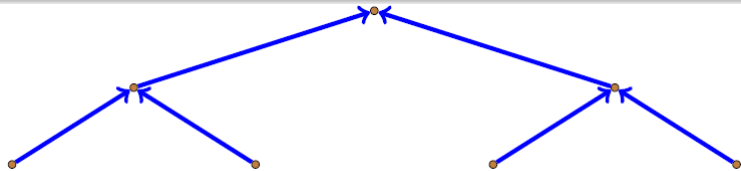


Zero-Knowledge Arguments for Lattice-Based Accumulators: Logarithmic-Size Ring Signatures and Group Signatures without Trapdoors



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EUROCRYPT 2016 - Vienna, Austria

- 1 Introduction
- 2 Our Accumulator and Its Supporting Zero-Knowledge Argument
- 3 Applications to Ring and Group Signatures

Cryptographic Accumulators

Accumulator [BdM'93]: a function hashing a large data set $R = \{d_0, \dots, d_{N-1}\}$ into a constant-size value u .

- For any $d \in R$, there is a short witness w that d was accumulated into u .
- It is infeasible to compute a valid witness w^* for some $d^* \notin R$.
- Numerous applications in authentication mechanisms.
- In many scenarios, a ZK proof of an input-witness pair (d, w) is desirable.

- 2 main families of number-theoretic accumulators: based on groups of hidden order, or on pairings (strong RSA and strong DH assumptions).
- A 3rd family relies on Merkle trees: hardly compatible with ZK proofs.
 - Known methods require non-standard assumptions in groups of hidden order [BCG'14] or non-falsifiable knowledge assumptions [BSCG+'14].
 - [PSTY'13]: SIS-based Merkle tree; supporting ZK proofs were not considered.

Our Results

First lattice-based accumulator supported by logarithmic-size ZK arguments.

- We build Merkle trees from a family of SIS-based CRHF

$$\mathcal{H} : D \times D \rightarrow D.$$

- We demonstrate in ZK the possession of a Merkle tree path (hash chain).

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- 1 First lattice-based logarithmic-size ring signature.

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Applications:

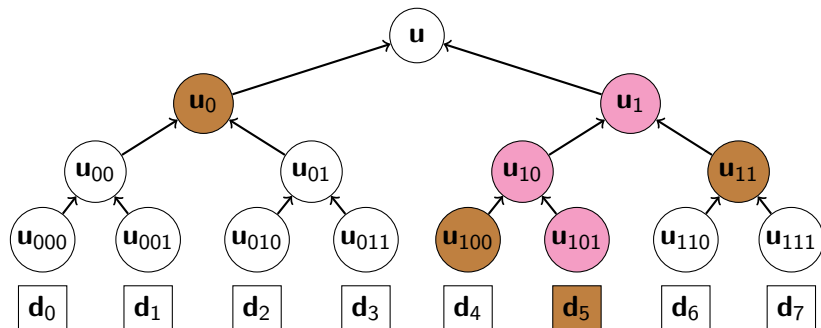
- 1 First lattice-based logarithmic-size ring signature.
- 2 First group signature without lattice trapdoors. Previous constructions [GKV'10,CNR'12,LLLS'13,LNW'15,NZZ'15] rely on trapdoors for key generation and/or for enabling tracing.

Being trapdoor-less: smaller parameters, shorter key and signature sizes.

User's signing key in our scheme has size of several KBs, compared with ≈ 90 GBs in [NZZ'15].

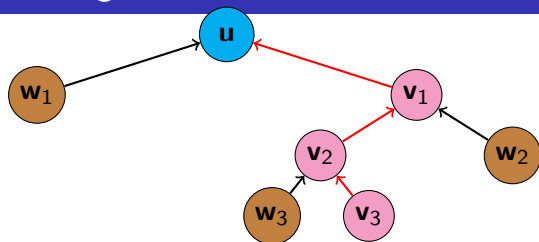
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From CRHF to Merkle-tree-style Accumulators



- A Merkle tree with $2^3 = 8$ leaves, which accumulates the data blocks $\mathbf{d}_0, \dots, \mathbf{d}_7$ into the value \mathbf{u} at the root.
- The value at each non-leaf node is the hash of its two children.
- The **brown** nodes together with the bit string $(j_3, j_2, j_1) = (1, 0, 1)$ form a witness to the fact that \mathbf{d}_5 is accumulated into \mathbf{u} .

