Online/Offline OR Composition of ∑-Protocols

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Proofs of Knowledge (PoKs)

A fundamental crypto tool with many applications

- Identification Schemes
- Simulation-Based Security
- E-Voting Systems
- …

Useful in cryptography when the witness is protected: Witness Indistinguishable (WI), Witness Hiding (WH), Zero Knowledge (ZK)

e.g., prove knowledge of one thing OR another thing OR …

Proofs of Knowledge (PoKs)

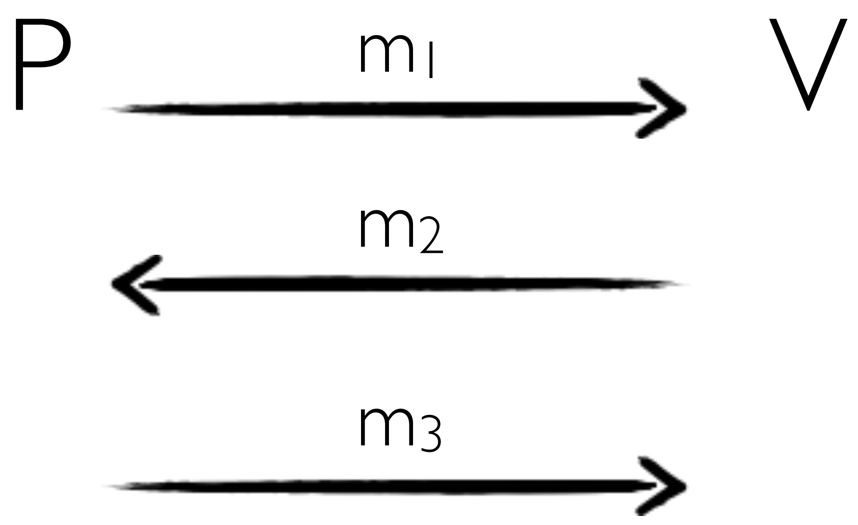
In theory

$x \in L$

$x \xrightarrow{\text{NP-reduction}} G$

"(G, C) in R_{HAM}"

WI Proof of Knowledge of Hamiltonicity
[Blum86, **L**apidot**S**hamir**90**]



In practice

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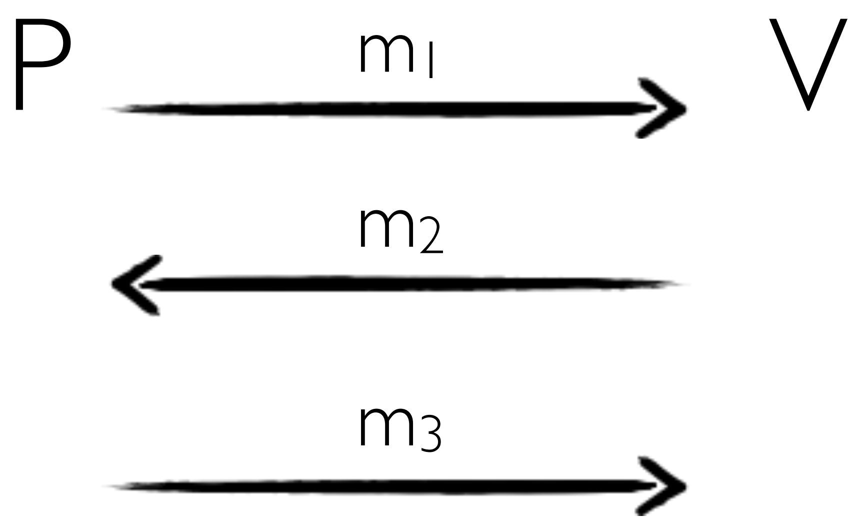
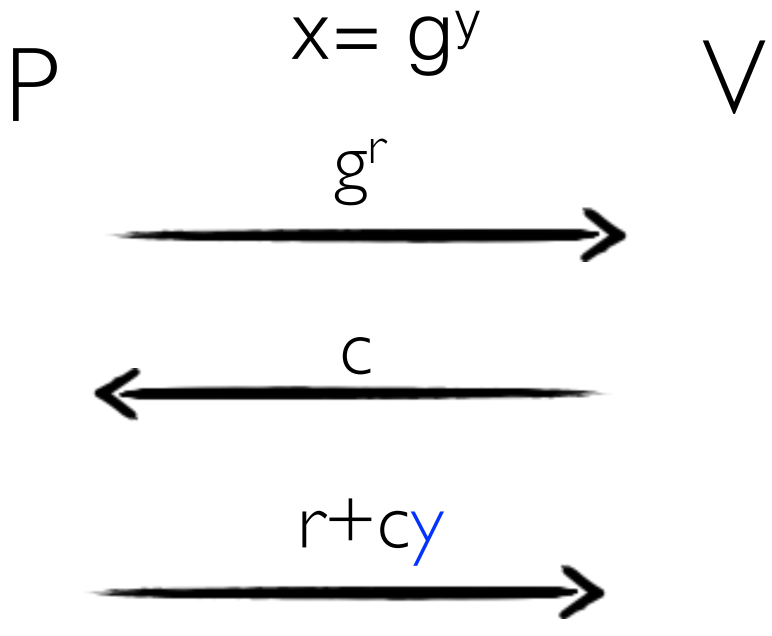


In practice

∑-protocol for $\mathbf{R}$

"(x, y) in $\mathbf{R}_{Dlog}$"

e.g. Discret Log [Schnorr89])



Proofs of K ledge (PoKs)

Observation: [LS90] and [Schnorr89] need the theorem and witness only in the last round

In theor

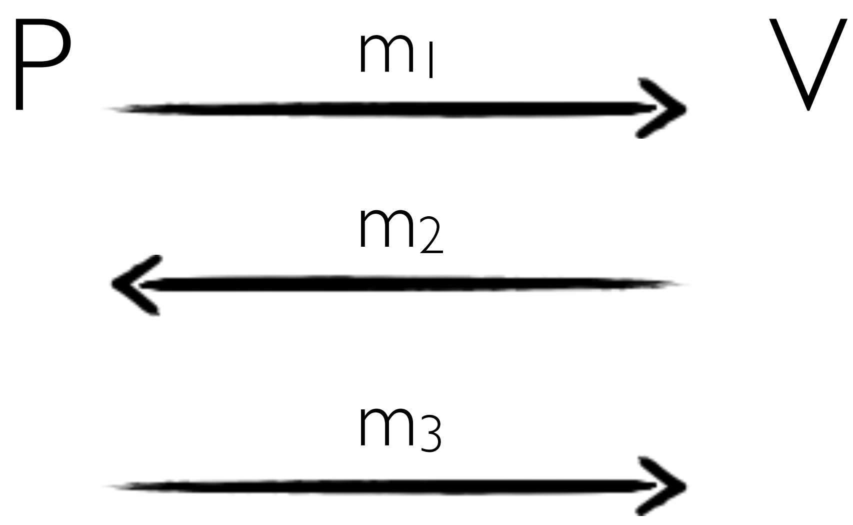
actice

x∈L

$x \xrightarrow{\text{NP-reduction}} G$

“(G, C) in $\mathbf{R}_{\text{HAM}}$”

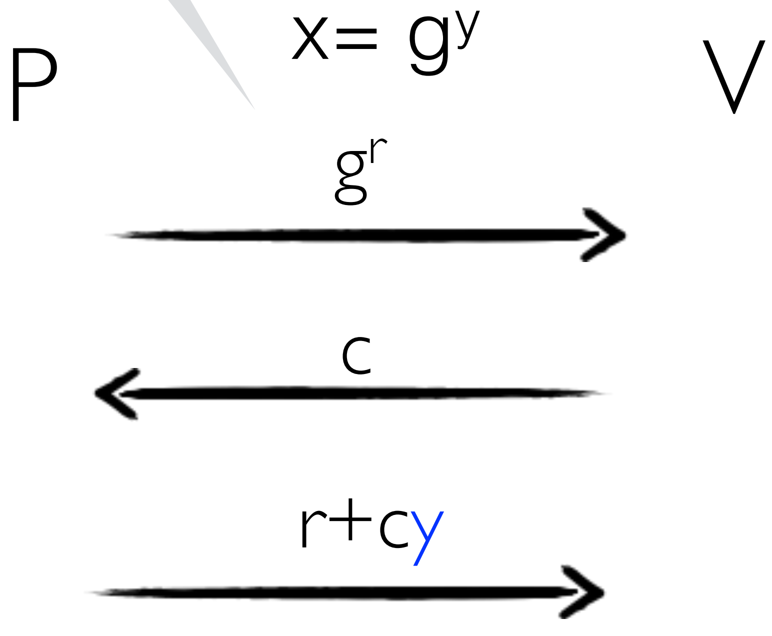
WI Proof of Knowledge of Hamilt onicity
[Blum86, **L**apidot**S**hamir**9**]



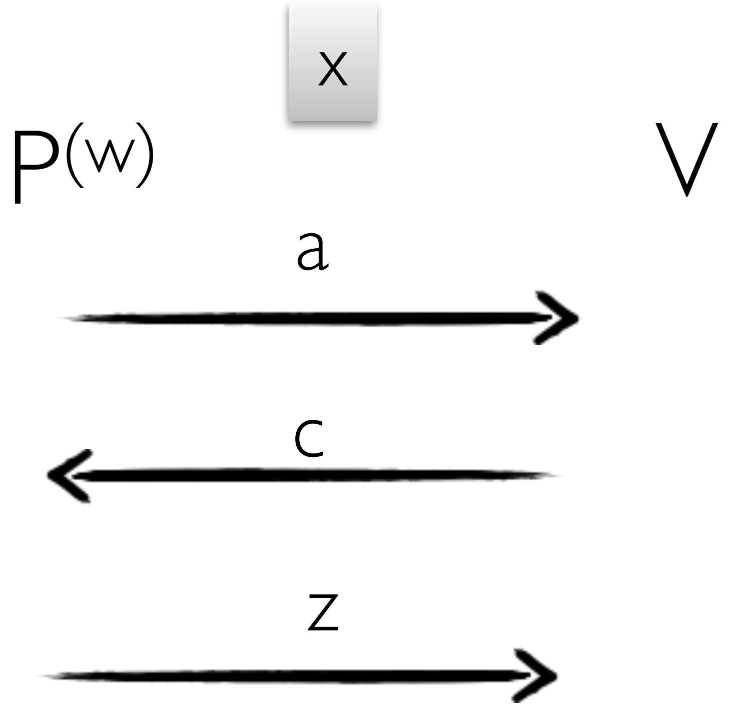
Σ-protocol for **R**

“(x, y) in $\mathbf{R}_{\text{Dlog}}$”

e.g. D scret Log [Schnorr89])



∑-protocol for relation **R**



x

$P^{(w)}$ V

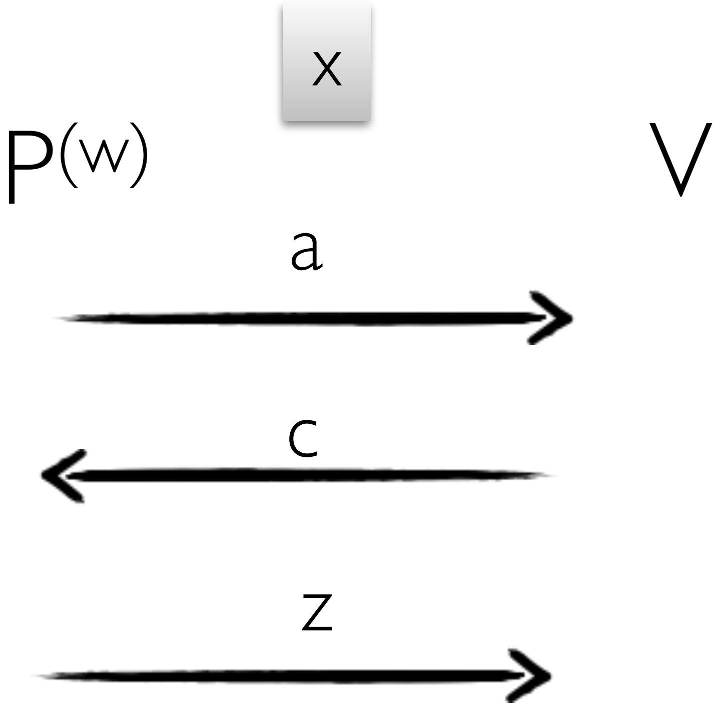
a

c

z

∑-protocol for relation R

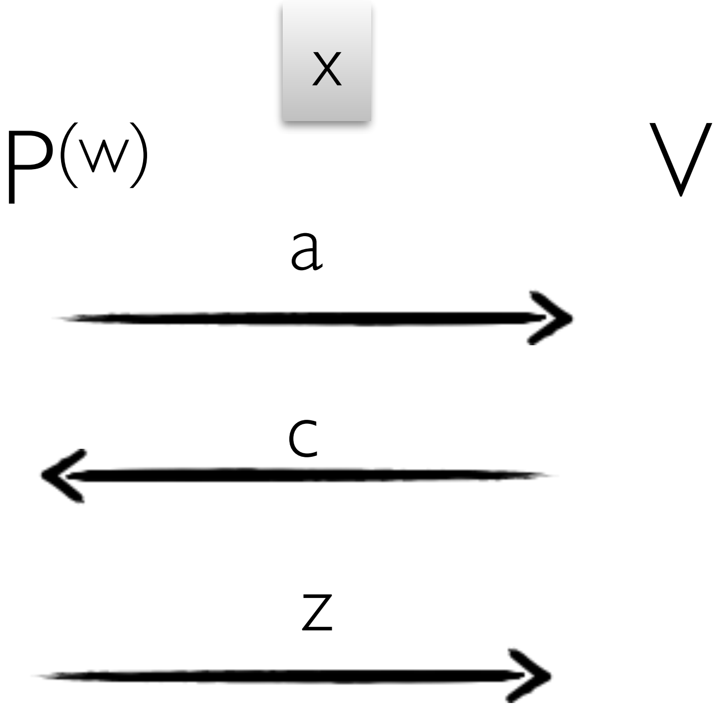
- **Completeness**



Σ-protocol for relation $\mathbf{R}$

- **Completeness**

- **SHVZK** Sim(x,c)⇒



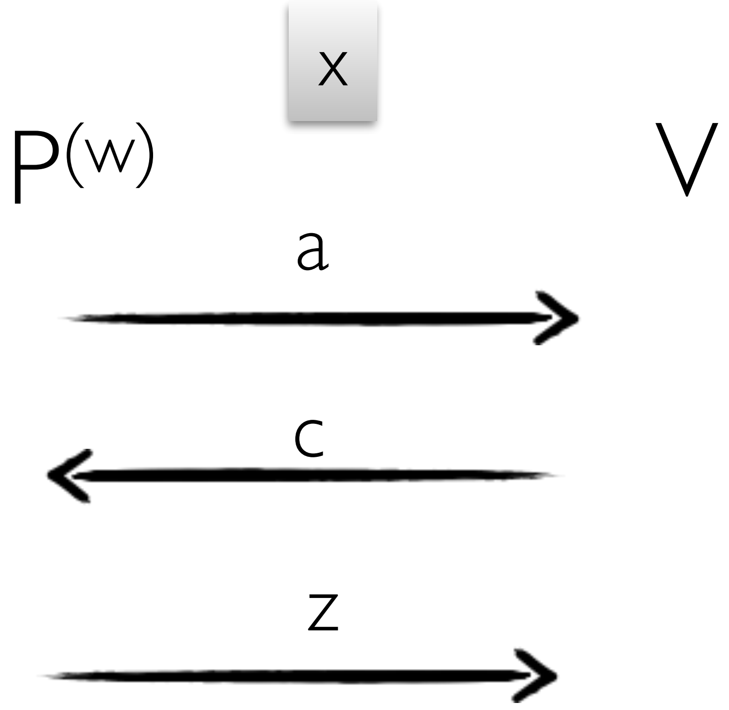
Σ-protocol for relation $\mathbf{R}$

- **Completeness**

a'

