Unified and Optimized Linear Collision Attacks and Their Application in a Non-Profiled Setting

Benoît Gérard and François-Xavier Standaert

CryptoGroup - Université catholique de Louvain - Belgium

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Power Analysis Attacks

Divide & Conquer

- Differential Power Analysis
- Correlation Power Analysis
- Template Attack
- ...

Alternatives

- Algebraic side-channel attacks
- Side-channel collision attacks

Motivations

- Getting rid of the leakage model
- Main idea

  same output ⇒ same leakage.

  leakage model choice → similarity metric choice

- [Schramm et al. '03] collision in the f function of DES
- [Bogdanov '07] collision between S-box computations
  - Software: table implementation of an S-boxes
  - Hardware: high area constraints → S-box reuse

This work

Target: linear collision attacks

1. Enhancing collision attacks
   - Handling errors generically
   - Exploiting non-colliding events

2. Collision-attack relevance
   - Comparison with **unprofiled** attacks
   - Software context

Overview

Linear Collision Attacks

Linear Collision and Coding Theory

Experiments

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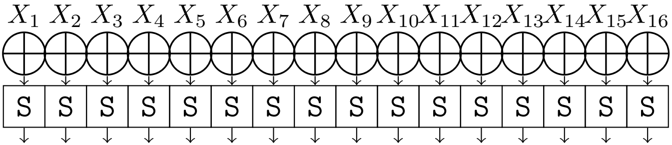
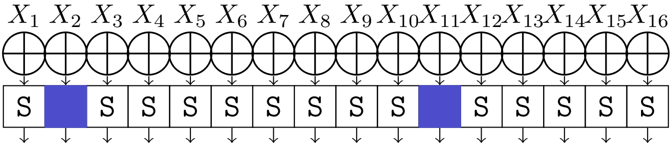
Linear Collision Attacks Principle

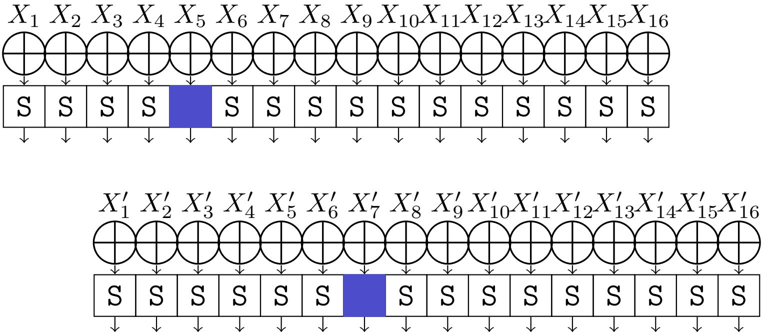


Linear Collision Attacks Principle



$$X_2 \oplus K_2 = X_{11} \oplus K_{11} \quad \Longrightarrow \quad K_2 \oplus K_{11} = X_2 \oplus X_{11}$$

Linear Collision Attacks Principle



$$X_5 \oplus K_5 = X'_7 \oplus K_7 \quad \Longrightarrow \quad K_5 \oplus K_7 = X_5 \oplus X'_7$$

Recovering the Key from Collisions

$$\left\{\begin{array}{lcl} K_1 \oplus K_5 & = & \Delta K_{1,5} \\ K_1 \oplus K_2 & = & \Delta K_{1,2} \\ K_2 \oplus K_8 & = & \Delta K_{2,8} \\ K_3 \oplus K_4 & = & \Delta K_{3,4} \\ K_6 \oplus K_7 & = & \Delta K_{6,7} \end{array}\right.$$

Recovering the Key from Collisions

$$\begin{cases} K_1 \oplus K_5 & = & \Delta K_{1,5} \\ K_1 \oplus K_2 & = & \Delta K_{1,2} \\ K_2 \oplus K_8 & = & \Delta K_{2,8} \\ K_3 \oplus K_4 & = & \Delta K_{3,4} \\ K_6 \oplus K_7 & = & \Delta K_{6,7} \end{cases}$$

