Constrained Pseudorandom Functions for Unconstrained Inputs

Apoorvaa Deshpande (Brown University)
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Pseudorandom Functions

(Goldreich-Goldwasser-Micali 84)

Pseudorandom Functions

(Goldreich-Goldwasser-Micali 84)

Keyed Function F

Key space $\mathcal{K}$

Numerous applications in Cryptography

Constrained PRFs

(Boneh-Waters, Boyle-Goldwasser-Ivan, Kiayias-Papadopoulos-Triandopoulos-Zacharias)

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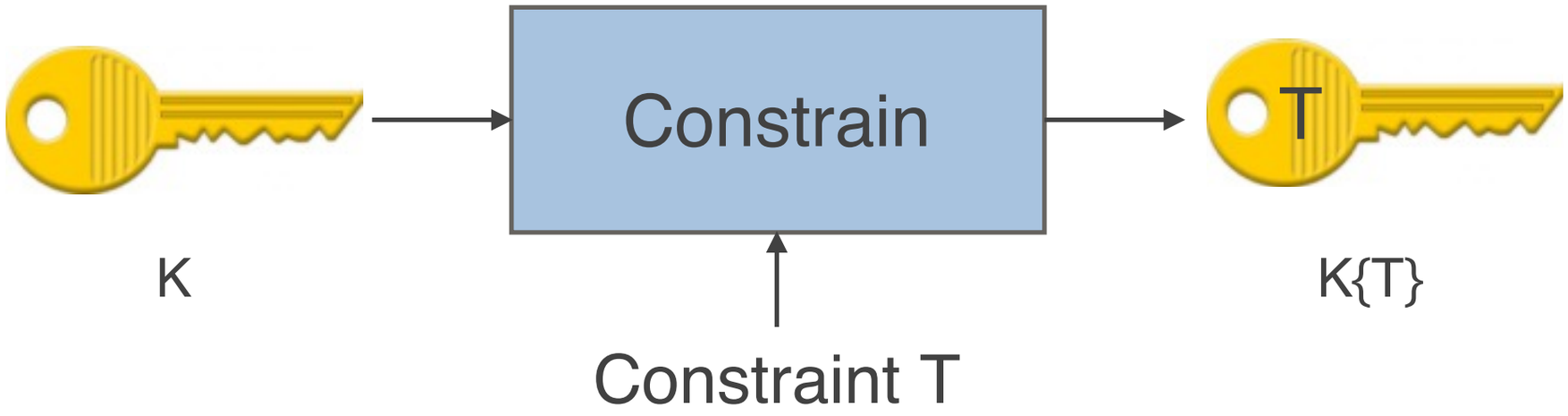

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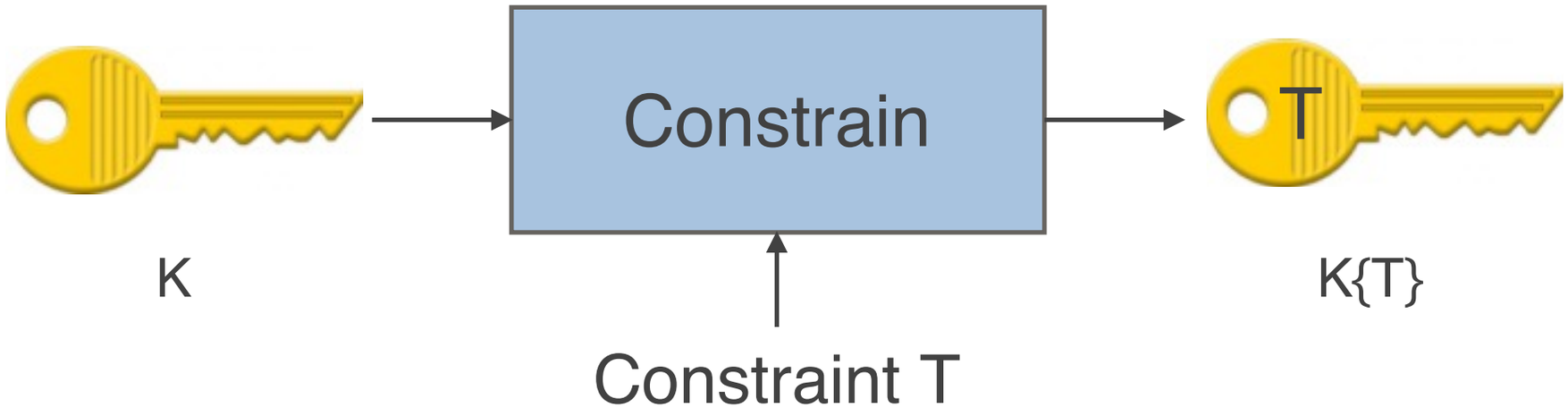

For all x s.t. x satisfies T, F(K , x) = F(K{T} , x)

Constrained PRFs

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Families of Constraints:

Constrained PRFs

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Families of Constraints:

- Puncturable PRFs
  - Key can evaluate PRF at all points except ‘punctured point’
  - Goldreich-Goldwasswer-Micali 84 PRFs are puncturable PRFs
  - Punctured programming approach (Sahai-Waters 14)

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  - Goldreich-Goldwasswer-Micali 84 PRFs are puncturable PRFs
  - Punctured programming approach (Sahai-Waters 14)
- Bit Fixing PRFs
  - Key for a string *s* in $\{0, 1, \perp\}^n$: can evaluate PRF at all points fixed by *s*
  - Multilinear maps based construction (Boneh-Waters 13)
  - Optimal broadcast encryption

Constrained PRFs

(Boneh-Waters, Boyle-Goldwasser-Ivan, Kiayias-Papadopoulos-Triandopoulos-Zacharias)

Families of Constraints:

- Circuit Constrained PRFs
  - Key corresponding to circuit C : can evaluate PRF at input x if C(x) = 1
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Circuits can handle only bounded length inputs!

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for bounded number of users

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Constrained PRFs for Unconstrained Inputs

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Turing Machine Constrained PRFs

Abusalah, Fuchsbauer, Pietrzak 14

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Turing Machine Constrained PRFs

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- Identity based Noninteractive Key Exchange : unbounded users
- Broadcast encryption : unbounded users

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Construction based on knowledge-type assumption

Code Obfuscation

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Goal: Make programs maximally unintelligible.

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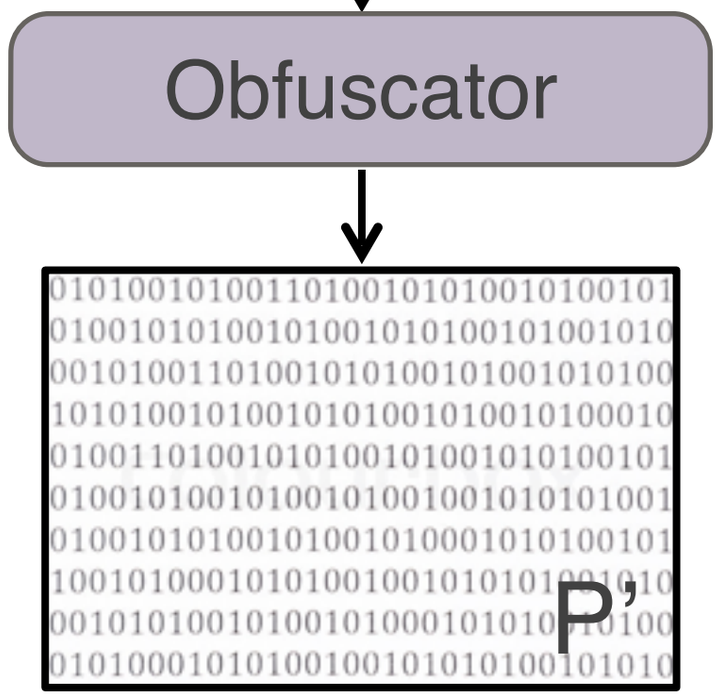

P(x) = P’(x) for all inputs x

Code Obfuscation

Security for obfuscation

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Virtual Black Box obfuscation (VBB)

Code Obfuscation

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Virtual Black Box obfuscation (VBB)

Obfuscated code

≈

Oracle access to code

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Impossibility results

(Barak et al. 2001)

Code Obfuscation

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	Differing inputs obfuscation (diO)	**Virtual Black Box obfuscation (VBB)**
		Obfuscated code ≈ Oracle access to code
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Code Obfuscation

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Differing inputs obfuscation (diO)

If $diO(P_1)$ and $diO(P_2)$ are distinguishable, then one can extract differing input.

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Public coins differing inputs obfuscation (pcdiO)

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Code Obfuscation

Security for obfuscation

Indistinguishability obfuscation (iO)

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Obfuscated code

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Code Obfuscation

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If P_1 and P_2 functionally identical, then $iO(P_1) \approx iO(P_2)$

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Constrained PRFs for Unconstrained Inputs

Turing Machine
Constrained PRFs

Abusalah, Fuchsbauer, Pietrzak 14

- Identity based Noninteractive Key Exchange : unbounded users
- Broadcast encryption : unbounded users

Construction based on
public coins differing inputs obfuscator

Can we build a constrained PRF scheme for Turing machines based on indistinguishability obfuscation (iO)?

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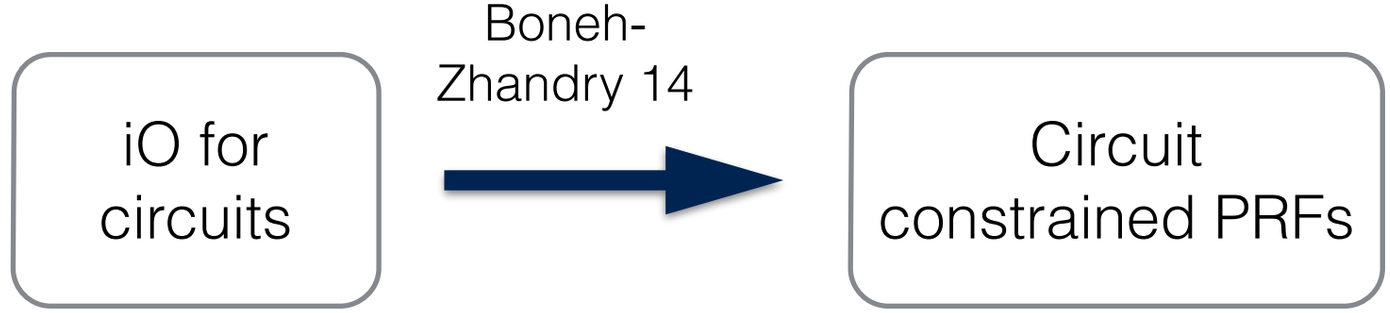

