Provably Secure Higher-Order Masking of AES

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Outline

1. Introduction
   - Higher-Order Masking
   - ISW Scheme (CRYPTO'03)
2. Our Scheme
   - Masking the S-box
   - Masking the Whole AES
   - Security
   - Implementation Results
3. Conclusion

Outline

Higher-Order Masking
Basic principle

- Every key-dependent variable x is shared into $d+1$ variables

$$x_0 \perp x_1 \perp \cdots \perp x_d = x$$

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- Computation carried out by processing the shares separately

Higher-Order Masking
Soundness

[Chari-Jutla-Rao-Rohatgi CRYPTO'99]

- Bit x masked $\mapsto x_0, x_1, \ldots, x_d$
- Leakage : $L_i \sim x_i + \mathcal{N}(\mu, \sigma^2)$

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Higher-order masking is sound in the presence of noisy leakage!

Higher-Order Masking Schemes

Definition

A *dth-order masking scheme* for an encryption algorithm $c \leftarrow \mathcal{E}(m, k)$ is an algorithm

$$(c_0, c_1, \ldots, c_d) \leftarrow \mathcal{E}'\big((m_0, m_1, \ldots, m_d), (k_0, k_1, \ldots, k_d)\big)$$

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For SPN (*eg.* DES, AES) the main issue is masking the S-box.

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Software implementations:

- [Schramm-Paar CT-RSA'06]
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Hardware implementations:

- [Ishai-Sahai-Wagner CRYPTO'03]
  - every wire/logic gate is masked at an arbitrary order d
  - wires values $\equiv$ intermediate variables

    $\Rightarrow$ dth-order masking scheme

Ishai-Sahai-Wagner (ISW) Scheme

Principle

- AND gates encoding:
  - Input: $(a_i)_i$, $(b_i)_i$ s.t. $\bigoplus_i a_i = a$, $\bigoplus_i b_i = b$
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  - We prove dth-order security

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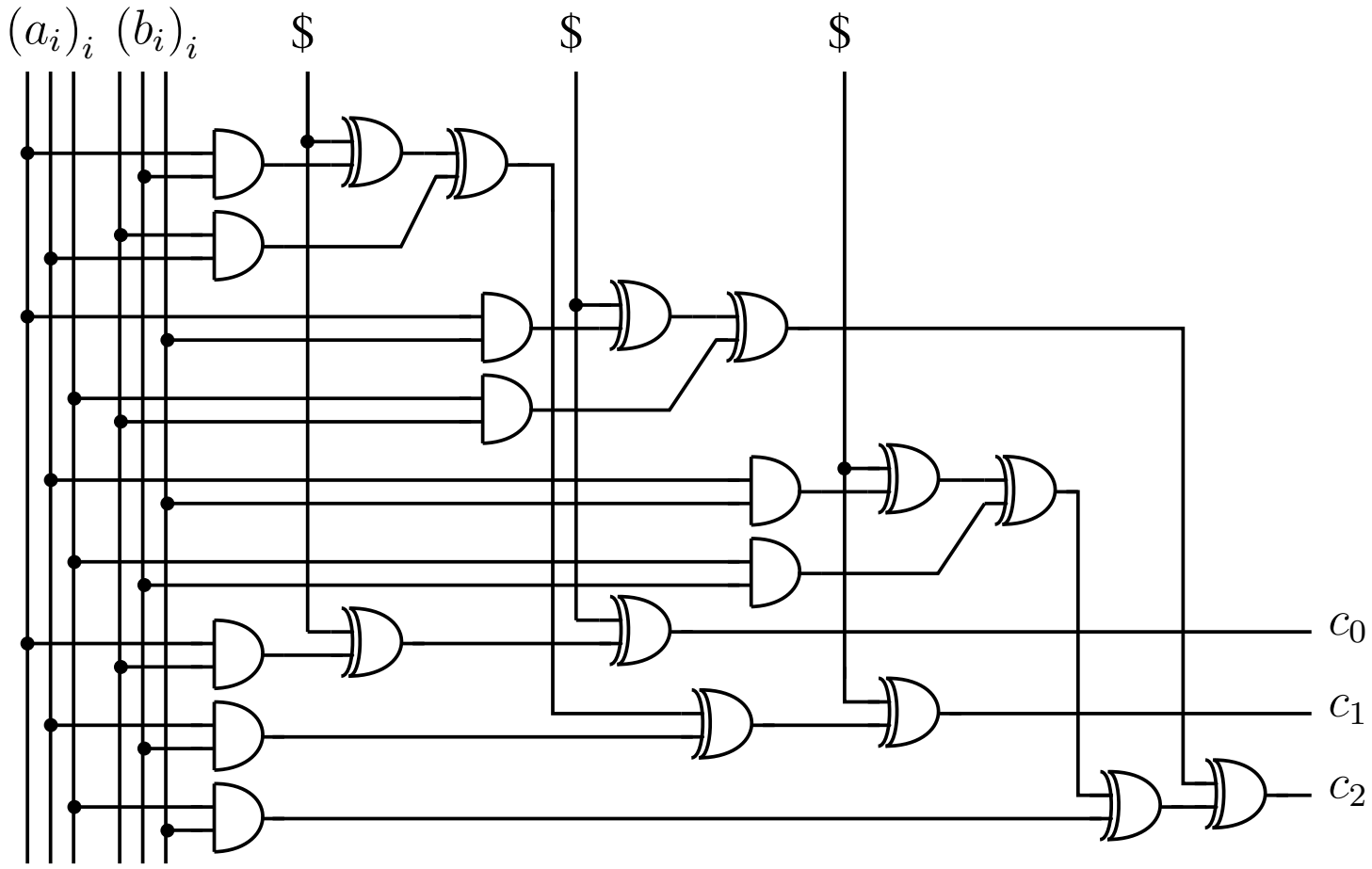
Example: AND gate for $d = 2$



