On the Impossibility of Tight Cryptographic Reductions

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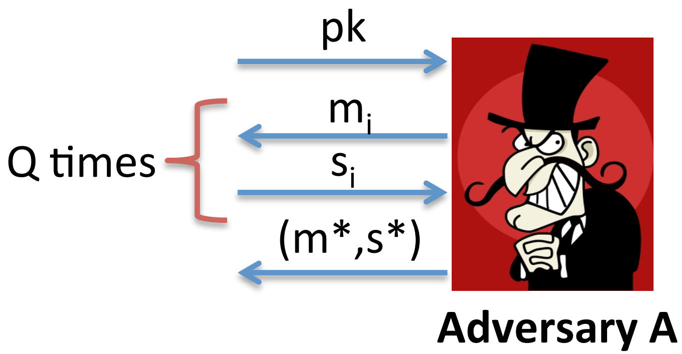
“Tight” Cryptographic Reductions

1. Define a **security model**



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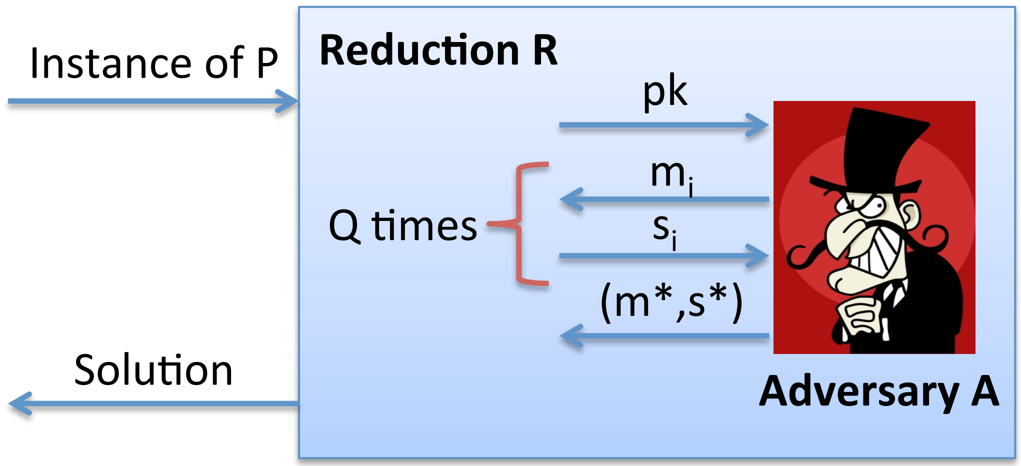
2. Prove: efficient **adversary A** implies efficient algorithm **R** that solves a **"hard" problem P**



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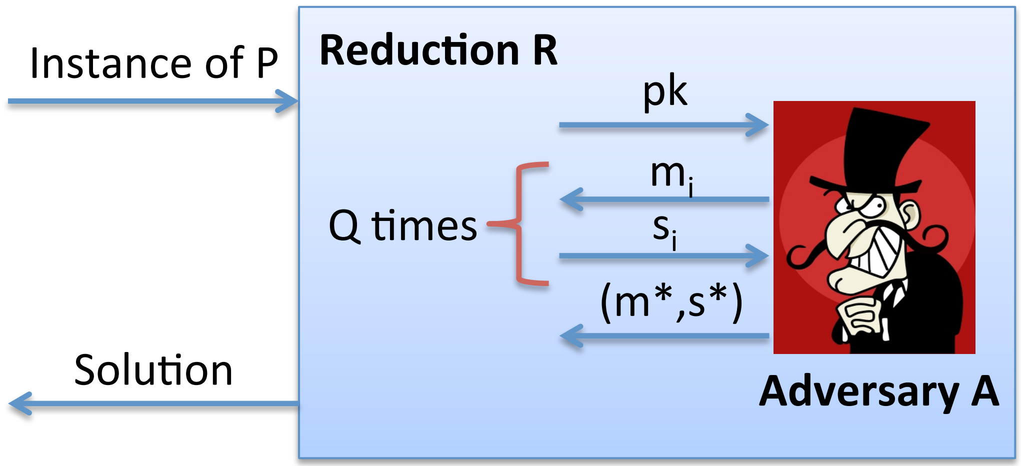
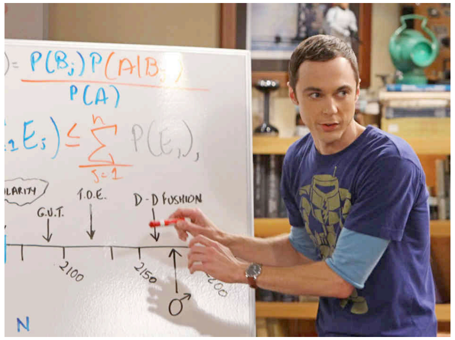


Reduction R is **tight**, if

$\mathbf{t_R} \approx \mathbf{t_A}$ and $\mathbf{succ_R} \approx \mathbf{succ_A}$

Why is tight security interesting?

- Do schemes with **tight security exist**?
  - Inherent tightness lower bounds?



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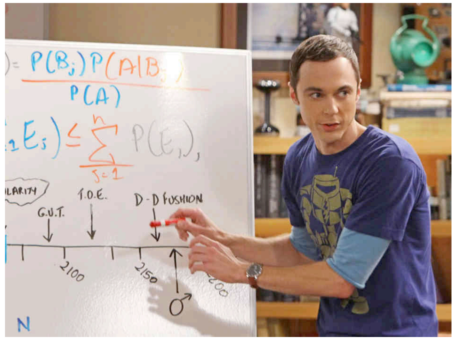
- Do schemes with **tight security exist**?
  – Inherent tightness lower bounds?
- Relevant for **theoretically-sound** selection of parameters
  – “Non-tight“ reduction ⇒ large parameters
  – Tight reduction ⇒ smaller parameters



Many Tightly-Secure Cryptosystems

Identity-based Encryption
- Chen, Wee (Crypto 2013)
- Blazy, Kiltz, Pan (Eurocrypt 2014)
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Public-Key Encryption
- Bellare, Boldyreva, Micali (Eurocrypt 2000)
- Hofheinz, Jager (Crypto 2012)
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Key Exchange
- Bader, Hofheinz, Jager, Kiltz, Li (TCC 2015)

Digital Signatures
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Pseudorandom Functions
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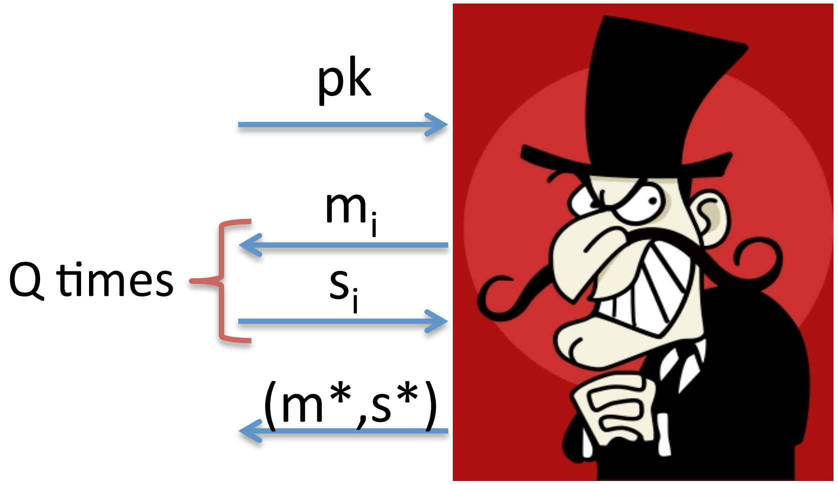
Which **properties** must a cryptosystem (not) have to allow for a **tight security** proof?