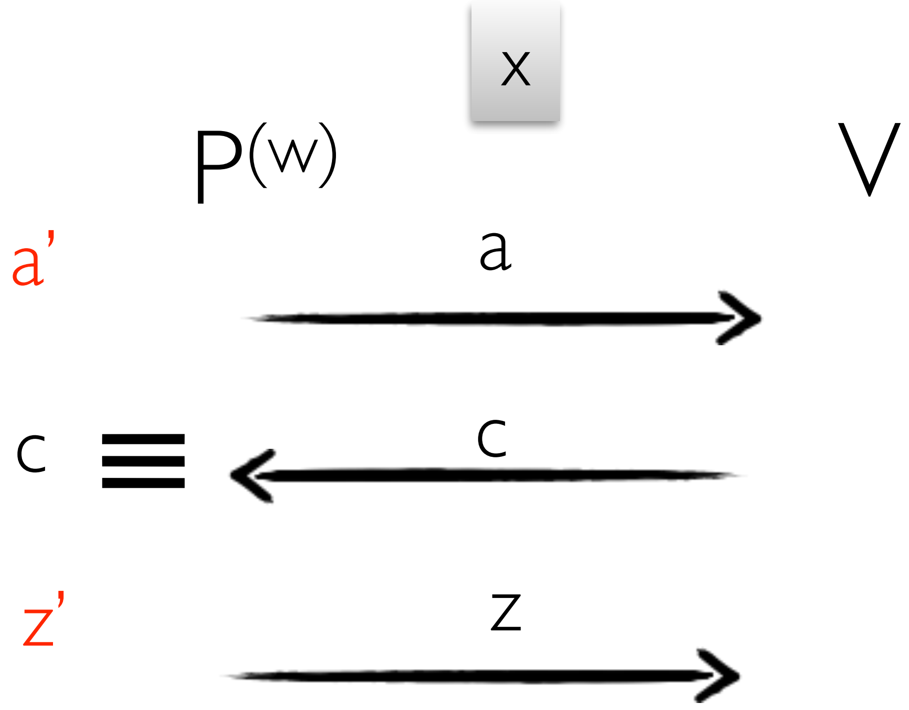
- **SHVZK** Sim(x,c)⇒ c

z'



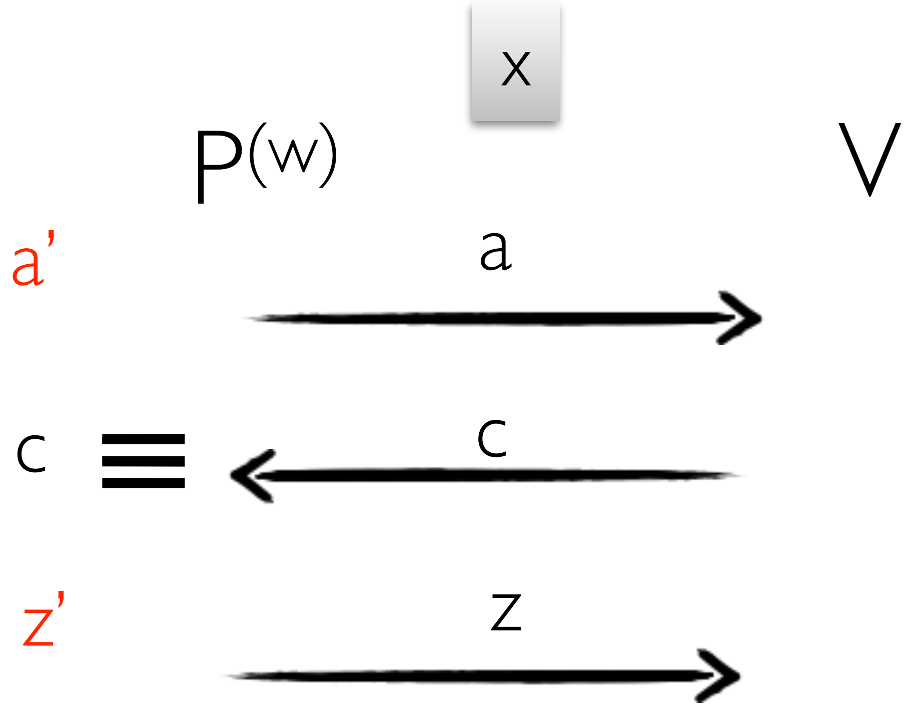
∑-protocol for relation $\mathbf{R}$

- **Completeness**
- **SHVZK** Sim(x,c)⇒



Σ-protocol for relation $\mathbf{R}$

- **Completeness**
- **SHVZK** Sim(x,c)⇒
- **Special Soundness**



x

$P^{(w)}$

V

a'

a

c ≡

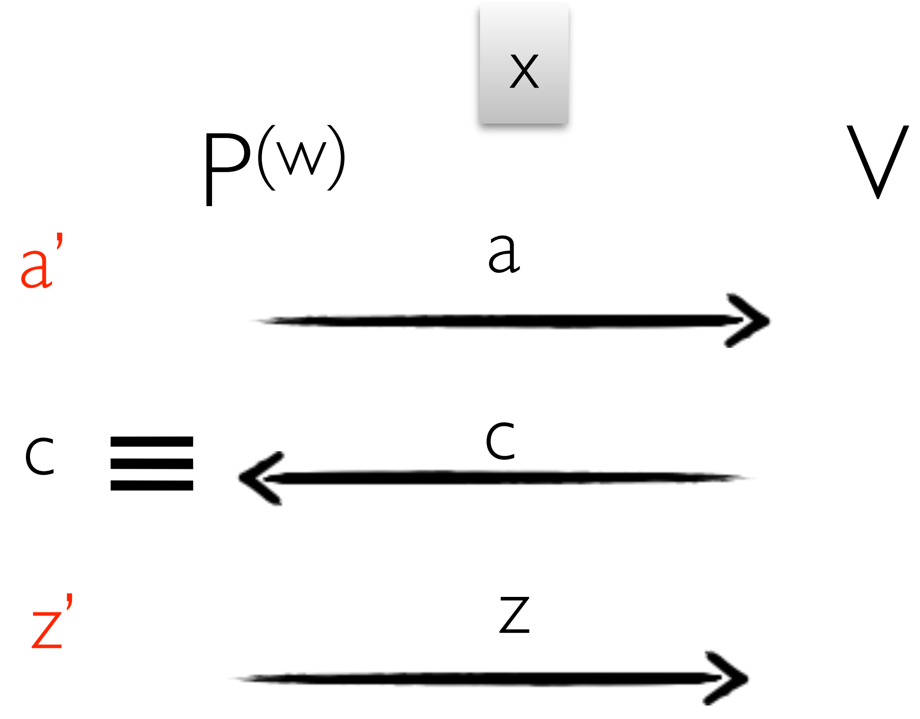
c

z'

z

∑-protocol for relation R

- **Completeness**
- **SHVZK** Sim(x,c)⇒
- **Special Soundness**



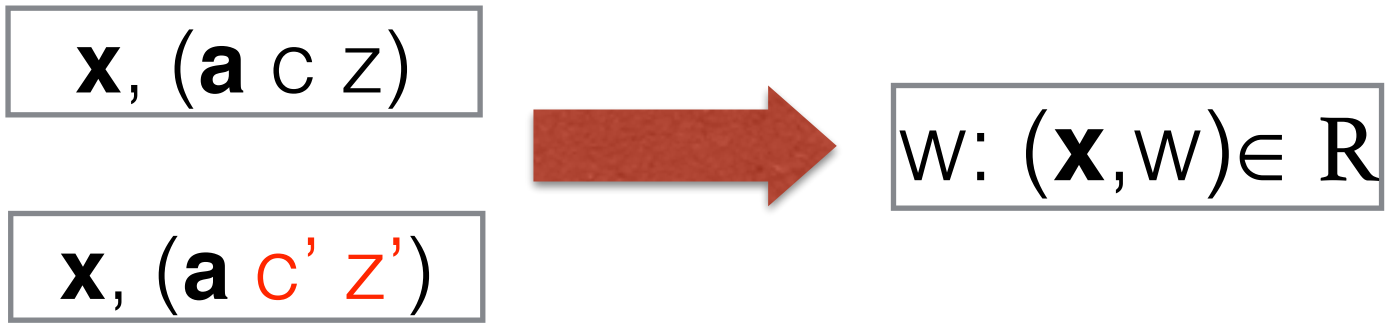
x

$P^{(w)}$ V

a' a

c ≡ c

z' z



x, (**a** c z)

x, (**a** c' z')

w: (**x**,w)∈ R

R_0 OR R_1

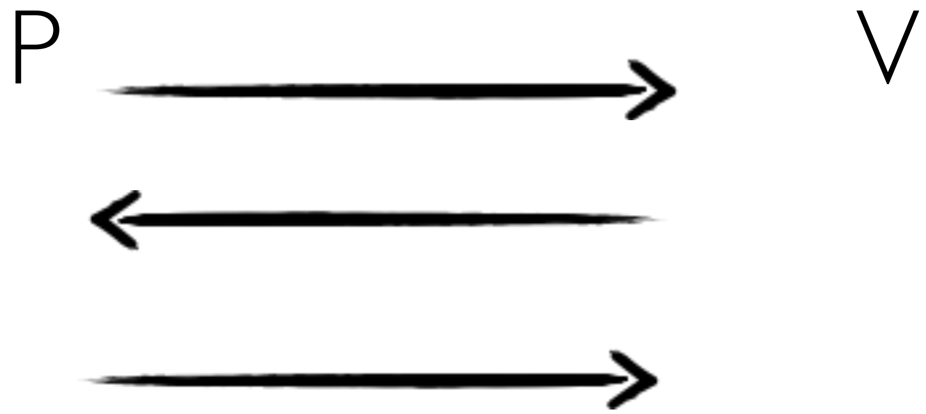
R_0 OR R_1

In theory

$$(x_0 \vee x_1) \xrightarrow{\text{NP-reduction}} G$$

"(G, C) in $\mathrm{R}_{\mathrm{HAM}}$"

WI Proof of Knowledge of Hamiltonicity
[Blum86, LS90]



In practice

Consider the ∑-protocols Σ_0 and Σ_1 for R_0 and R_1 and compile them using

[**C**ramer**D**amgard**S**choenmakers**94**]

In both cases you get 3 rounds, WI and PoK

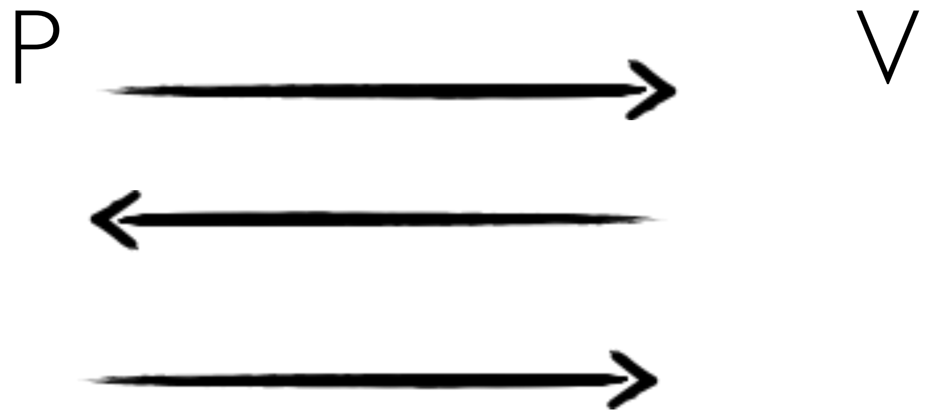
$\mathbf{R}_0$ OR $\mathbf{R}_1$: The Gap

In theory

$$(x_0 \vee x_1) \xrightarrow{\text{NP-reduction}} G$$

"(G, C) in $\mathbf{R}_{\mathrm{HAM}}$"

WI Proof of Knowledge of Hamiltonicity
[Blum86, **LS90**]



In practice

Consider the ∑-protocols Σ_0 and Σ_1 for $\mathbf{R}_0$ and $\mathbf{R}_1$ and compile them using

[**C**ramer**D**amgard**S**choenmakers**94**]

R_0 OR R_1: The Gap

In theory

$$(x_0 \vee x_1) \xrightarrow{\text{NP-reduction}} G$$

"(G, C) in R_{HAM}"

WI Proof of Knowledge of Hamiltonicity
[Blum86, **LS90**]

$$P \longrightarrow V$$

No need to know any theorem already at the 1rd round

In practice

Consider the Σ-protocols Σ_0 and Σ_1 for R_0 and R_1 and compile them using [**C**ramer**D**amgard**S**choenmakers**94**]

x_0 and x_1 are needed already at the 1rd round

R_0 OR R_1: The Gap

In theory

[LS90]