2^{24} values for (K_1, K_3, K_6)

$\Downarrow$

2^{24} keys $(K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8)$

instead of 2^{64}

Limitations

1. Information available only if a collision occurs
   - Non-colliding event also brings information
2. Errors
   - Inconsistency in the system
   - Undetectable erroneous system

Techniques to enhance the attack

- [Bogdanov '08] binary and ternary voting
- [Moradi et al. '10] determining $\Delta K_{a,b}$ using correlation

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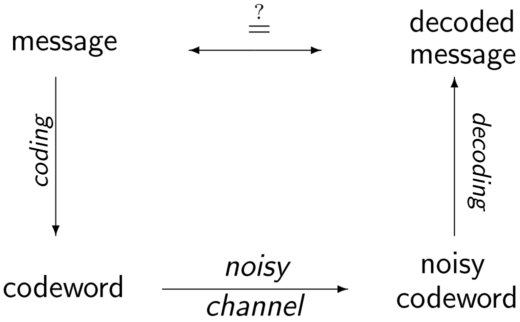
Decoding Problem



Coding: adding redundancy to the message

Collision Attacks as a Decoding Problem

key
$(K_1, \ldots, K_{16})$

Collision Attacks as a Decoding Problem

key

$(K_1, \ldots, K_{16})$

↓

$(\Delta K_{a,b})_{1 \leq a < b \leq 16}$

Collision Attacks as a Decoding Problem

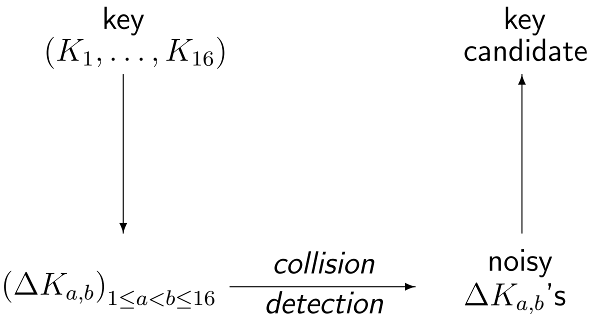
key

$(K_1, \ldots, K_{16})$

↓

$(\Delta K_{a,b})_{1 \leq a < b \leq 16}$ —*collision detection*→ noisy $\Delta K_{a,b}$'s

Collision Attacks as a Decoding Problem



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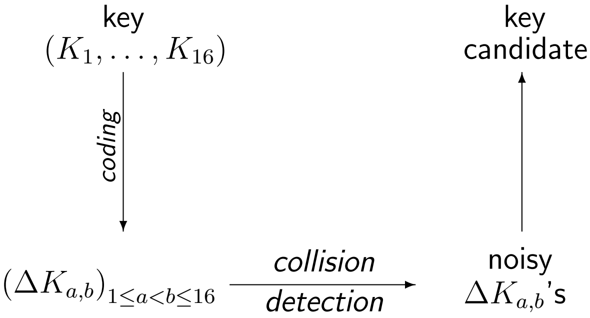

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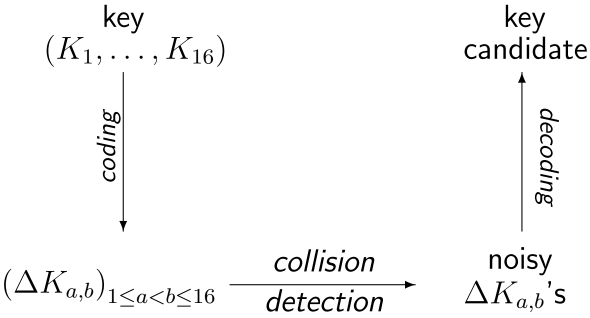