Coron's Result* (1/2)

(Eurocrypt 2002)

- Digital signatures
  - **single-user** setting
  - **unique** signatures**



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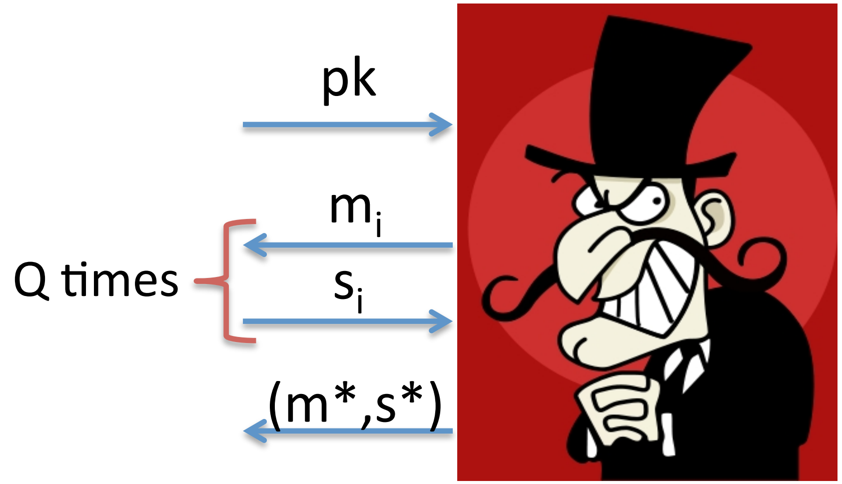

Result:
If a signature scheme has **unique signatures**, then any security reduction "loses" a factor of at least 1/Q.

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Coron's Result (2/2)

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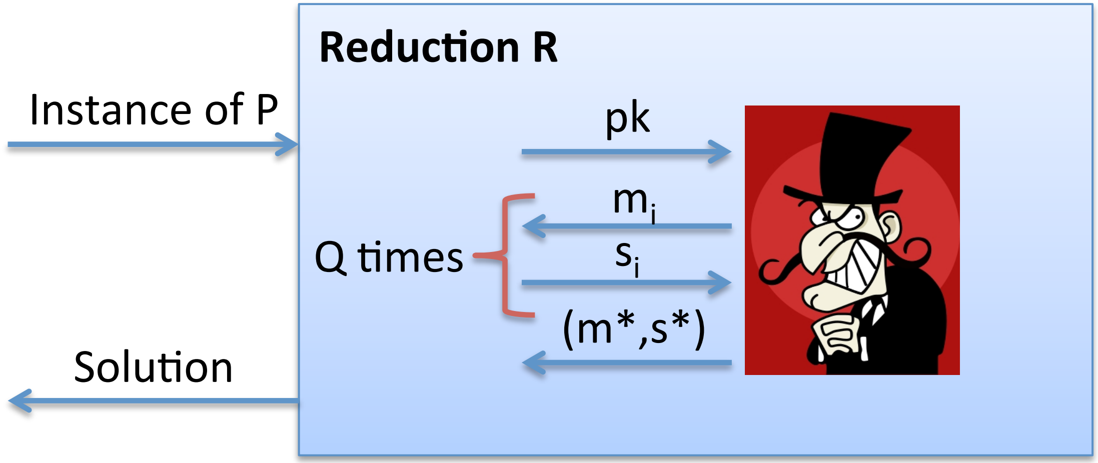

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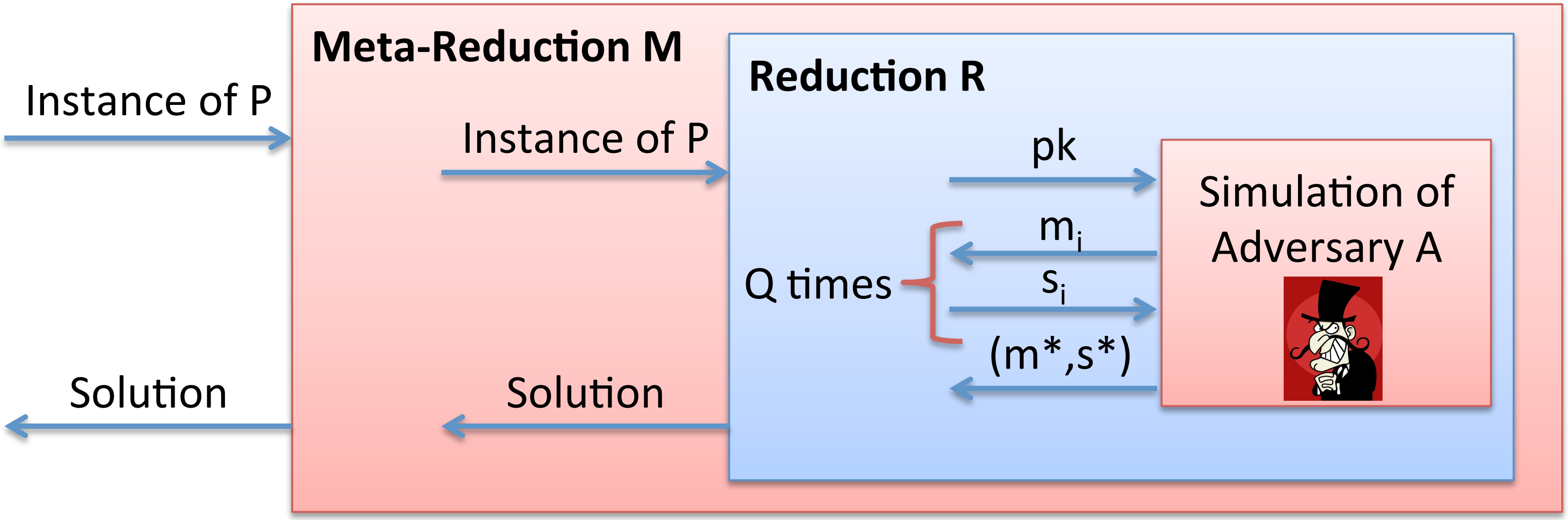