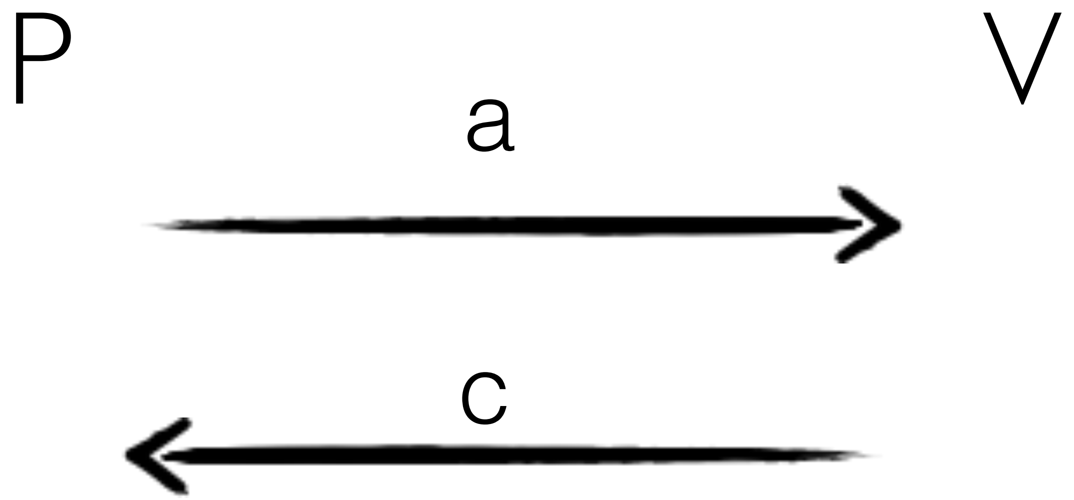
- **Delayed-Input Completeness**

In practice

[CDS94]

- **Completeness**

Delayed-Input Completeness



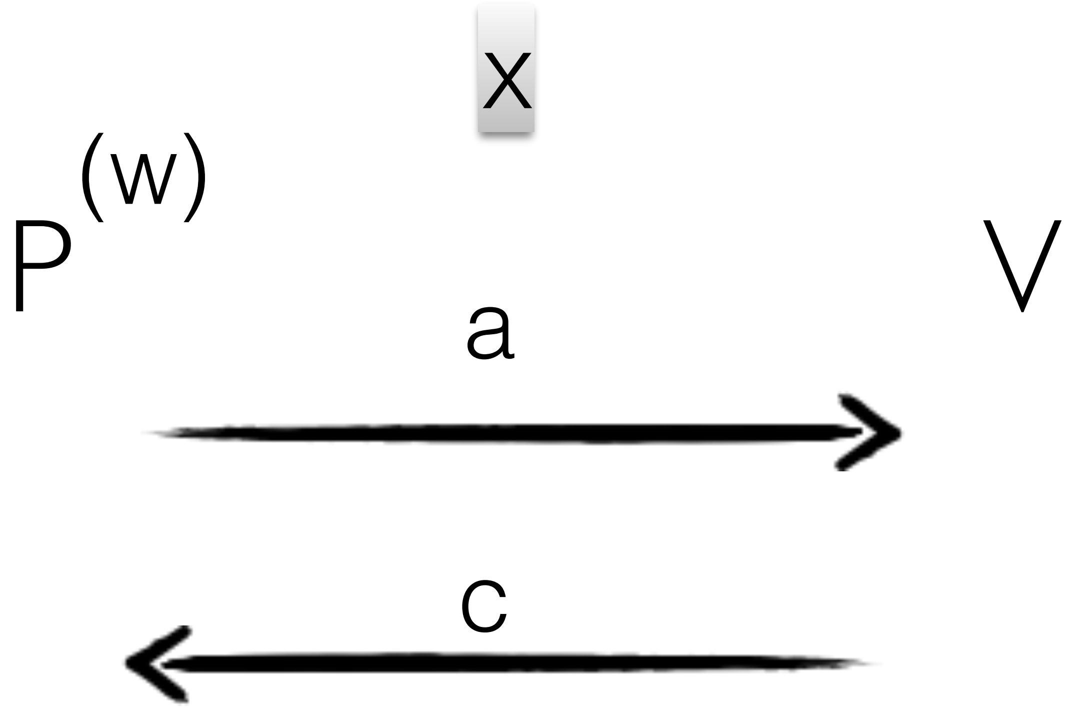
P

V

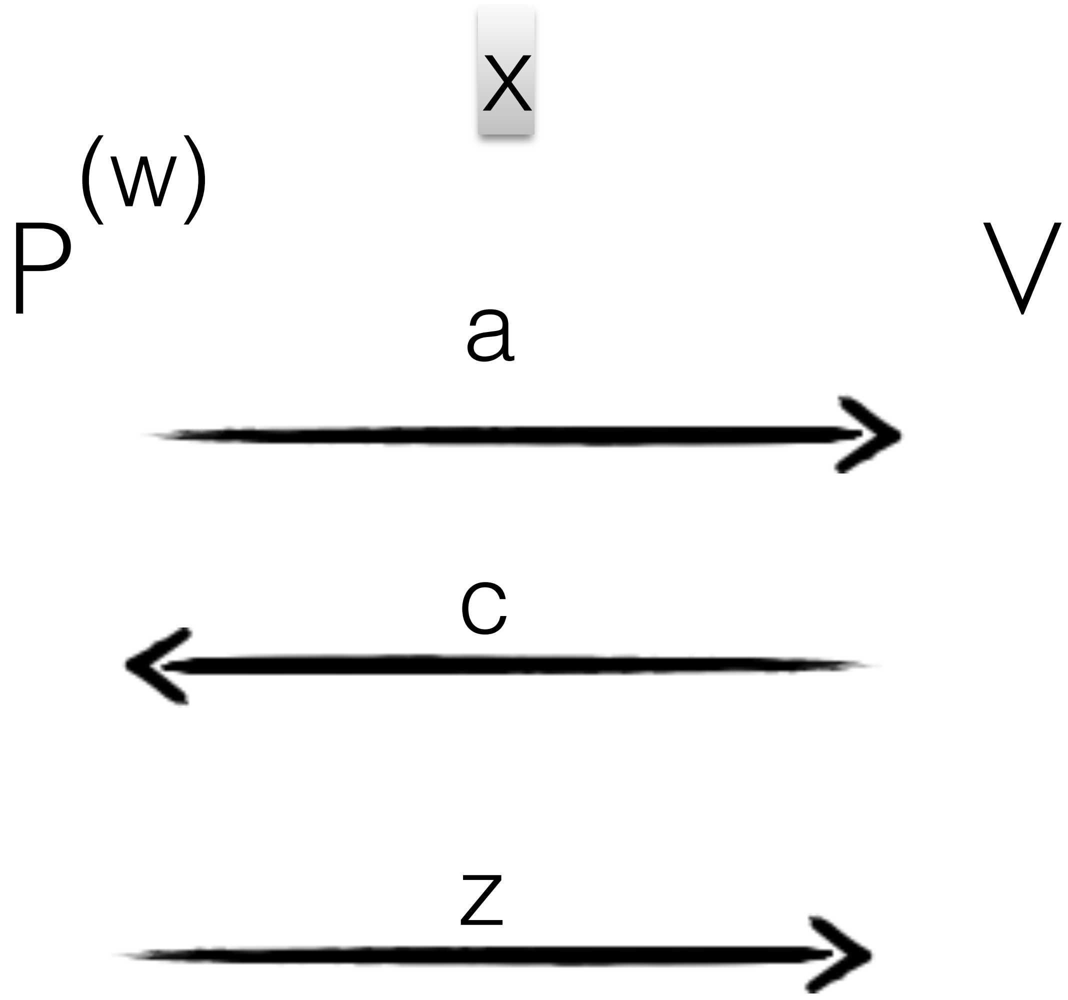
a

c

Delayed-Input Completeness



Delayed-Input Completeness



x

P^(w) V

a

c

z

R_0 OR R_1: The Gap

In theory

[LS90]

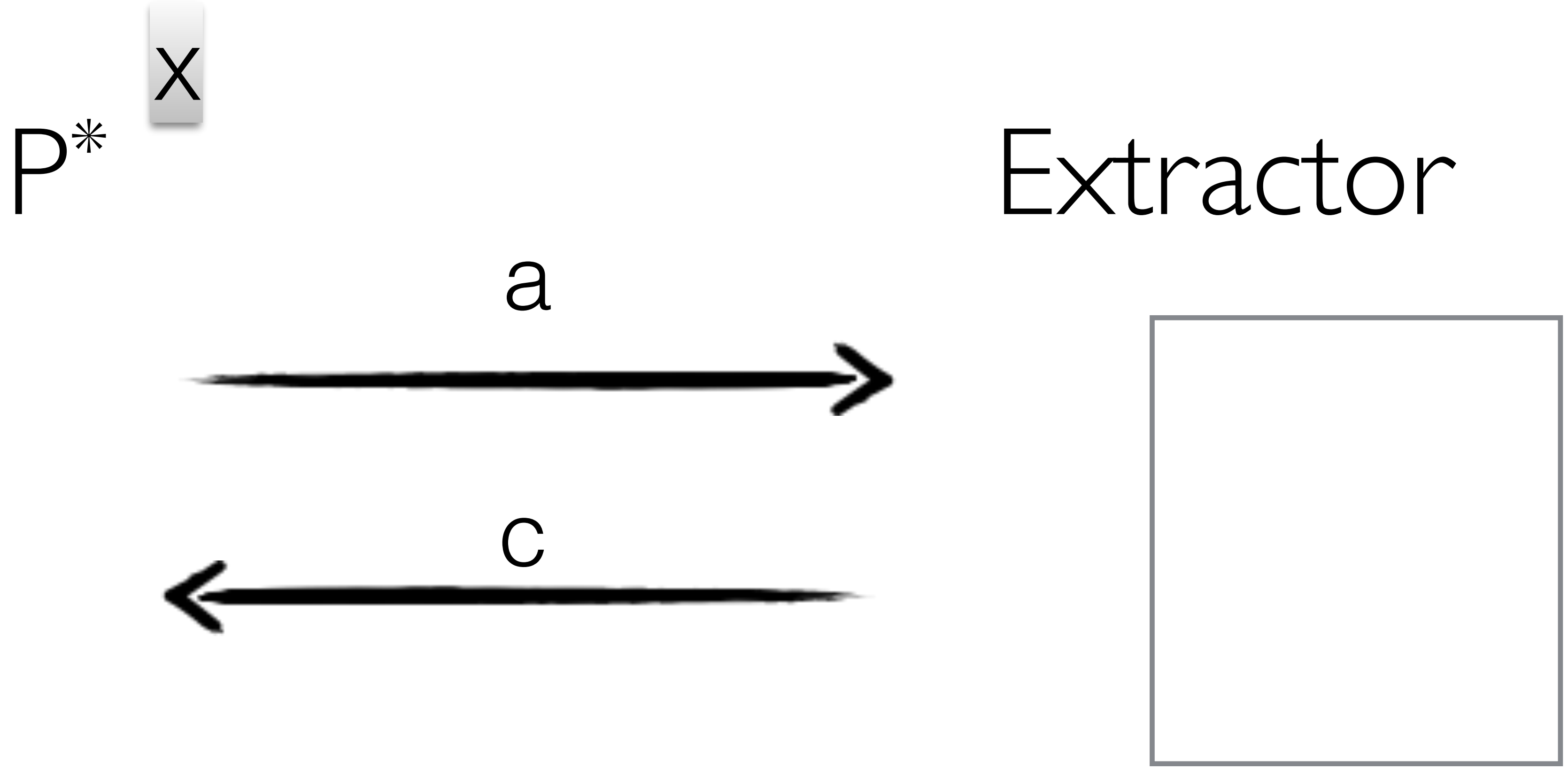
- **Delayed-Input Completeness**
- **Adaptive-Input Proof of Knowledge**

In practice

[CDS94]