Ishai-Sahai-Wagner (ISW) Scheme

Practical Issues

- Important area overhead for the masked circuit
  - A wire is encoded by $d+1$ wires
  - One AND gate encoded by
    - $(d+1)^2$ ANDs $+$ $2d(d+1)$ XORs $+$ $d(d+1)/2$ \$

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- Not suitable for software implementations

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  - Exp : $x \mapsto x^{254}$ over $\mathbb{F}_{256}$

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- Masking Af is easy:

$$\mathsf{Af}(x) = \mathsf{Af}(x_0) \oplus \mathsf{Af}(x_1) \oplus \cdots \oplus \mathsf{Af}(x_d) \oplus \texttt{0x63} \quad \text{iff } d \text{ is odd}$$

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- For Exp we use an exponentiation algorithm
  - approach used for 1st-order masking in [Blömer-Merchan-Krummel SAC'04]
  - we want to design a dth-order secure exponentiation
  - we need dth-order secure square and multiplication

Masking the S-box

- dth-order secure square
  - squaring is linear over $\mathbb{F}_{256}$

$$x_0^2 \oplus x_1^2 \oplus \cdots \oplus x_d^2 = x^2$$

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$$x_0^{2^j} \oplus x_1^{2^j} \oplus \cdots \oplus x_d^{2^j} = x^{2^j}$$

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- dth-order secure multiplication
  - we generalize the ISW scheme to $\mathbb{F}_{256}$
    - AND $\Rightarrow$ $\mathbb{F}_{256}$ multiplication
    - XOR $\Rightarrow$ $\mathbb{F}_{256}$ addition (8-bit XOR)
    - $\$_1 \Rightarrow \$_8$ (random 8-bit value)

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- dth-order secure multiplication
  - we generalize the ISW scheme to $\mathbb{F}_{256}$
    - AND $\Rightarrow$ $\mathbb{F}_{256}$ multiplication
    - XOR $\Rightarrow$ $\mathbb{F}_{256}$ addition (8-bit XOR)
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- Complexity:
  - secure square: $d+1$ squares
  - secure mult: $(d+1)^2$ mult, $2d(d+1)$ XOR, $d(d+1)/2$ $\$_8$

Masking the S-box

- dth-order secure square
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  - secure square: $d+1$ squares
  - secure mult: $(d+1)^2$ mult, $2d(d+1)$ XOR, $d(d+1)/2$ $\$_8$
- Our goal: minimize the number of multiplications which are not squares

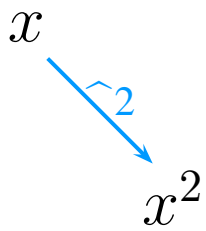
Masking the S-box

The proposed addition chain:

$$x$$

Masking the S-box

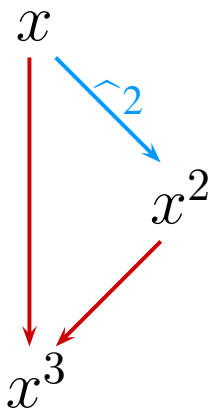
The proposed addition chain:



- one square

Masking the S-box

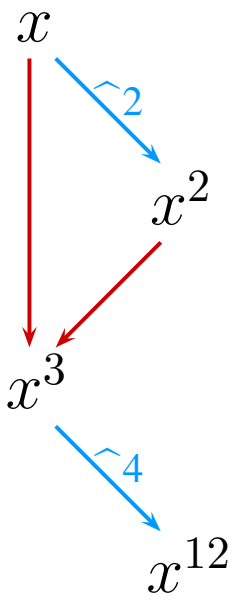
The proposed addition chain:



- one square
- one mult

Masking the S-box

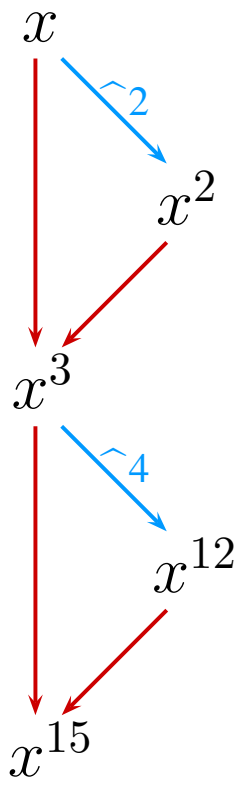
The proposed addition chain:



- one square
- one mult
- one ^4 (two squares)

Masking the S-box

The proposed addition chain:



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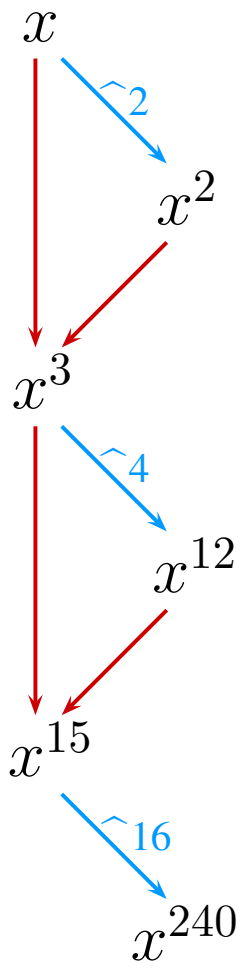
- one mult
- one ^4 (two squares)
- one mult

Masking the S-box

The proposed addition chain:



- one square
- one mult
- one ^4 (two squares)
- one mult
- one ^16 (four squares)

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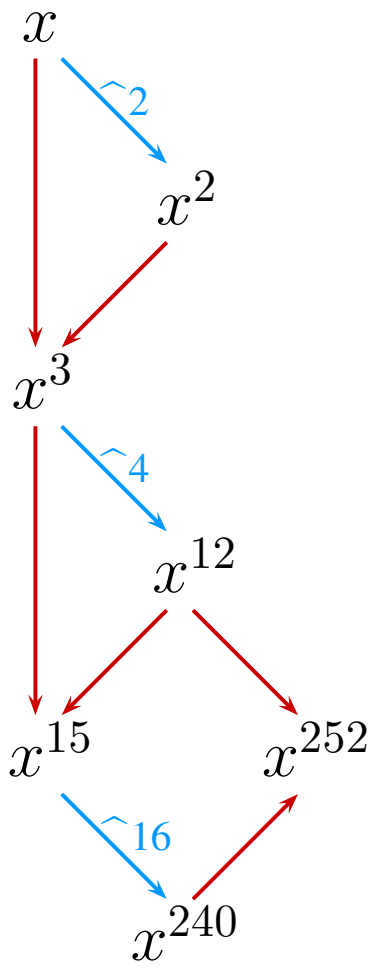

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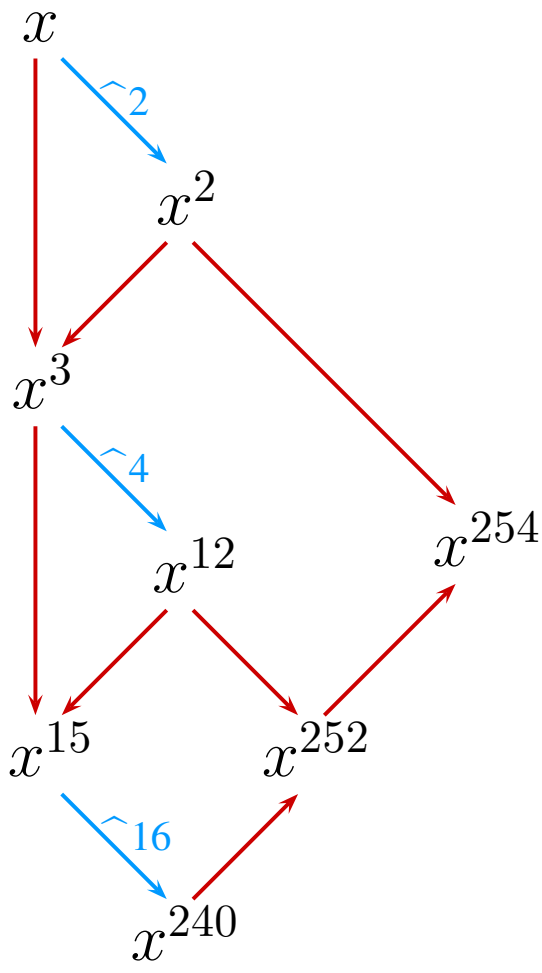