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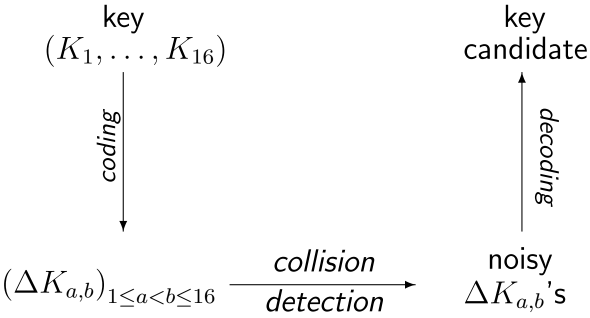
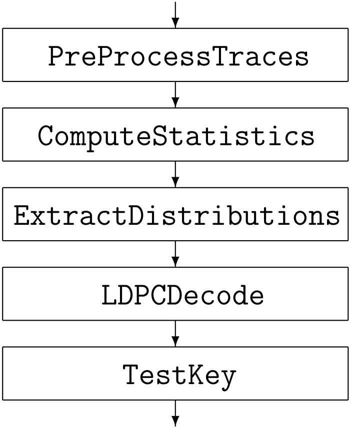


$\Rightarrow$ why not using a decoding algorithm?

Contributions

1. General framework
2. Enhancement of current tools
   - LDPC **soft** decoding
   - Bayesian extensions
3. Experiments on software implementations

General Framework for Collision Attacks



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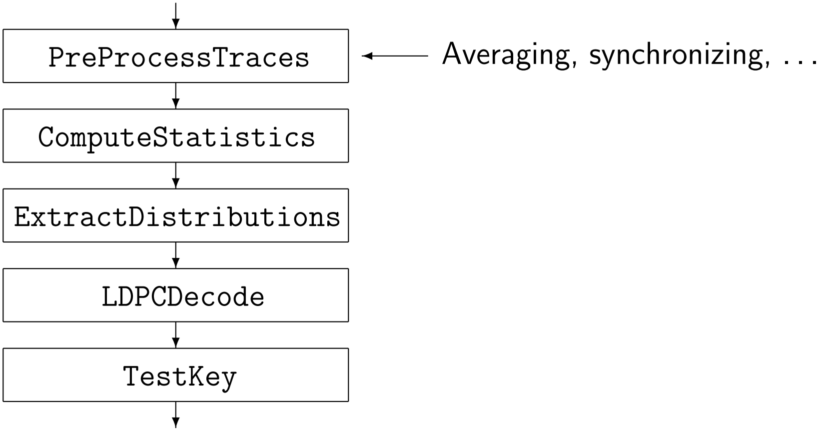

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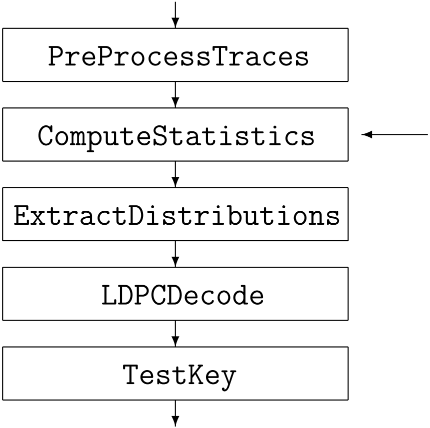
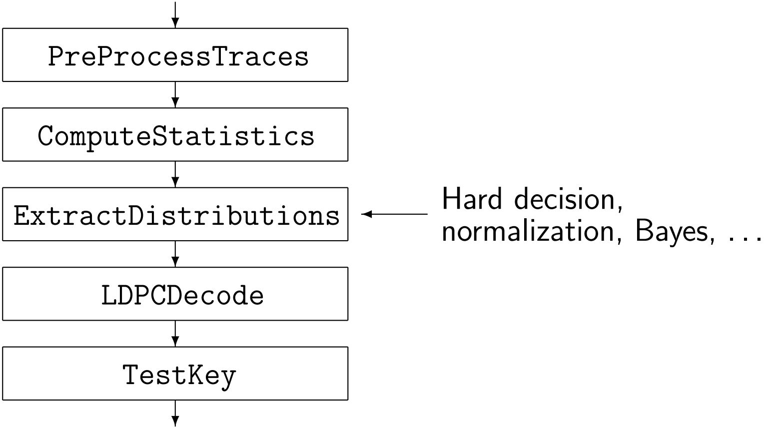