- **Completeness**
- **Proof of Knowledge**

Adaptive-Input PoK



x

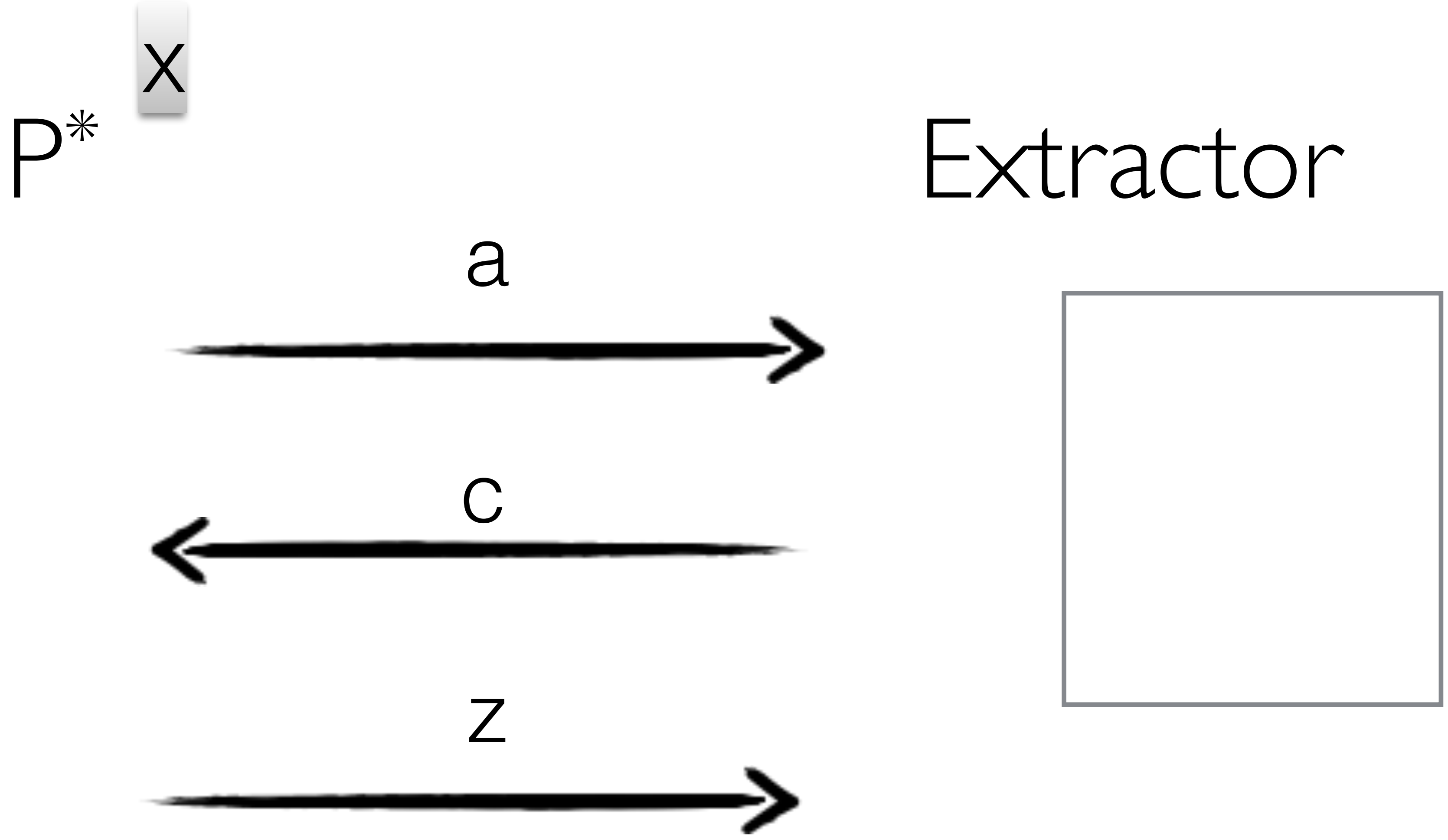
P*

Extractor

a

c

Adaptive-Input PoK



x

P*

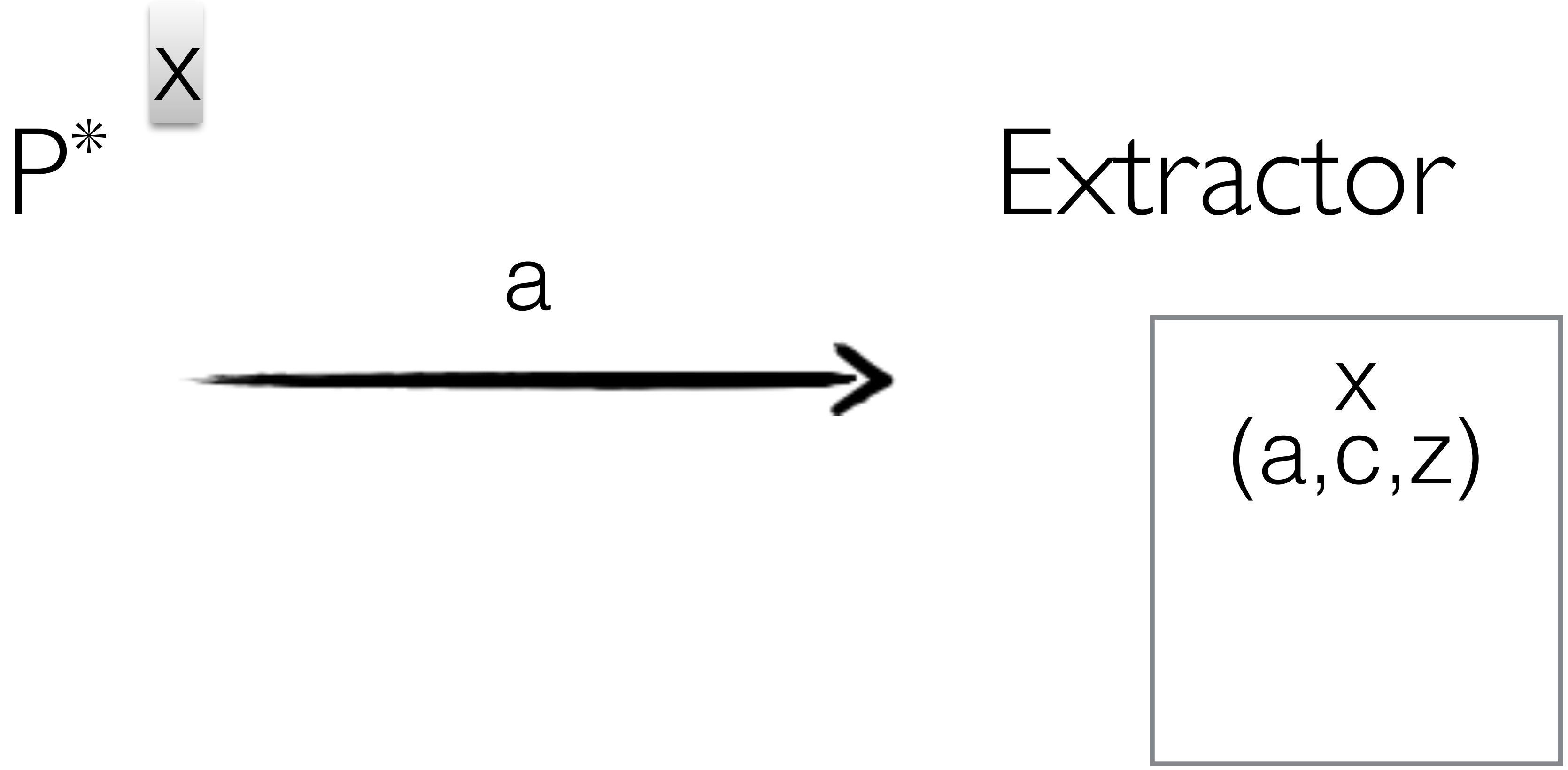
Extractor

a

c

z

Adaptive-Input PoK



P*

x

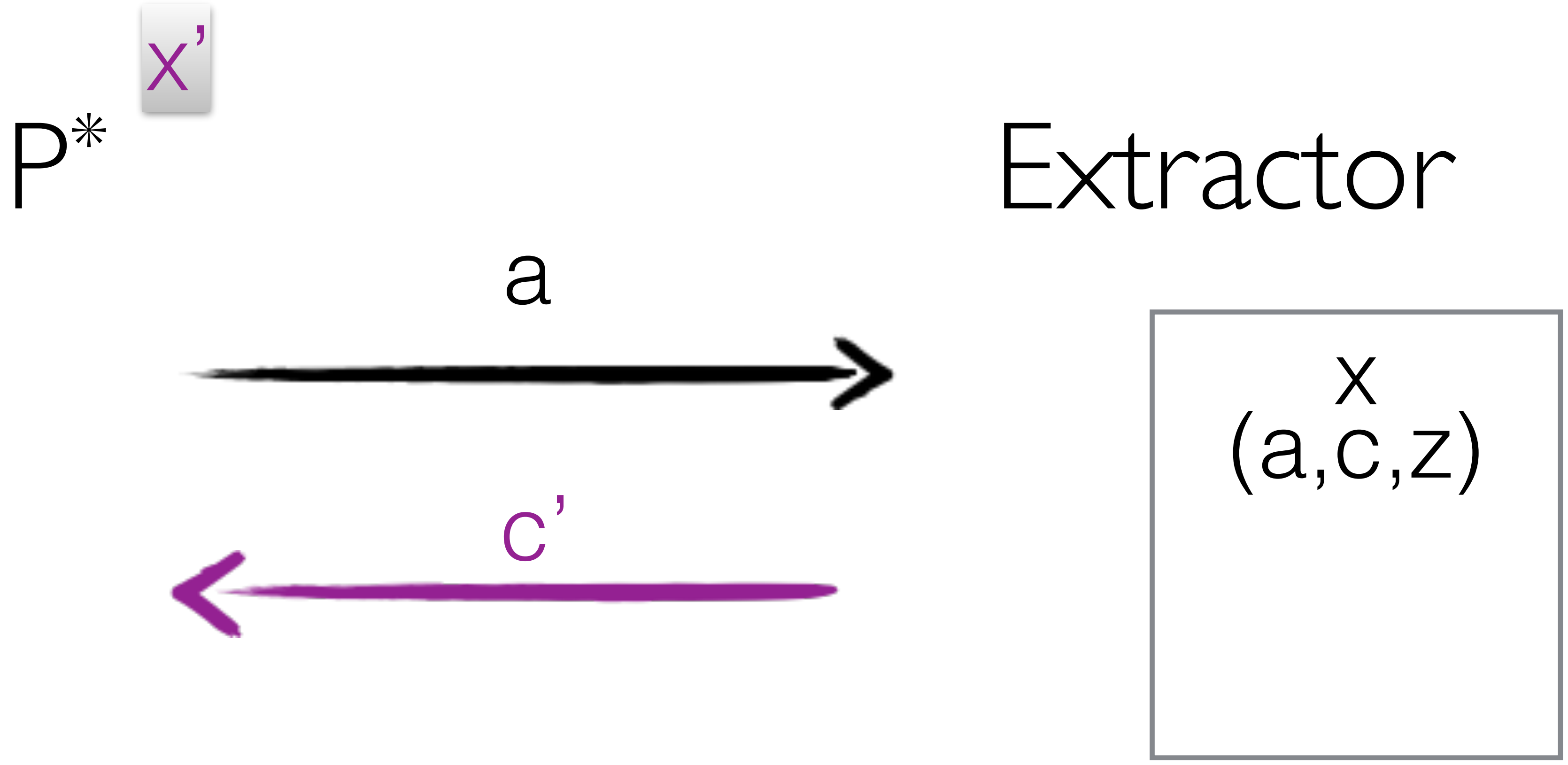
Extractor

a

x

(a,c,z)

Adaptive-Input PoK



x’

P*

Extractor

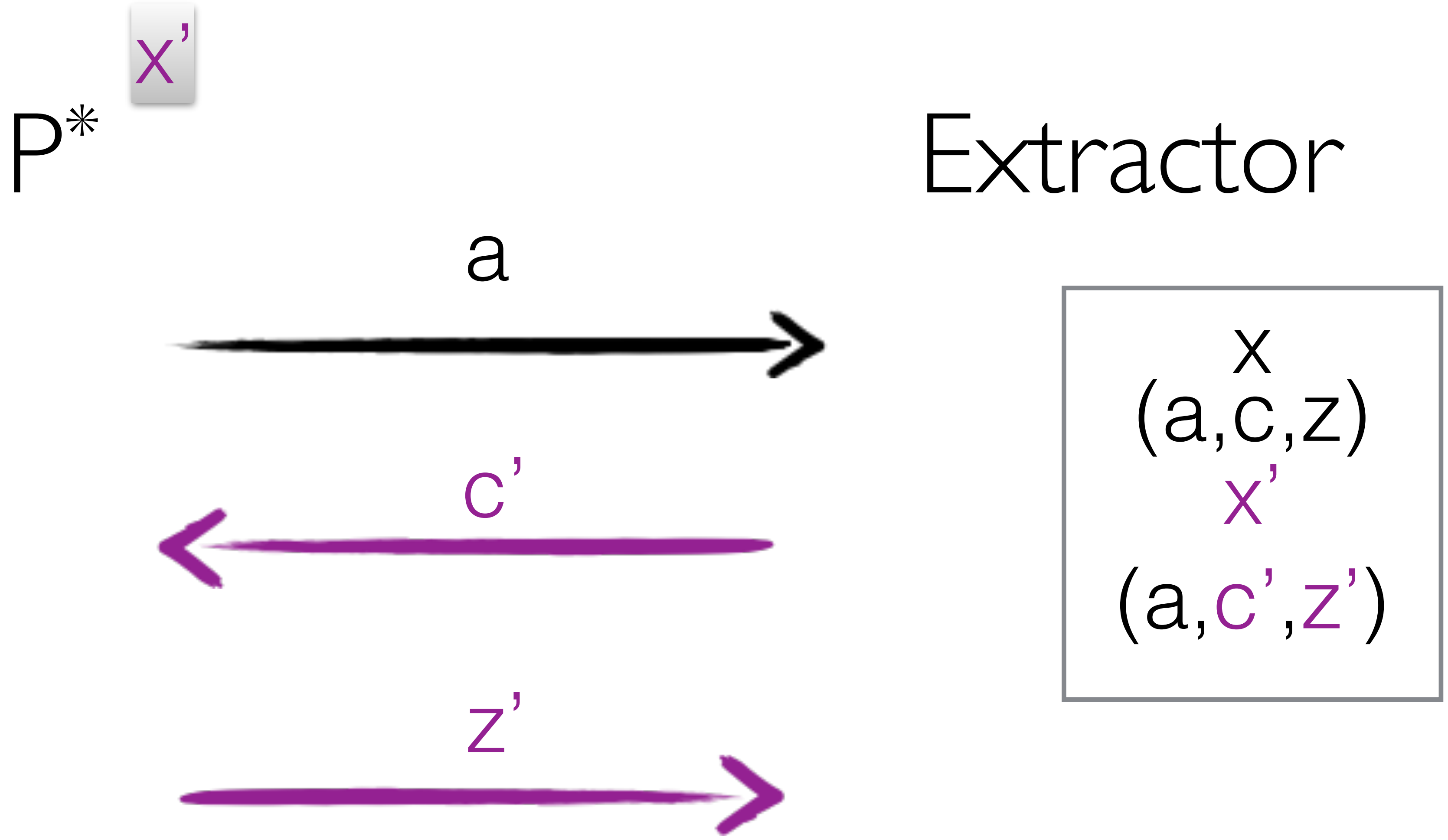
a

c’

x

(a,c,z)

Adaptive-Input PoK



x’

P*

Extractor

a

c’

z’

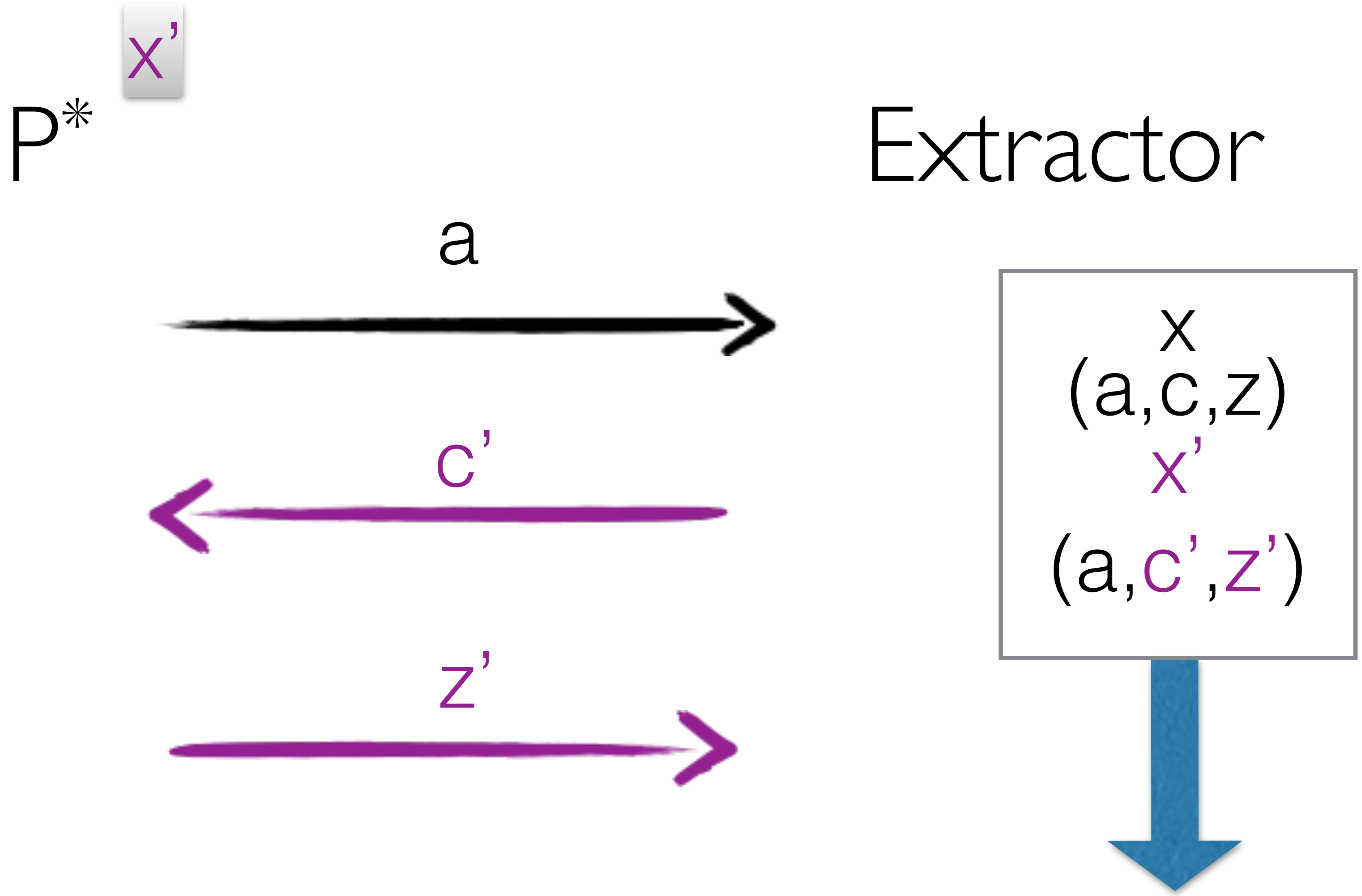
x

(a,c,z)

x’

