Functional Encryption for Regular Languages

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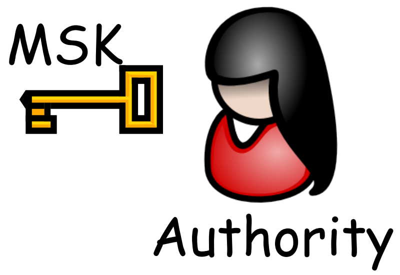
Public Key Encryption [DH76,M78,RSA78,GM84]

Avoid Prior Secret Exchange

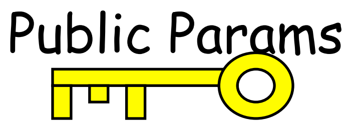
Functional Encryption [SW05…]

Functionality: f(¢ , ¢)

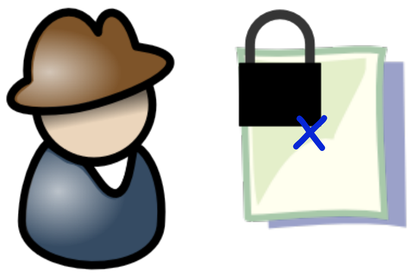


Key: y 2 {0,1}*

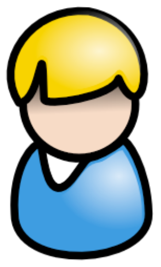
CT: x 2 {0,1}*



Security: "Can only learn f(x,y)"



f(x,y)



"Key Policy" ABE [GPSW06]

Key: $y = \phi \longleftarrow$ Boolean Formula (or circuit)

CT: $x = (m, \vec{X} \in \{0,1\}^n) \longleftarrow$ Variables

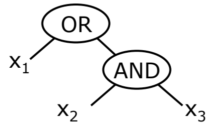
$$f(x = (m, \vec{X}), y) \rightarrow \quad m, \vec{X} \text{ if } \phi(\vec{X}) = \text{true}$$

$$\vec{X} \text{ if } \phi(\vec{X}) = \text{false}$$

"Public Index" $\nearrow$

Functionality: Evaluate formula, if true give message

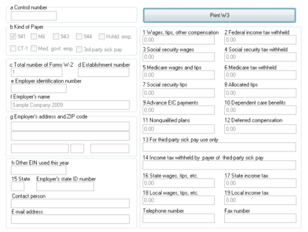
Limitations

Key is a single formula/circuit $\phi =$ 

Operates over fixed sized input

Fixed Size:

Form



a Control number

Print W3

b Kind of Payer
[✓ 941] Mil. 943 944 Hshld. emp.
CT-1 Med. govt. emp. 3rd-party sick pay

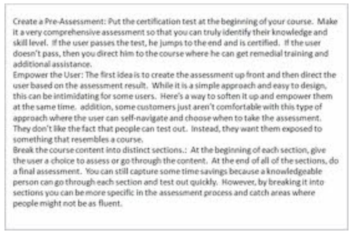
c Total number of Forms W-2: 1
d Establishment number
e Employer identification number
f Employer's name: Sample Company 2009
g Employer's address and ZIP code
h Other EIN used this year
15 State
Employer's state ID number
Contact person
E-mail address

1 Wages, tips, other compensation: 0.00
2 Federal income tax withheld: 0.00
3 Social security wages: 0.00
4 Social security tax withheld: 0.00
5 Medicare wages and tips: 0.00
6 Medicare tax withheld: 0.00
7 Social security tips: 0.00
8 Allocated tips: 0.00
9 Advance EIC payments: 0.00
10 Dependent care benefits: 0.00
11 Nonqualified plans: 0.00
12 Deferred compensation: 0.00
13 For third-party sick pay use only
14 Income tax withheld by payer of third-party sick pay
16 State wages, tips, etc.: 0.00
17 State income tax: 0.00
18 Local wages, tips, etc.: 0.00
19 Local income tax: 0.00
Telephone number
Fax number

Image

Arbitrary Length:

Text



Create a Pre-Assessment: Put the certification test at the beginning of your course. Make it a very comprehensive assessment so that you can truly identify their knowledge and skill level. If the user passes the test, he jumps to the end and is certified. If the user doesn't pass, then you direct him to the course where he can get remedial training and additional assistance.
Empower the User: The first idea is to create the assessment up front and then direct the user based on the assessment result. While it is a simple approach and easy to design, this can be intimidating for some users. Here's a way to soften it up and empower them at the same time. addition, some customers just aren't comfortable with this type of approach where the user can self-navigate and choose when to take the assessment. They don't like the fact that people can test out. Instead, they want them exposed to something that resembles a course.
Break the course content into distinct sections.: At the beginning of each section, give the user a choice to assess or go through the content. At the end of all of the sections, do a final assessment. You can still capture some time savings because a knowledgeable person can go through each section and test out quickly. However, by breaking it into sections you can be more specific in the assessment process and catch areas where people might not be as fluent.

Video

Goal: Functional Enc. for arbitrary length inputs

Regular Languages

Language is regular iff

strings accepted some Deterministic Finite Automata (DFA)

<u>Applications</u>

Search `<[^>]*>`

Firewall Rules `(?i)^([^./]+\.)*(grooveshark\.com|gs-cdn\.net)(?![^/])`

Determinstic Finite Automata (DFA)

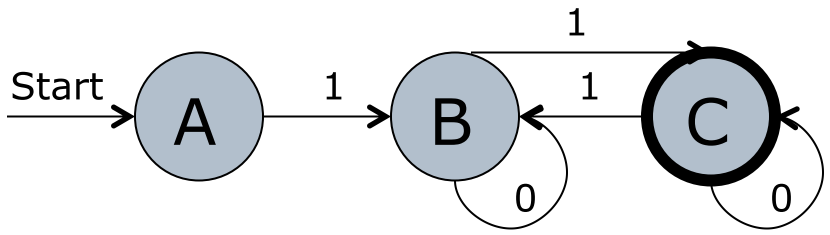
$$M = (Q, \Sigma, \delta, q_0, F)$$

Q	Set of states	$q_0 \in Q$	Start state
Σ	Alphabet	$F \in Q$	Accept states
$\delta : Q \times \Sigma \to Q$	Transition		

Note: Some Regular Expressions not efficiently expressible as DFAs.