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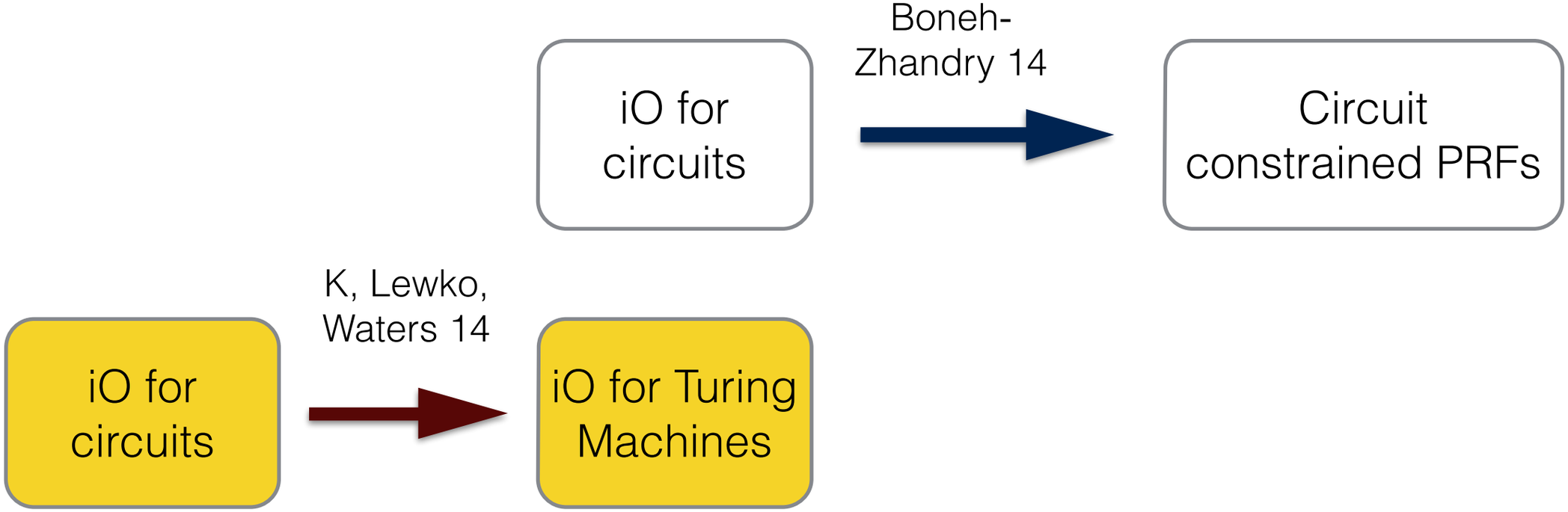

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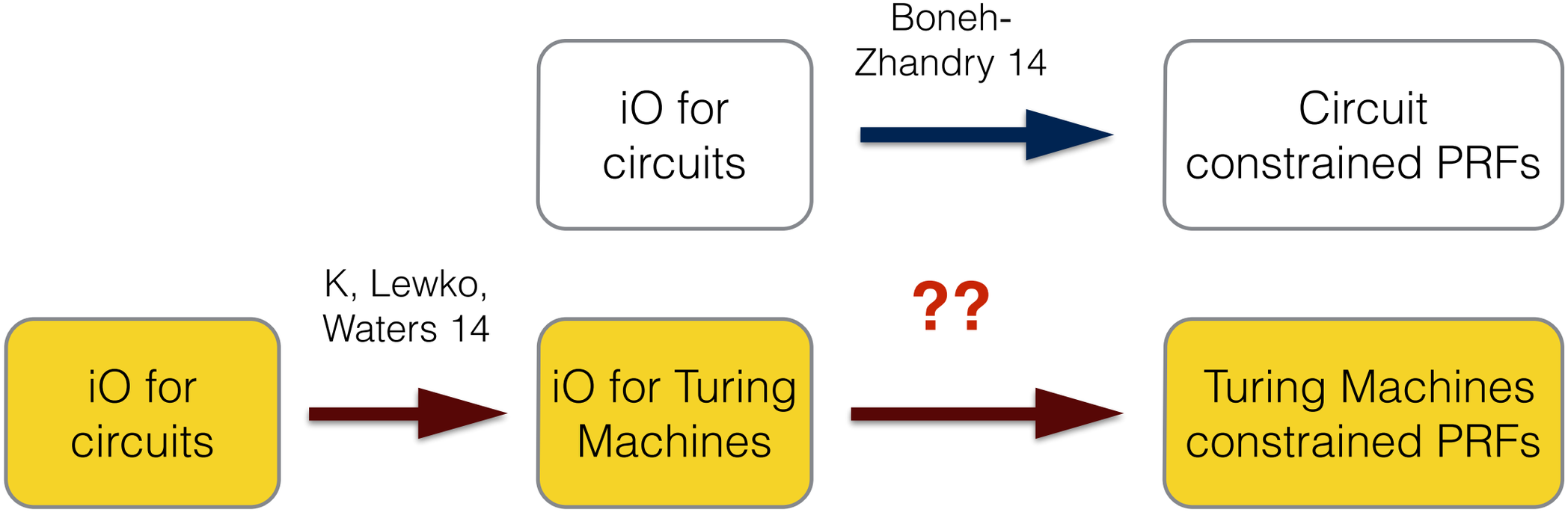

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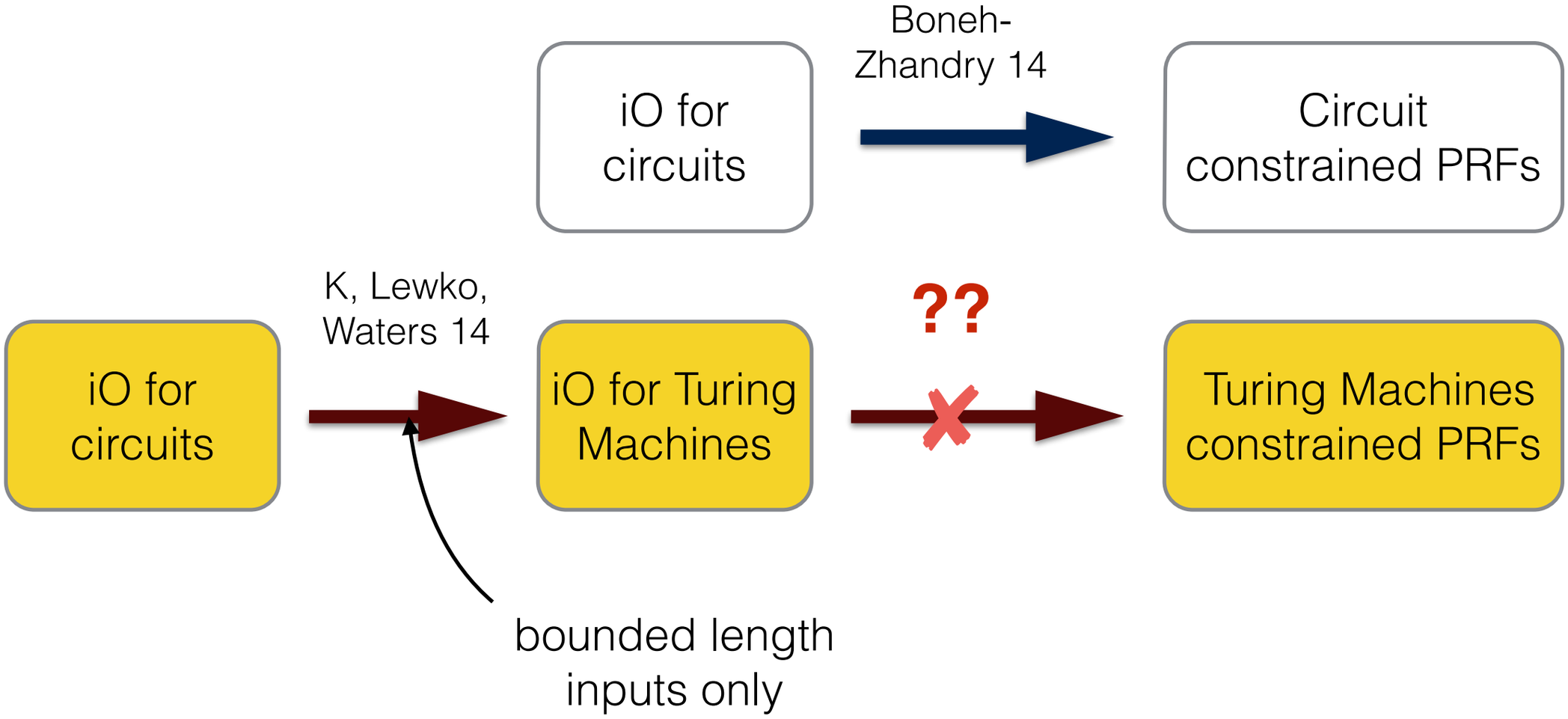

Our Results

Assuming iO (and one way functions), we show a constrained PRF scheme for Turing machines.

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Assuming iO (and one way functions), we show a constrained PRF scheme for Turing machines.

Assuming iO (and one way functions), we show an Attribute Based Encryption scheme for Turing machines.

FREE !!

Security of Constrained PRFs

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Selective Security

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Chooses PRF key K.

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$\leftarrow$ x^*

$y^* = PRF(K, x^*)$ $\rightarrow$
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x^*

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M_i

$K\{M_i\}$

$M_i(x^*) = 0$

Security of Constrained PRFs

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Guess PRF/random

Indistinguishability Obfuscation for Circuits

Indistinguishability Obfuscator

C_0, C_1 functionally identical circuits.

$iO(C_0) \approx iO(C_1)$

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Candidate iO schemes for circuits:

Garg-Gentry-Halevi-Raykova-Sahai-Waters 13

Barak-Garg-Kalai-Paneth-Sahai 14

Zimmerman 14

…

PRFs with Unbounded Inputs

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PRF F with bounded length inputs

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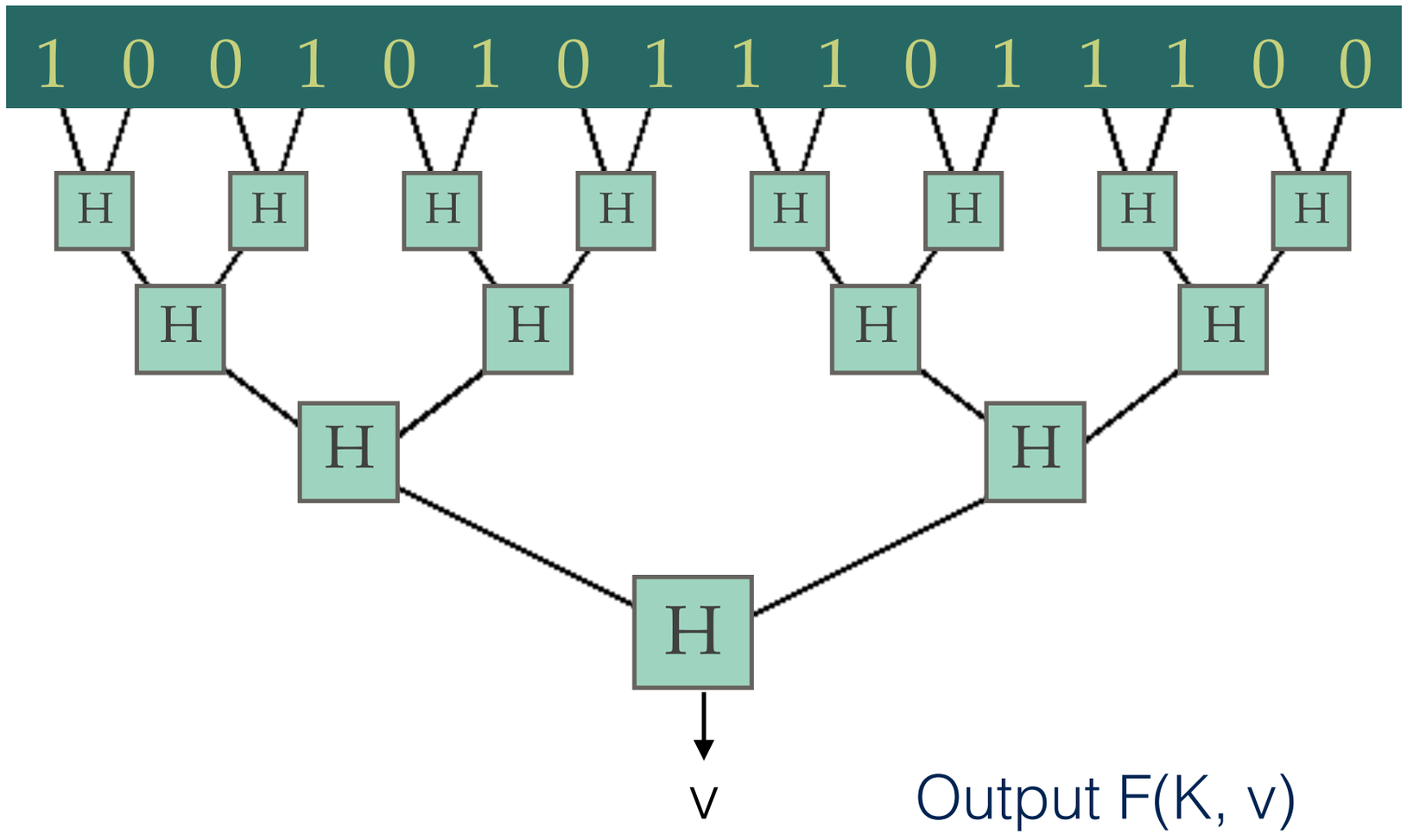
For unbounded inputs : Choose PRF key K, hash function H