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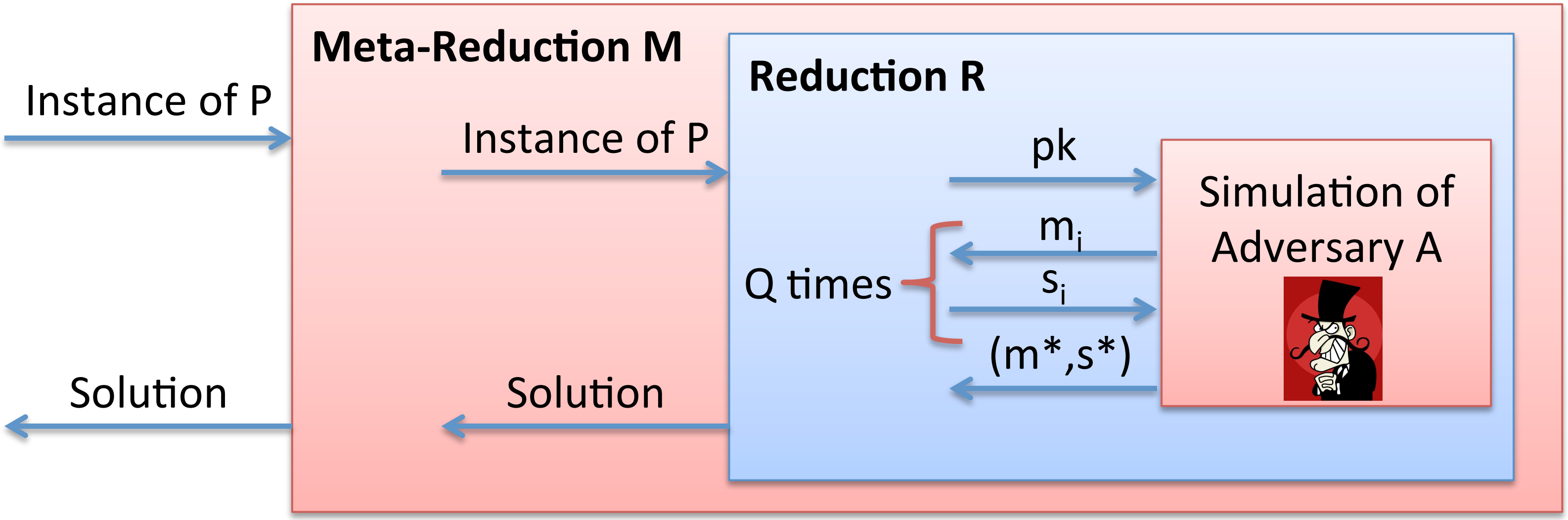

Coron shows:

If a signature scheme has **unique signatures**, then any reduction **R implies an algorithm M that solves P**

- In time $t_M \approx t_R$
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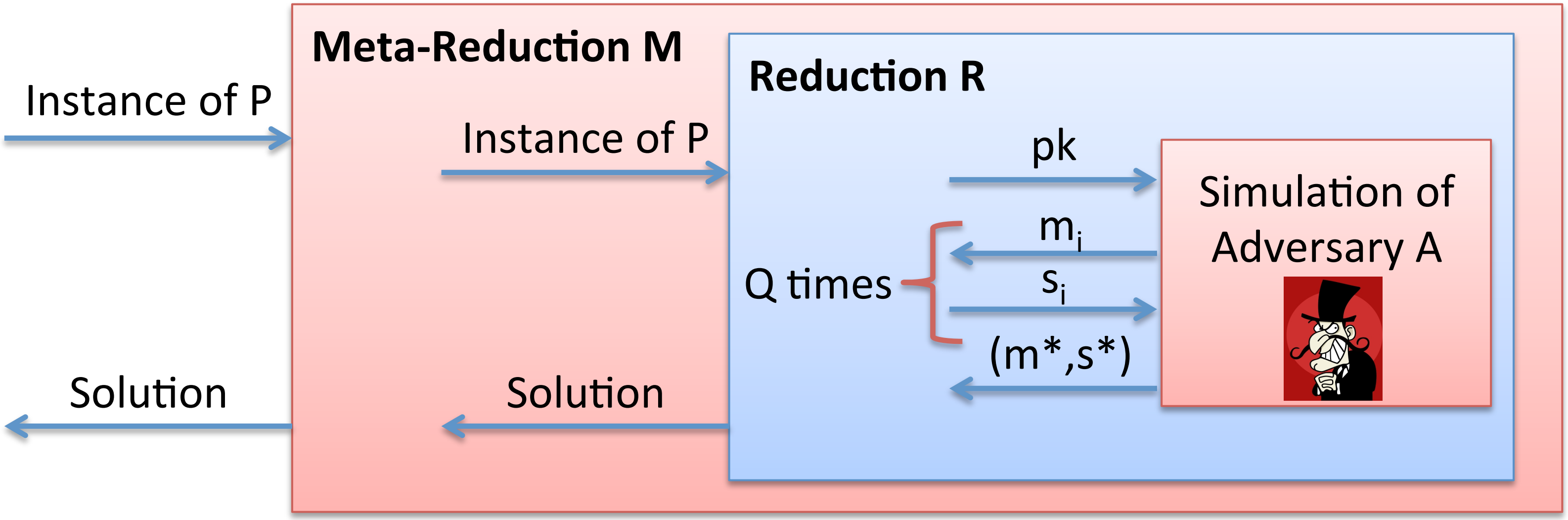

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"Annoying term"

Limitations of Coron‘s Technique

- **Restricted but reasonable** class of reductions:
  - Treat adversary A as a **black-box**
  - Few advanced capabilities (e.g. seq. rewinding)
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- Only useful in settings where **Q << |MsgSpace|**
  - Acceptable for [C`02, KK`12, HJK`12]
  - Makes **application to other settings difficult**

Multi-User Security of Signatures

- A receives **N public keys**

$pk_1, \ldots, pk_N$

(m^*, s^*)

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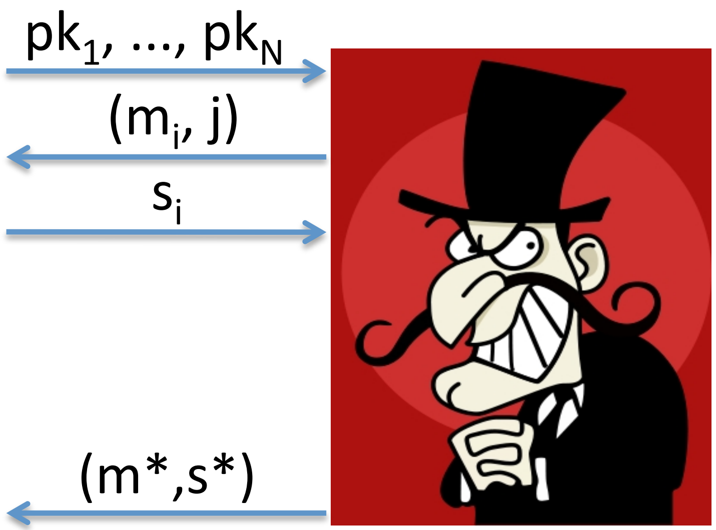