"Collision detection":

Pearson correlation
Euclidean distance

General Framework for Collision Attacks



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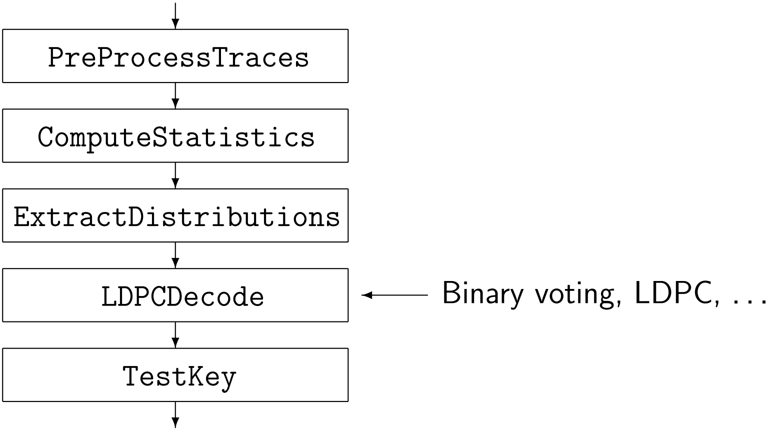

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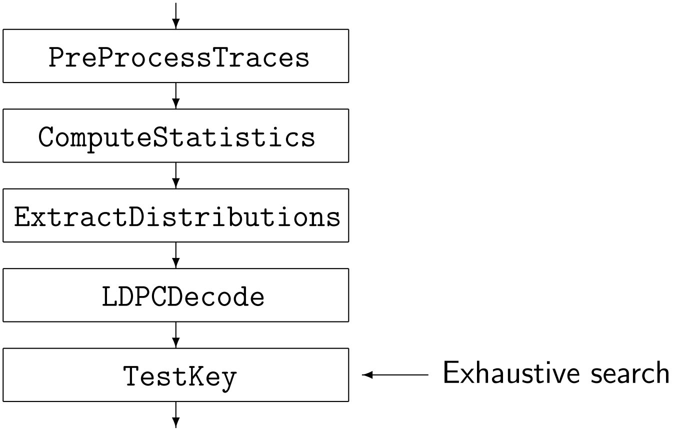

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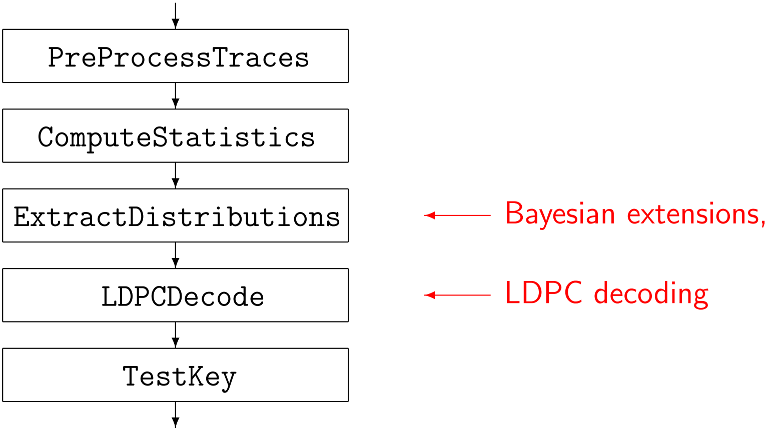

