implementations have been submitted to eBACS