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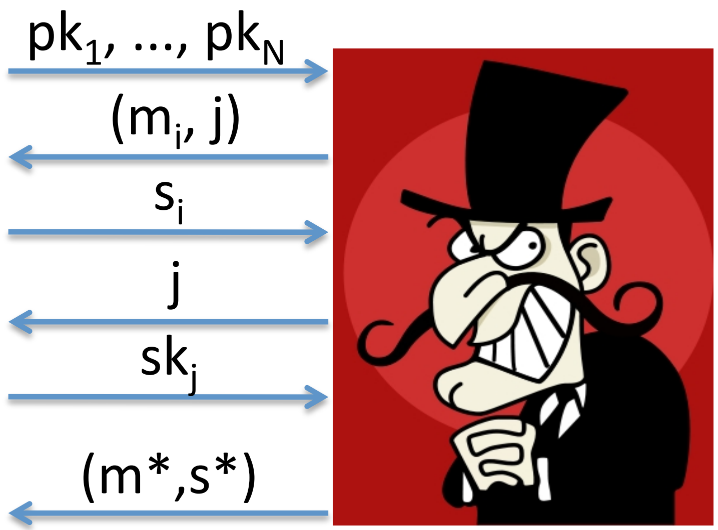

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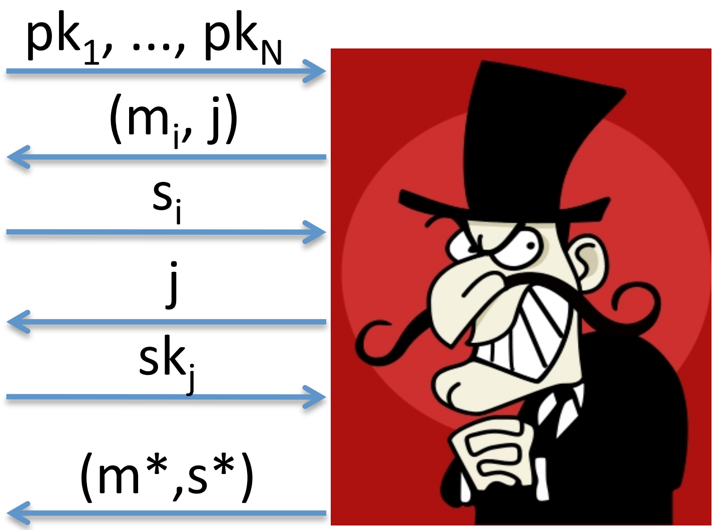

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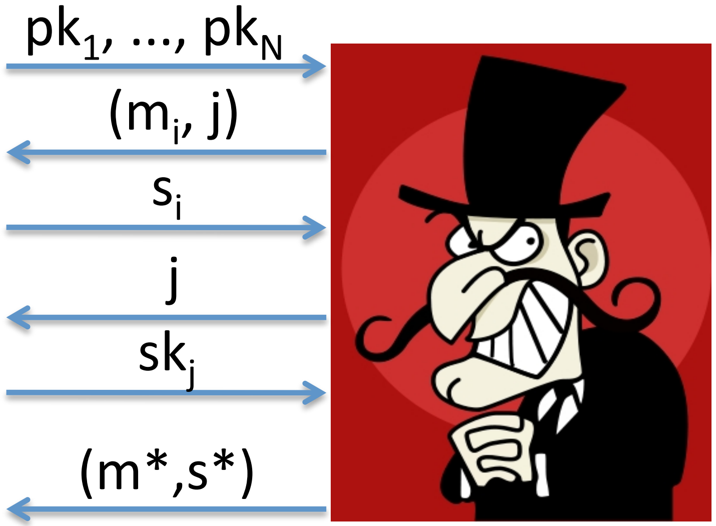

Single-user security ⇒ multi-user security

But the reduction is **not tight**, loses a factor 1/N

Applying Coron's technique to the multi-user setting

- To show that this loss is impossible to avoid:

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Trivial bound, because of the "annoying term"

Our approach

<u>Goal:</u> Prove that 1/N-loss is impossible to avoid

1. Define a *weaker* security definition
   – **Counterintuitive:** Should be **more difficult** to prove impossibility of tight reductions!
2. New meta-reduction technique
   – No "annoying term"
   – Weakness of security definitions enables **simple and clean** analysis
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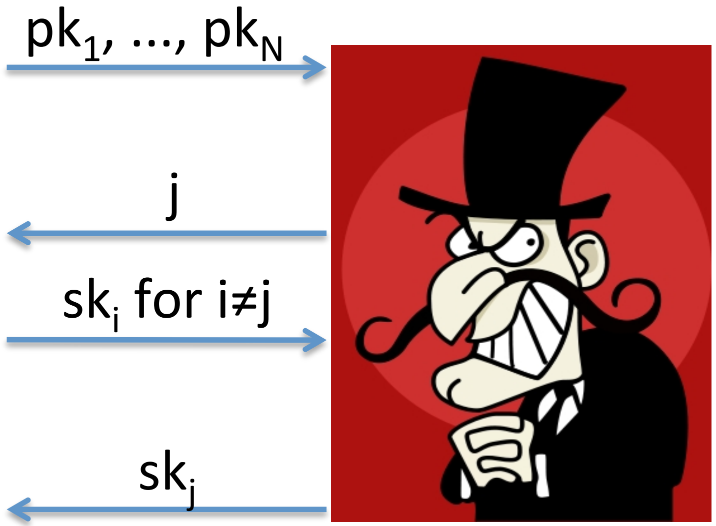

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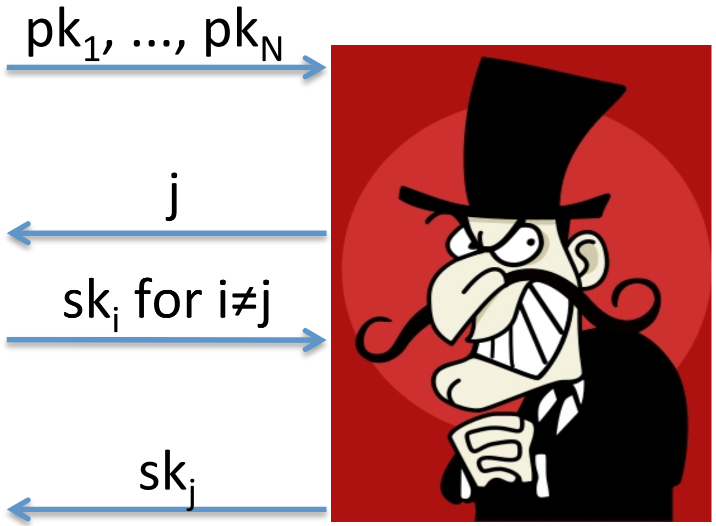

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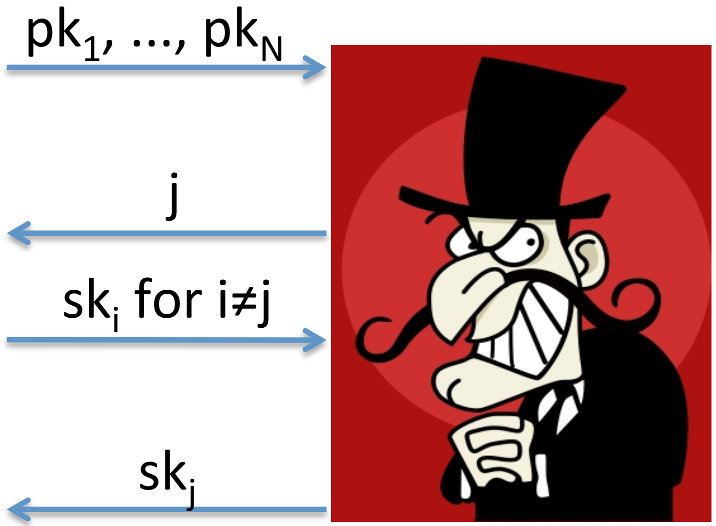

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Makes sense for any public-key scheme!