Collision-Attack LDPC Code

$$\Delta K \stackrel{\text{def}}{=} (\Delta K_{1,2}, \ldots, \Delta K_{15,16})$$

- 120 $\Delta K's \rightarrow$ dimension-15 subspace
- Constraints involving 3 positions

$$\underbrace{\Delta K_{a,b}}_{\cancel{K_a} \oplus K_b} \oplus \underbrace{\Delta K_{a,c}}_{\cancel{K_a} \oplus K_c} = \underbrace{\Delta K_{b,c}}_{K_b \oplus K_c}$$

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Sparse
constraints

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Sparse constraints $\Longrightarrow$ LDPC code

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Sparse constraints $\Longrightarrow$ LDPC code $\Longrightarrow$ efficient decoding algorithm

Bayesian extensions

$$\left.\begin{array}{l}\text{scores } (s_1, \ldots, s_n) \\ \Pr[S|\text{coll}] \\ \Pr[S|\text{non-coll}]\end{array}\right\} \xrightarrow{\textit{Bayes}} \Pr[\Delta K_{a,b} = \delta]$$

Unprofiled setting:

- Theoretical models
- On-line parameter estimation

Bayesian extensions

1. Euclidean distance (normalized distance)

$$\mathrm{NED}(T, T') = \sum_j \frac{\left(T_j - T'_j\right)^2}{2\sigma_j^2}$$

2. Correlation-enhanced (Fisher transform)

$$\mathrm{Fisher}(c) = \mathrm{arctanh}(c)$$

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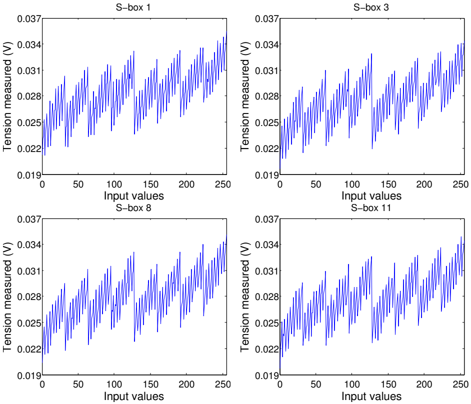
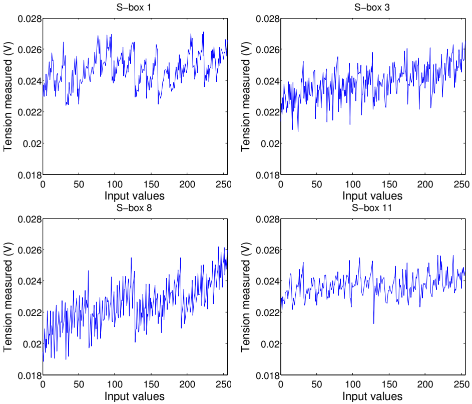
Reference implementation