PRFs with Unbounded Inputs

PRF F with bounded length inputs

For unbounded inputs : Choose PRF key K, hash function H

Merkle Tree



Output F(K, v)

Our Constrained PRF Construction

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Puncturable PRF F with bounded length inputs

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Choose puncturable PRF key K, special hash function H

Our scheme's PRF key : (K, H)

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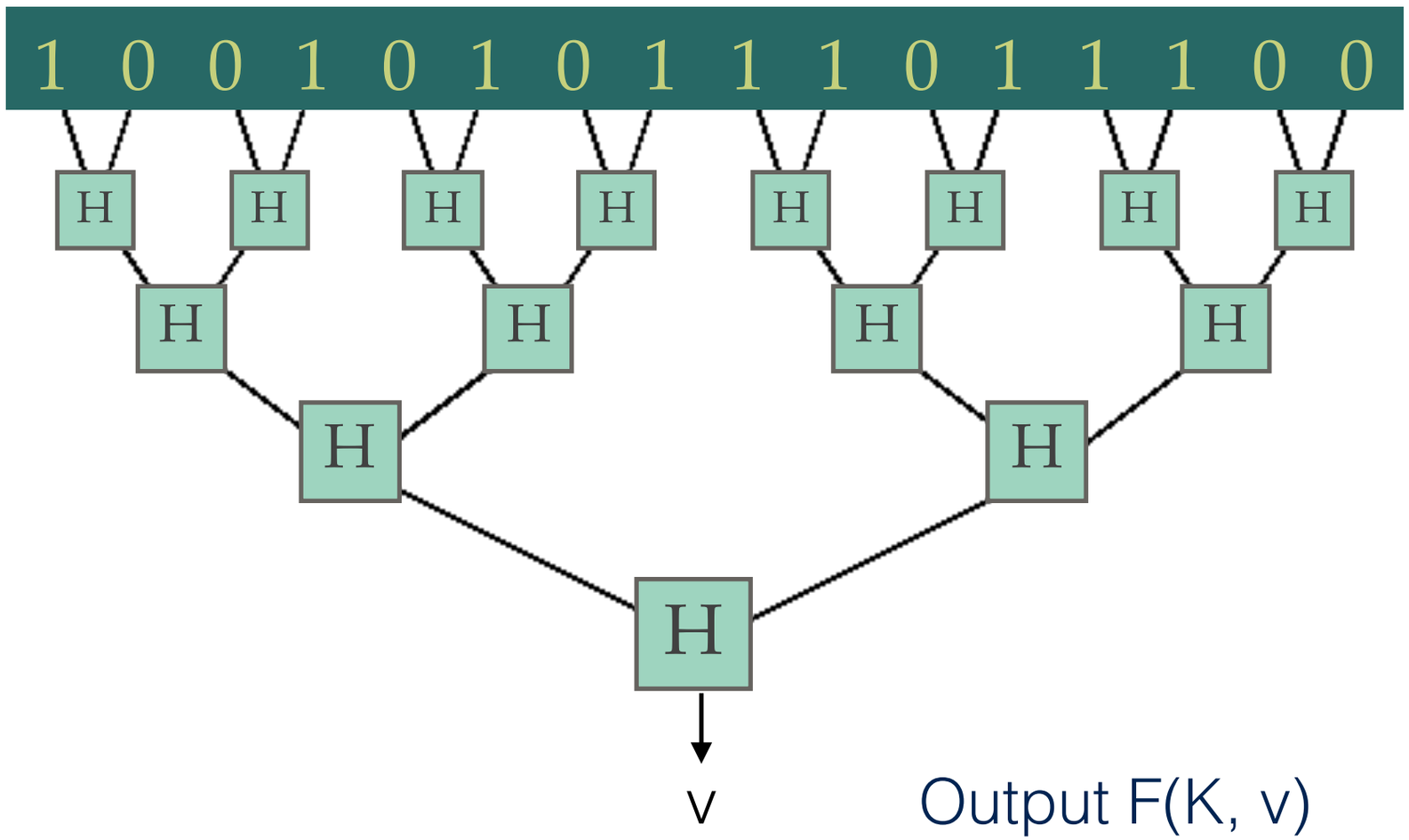

Our Constrained PRF Construction

Constrained key for Turing machine M

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K{M} = H, iO(Prog-Iterate), iO(Prog-Start)

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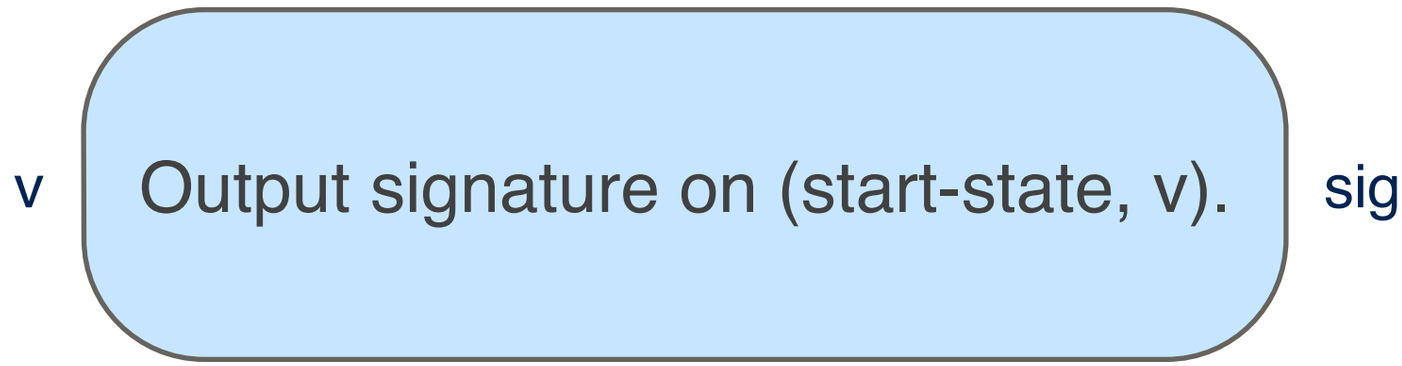
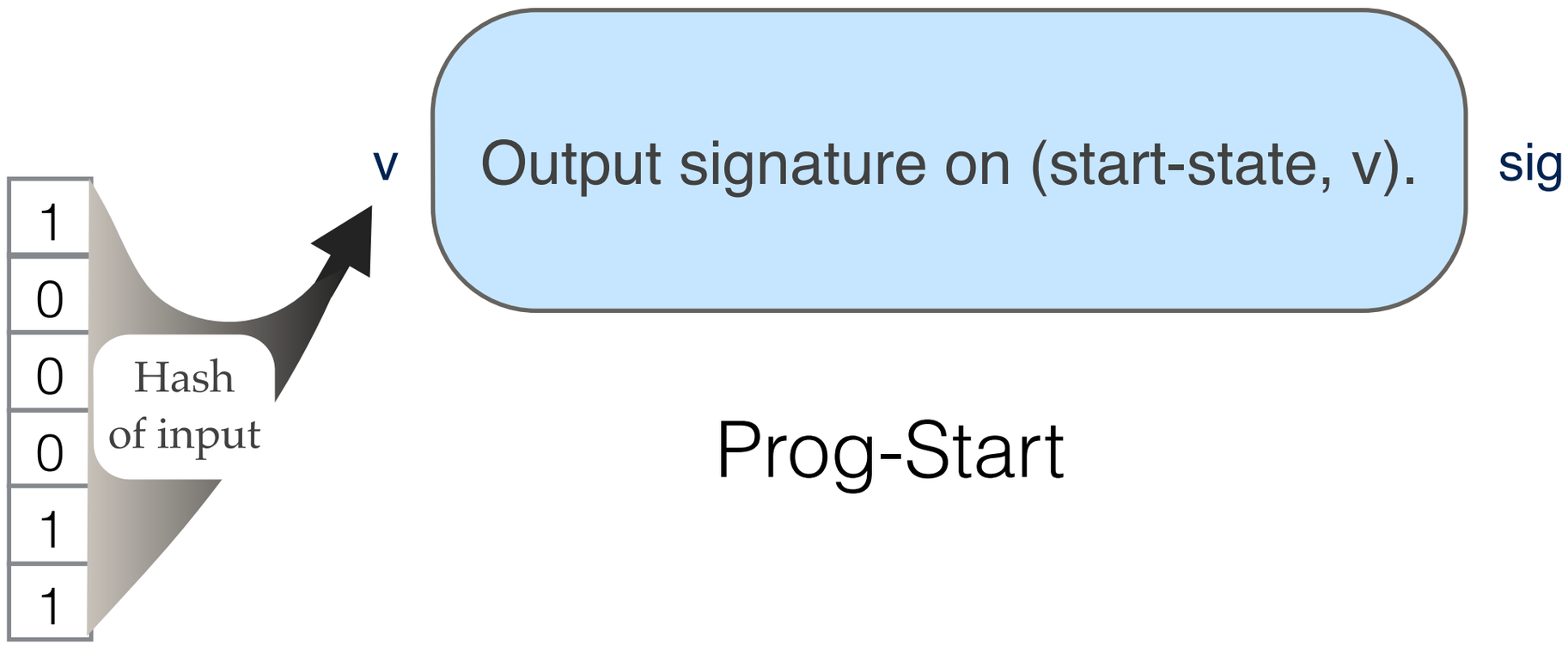


Prog-Start

K{M} = H, iO(Prog-Iterate), iO(Prog-Start)