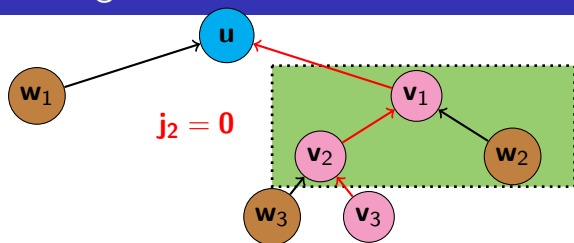
Proving Knowledge of an Accumulated Value



- **Public input:** $\mathbf{A}; \mathbf{u} = \mathbf{v}_0$.
Secret input: $(\mathbf{w}_\ell, \dots, \mathbf{w}_1), (\mathbf{v}_\ell, \dots, \mathbf{v}_1), (j_\ell, \dots, j_1)$.
- **Prover's goal:** Proving that

$$\forall i \in \{\ell - 1, \dots, 1, 0\} : \mathbf{v}_i = \begin{cases} h_{\mathbf{A}}(\mathbf{v}_{i+1}, \mathbf{w}_{i+1}), & \text{if } j_{i+1} = 0; \\ h_{\mathbf{A}}(\mathbf{w}_{i+1}, \mathbf{v}_{i+1}), & \text{if } j_{i+1} = 1. \end{cases}$$

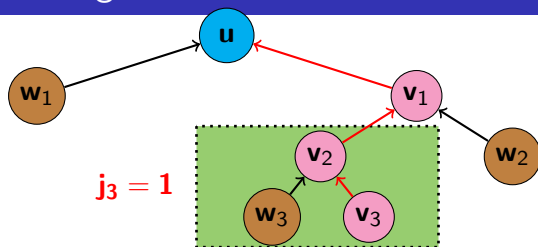
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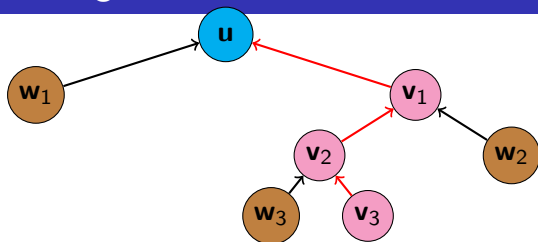
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✗ Previous protocols for SIS-based hash functions ([Lyu'08,09,12], [LNSW'13]) only prove knowledge of a hidden preimage for a given image.

? Here, we essentially need to prove knowledge of
“ ℓ hidden preimage-image pairs nested along a hidden path.”

Transformations

For any bit b and binary vector \mathbf{v} , define $\bar{b} = 1 - b$ and $\text{ext}(b, \mathbf{v}) = \begin{pmatrix} \bar{b} \cdot \mathbf{v} \\ b \cdot \mathbf{v} \end{pmatrix}$.

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Observe that

$$\mathbf{v}_i = \begin{cases} h_{\mathbf{A}}(\mathbf{v}_{i+1}, \mathbf{w}_{i+1}), & \text{if } j_{i+1} = 0; \\ h_{\mathbf{A}}(\mathbf{w}_{i+1}, \mathbf{v}_{i+1}), & \text{if } j_{i+1} = 1. \end{cases}$$

is equivalent to:

$$\mathbf{v}_i = \bar{j}_{i+1} \cdot h_{\mathbf{A}}(\mathbf{v}_{i+1}, \mathbf{w}_{i+1}) + j_{i+1} \cdot h_{\mathbf{A}}(\mathbf{w}_{i+1}, \mathbf{v}_{i+1})$$

$$\Leftrightarrow \bar{j}_{i+1} \cdot (\mathbf{A}_0 \cdot \mathbf{v}_{i+1} + \mathbf{A}_1 \cdot \mathbf{w}_{i+1}) + j_{i+1} \cdot (\mathbf{A}_0 \cdot \mathbf{w}_{i+1} + \mathbf{A}_1 \cdot \mathbf{v}_{i+1}) = \mathbf{G} \cdot \mathbf{v}_i \pmod{q}$$

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Developing Stern's Protocol

Now, the task is to prove in ZK the possession of $\{j_i, \mathbf{v}_i, \mathbf{w}_i\}_{i=1}^{\ell}$ s.t.

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Stern's protocol [Stern'96]: Main ideas

Proving in ZK the possession of a binary vector \mathbf{s} with fixed Hamming weight t , s.t. $\mathbf{M} \cdot \mathbf{s} = \mathbf{u} \pmod{q}$, for given (\mathbf{M}, \mathbf{u}) .

- 1 Proving the linear equation: show that $\mathbf{M}(\mathbf{s} + \mathbf{r}) = \mathbf{u} + \mathbf{M} \cdot \mathbf{r} \pmod{q}$, for random \mathbf{r} .
- 2 Proving the constraint of \mathbf{s} : show that $\pi(\mathbf{s})$ has weight t , for random π .