Reference
mov SR, STxy
mov ZL, SR
lpm SR, Z
mov STxy,SR

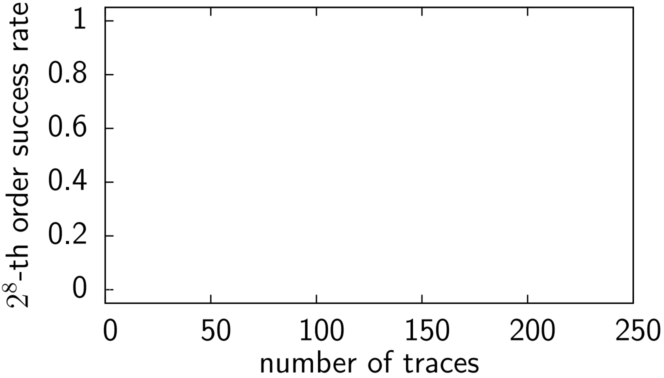


Furious implementation

Furious
mov H1, ST21
mov ZL, ST22
lpm ST21, Z
mov ZL, ST23
lpm ST22, Z



Attacking the Reference Implementation



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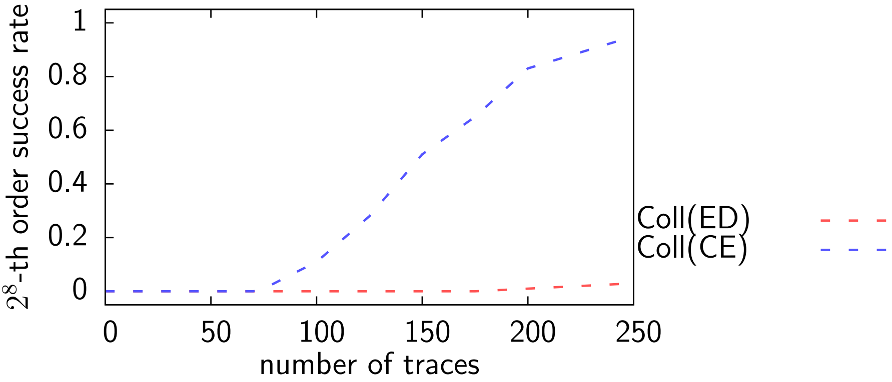

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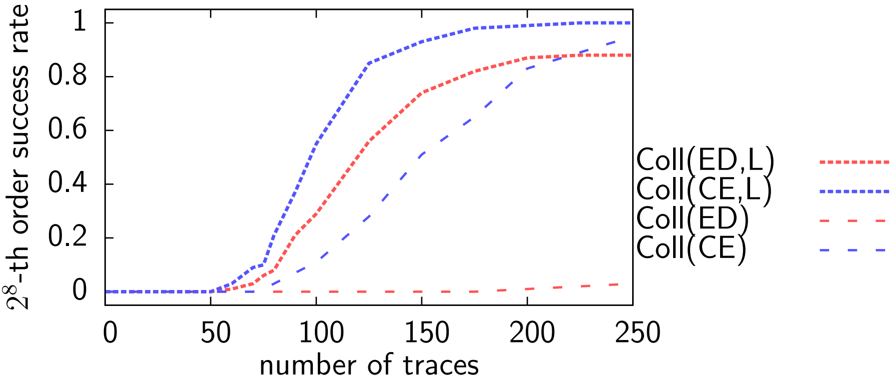

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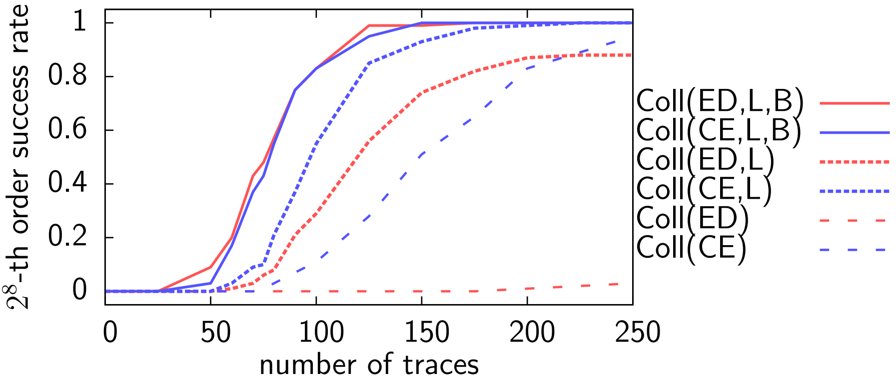