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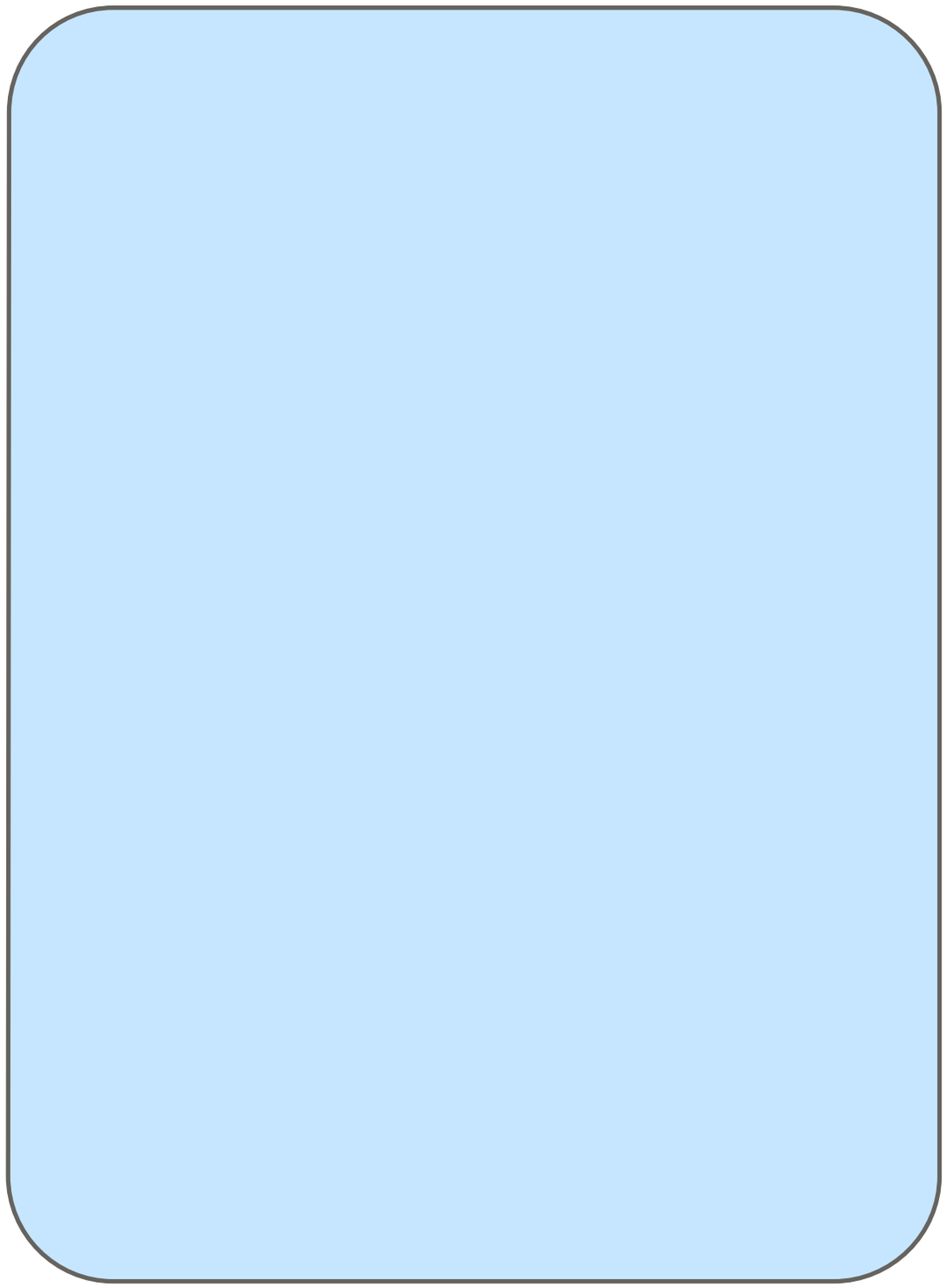

Prog-Iterate

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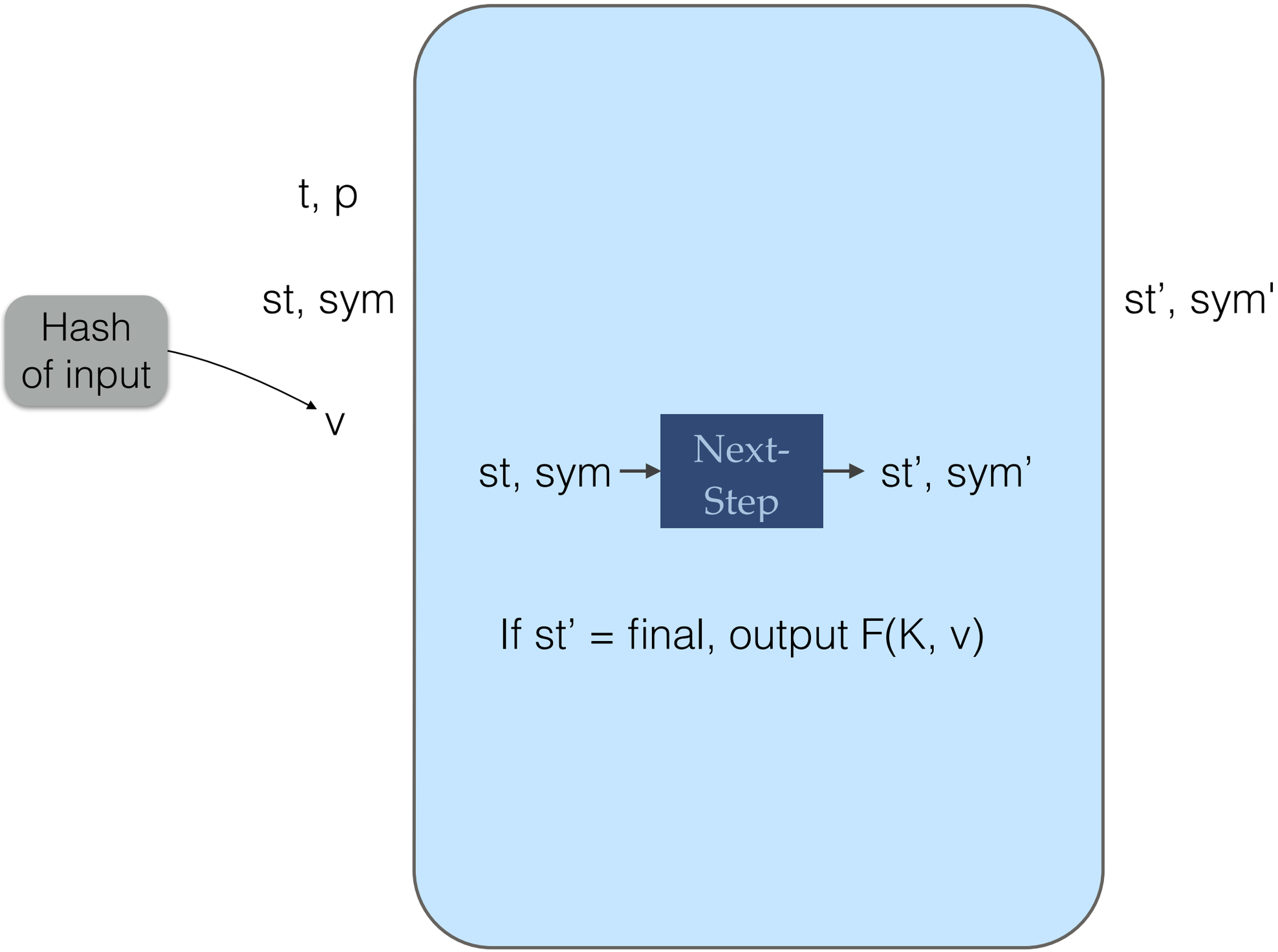

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Adversary not bound to correct execution.

t, p

st, sym

Hash of input

v

st', sym'

st, sym → Next-Step → st', sym'

If st' = final, output F(K, v)

Prog-Iterate

K{M} = H, iO(Prog-Iterate), iO(Prog-Start)

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Hash of work tape

h

Using h, verify sym at position p

st, sym → Next-Step → st', sym'

If st' = final, output F(K, v)

Using sym', update h to h'

st', sym'

h'

Prog-Iterate

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Adversary not bound to correct execution.

t, p

st, sym

Hash of input

v

Hash of work tape

h

sig

Using h, verify sym at position p

Verify sig on (st, h)

st, sym → Next-Step → st', sym'

If st' = final, output F(K, v)

Using sym', update h to h'

Output signature on (st', h')

st', sym'

h'

sig'

Prog-Iterate

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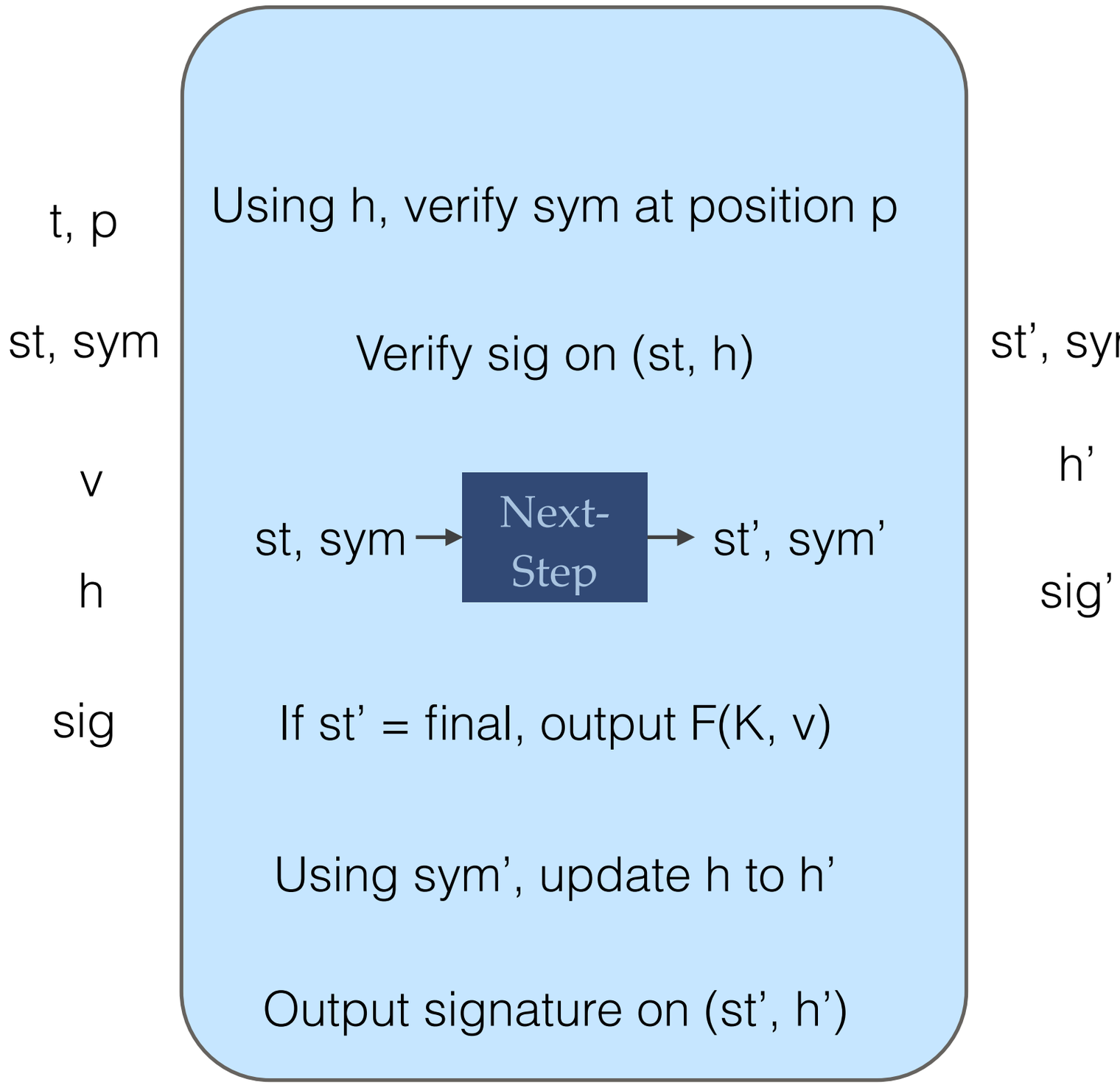

