Q: Must you know the **code** of f to securely compute f?

Mike Rosulek | The University of Montana | CRYPTO 2012

black-box reductions

Reduction

X has an algorithm $\Rightarrow$ Y has an algorithm

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When do black-box constructions exist?

Black-box constructions tend to be more practical (efficient & modular).

secure computation. . .

Several parties wish to carry out an agreed-upon computation.

- Parties have individual inputs / output
- Security guarantees:
  - Privacy (learn no more than your prescribed output)
  - Input independence
  - Output consistency, etc..
- Parties are mutually distrusting, some possibly malicious

black-box secure computation

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If trapdoor functions exist, then for **every** f, there is a secure (in some model) protocol for evaluating f.

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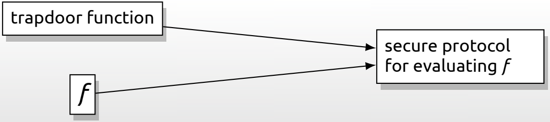

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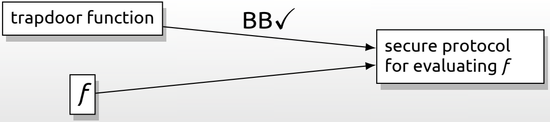

Protocol can be black-box in its usage of underlying primitives!

- [Ishai+06, LindellPinkas07, Haitner08, IshaiPrabhakaranSahai08, Choi+09, PassWee09, ..]

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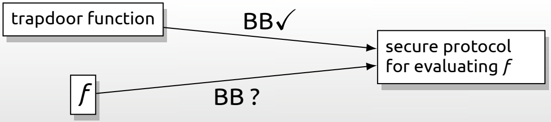

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What about *usage of f?* Typical approach (since [Yao86,GMW87]):

- Express f as a circuit, and evaluate it gate-by-gate — non-black-box!

the model

the model (2-party SFE)

Let $\mathcal{C}$ be a class of 2-input functions.

Definition

Functionality-black-box (FBB) secure evaluation of $\mathcal{C}$ means:

- $\exists$ oracle machines π_A, π_B:
- $\forall f \in \mathcal{C}$:
- $\pi_A^f(x) \rightleftarrows \pi_B^f(y)$ is a secure protocol for evaluating $f(x, y)$

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FBB secure evaluation of $\mathcal{C}$ is *trivial* if:

- $|\mathcal{C}| = 1$ (protocol could "know" code of f)
- $\mathcal{C}$ is exactly learnable via oracle queries (learn code of f, then proceed in non-black-box way)

autoreducibility

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How much "structure" does a set/function L have?

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Basic Definition

L is autoreducible if there exists efficient M:

1. $M^L(x) = L(x)$
2. M doesn't simply query its oracle on x

autoreducibility examples

Discrete log problem in $\langle g \rangle$ is autoreducible:

autoreducibility examples

Discrete log problem in $\langle g \rangle$ is autoreducible:

$dlog_g(x)$: // find d such that $g^d = x$

1. *Choose $a \leftarrow \mathbb{Z}_n$, where $n = \mathrm{ord}(g)$.*
2. *Output: $dlog_g(x \cdot g^a) - a$ (mod n)*

autoreducibility examples

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autoreducibility examples

Discrete log problem in $\langle g \rangle$ is instance-hiding autoreducible:

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"Instance-hiding" autoreducible [BeaverFeigenbaum90]

Oracle queries of $M^L(x)$ distributed independent of x.

semi-honest adversaries

characterization

Definition

A class $\mathcal{C}$ is 2-hiding autoreducible if there exists efficient M:

1. $M^{f,f}(x, y) = f(x, y)$, for all $f \in \mathcal{C}$

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Discussion:

- Same M must work for every $f \in \mathcal{C}$.
- Distinction between x and y.

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Theorem

FBB secure computation of $\mathcal{C}$ is possible in $\mathcal{F}_{\text{ot}}$-hybrid (against semi-honest adversaries) *if and only if* $\mathcal{C}$ is 2-hiding autoreducible

proof: fbb $\Rightarrow$ autoreducible

Given FBB protocol, construct M for autoreducibility:

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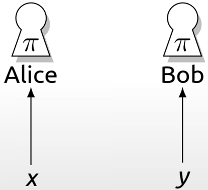

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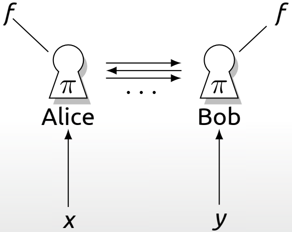

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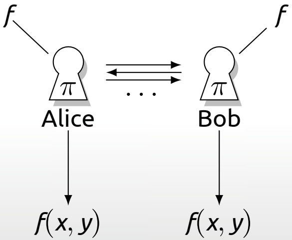