(a,c’,z’)

Adaptive-Input PoK



x'

P*

Extractor

a

c'

z'

x

(a,c,z)

x'

(a,c',z')

w witness for x

R_0 OR R_1: The Gap

In theory

[LS90]

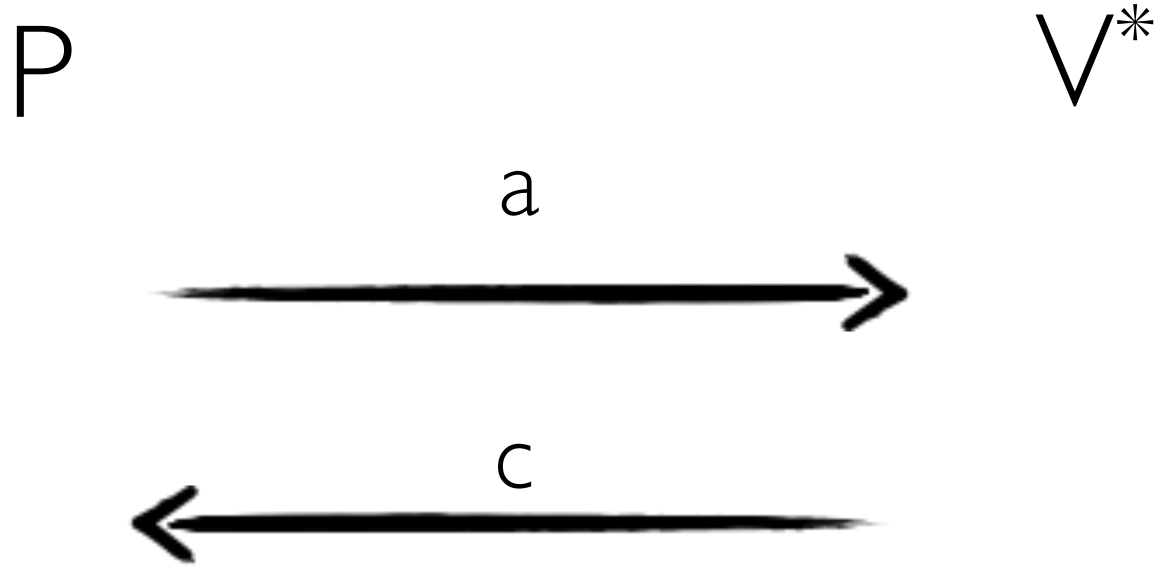
- **Delayed-Input Completeness**
- **Adaptive-Input Proof of Knowledge**
- **Adaptive-Input Witness Indistinguishable**

In practice

[CDS94]

- **Completeness**
- **Proof of Knowledge**
- **Witness Indistinguishable**

Adaptive-Input WI



P

V^*

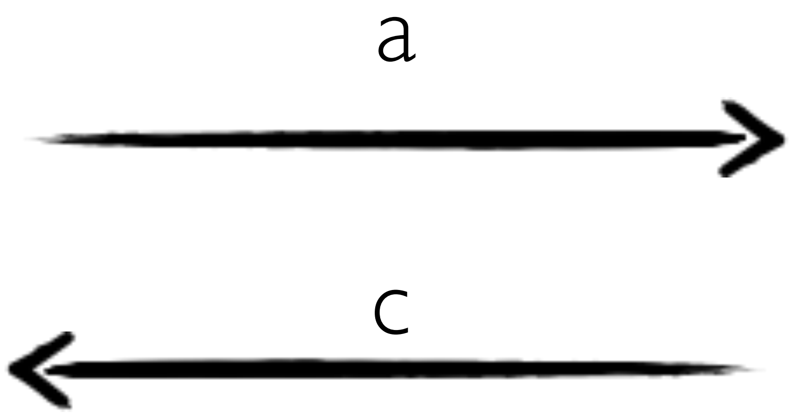
a

c

Adaptive-Input WI

(x, w_1, w_2)

$P^{(w_b)}$ V^*

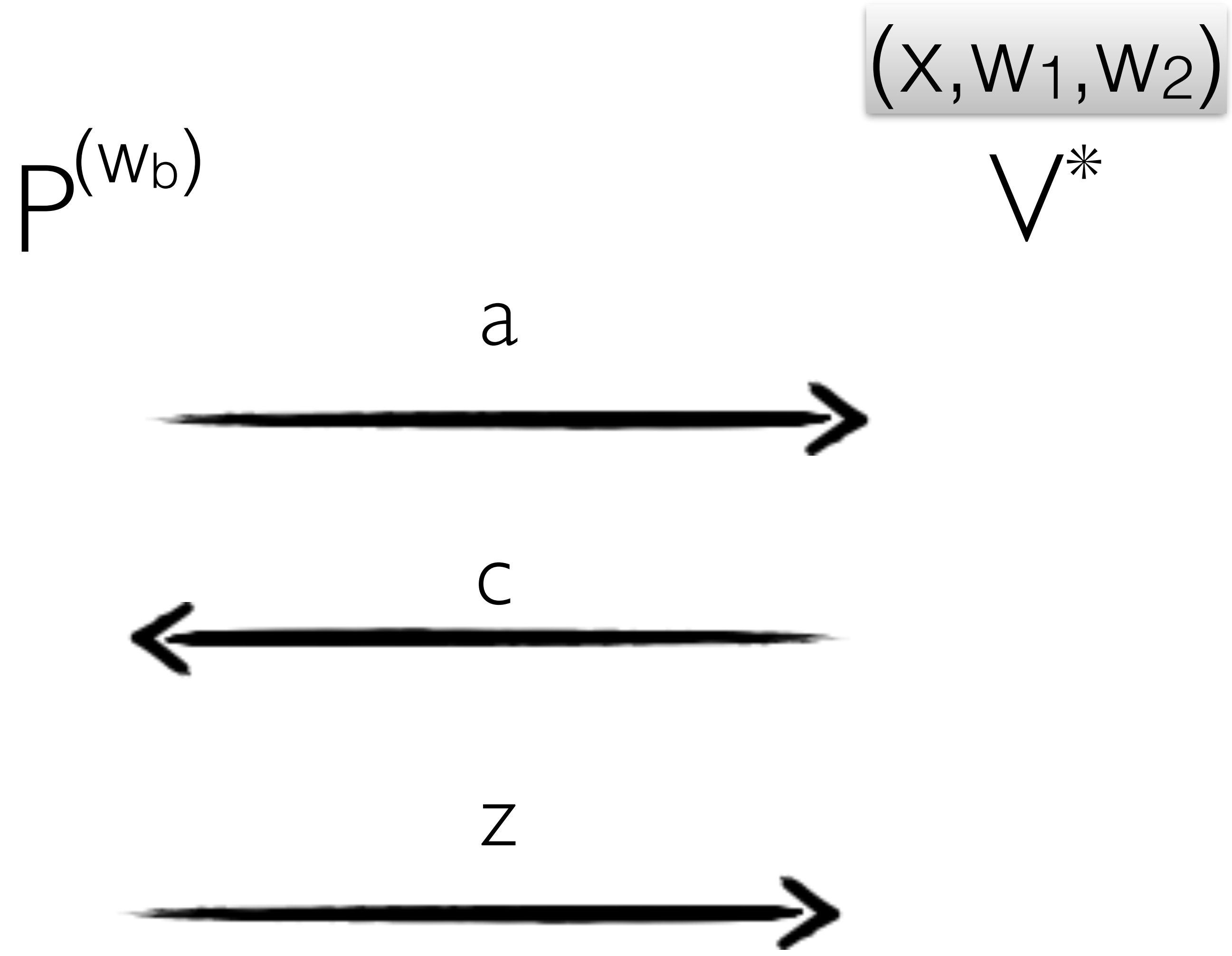


a

c

w_1, w_2 witnesses for x

Adaptive-Input WI



(x,w_1,w_2)

$P^{(w_b)}$ V^*

a

c

z

w_1,w_2 witnesses for x

R_0 OR R_1: The Gap

In theory

[LS90]

- **Delayed-Input Completeness**
- **Adaptive-Input Proof of Knowledge**
- **Adaptive-Input Witness Indistinguishable**
- **Assumption:** OWP

In practice

[CDS94]

- **Completeness**
- **Proof of Knowledge**
- **Witness Indistinguishable**
- **Assumption:** none

R_0 OR R_1: The Gap

In theory

[LS90]

- **Delayed-Input Completeness**
- **Adaptive-Input Proof of Knowledge**
- **Adaptive-Input Witness Indistinguishable**
- **Assumption:** OWP
- **Requires NP-reduction and gives Computational WI**

In practice

[CDS94]

- **Completeness**
- **Proof of Knowledge**
- **Witness Indistinguishable**
- **Assumption:** none
- **No NP-reduction and gives Perfect WI**

R_0 OR R_1: The Gap

In theory

[LS90]

- Delayed-Input Completeness
- Adaptive-Input Proof of Knowledge
- Adaptive-Input Witness Indistinguishable
- Assumption: OWP
- Requires NP-reduction and gives Computational WI
- Applicable to All NP

In practice

[CDS94]

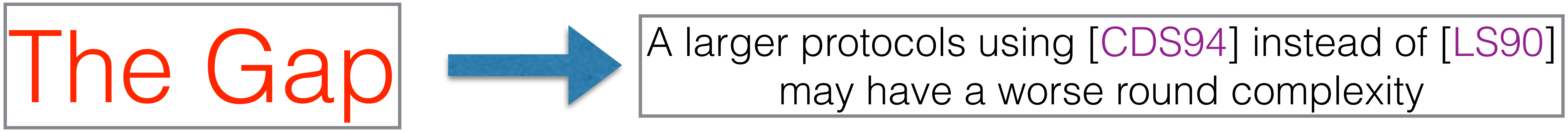
- Completeness
- Proof of Knowledge
- Witness Indistinguishable
- Assumption: none
- No NP-reduction and gives Perfect WI
- Restricted to ∑-protocols

R_0 OR R_1

The Gap →

A larger protocols using [CDS94] instead of [LS90] may have a worse round complexity

R_0 OR R_1



The Gap

A larger protocols using [CDS94] instead of [LS90] may have a worse round complexity

e.g. [Pass – Eurocrypt 03], [KaOs – Crypto 04], [YuZh – Eurocrypt 07][ScVi – Eurocrypt 12]…

R_0 OR R_1

The Gap → A larger protocols using [CDS94] instead of [LS90] may have a worse round complexity

e.g. [Pass – Eurocrypt 03], [KaOs – Crypto 04], [YuZh – Eurocrypt 07][ScVi – Eurocrypt 12]…

Recently Delayed-Input completeness is widely used

[GMPP16 – tomorrow], [Kiayias0Z15 – CCS15], [BBKPV16 – eprint]…

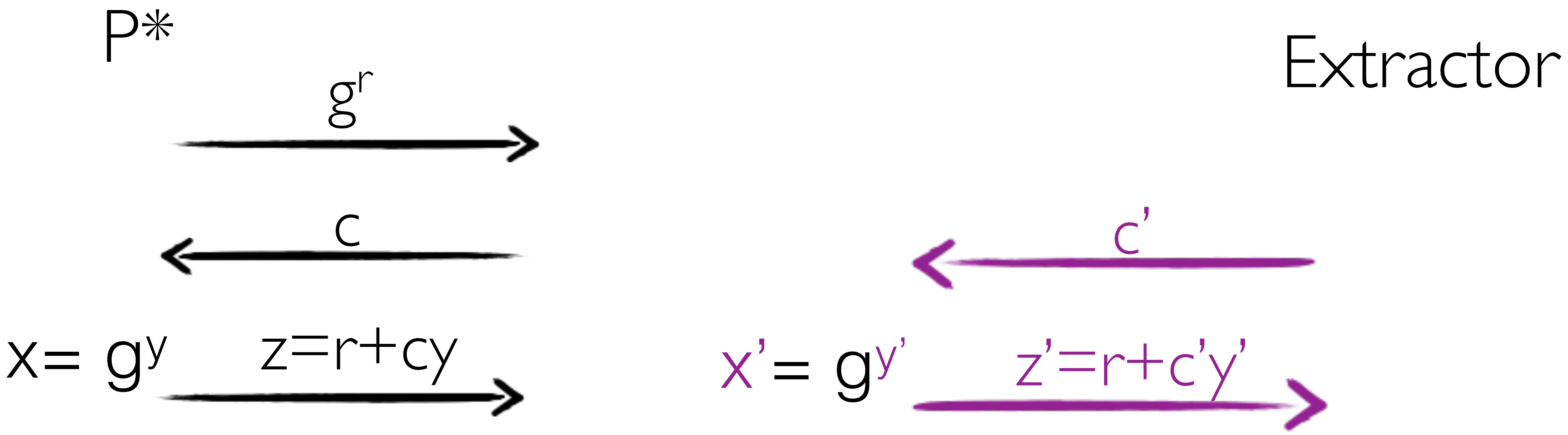
Our Results

1) From PoK to Adaptive-Input PoK

2) Bridging the gap

Our First Result: from PoK to Adaptive-Input PoK

Σ-Protocols (in general) are not Adaptive-Input PoK



P*

Extractor

g^r

c

c'