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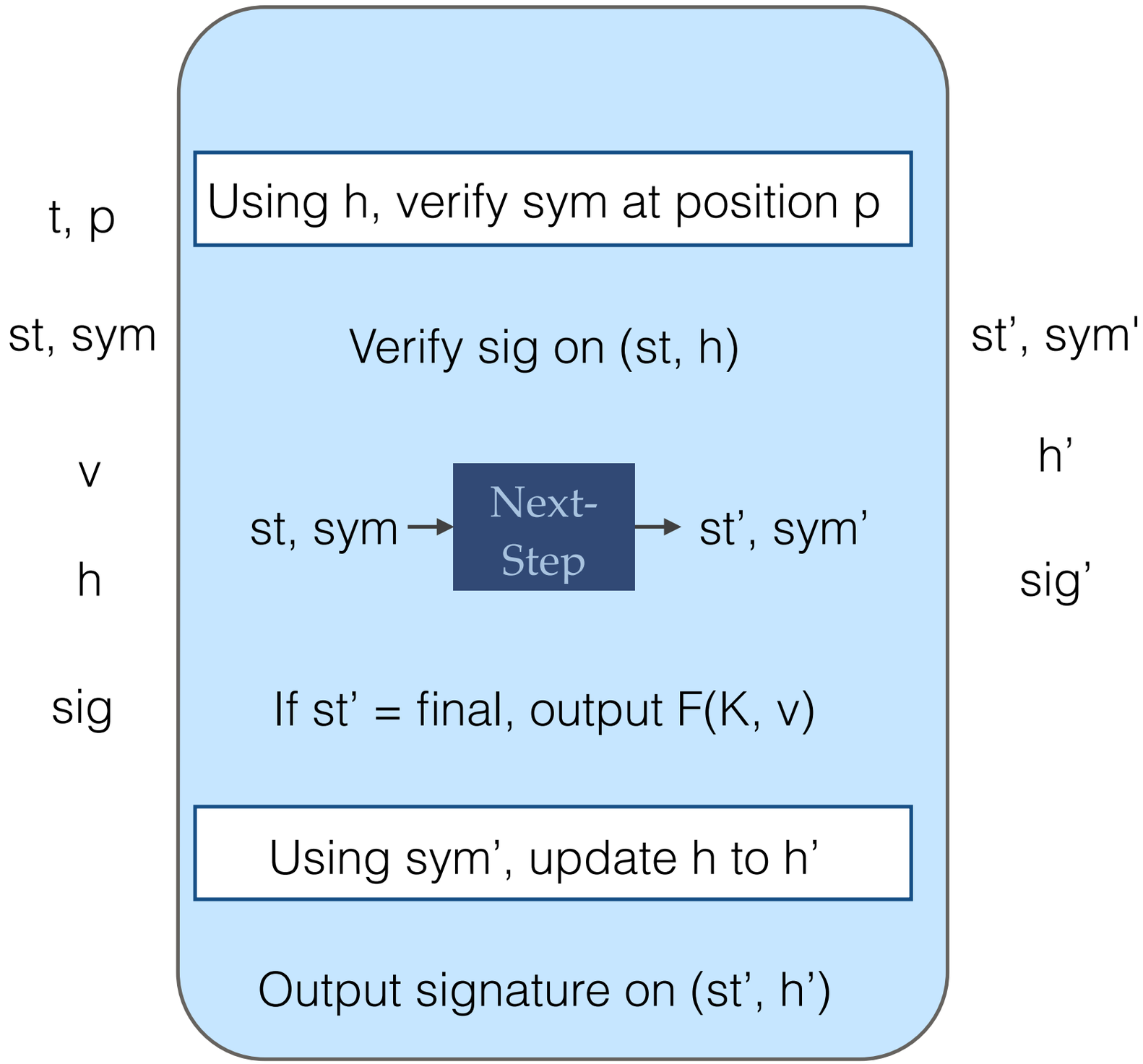
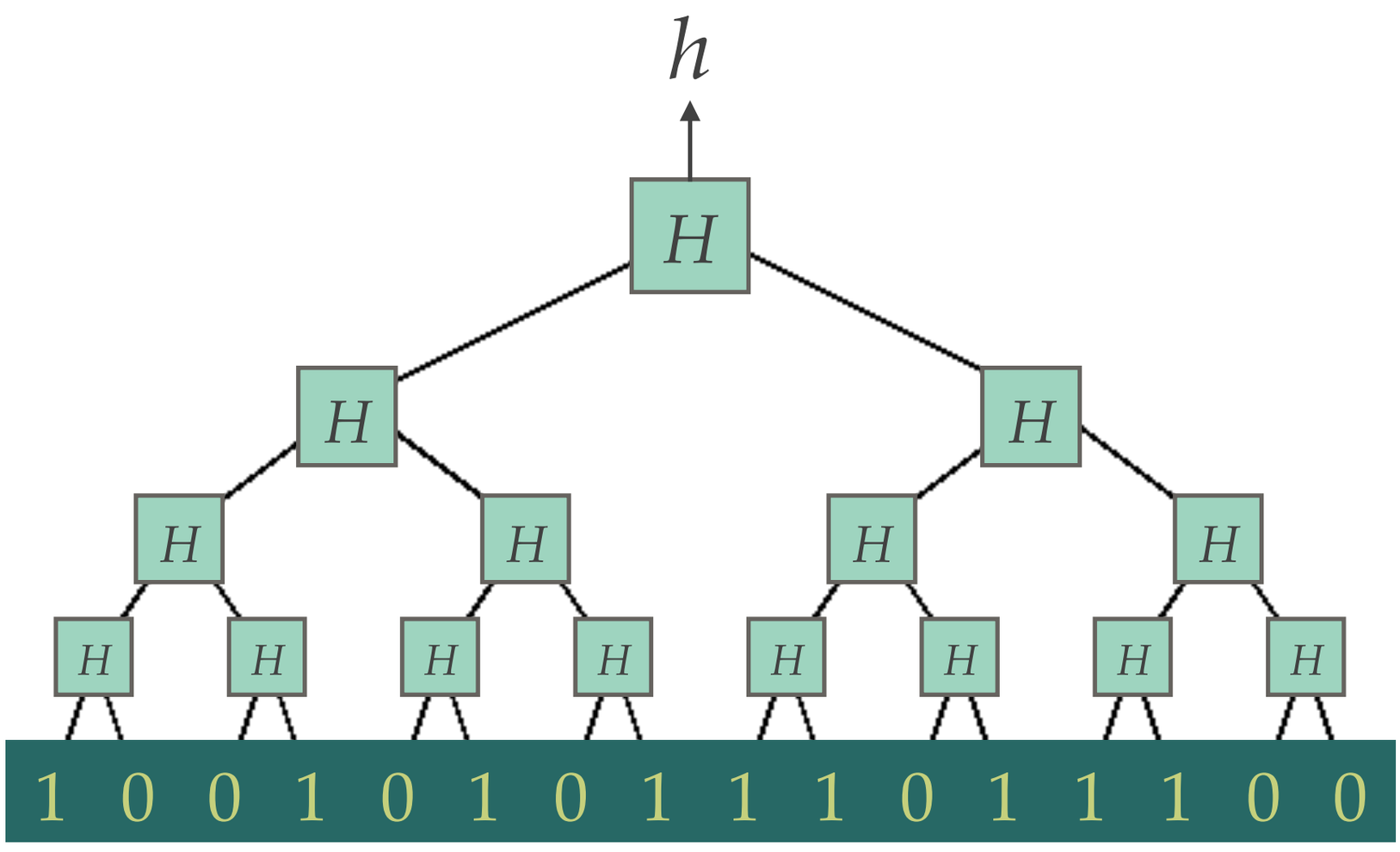