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Our result

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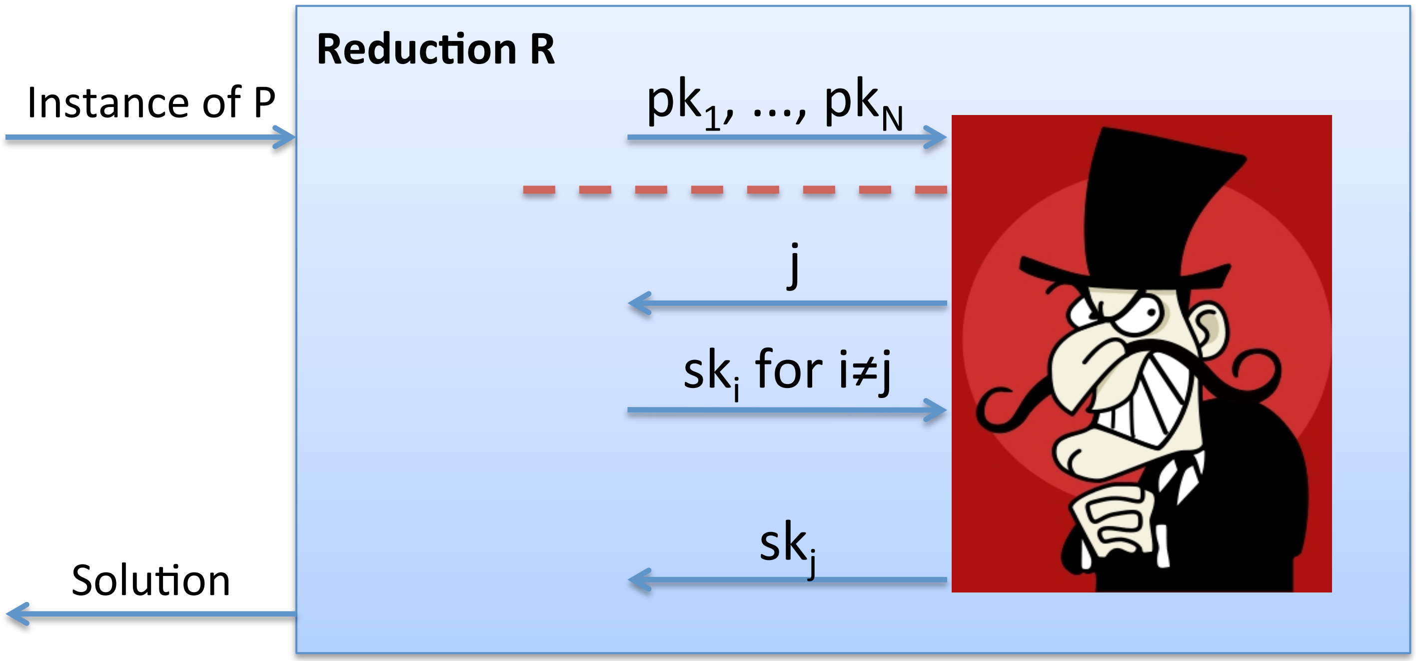
Tightness Bound: Intuition



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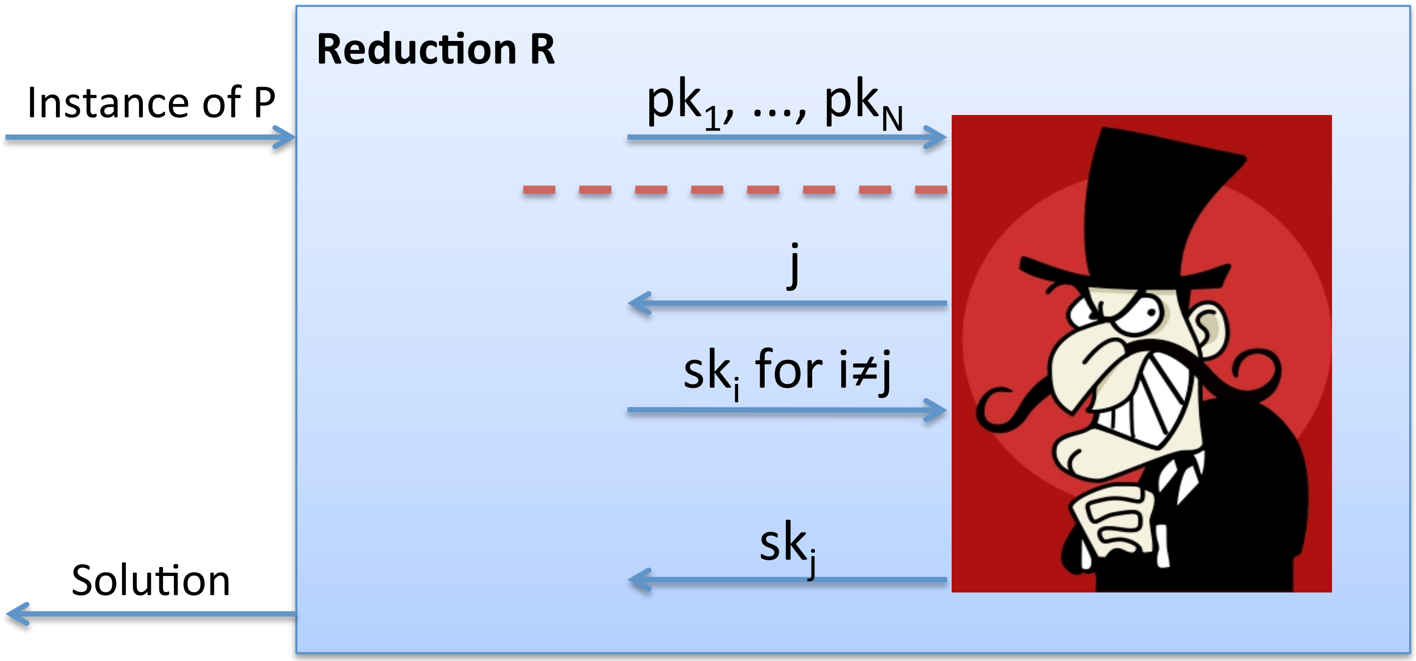