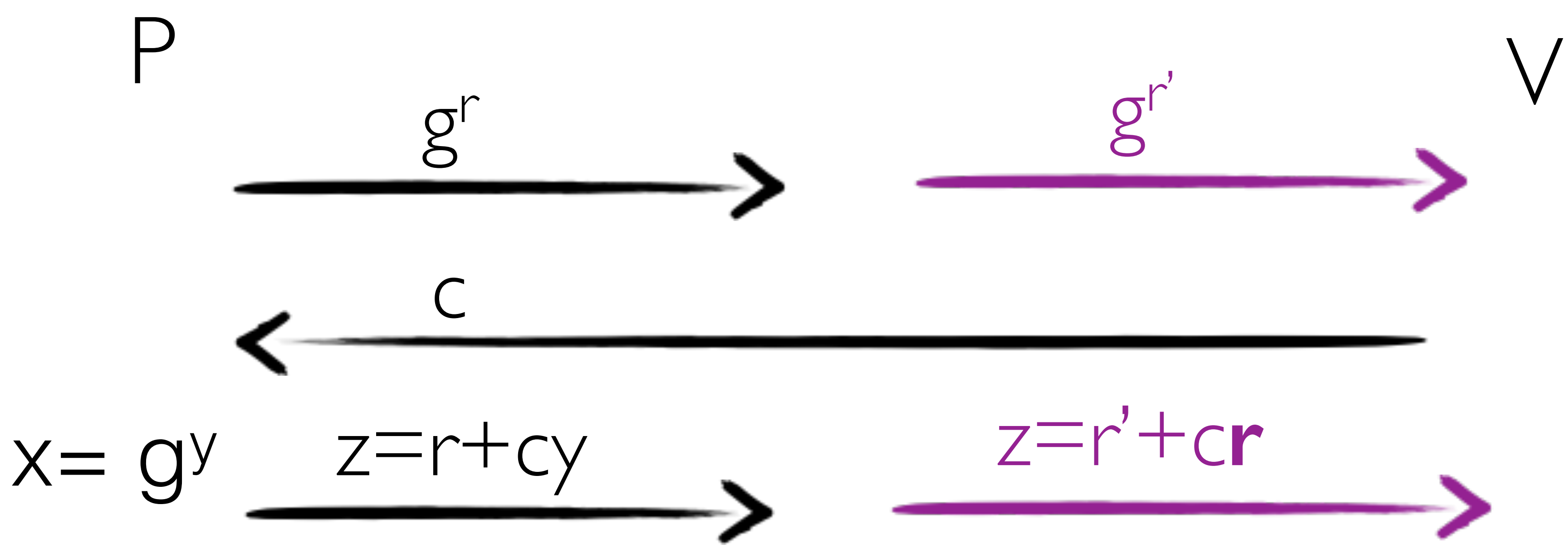
$x= g^y$ $z=r+cy$

$x'= g^{y'}$ $z'=r+c'y'$

Issue observed in [**B**ernhard**P**ereira**W**arinschi**12**] about the weak Fiat-Shamir transform

Our Transform

From PoK to Adaptive-Input PoK



P

V

g^r

$g^{r'}$

c

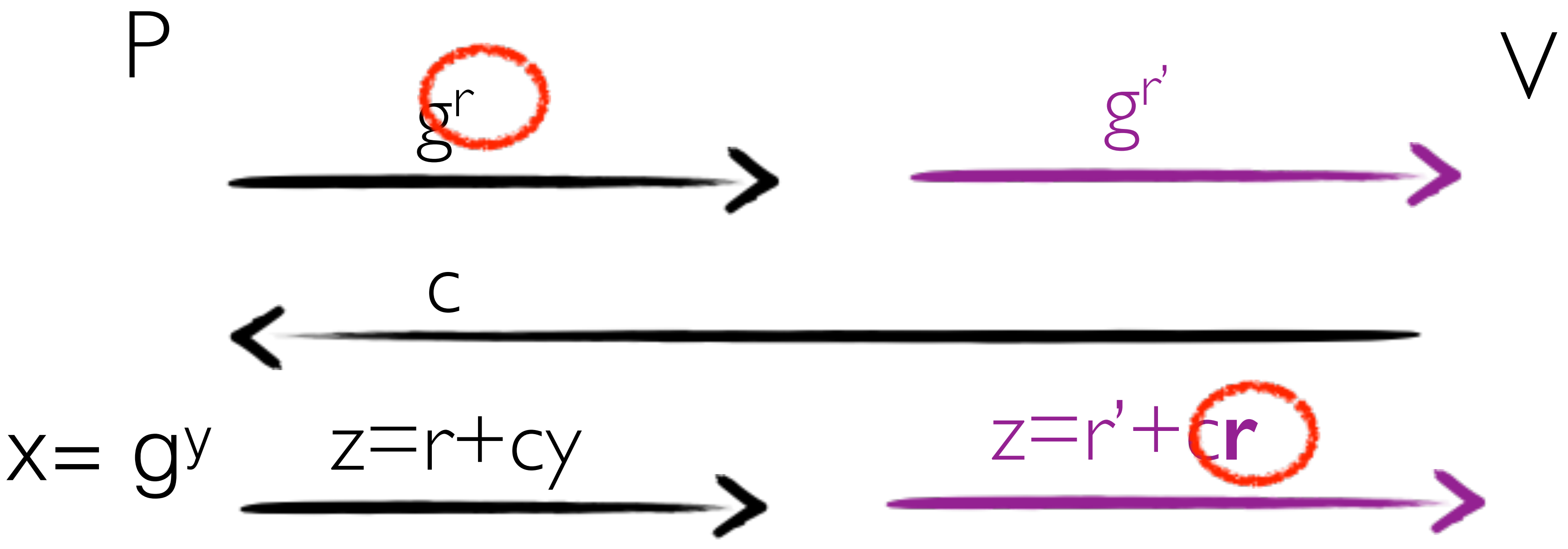
$x = g^y$

$z=r+cy$

$z=r'+c\mathbf{r}$

Our Transform

From PoK to Adaptive-Input PoK



P

V

g^r

$g^{r'}$

c

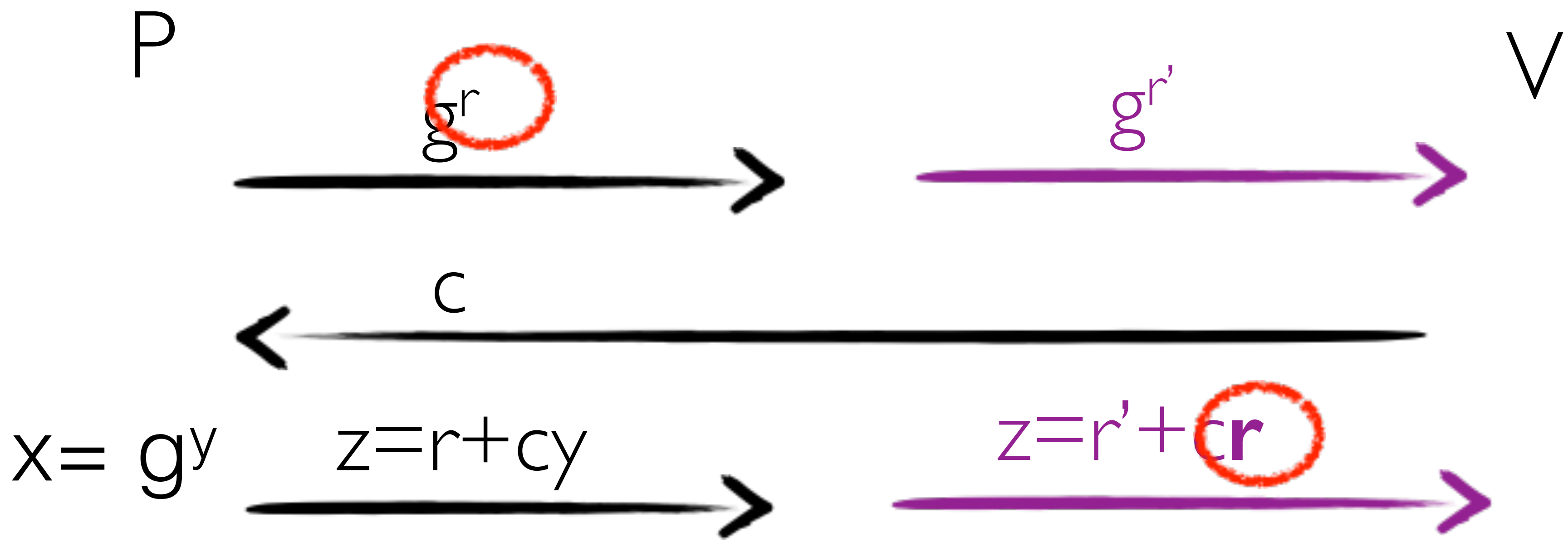
$x= g^y$

$z=r+cy$

$z=r'+c\mathbf{r}$

Our Transform

From PoK to Adaptive-Input PoK



P

V

g^r

$g^{r'}$

c

$x = g^y$

$z=r+cy$

$z=r'+c\mathbf{r}$

Our transform applies to the class described in [**C**ramer**96**, **M**aurer**15**, **C**ramer**D**amgard**98**]

e.g. Schnorr, Guillou–Quisquater, Diffie–Hellman, Multiplication proof for pedersen commitments, …

Our Results

1) From PoK to Adaptive-Input PoK

2) Bridging the gap

$\mathbf{R}_0$ OR $\mathbf{R}_1$: Bridging the Gap

In theory

[LS90]

In practice

[CDS94]

[CPS+ TCC 2016-A]

This work

$\mathbf{R}_0$ OR $\mathbf{R}_1$: Bridging the Gap

In theory

[LS90]

- Delayed-Input Completeness

In practice

[CDS94]

- Completeness

[CPS+ TCC 2016-A]

- Semi-Delayed Input Completeness

This work

- Delayed-Input Completeness: All input ∑-protocols have to be Delayed-Input

$\mathbf{R}_0$ OR $\mathbf{R}_1$: Bridging the Gap

In theory

[LS90]

- Delayed-Input Completeness
- Adaptive-Input PoK

In practice

[CDS94]

- Completeness
- Proof of Knowledge

[CPS+ TCC 2016-A]

- Semi-Delayed Input Completeness
- Proof of Knowledge

This work

- Delayed-Input Completeness: All input ∑-protocols have to be Delayed-Input
- Proof of Knowledge

R_0 OR R_1: Bridging the Gap

In theory

[LS90]

- Delayed-Input Completeness
- Adaptive-Input PoK
- Adaptive-Input WI

In practice

[CDS94]

- Completeness
- Proof of Knowledge
- Witness Indistinguishable

[CPS$^+$ TCC 2016-A]

- Semi-Delayed Input Completeness
- Proof of Knowledge
- Semi-Adaptive Input WI: one of two instances is adaptively chosen by V*

This work

- Delayed-Input Completeness: All input ∑-protocols have to be Delayed-Input
- Proof of Knowledge
- Adaptive-Input WI

R_0 OR R_1: Bridging the Gap

In theory

[LS90]

- Delayed-Input Completeness
- Adaptive-Input PoK
- Adaptive-Input WI
- Assumption: OWP

In practice

[CDS94]

- Completeness
- Proof of Knowledge
- Witness Indistinguishable
- Assumption: none

[CPS+ TCC 2016-A]

- Semi-Delayed Input Completeness
- Proof of Knowledge
- Semi-Adaptive Input WI: one of two instances is adaptively chosen by V*
- Assumption: none

This work

- Delayed-Input Completeness: All input ∑-protocols have to be Delayed-Input
- Proof of Knowledge
- Adaptive-Input WI
- Assumption: DDH

R_0 OR R_1: Bridging the Gap

In theory

[LS90]

- Delayed-Input Completeness
- Adaptive-Input PoK
- Adaptive-Input WI
- Assumption: OWP
- Works with multiple OR compositions

In practice

[CDS94]

- Completeness
- Proof of Knowledge
- Witness Indistinguishable
- Assumption: none
- Works with multiple OR compositions

[CPS$^+$ TCC 2016-A]

- Semi-Delayed Input Completeness
- Proof of Knowledge
- Semi-Adaptive Input WI: one of two instances is adaptively chosen by V*
- Assumption: none
- Works with only one OR composition

This work

- Delayed-Input Completeness: All input ∑-protocols have to be Delayed-Input
- Proof of Knowledge
- Adaptive-Input WI
- Assumption: DDH
- Works with multiple OR compositions

R_0 OR R_1: Bridging the Gap

In theory

[LS90]

- Delayed-Input Completeness
- Adaptive-Input PoK
- Adaptive-Input WI
- Assumption: OWP
- Works with multiple OR compositions
- Requires NP-reduction and gives Computational WI

In practice

[CDS94]

- Completeness
- Proof of Knowledge
- Witness Indistinguishable
- Assumption: none
- Works with multiple OR compositions
- No NP-reduction and gives Perfect WI

[CPS+ TCC 2016-A]

- Semi-Delayed Input Completeness
- Proof of Knowledge
- Semi-Adaptive Input WI: one of two instances is adaptively chosen by V*
- Assumption: none
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This work

- Delayed-Input Completeness: All input ∑-protocols have to be Delayed-Input
- Proof of Knowledge
- Adaptive-Input WI
- Assumption: DDH
- Works with multiple OR compositions
- No NP-reduction and gives Computational WI

R_0 OR R_1: Bridging the Gap

In theory

[LS90]

- Delayed-Input Completeness
- Adaptive-Input PoK
- Adaptive-Input WI
- Assumption: OWP
- Works with multiple OR compositions
- Requires NP-reduction and gives Computational WI
- Applicable to All NP

In practice

[CDS94]

- Completeness
- Proof of Knowledge
- Witness Indistinguishable
- Assumption: none
- Works with multiple OR compositions
- No NP-reduction and gives Perfect WI
- Restricted to Σ-protocols

[CPS^{+} TCC 2016-A]

- Semi-Delayed Input Completeness
- Proof of Knowledge
- Semi-Adaptive Input WI: one of two instances is adaptively chosen by V*
- Assumption: none
- Works with only one OR composition
- No NP-reduction and gives Perfect WI
- Restricted to (a large class of) Σ-protocols