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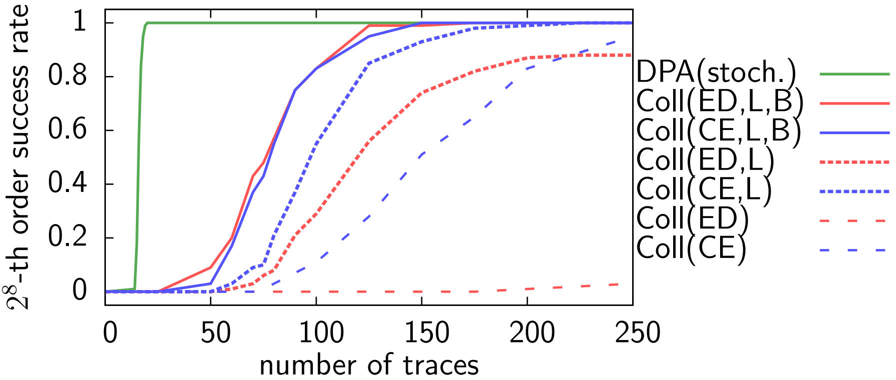

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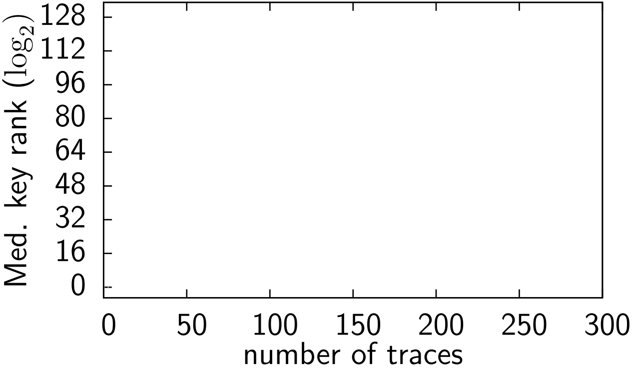

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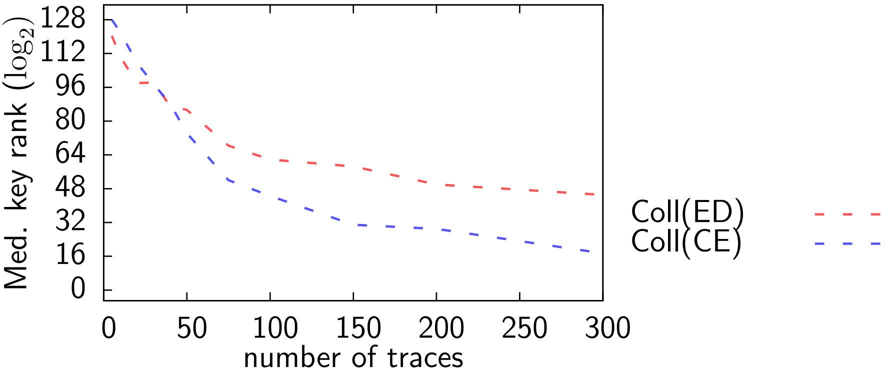

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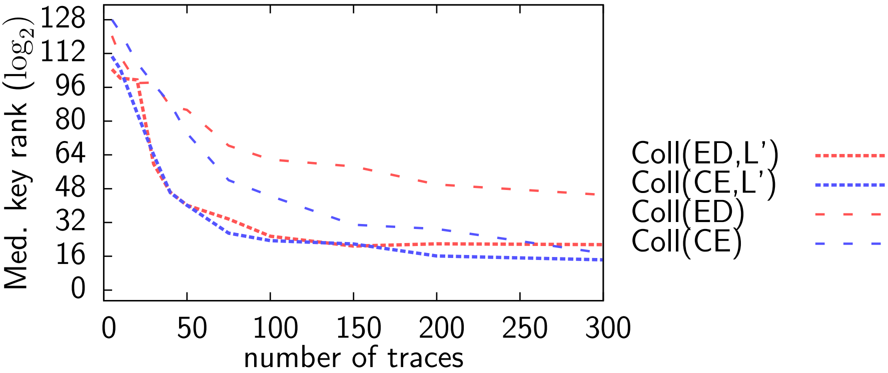