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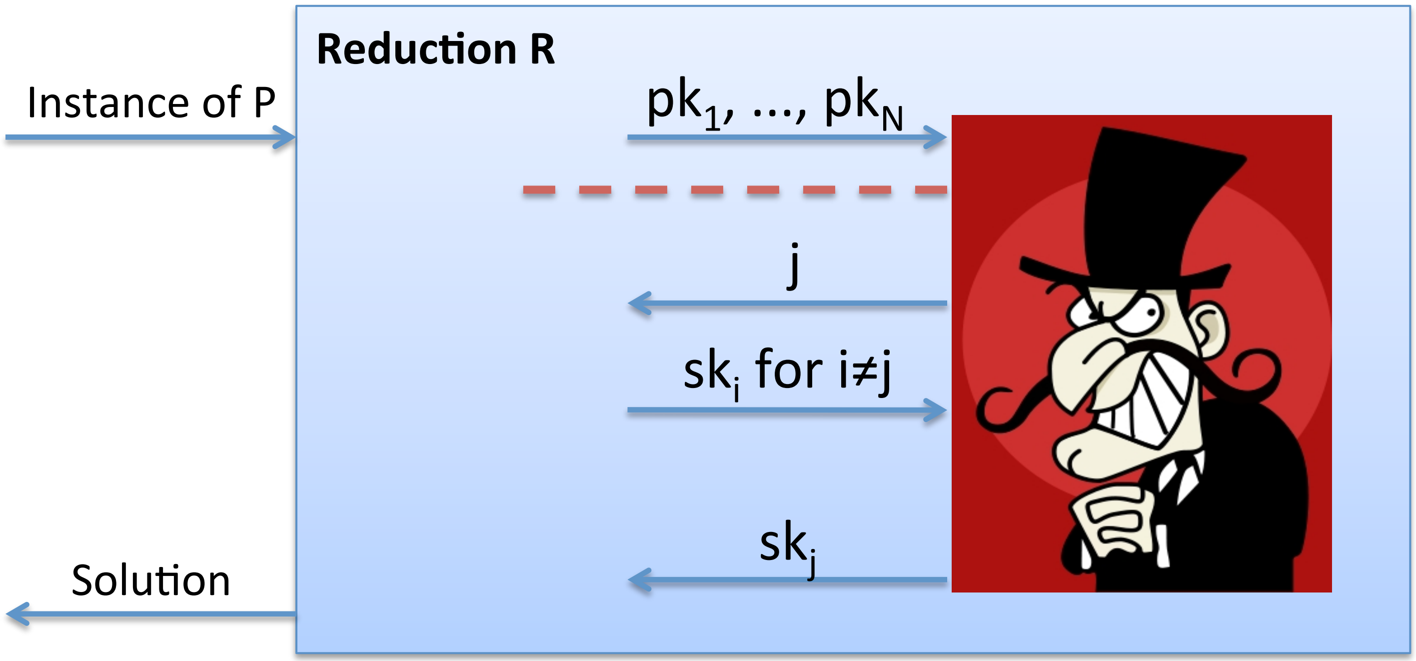

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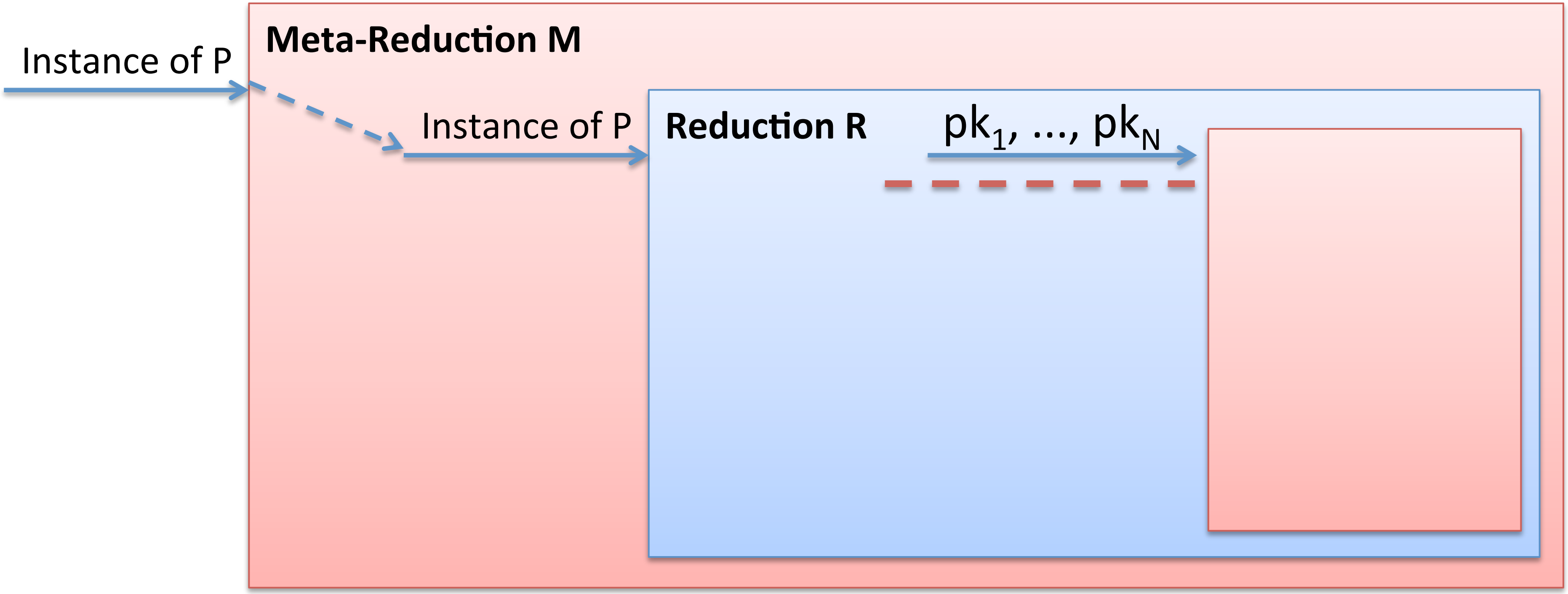

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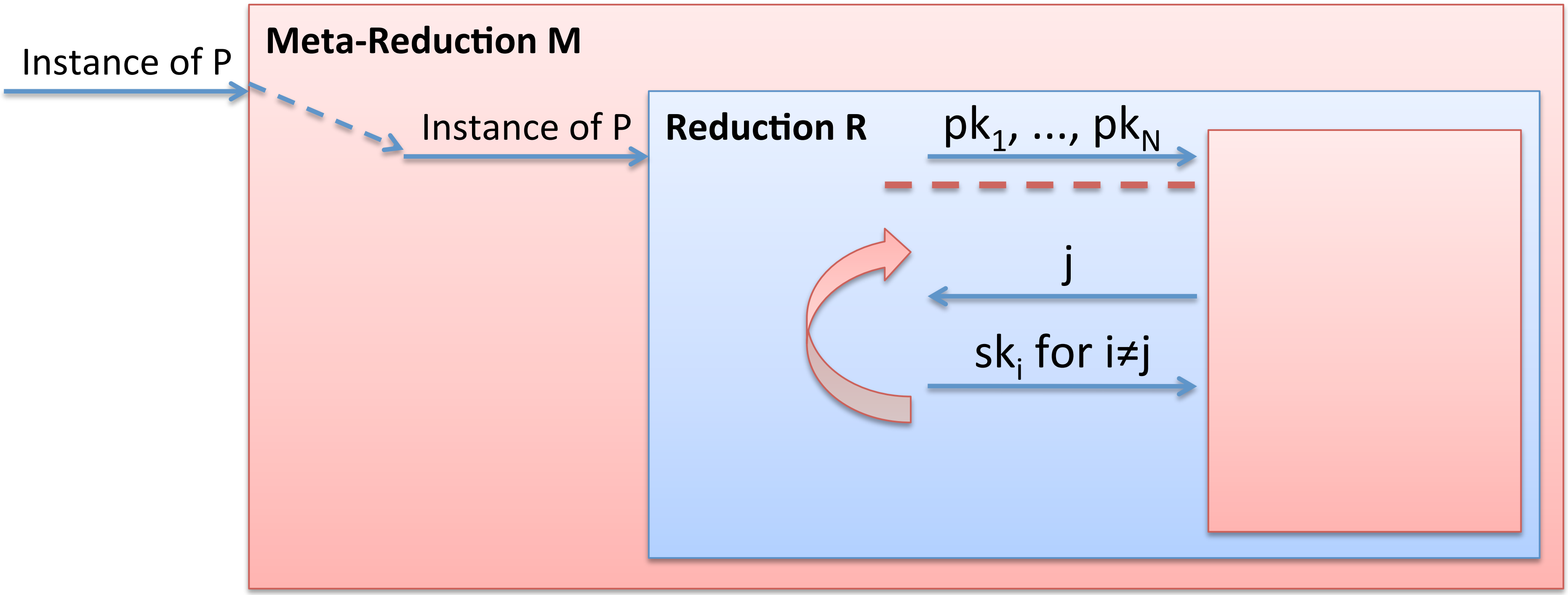