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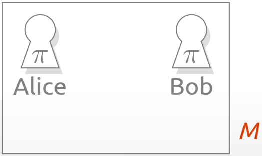

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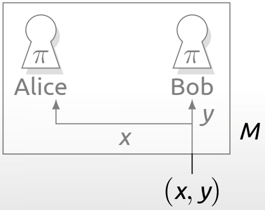

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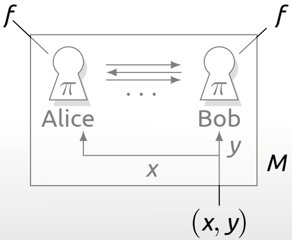

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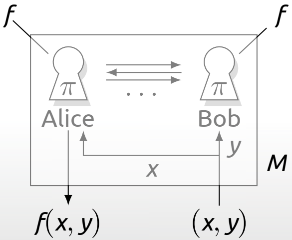

Correctness of protocol:

$\Rightarrow$ Output is $f(x, y)$

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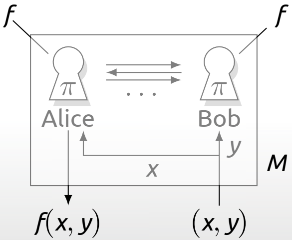

Correctness of protocol:

⇒ Output is $f(x, y)$

Security of protocol:

⇒ Alice's view (incl. oracle queries) "doesn't depend on" y.

⇒ Bob's view (incl. oracle queries) "doesn't depend on" x.

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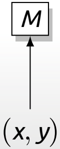

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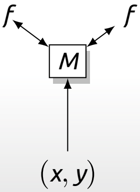

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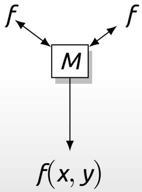

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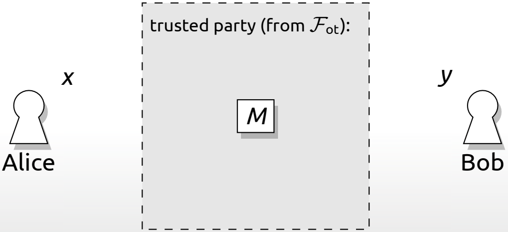

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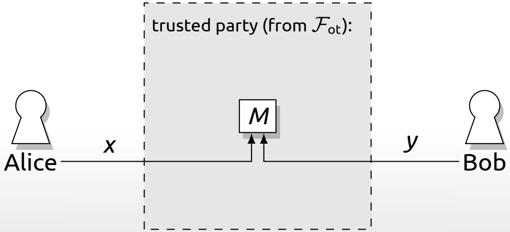

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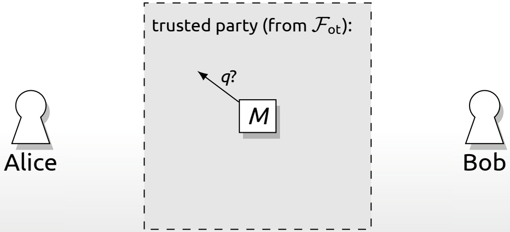

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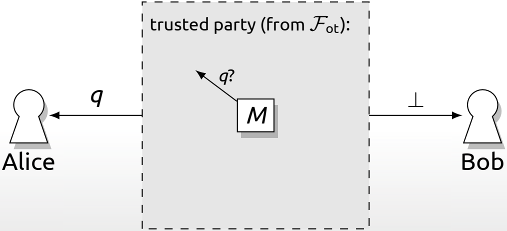

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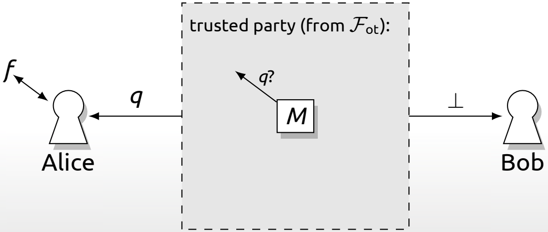

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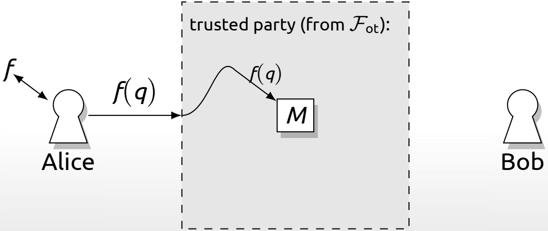

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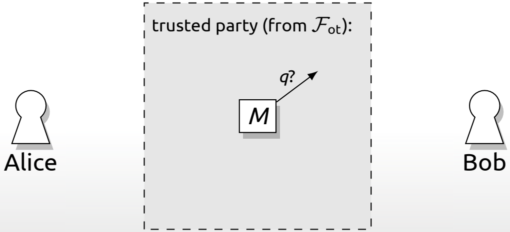