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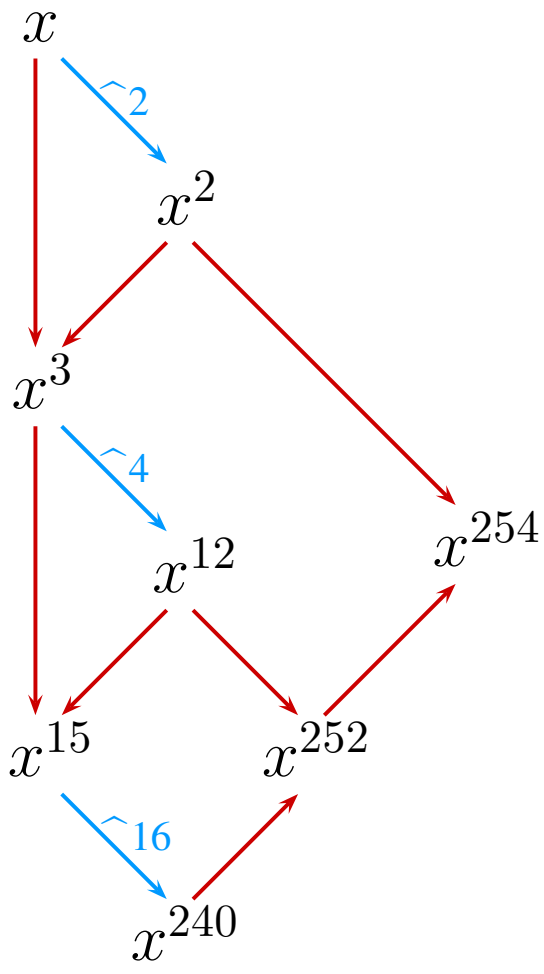

- one square
- one mult
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- one mult
- one ^ 16 (four squares)
- one mult
- one mult
- Total: 4 mult and 7 squares

Masking the S-box

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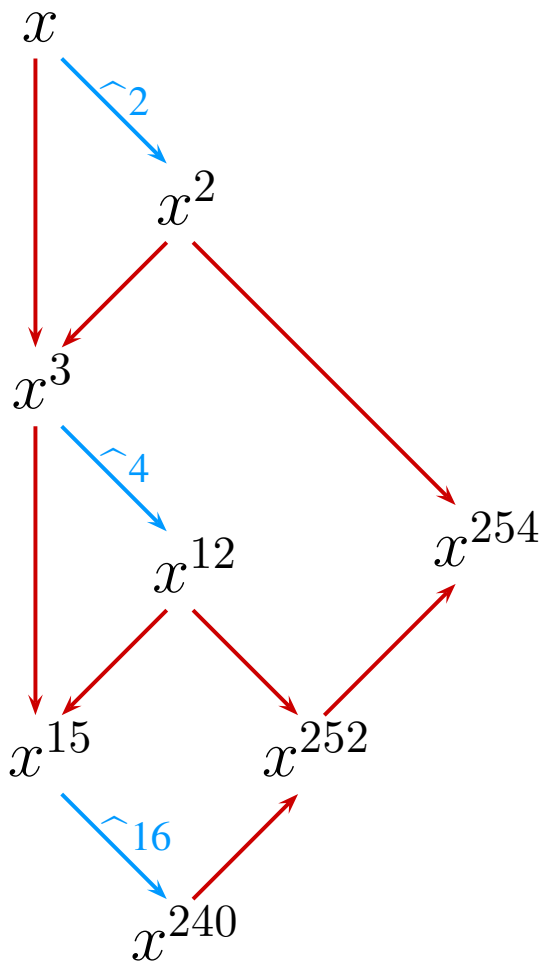