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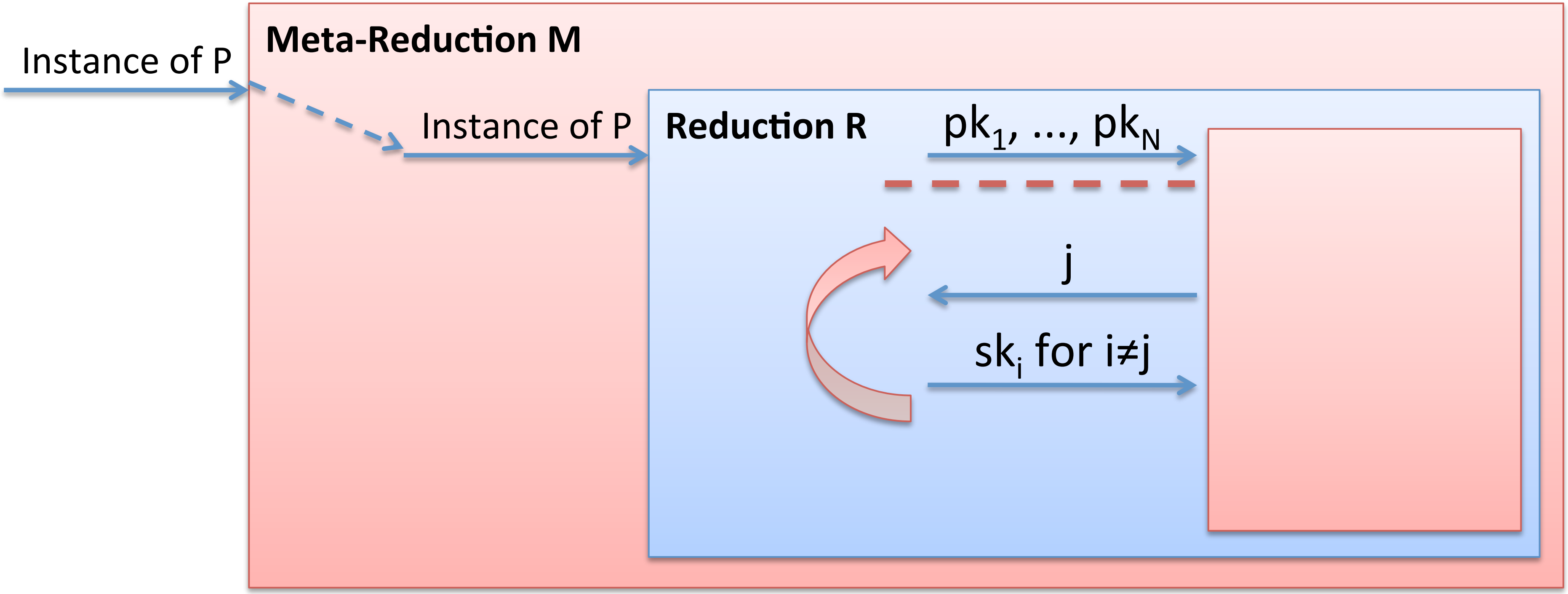

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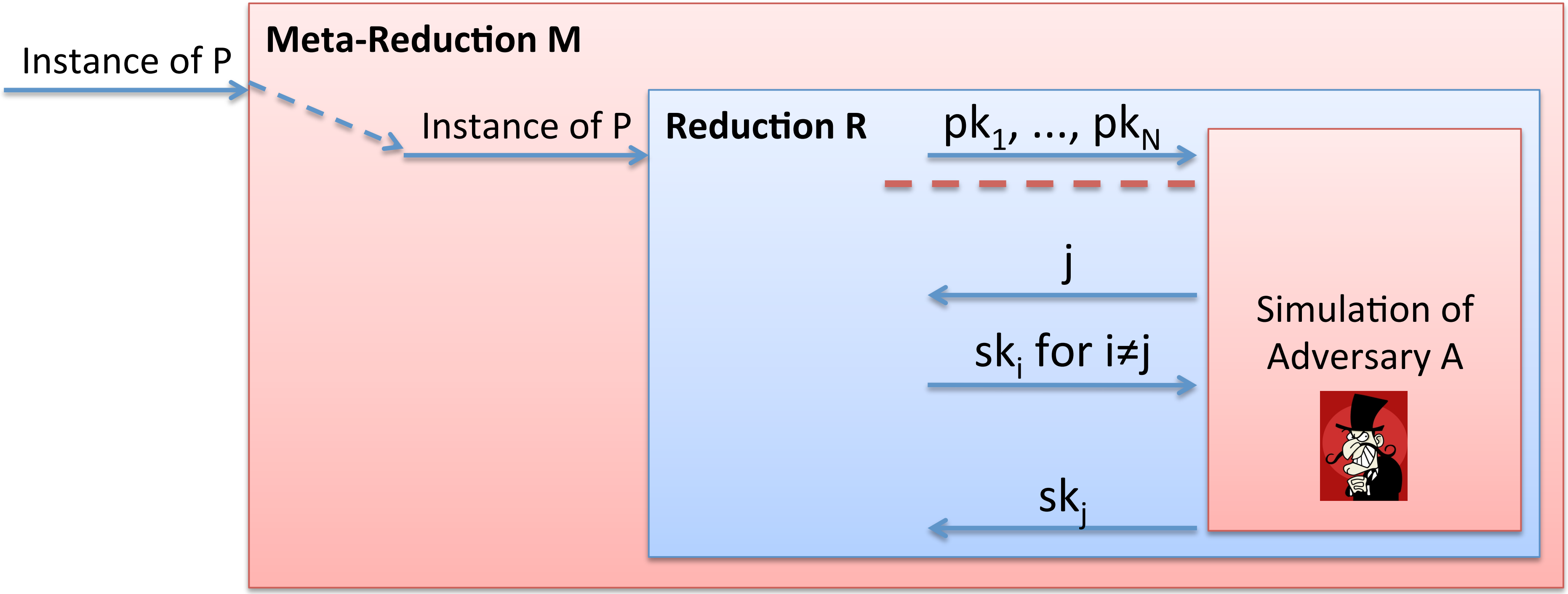
⇒ M learns **all** secret keys