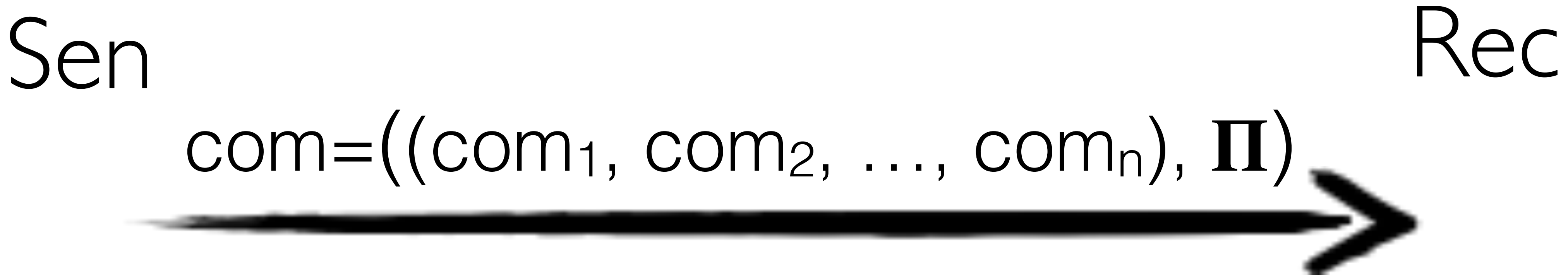
This work

- Delayed-Input Completeness: All input Σ-protocols have to be Delayed-Input
- Proof of Knowledge
- Adaptive-Input WI
- Assumption: DDH
- Works with multiple OR compositions
- No NP-reduction and gives Computational WI
- Restricted to (a large class of) Σ-protocols

Comparison: Summary

	Assumption	Completeness	Adaptive WI	Adaptive PoK	Online Efficiency
[LS90]	OWP	Delayed-Input	k out of n (all adaptive)	k out of n	NP-reduction
[CDS94]	/	/	/	k out of n*	Entire protocol
[CPSSV16]	/	Semi-Delayed Input	1 out of 2 (1 adaptive)	k out of n*	Entire protocol
This work	**DDH**	Delayed-Input	k out of n (all adaptive)	k out of n*	≤ CDS94

Our Construction: Tools



Sen

Rec

com=((com$_1$, com$_2$, …, com$_n$), $\mathbf{\Pi}$)

- (K,N) Trapdoor Commitment

Our Construction: Tools

At least k are perfectly binding

Sen

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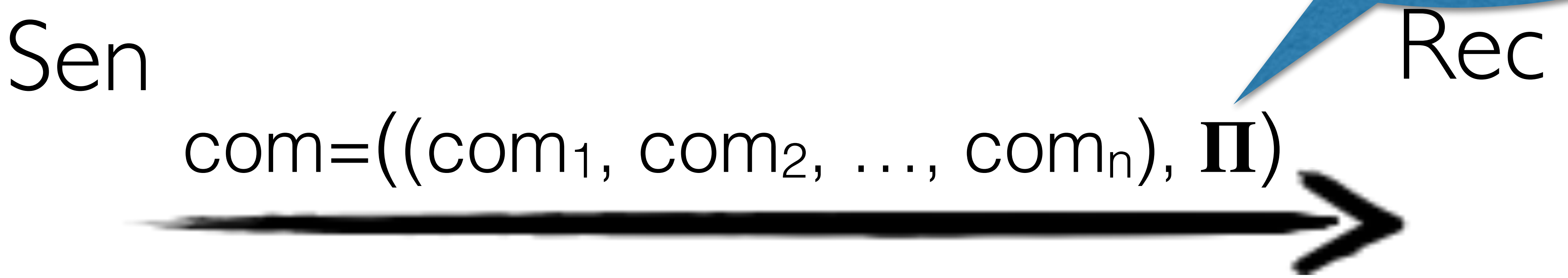
Our Construction: Tools

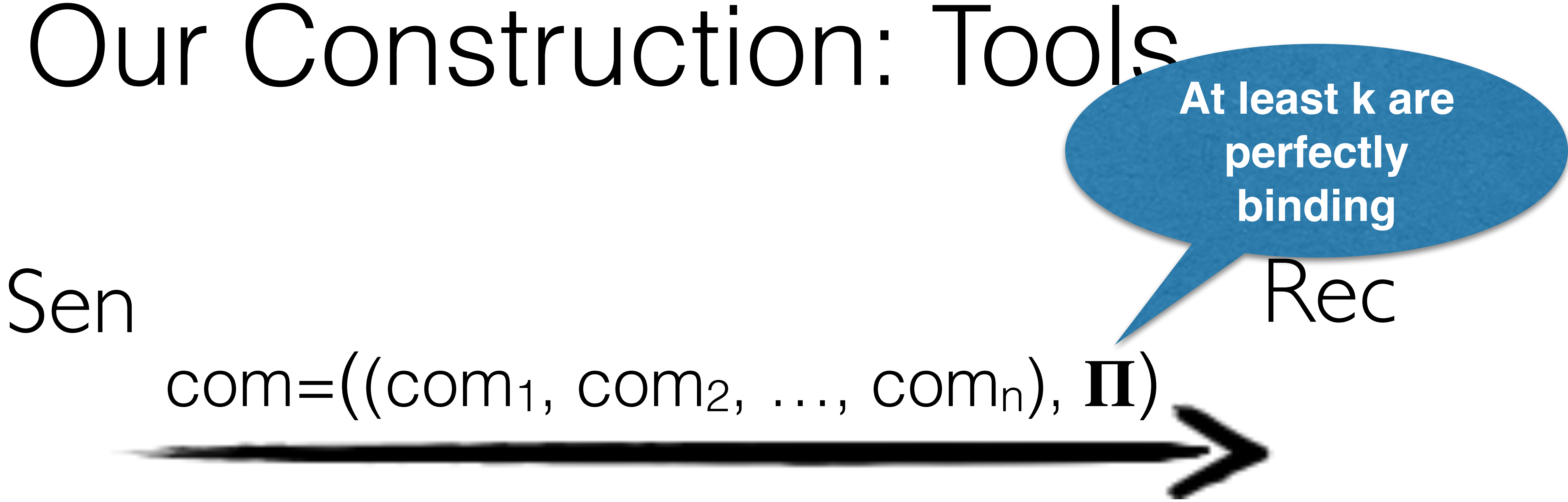


At least k are perfectly binding

- (K,N) Trapdoor Commitment
  - $\mathrm{Com}_{KN}(m_1, m_2, \ldots, m_n)$ → com
  - $\mathrm{Open}_{KN}(\mathrm{com}, m_1^*, m_2^*, \ldots, m_{n-k}^*)$ → dec

n-k commitments can be equivocated

Our Construction: Tools



At least k are perfectly binding

Sen

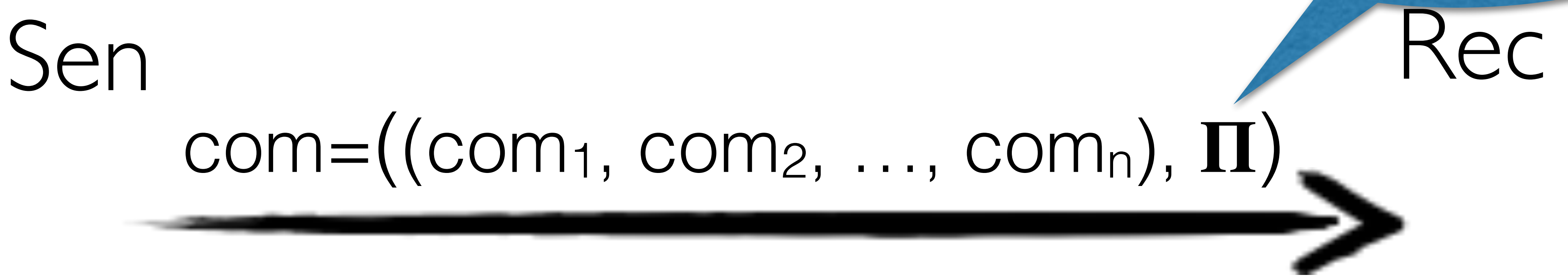
Rec

com=((com$_1$, com$_2$, …, com$_n$), $\mathbf{\Pi}$)

- (K,N) Trapdoor Commitment
  - Com$_{KN}$(m_1, m_2, …, m_n) → com
  - Open$_{KN}$(com, m_1^*, m_2^*, …, $m_{n\text{-}k}^*$) → dec
- $\sum$: Delayed-Input Σ-protocol for the relation $\mathbf{R}$

n-k commitments can be equivocated

Our Construction: Tools



Sen

Rec

com=((com$_1$, com$_2$, …, com$_n$), $\mathbf{\Pi}$)

At least k are perfectly binding

- (K,N) Trapdoor Commitment
  - Com$_{KN}$(m$_1$, m$_2$, …, m$_n$) → com
  - Open$_{KN}$(com, m$_1^*$, m$_2^*$, …, m$_{n-k}^*$) → dec

n-k commitments can be equivocated

- ∑: Delayed-Input ∑-protocol for the relation R
- Sim$_∑$: SHVZK simulator for ∑