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- ✓ The first idea can be generalized to prove all ℓ linear equations in (1) hold.
- ? We'd like to prove the constraints of

$$\mathbf{v}_i \in \{0, 1\}^{nk}, \quad \mathbf{w}_i \in \{0, 1\}^{nk}, \quad \mathbf{z}_i = \text{ext}(j_i, \mathbf{v}_i) \text{ and } \mathbf{y}_i = \text{ext}(\bar{j}_i, \mathbf{w}_i)$$

using random permutations. How?

Extensions and Permutations

Proving in ZK that $\mathbf{v}_i, \mathbf{w}_i \in \{0, 1\}^{nk}$

- 1 Extend to $\mathbf{v}_i^*, \mathbf{w}_i^* \in B_m^{nk}$, res., where $B_m^{nk} := \{\mathbf{x} \in \{0, 1\}^m : \text{wt}(\mathbf{x}) = nk\}$.
- 2 Show the verifier that $\pi(\mathbf{v}_i^*), \phi(\mathbf{w}_i^*) \in B_m^{nk}$, where $\pi, \phi \xleftarrow{\$} \mathcal{S}_m$.

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Proving in ZK that $\mathbf{z}_i^* = \text{ext}(j_i, \mathbf{v}_i^*)$ and $\mathbf{y}_i^* = \text{ext}(\bar{j}_i, \mathbf{w}_i^*)$

- 1 For $b \in \{0, 1\}$, for $\pi \in \mathcal{S}_m$, we define the permutation $F_{b,\pi}$ that transforms vector $\mathbf{z} = \begin{pmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \end{pmatrix} \in \mathbb{Z}_q^{2m}$ to vector $F_{b,\pi}(\mathbf{z}) = \begin{pmatrix} \pi(\mathbf{z}_b) \\ \pi(\mathbf{z}_{\bar{b}}) \end{pmatrix}$.

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- 2 For all b, π, ϕ , we have:
$$\mathbf{z}_i^* = \text{ext}(j_i, \mathbf{v}_i^*) \iff F_{b,\pi}(\mathbf{z}_i^*) = \text{ext}(j_i \oplus b, \pi(\mathbf{v}_i^*))$$
$$\mathbf{y}_i^* = \text{ext}(\bar{j}_i, \mathbf{w}_i^*) \iff F_{\bar{b},\phi}(\mathbf{y}_i^*) = \text{ext}(j_i \oplus b, \phi(\mathbf{w}_i^*)).$$
- 3 $j_i \oplus b$ perfectly hides j_i , if b is a random bit.

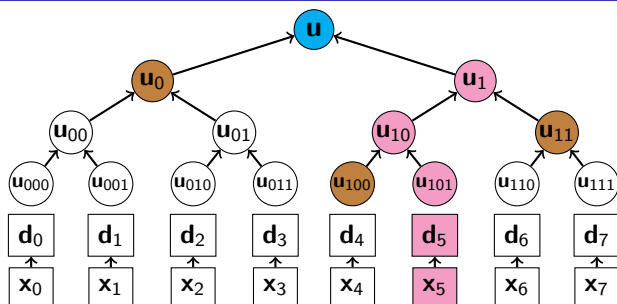
Summary of Our ZK Argument

Putting everything together, in the framework of Stern's protocol, we obtain a ZK argument system for our accumulator.

- When extending the secret vectors, we also extend the public matrices **A**, **G** (by inserting zero-columns) to preserve the equations.
- To prove that the same \mathbf{v}_i is “nested” in 2 equations, we use the same permutation at both places.
- Each round has communication cost $\tilde{O}(\ell \cdot n) = \tilde{O}(\log N \cdot n)$.
- Each round has soundness error $2/3$, which can be made negligible by repeating $\kappa = \omega(\log n)$ times in parallel.

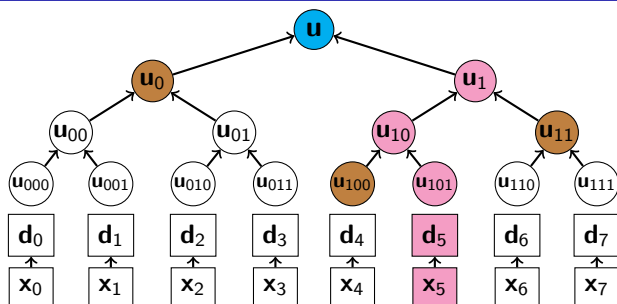
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From ZK-for-Accumulator to Ring Signatures



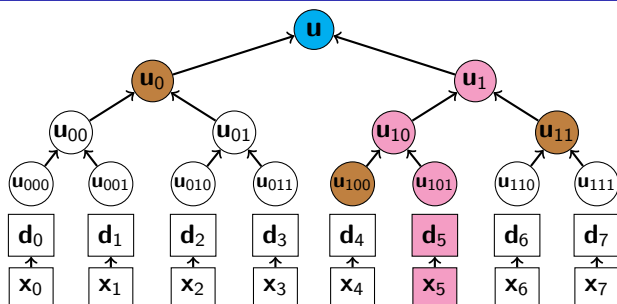
- One more hashing layer is added: Each user picks $sk = x \xleftarrow{\$} \{0, 1\}^m$, and outputs $pk = d = \text{bin}(\mathbf{A} \cdot x \text{ mod } q) \in \{0, 1\}^{nk}$.