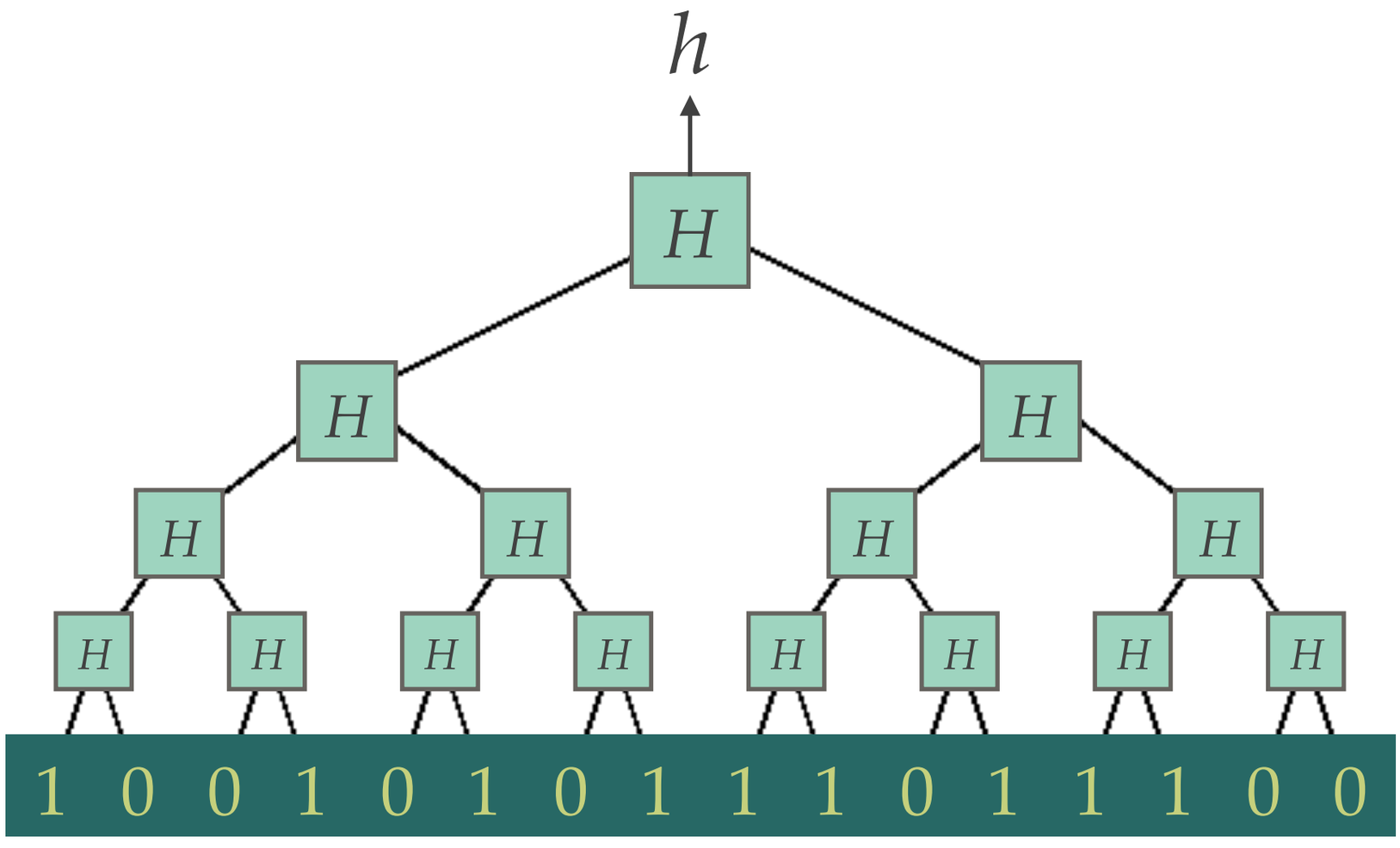
Prog-Iterate

Merkle Trees: Succinct Proof



Merkle Trees: Succinct Proof

Prove m is at position p



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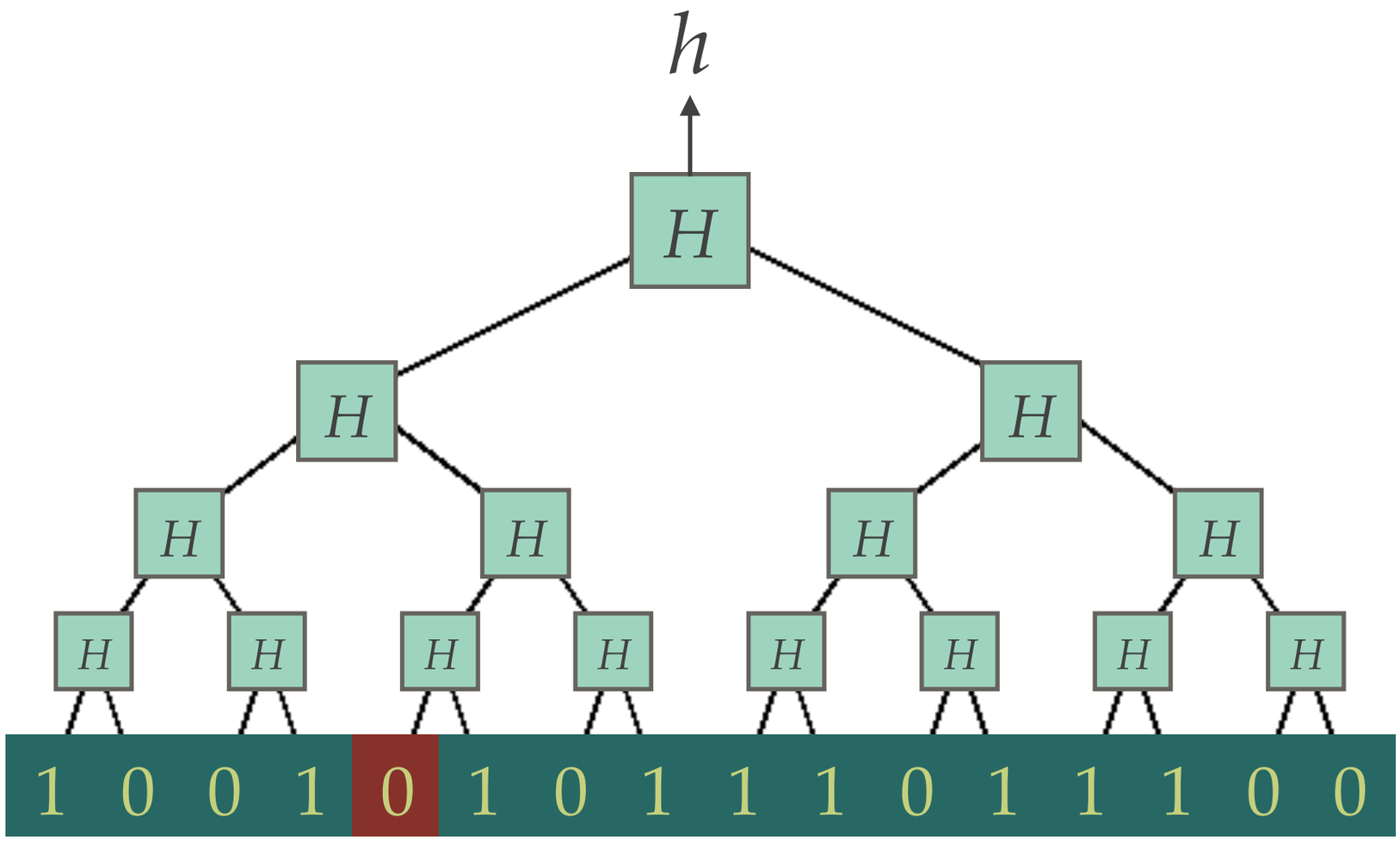

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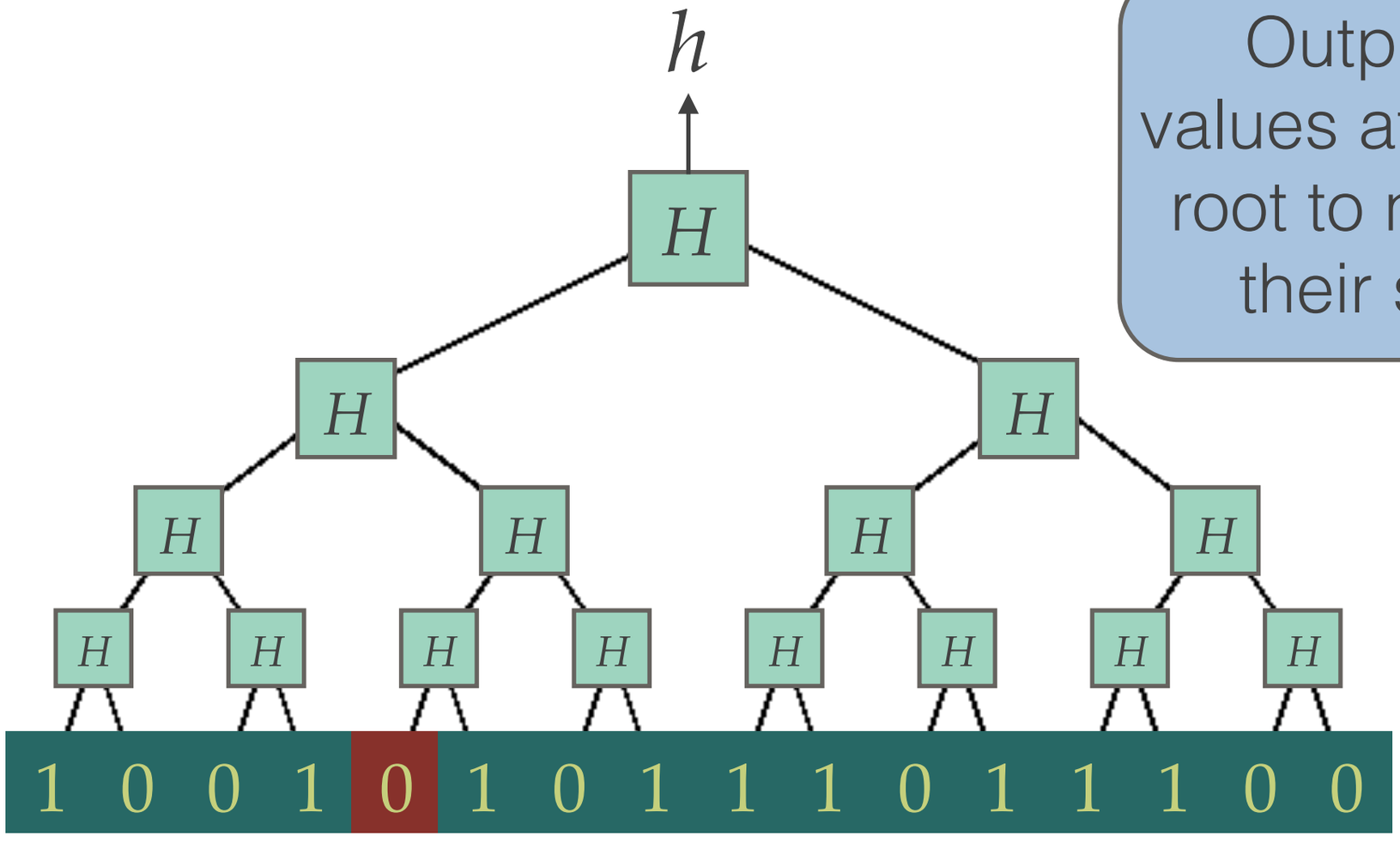

Output hash values at path from root to node, and their siblings

Merkle Trees: Succinct Proof

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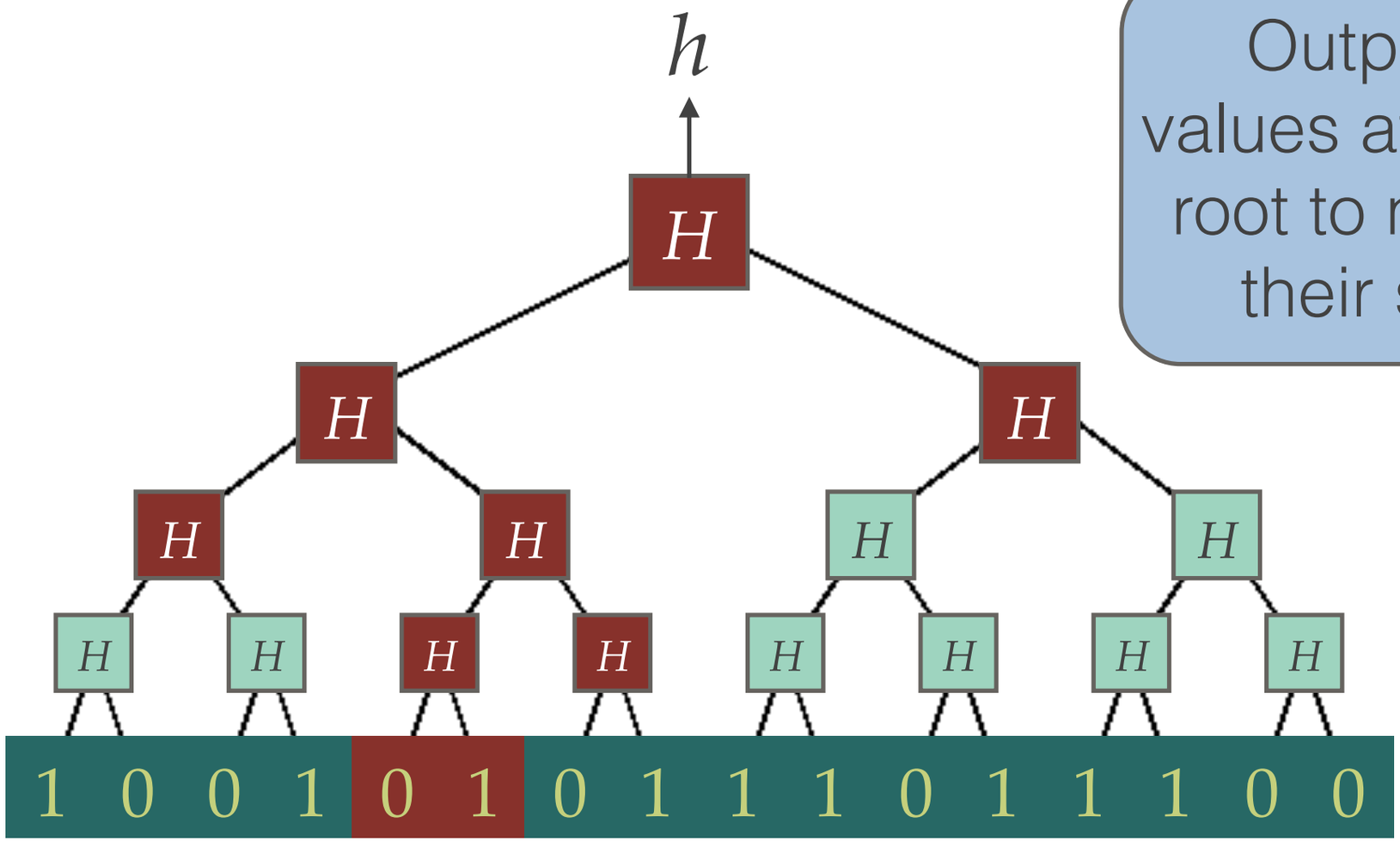