A Simple Example

Language = "Begins with 1 and has even parity"


$$\begin{tikzcd}
\arrow[r, "Start"] & A \arrow[r, "1"] & B \arrow[r, bend left, "1"] \arrow[loop below, "0"] & \boxed{C} \arrow[l, "1"] \arrow[loop below, "0"]
\end{tikzcd}$$

✓ Accept(M,w)

w = 1 0 1 0

DFA-Based F.E. System

Key: $M = (Q, \Sigma, \delta, q_0, F)$ $\longleftarrow$ DFA

CT: $x = (m, w \in \Sigma^*)$ $\longleftarrow$ Arbitrary length string

$f(x = (m, \vec{X}), M) \rightarrow$ m, w if Accept(M, w)

w if Reject(M, w)

"Public Index" $\nearrow$

Functionality: Evaluate DFA M on w, if accepts give message

System Overview

Setting: Bilinear group G of order p

Key: $|Q|$ states, $D_0, \ldots, D_{|Q|-1} \overset{R}{\leftarrow} G$

CT: w: ℓ-symbol string, $s_0, \ldots, s_\ell \overset{R}{\leftarrow} \mathbb{Z}_p$

Decrypt: $e(g, D_x)^{s_j}$ $\longleftarrow$ At state x after j symbols

<u>Three Mechanisms</u>

Initialization: Compute $e(g, D_0)^{s_0}$

Transition: $e(g, D_x)^{s_j} \rightarrow e(g, D_y)^{s_{j+1}}$ if $\delta(x, w_j) = y$

Completion: Recover message using $e(g, D_x)^{s_\ell}$ if $q_x \in F$

Setup

Input: Σ

1) Choose Bilinear group G of order p

2) $\alpha \overset{R}{\leftarrow} \mathbb{Z}_p \quad g, z, h_{\mathsf{start}}, h_{\mathsf{end}}, \forall_{\sigma \in \Sigma}\ h_\sigma \overset{R}{\leftarrow} G$

Public Parameters: $e(g,g)^\alpha, g, z, h_{\mathsf{start}}, h_{\mathsf{end}}, \forall_{\sigma \in \Sigma}\ h_\sigma$

Master Secret: g^α

Encryption

Input: Message m, w: ℓ-symbol string

$s_0, \ldots, s_\ell \xleftarrow{R} \mathbb{Z}_p$

For $i = 1$ to ℓ $\quad C_{i,1} = g^{s_i}, \qquad C_{i,2} = (h_{w_i})^{s_i} z^{s_{i-1}}$

"Linking"

Note: Only showing components for transition mechanism!

Key Generation

Input: $M = (Q, \delta, q_0, F)$

Define $(x, y, \sigma) \in \mathcal{T}$ if $\delta(x, \sigma) = y$

$$D_0, \ldots, D_{|Q|-1} \xleftarrow{R} G \qquad \forall t \in \mathcal{T} \; r_t \xleftarrow{R} \mathbb{Z}_p$$

$$\forall t = (x, y, \sigma) \in \mathcal{T}$$

$$K_{t,1} = D_x^{-1} z^{r_t}, \; K_{t,2} = g^{r_t}, \; K_{t,3} = D_y (h_\sigma)^{r_t}$$

Note: Only showing components for transition mechanism!

Transition Mechanism (of decryption)

Suppose $t = (x, y, \sigma) \in \mathcal{T}$ and $w_i = \sigma$

Compute:

$$e(C_{i-1,1}, K_{t,1}) e(C_{i,2}, K_{t,2})^{-1} e(C_{i,1}, K_{t,3})$$

$$= e(g^{s_{i-1}}, D_x^{-1} z^{r_t}) e((h_{w_i})^{s_i} z^{s_{i-1}}, g^{r_t})^{-1} e(g^{s_i}, D_y (h_\sigma)^{r_t})$$

$$= e(g, D_y)^{s_i} / e(g, D_x)^{s_{i-1}}$$

Transition: $e(g, D_x)^{s_{i-1}}$ to $e(g, D_y)^{s_i}$

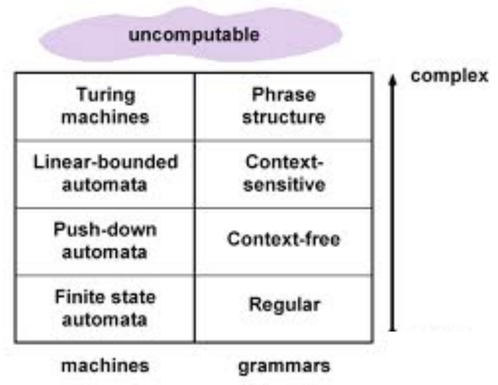
Summary & Three Problems

Functional Enc. for arbitrary length inputs: Achieved DFAs

Problems

(1) Support Non-deterministic Finite Automata (NFA)

(2) Climb the Chomsky Hierarchy



machines	grammars
Turing machines	Phrase structure
Linear-bounded automata	Context-sensitive
Push-down automata	Context-free
Finite state automata	Regular

(3) Move past public index model

Thank you