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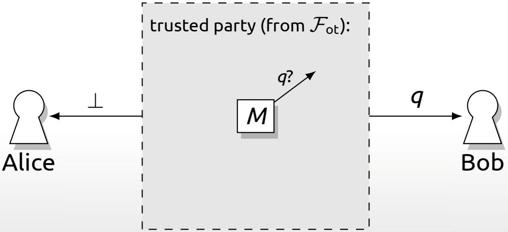

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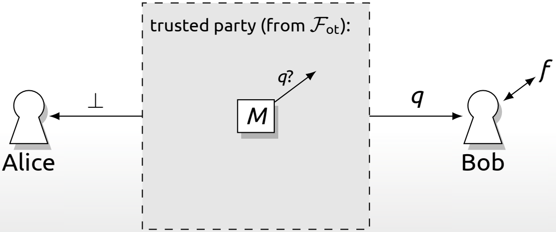

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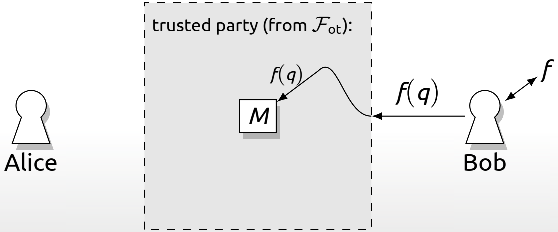

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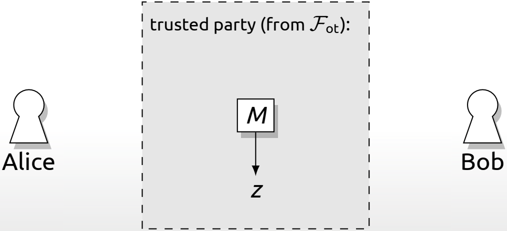

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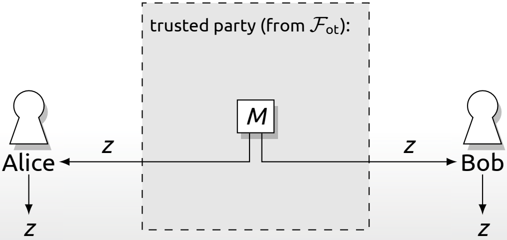

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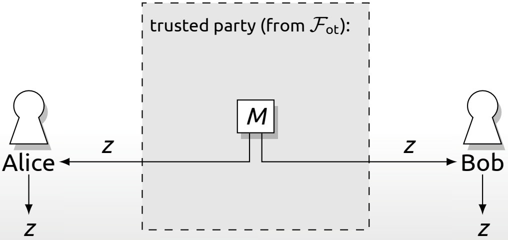

- Entire protocol treats f as black-box.
- Protocol output is correct (when protocol is followed!)
- Alice sees only output & M's left oracle queries.
    - These "don't depend on" Bob's input y.
- Bob's sees only output & M's right oracle queries.
    - These "don't depend on" Alice's input x.

using the characterization:

Positive example

There is a class $\mathcal{C}$ that is 2-hiding autoreducible, but *not learnable* via oracle queries.

- $\Rightarrow$ *Non-trivial* FBB secure computation!
- ☹ Class $\mathcal{C}$ is not especially interesting.

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Negative example

Class of all PRFs is **not** 2-hiding autoreducible.

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Negative example

Class of all PRFs is **not** 2-hiding autoreducible.

- ⇒ Can't securely evaluate PRFs in FBB way (Alice holds seed, Bob holds input)
- … even against semi-honest adversaries.
- … even with arbitrarily powerful trusted setup

malicious adversaries

malicious adversaries

Definition

A class $\mathcal{C}$ is 1-hiding autoreducible if there exists efficient M:

1. $M^f(x, y) = f(x, y)$, for all $f \in \mathcal{C}$
2. M's queries to oracle "don't depend on" (x, y)

Compare to "instance hiding" [BeaverFeigenbaum90]

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Proof sketch:

- Securely simulate M
- Send its oracle queries to *both* parties
- Securely check for agreement of oracle responses

wrap-up. . .

Also in the paper:

- Definition of FBB for more than just function evaluation
- Impossibility of ZK for membership in $\mathrm{im}(f)$, for f OWF

wrap-up...

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- Definition of FBB for more than just function evaluation
- Impossibility of ZK for membership in $\mathrm{im}(f)$, for f OWF

Summary:

- Definitions for MPC protocol that has "black-box usage of functionality"
- Characterizations based on autoreducibility
- It is possible to "evaluate f without knowing the code of f"
- ... but definitely not in general.

The End