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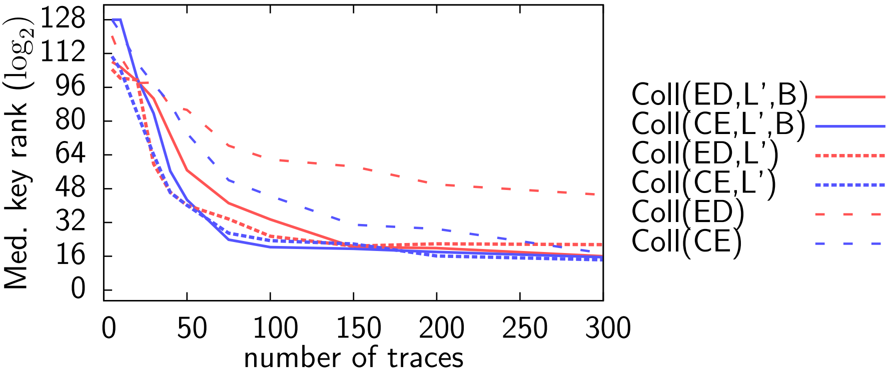

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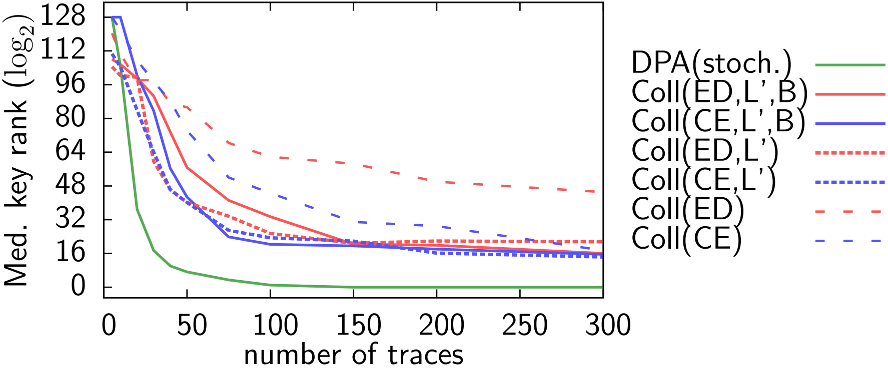
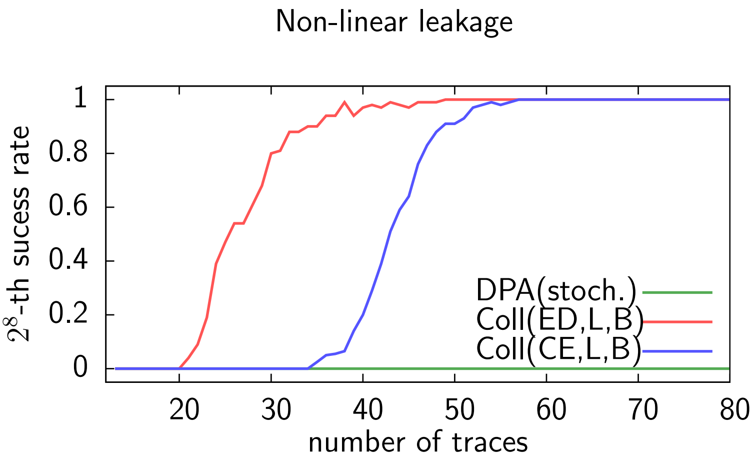


Theoretical context



Conclusions

Collision attacks as a decoding problem

- General framework
- Improvements of former attacks
  - Soft decoding algorithm
  - Bayesian extensions

Experiments performed

- Usually less efficient than stochastic DPA
- May be useful in challenging implementation contexts

Perspectives

- List decoding
- Application to masked implementations
- Application to non-linear collisions