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- Total: 4 mult and 7 squares
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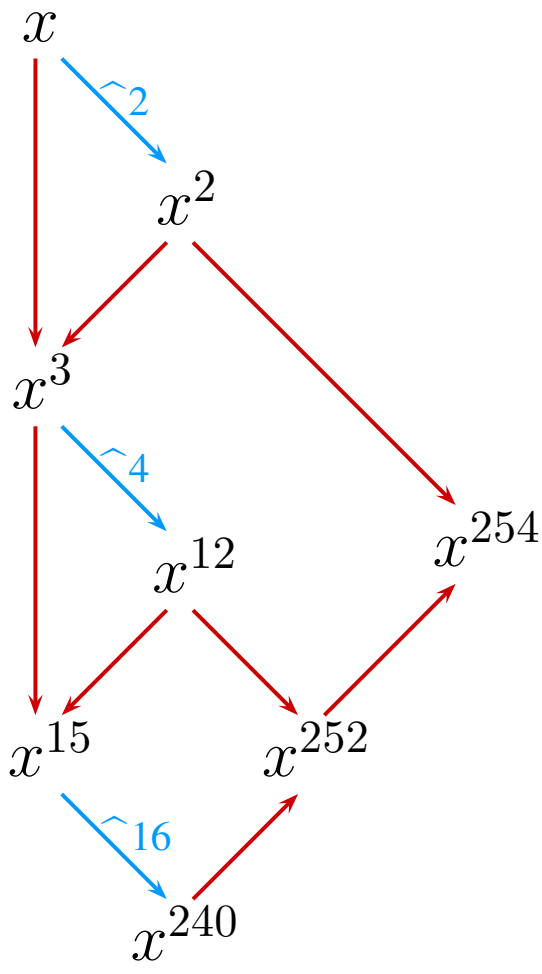

- one square
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- one ^4 (two squares)
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- one mult
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- Total: 4 mult and 7 squares
- Memory: 3 registers
- LUT for ^2, ^4 and ^16

Masking the S-box

Algorithmic description:

Input: shares x_i s.t. $\bigoplus_i x_i = x$
Output: shares y_i s.t. $\bigoplus_i y_i = x^{254}$

1.	$(z_i)_i \leftarrow (x_i^2)_i$	$[\bigoplus_i z_i = x^2]$
2.	RefreshMasks$\big((z_i)_i\big)$	
3.	$(y_i)_i \leftarrow$ SecMult$\big((z_i)_i, (x_i)_i\big)$	$[\bigoplus_i y_i = x^3]$
4.	$(w_i)_i \leftarrow (y_i^4)_i$	$[\bigoplus_i w_i = x^{12}]$
5.	RefreshMasks$\big((w_i)_i\big)$	
6.	$(y_i)_i \leftarrow$ SecMult$\big((y_i)_i, (w_i)_i\big)$	$[\bigoplus_i y_i = x^{15}]$
7.	$(y_i)_i \leftarrow (y_i^{16})_i$	$[\bigoplus_i y_i = x^{240}]$
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- Linear operations of encryption/key schedule (ShiftRows, MixColumns, RotWord) processed on every share independently

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- Key addition performed by adding each key-share to one single state-share

$$\left(\bigoplus_i s_i\right) \oplus \left(\bigoplus_i k_i\right) = \bigoplus_i (s_i \oplus k_i)$$

Security

dth-order security

$$\forall (iv_1, iv_2, \ldots, iv_d) \in \{\text{intermediate var. of } \mathcal{E}'\}^d :$$
$$\mathrm{MI}\big((iv_1, iv_2, \ldots, iv_d), (m, k)\big) = 0$$

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    - $d/2 \rightarrow d$
- Local security for every transformation implies global security for the whole algorithm

Implementation Results (8051)

Method	K cycles	ms (31MHz)	RAM (bytes)	ROM (bytes)
Unprotected Implementation				
Na.	3	0.1	32	1150
First-Order Masking				
[Messerges FSE'00]	10	0.3	256+35	1553
[Oswald+ FSE'05]	77	2.5	42	3195
Our scheme (d=1)	129	4	73	3153
Second-Order Masking				
[Schramm+ CT-RSA'06]	594	19	512+90	2336
[Rivain+ FSE'08]	672	22	256+86	2215
Our scheme (d=2)	271	9	79	3845
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- Interpolation: $30d^2 + 50d + 50$ K cycles
  - $d = 4$: 730 Kc / 24 ms
  - $d = 5$: 1050 Kc / 34 ms

Outline

Conclusion

- First masking scheme for software implementations of AES with provable security at any order
- Based on the work [Ishai-Sahai-Wagner CRYPTO'03]
- Generalization: secure field multiplication in software
- Improved security proof ($d/2 \rightarrow d$), significant in practice
- On-going work:
  - generalization to any S-box/SPN
  - formal security model for dth-order secure implementations