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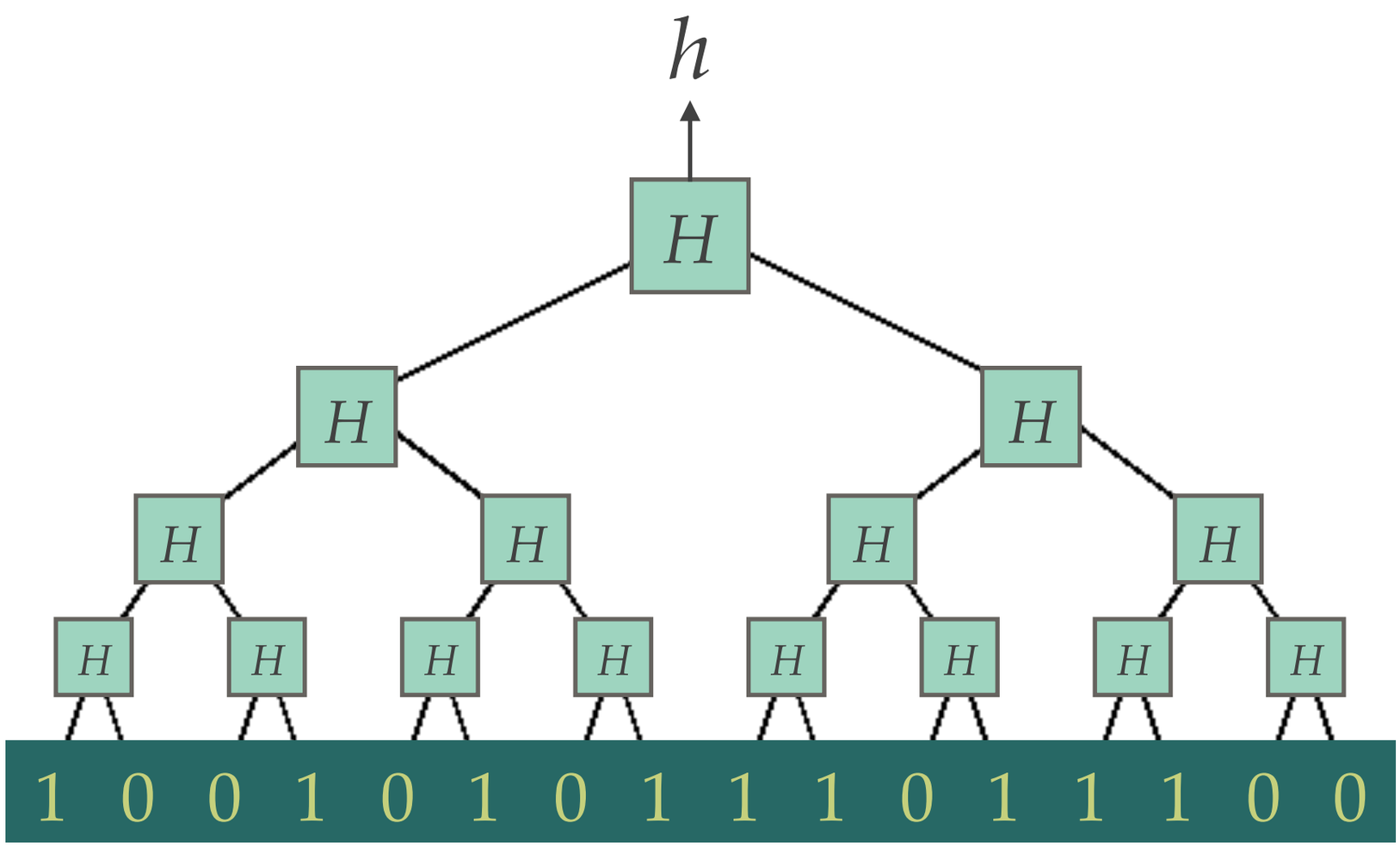
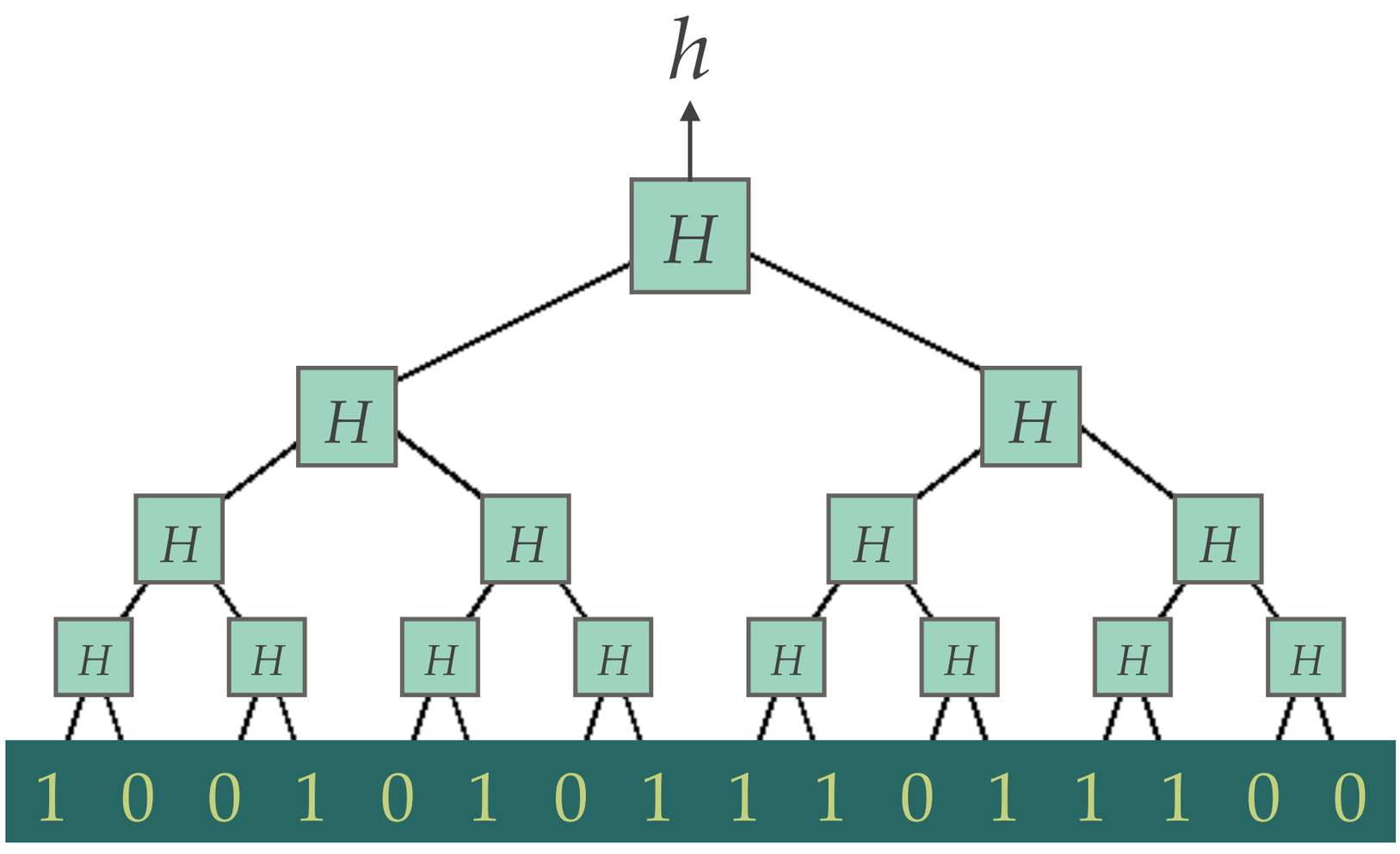
Merkle Trees: Update Hash



Merkle Trees: Update Hash

Update h to write m at position p



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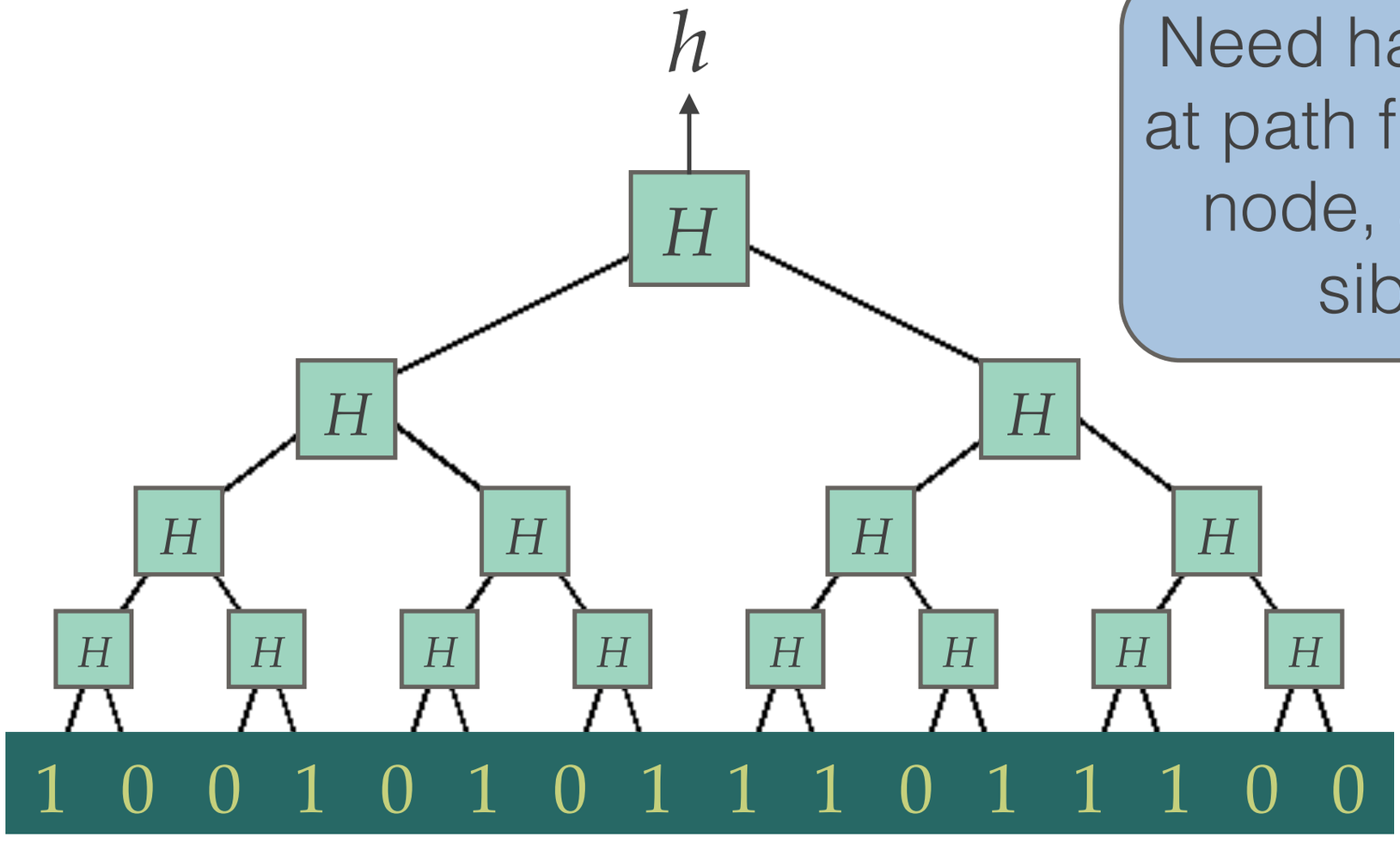