Tightness Bound: Proof Sketch (2/2)



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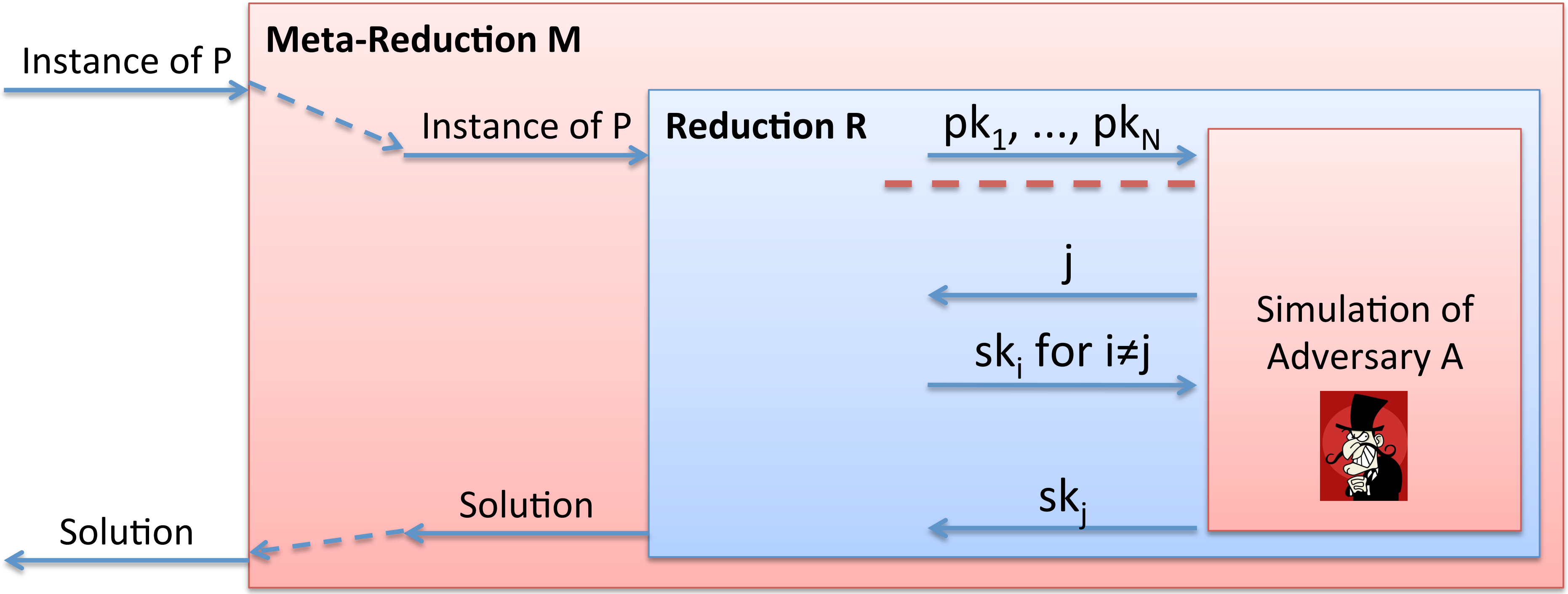

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Perfect simulation of a successful adversary

Requirements on the public-key scheme

- For each pk there is only one **unique** sk (*)
- One can **efficiently verify** that a given sk belongs to a given pk
- Holds for **many** known constructions

(* In the paper: generalized to re-randomizable keys)

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Result:
A public-key scheme that satisfies the above conditions **cannot have a tight security proof** in the multi-user setting with corruptions.

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Goal: easy applicability

Generalized experiment

“Master theorem”

Multi-user security with corruptions

Strengthened versions of [C02, KK12, HJK12]

Security of non-interactive key exchange

Special cases

Generalization to Abstract Relations

$S = \{(x_1, w_1), (x_2, w_2), \ldots, (x_N, w_N)\}$

$x_1, \ldots, x_N$

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w_i for $i \neq j$

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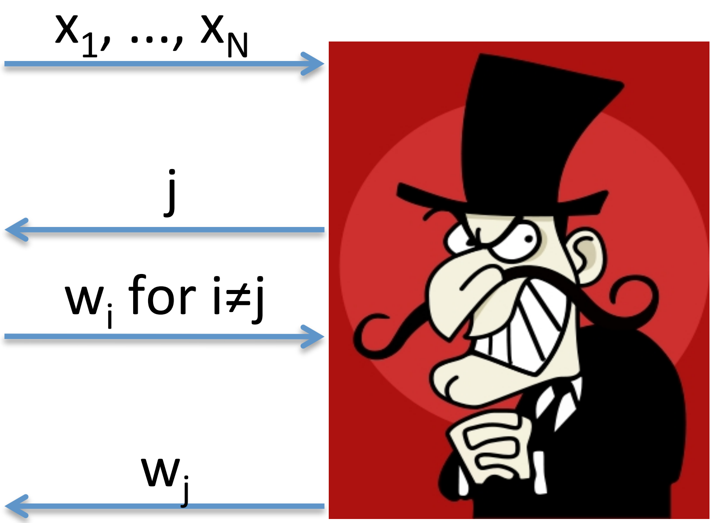

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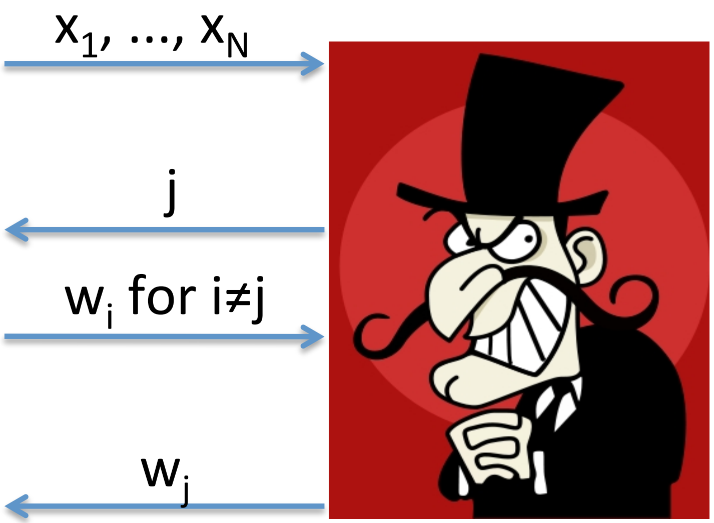

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- Signatures in the single-user setting:
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Summary

New techniques to prove **inexistence of tight reductions**

- **Stronger results** but **simpler proof**
- More **applications**
- **Easy to check** whether a construction can have a tight security proof
- **Easy to adapt** to other applications via generic "master theorem"

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