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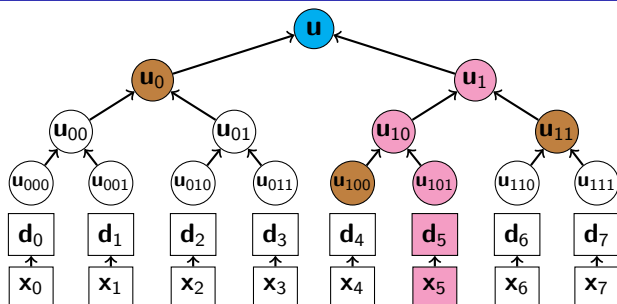
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- Signing w.r.t. a ring $R = (pk_0, \dots, pk_{N-1})$ using $sk = x$ s.t. $pk \in R$:
 - 1 Accumulate R into u .
 - 2 Extend the ZK-argument-for-accumulator to additionally prove knowledge of x s.t. the value at the secret leaf is $\text{bin}(\mathbf{A} \cdot x \bmod q)$.
 - 3 The argument is transformed into a signature via Fiat-Shamir.

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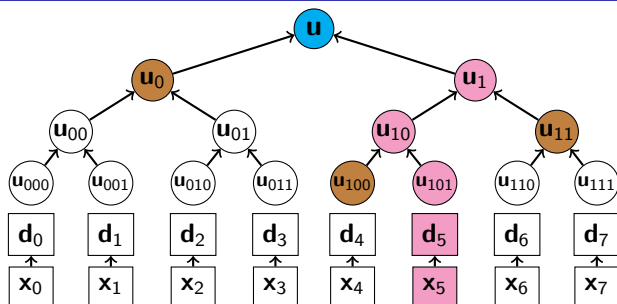
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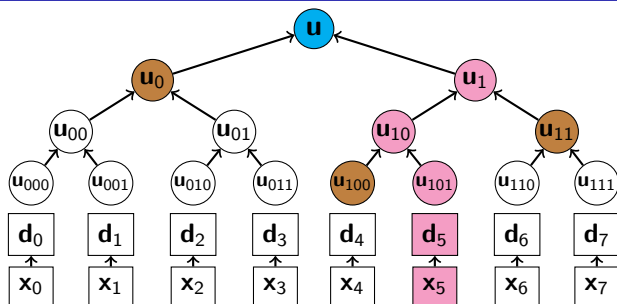
- Fix $N = 2^\ell$. The manager samples $\mathbf{x}_0, \dots, \mathbf{x}_{N-1}$, computes $\mathbf{d}_0, \dots, \mathbf{d}_{N-1}$ and the accumulator \mathbf{u} . The sk of user j is \mathbf{x}_j and the witness for \mathbf{d}_j .

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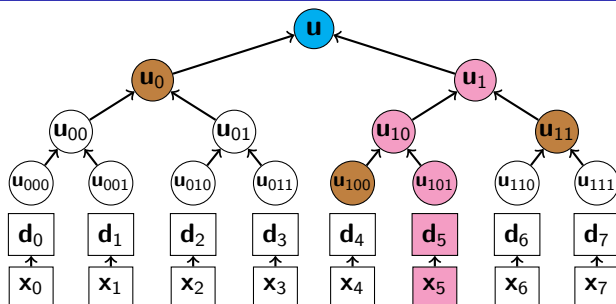
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- To be trapdoor-less: Use the Naor-Yung double-encryption paradigm [NY'90] with the multi-bit version of Regev's LWE-based encryption [Reg'05].

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- To be trapdoor-less: Use the Naor-Yung double-encryption paradigm [NY'90] with the multi-bit version of Regev's LWE-based encryption [Reg'05].
- The argument system for the ring signature is extended to additionally prove that the two ciphertexts correspond to the same plaintext (j_1, \dots, j_ℓ) .

We propose:

- A Merkle-tree-style lattice-based accumulator, supported by short zero-knowledge argument.
- The first lattice-based RS with logarithmic-size signatures.
- The first lattice-based GS without trapdoors.
Also the first logarithmic-size GS in the [BMW'03] model that does not use a full-fledged digital signature for generating group members' private keys.

Thank you!