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Merkle Trees: Update Hash

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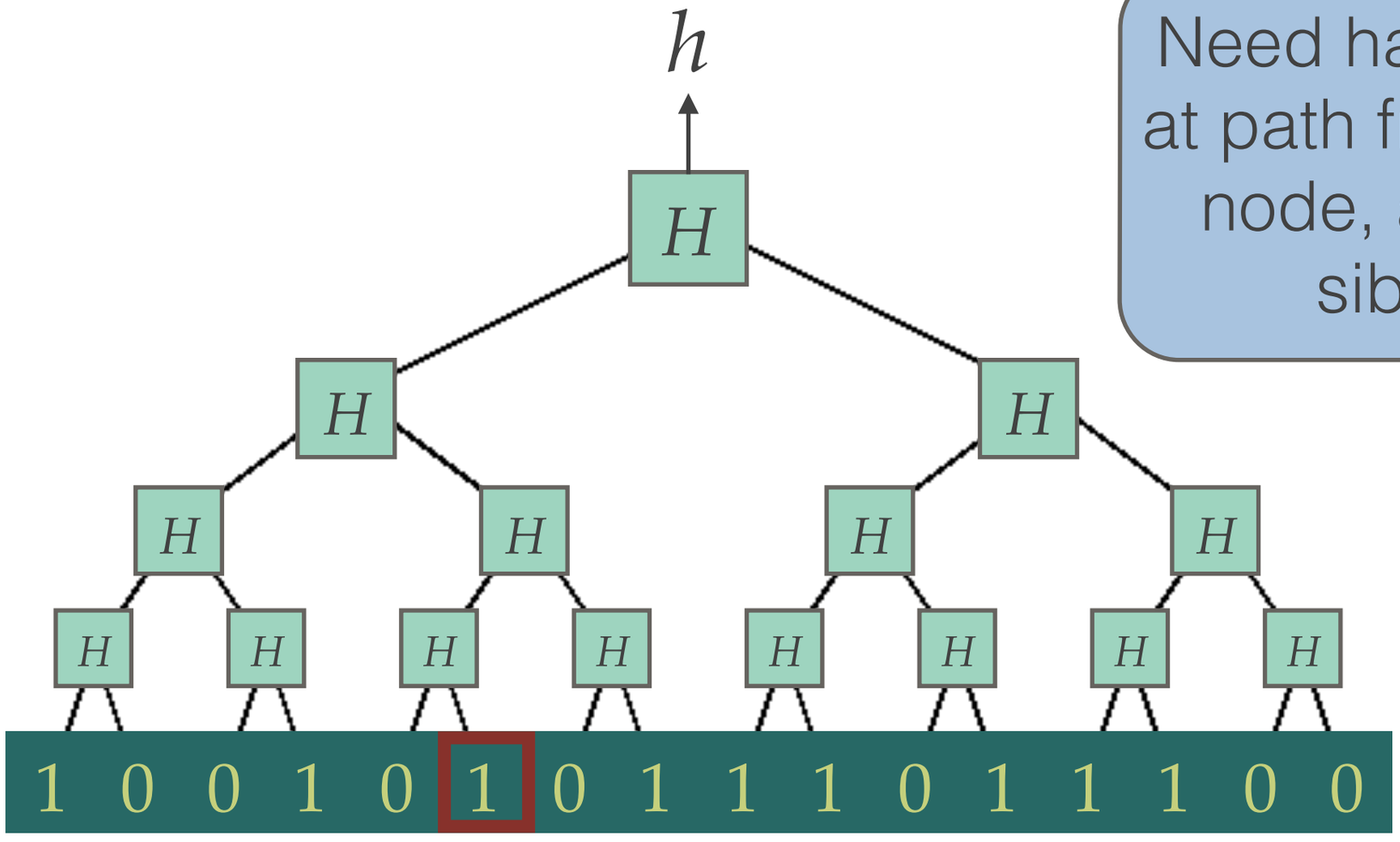

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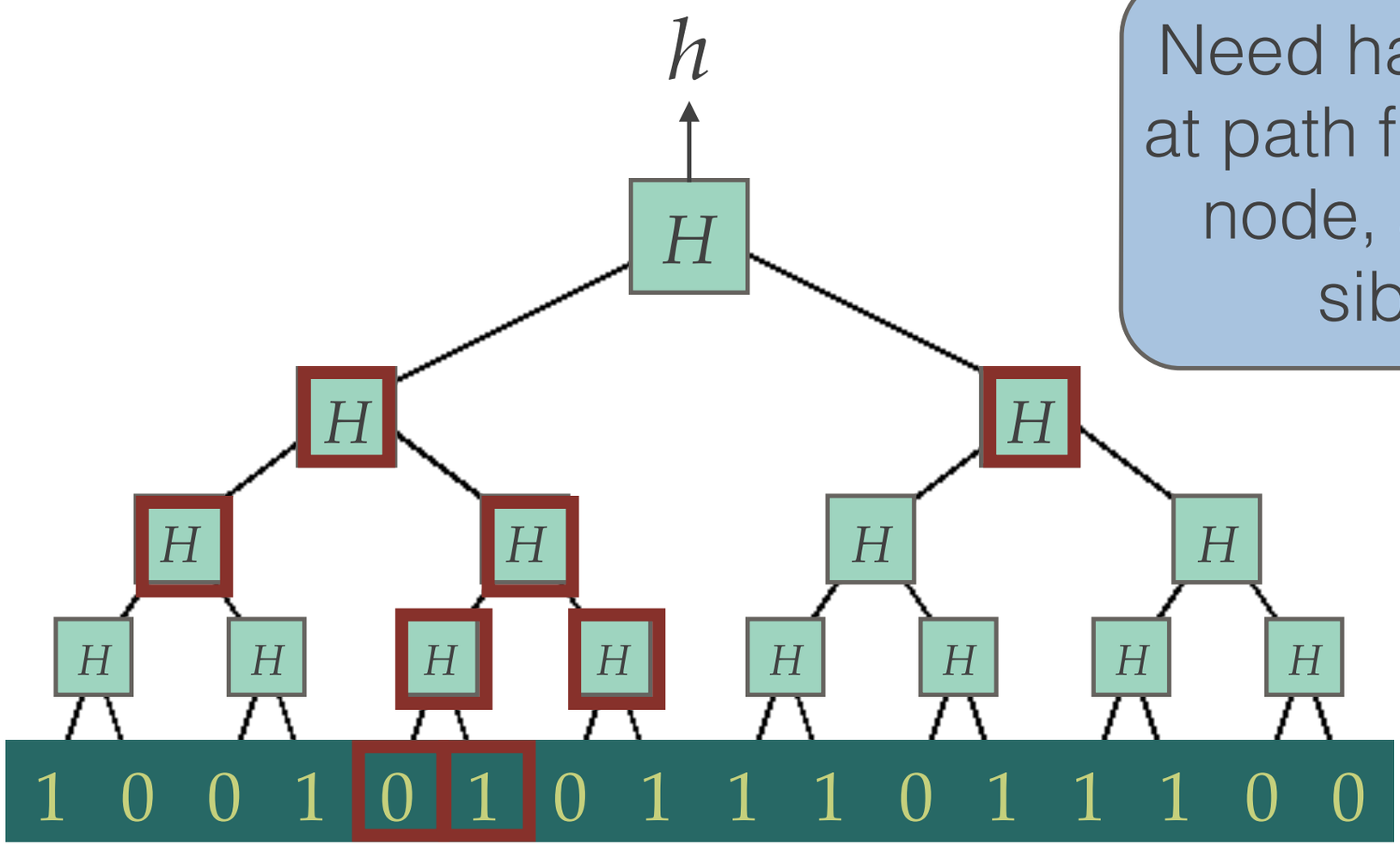

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Proving Selective Security

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Chooses PRF key K.

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x^* ←

$y^* = F(K, \text{hash of } x^*)$ or random →

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Chooses PRF key K.

v^*

x^*

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M (s.t. $M(x^*) = 0$)

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= (H, iO(Prog-Start),
iO(Prog-Iterate))

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Chooses PRF key K.

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x^*

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M (s.t. $M(x^*) = 0$)

$K\{M\}$
$= (H, iO(\text{Prog-Start}),$
$iO(\text{Prog-Iterate}))$

Guess PRF/random

Proving Selective Security

Chooses PRF key K.

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or random

x^*

M (s.t. $M(x^*) = 0$)

$K\{M\}$
$= (H, iO(\text{Prog-Start}),$
$iO(\text{Prog-Iterate}))$

Contains PRF key K

Guess PRF/random

Proving Selective Security

Using h, verify sym at position p

Verify sig on st and h

If st' = final, output F(K, v)

Using sym', update h to h'

Output signature on st' and h'

Prog-Iterate

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Prog-Iterate

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If st' = final and v=v*, output ⊥

Else if st' = final, output F(K{v*}, v)

Using sym', update h to h'

Output signature on st' and h'

Prog-Iterate'

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???

≈

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Prog-Iterate'

Need iO-Compatible Primitives

Primitives Required for our Construction:

(Collision resistant) Hash functions

Signature Schemes

Indistinguishability Obfuscation

Need iO-Compatible Primitives

Primitives Required for our Construction:

~~(Collision resistant) Hash functions~~

Positional Accumulators

~~Signature Schemes~~

Splittable Signature Schemes

Indistinguishability Obfuscation

Need iO-Compatible Primitives

Primitives Required for our Construction:

~~(Collision resistant) Hash functions~~

Positional Accumulators

~~Signature Schemes~~

Splittable Signature Schemes

Indistinguishability Obfuscation

iO for Turing Machines
(K, Lewko, Waters 14)

Proving Selective Security

Using h, verify sym at position p

Verify sig on st and h

st, sym → Next-Step → st’, sym’

If st’ = final, output F(K, v)

Using sym’, update h to h’

Output signature on st’ and h’

Prog-Iterate

KLW techniques

≈

Using h, verify sym at position p

Verify sig on st and h

st, sym → Next-Step → st’, sym’

If st’ = final and v=v*, output ⊥

Else if st’ = final, output F(K{v*}, v)

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Output signature on st’ and h’

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Conclusions

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- Constrained PRFs for Turing machines based on iO and one way functions

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- Unbounded broadcast encryption, unbounded ID-NIKE from iO and OWFs
  - unbounded broadcast encryption - Zhandry 14
  - unbounded ID-NIKE - Khurana, Rao, Sahai 15

Conclusions

- Constrained PRFs for Turing machines based on iO and one way functions
- Unbounded broadcast encryption, unbounded ID-NIKE from iO and OWFs
    - unbounded broadcast encryption - Zhandry 14
    - unbounded ID-NIKE - Khurana, Rao, Sahai 15
- Attribute Based Encryption for Turing machines
    - Functional Encryption for Turing machines - Ananth, Sahai 15

Danke!