Our Construction: Main Idea

e.g. k=1, n=2

$\mathbf{R}$ OR $\mathbf{R}$

P

V

$P_{\sum} \rightarrow a_1, a_2$

Our Construction: Main Idea

e.g. k=1, n=2

$\mathbf{R}$ OR $\mathbf{R}$

P V

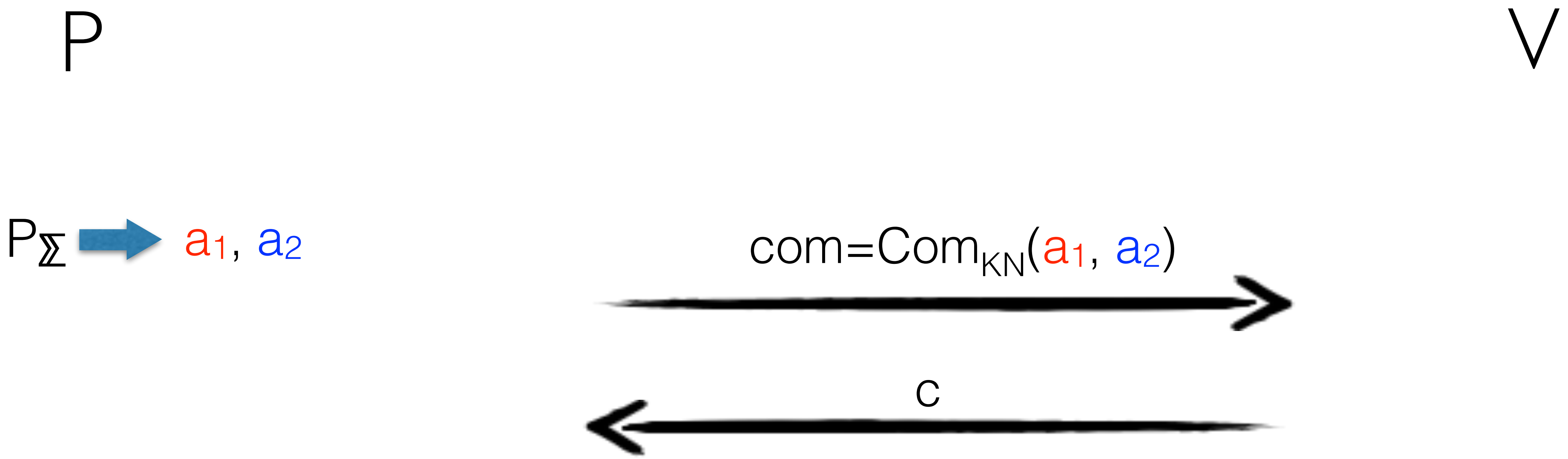
$P_{\Sigma} \rightarrow a_1, a_2$

$$\xrightarrow{\text{com}=\text{Com}_{KN}(a_1,\, a_2)}$$

Our Construction: Main Idea

e.g. k=1, n=2

$\mathbf{R}$ OR $\mathbf{R}$



P

V

P_Σ → a_1, a_2

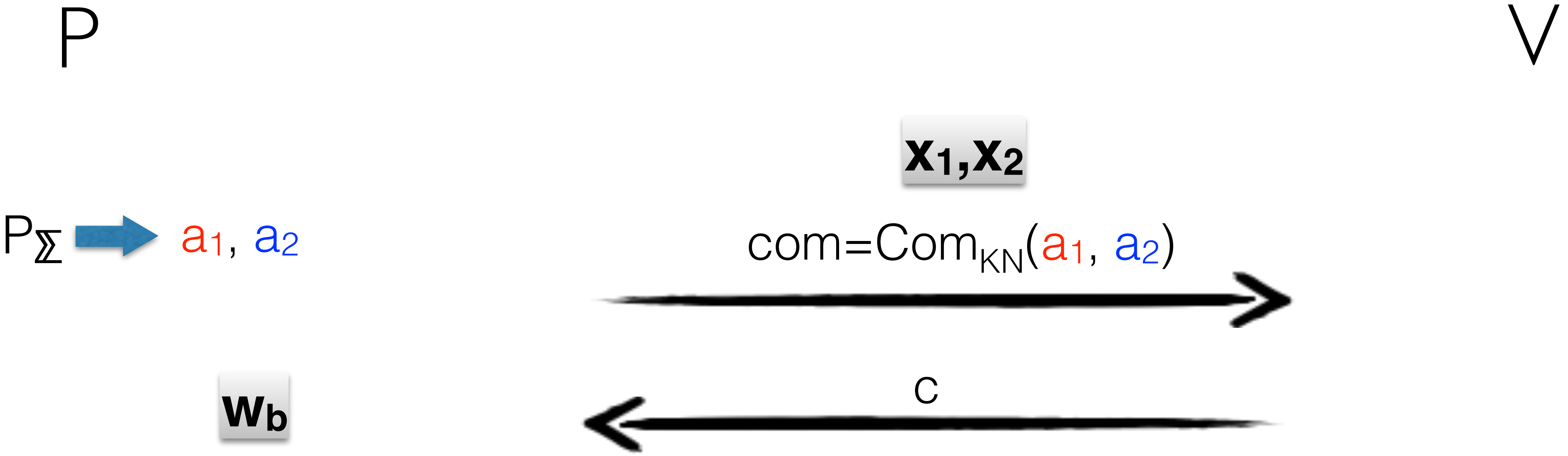
com=$\mathrm{Com}_{KN}(a_1, a_2)$

c

Our Construction: Main Idea

e.g. k=1, n=2

R OR R



P

V

x_1, x_2

P_Σ → a_1, a_2

com=$\mathrm{Com}_{KN}(a_1, a_2)$

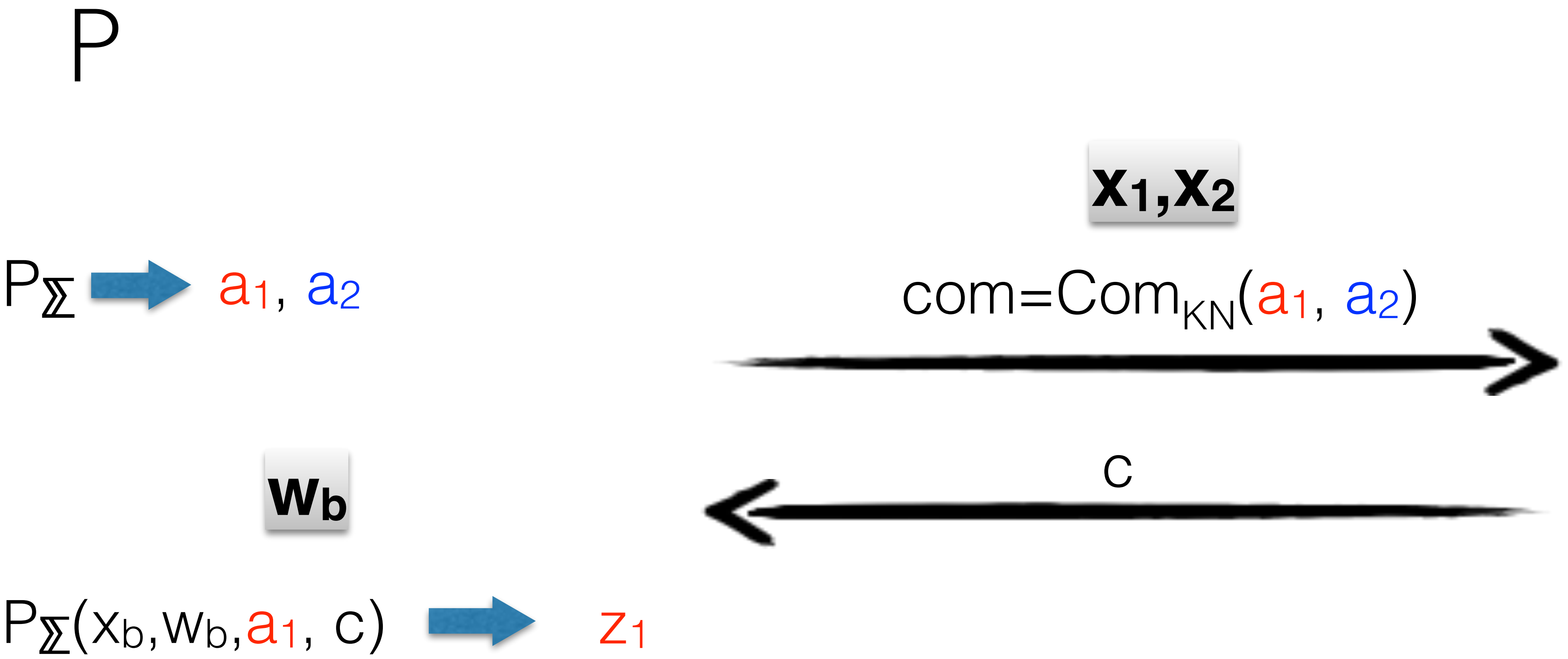
c

w_b

Our Construction: Main Idea

e.g. k=1, n=2

$\mathbf{R}$ OR $\mathbf{R}$



P

V

$\mathbf{x_1,x_2}$

$P_\Sigma \rightarrow a_1, a_2$

$com = Com_{KN}(a_1, a_2)$

c

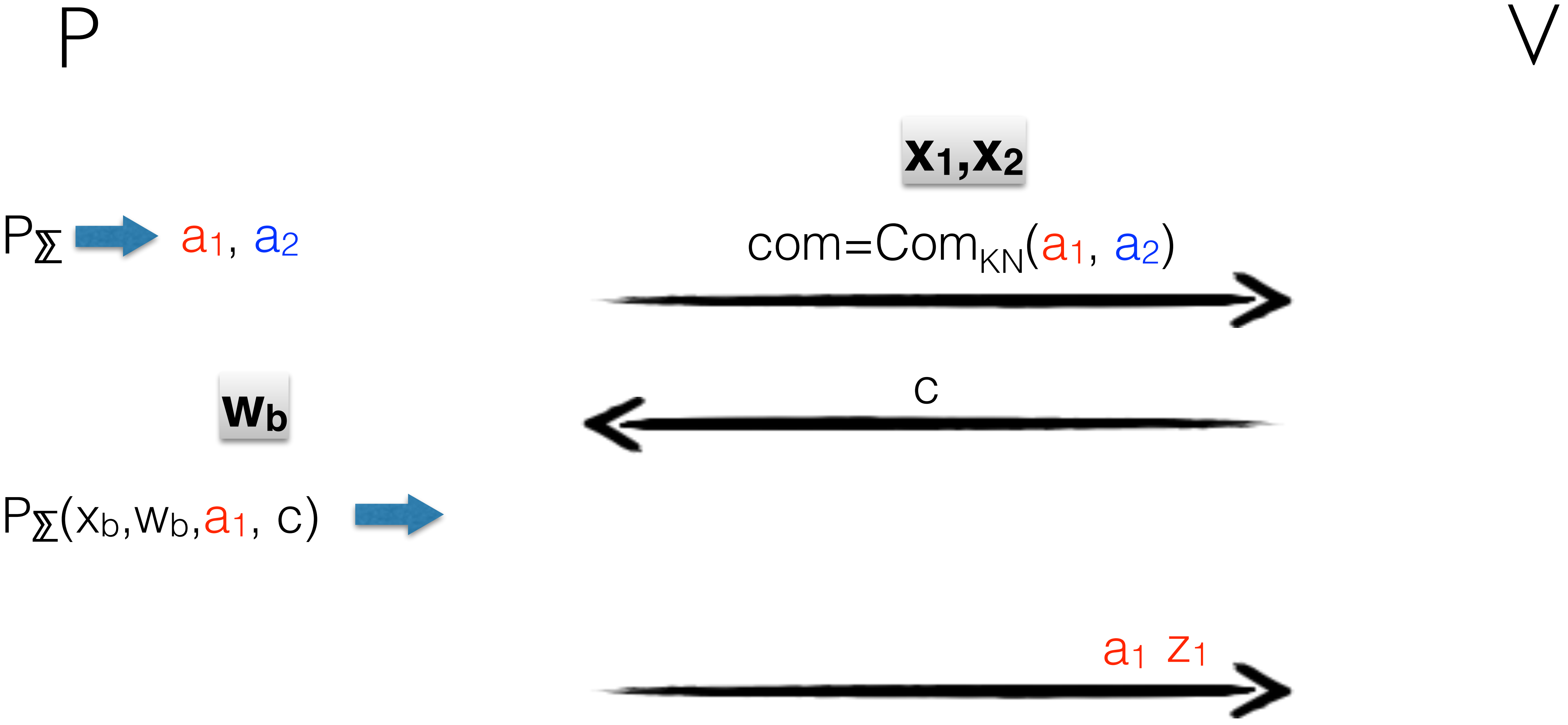
$\mathbf{w_b}$

$P_\Sigma(x_b, w_b, a_1, c) \rightarrow z_1$

Our Construction: Main Idea

e.g. k=1, n=2

R OR R



P

V

x_1,x_2

P_Σ → a_1, a_2

com=$\text{Com}_{KN}(a_1, a_2)$

c

w_b

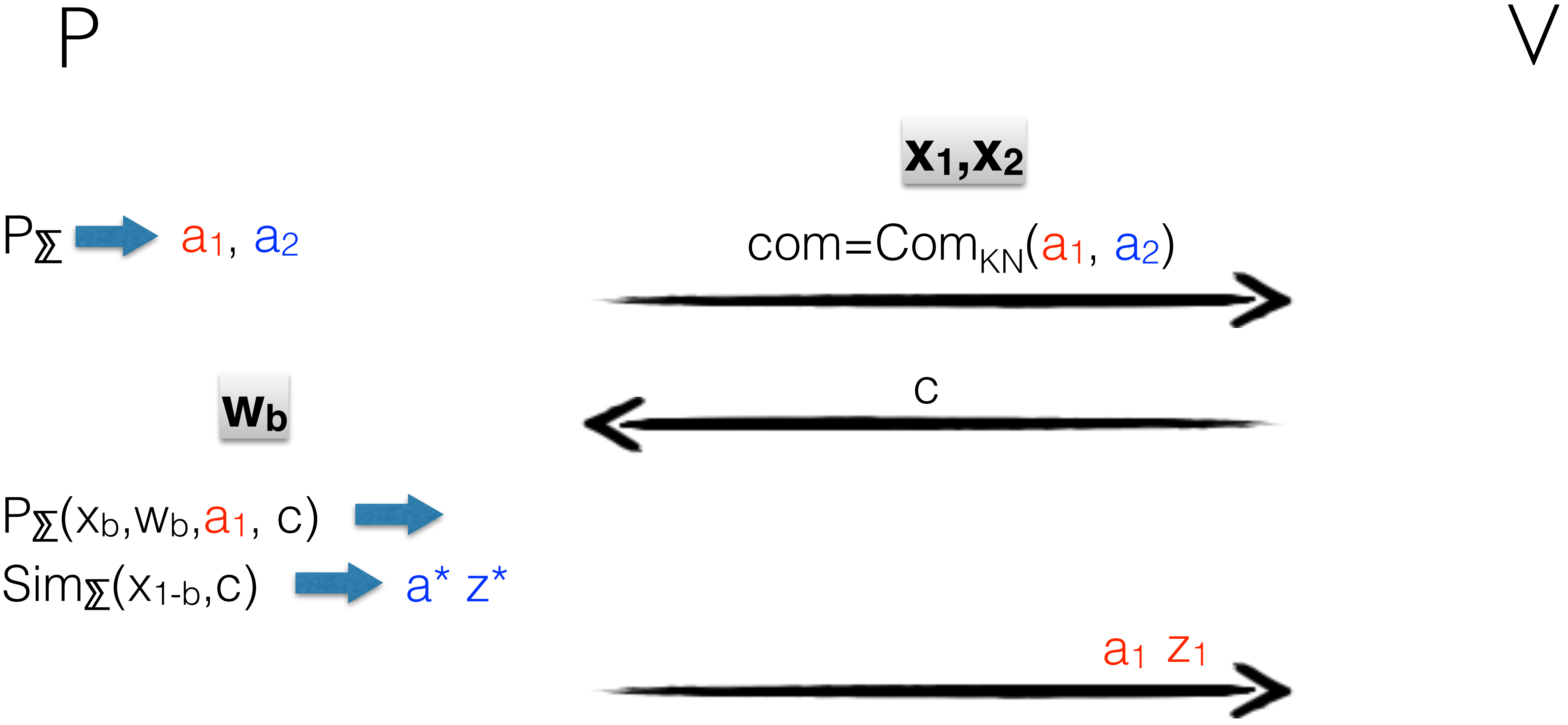
$P_\Sigma(x_b,w_b,a_1, c)$ →

a_1 z_1

Our Construction: Main Idea

e.g. k=1, n=2

$\mathbf{R}$ OR $\mathbf{R}$



P

V

x_1,x_2

P_Σ → a_1, a_2

com=$\mathrm{Com}_{KN}(a_1, a_2)$

c

w_b

$P_\Sigma(x_b,w_b,a_1, c)$ →

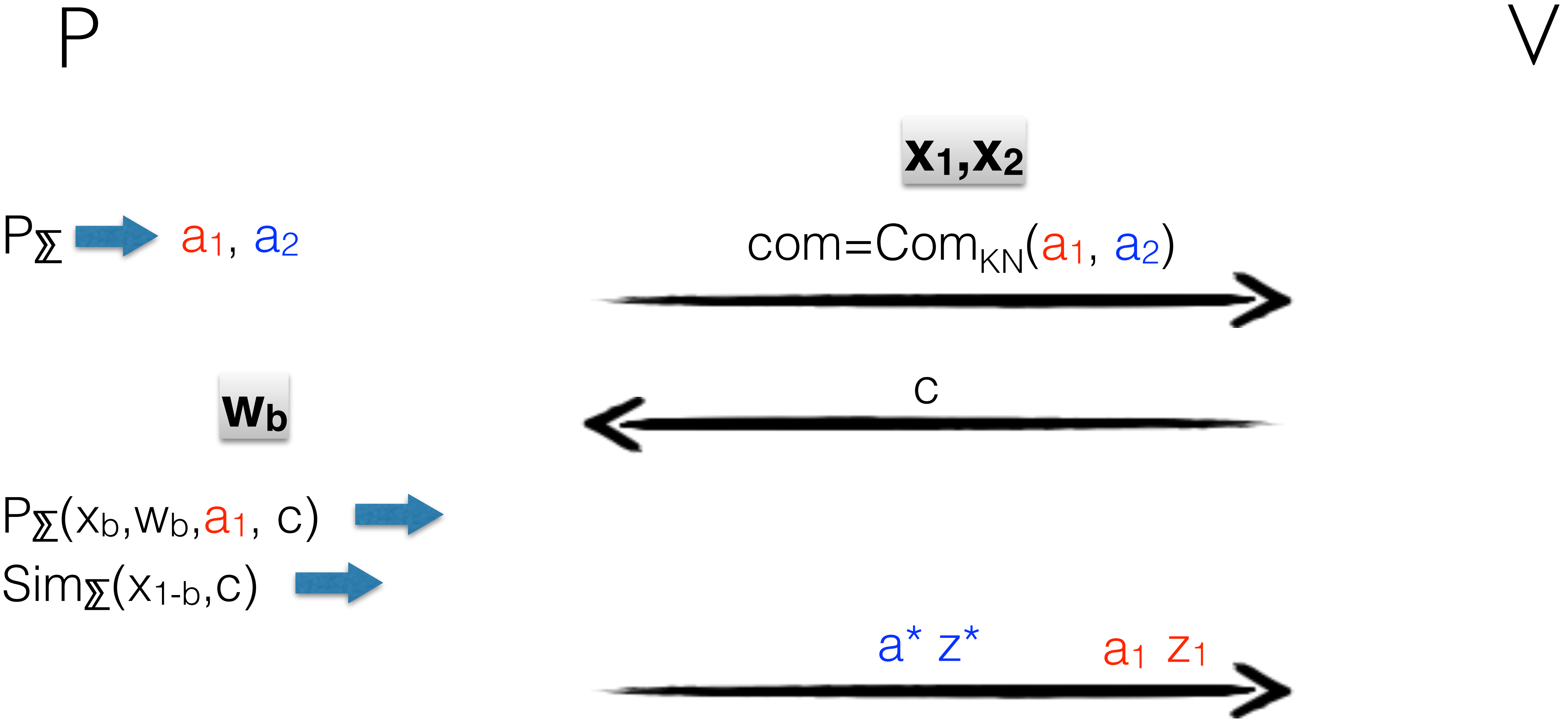
$\mathrm{Sim}_\Sigma(x_{1-b},c)$ → a^* z^*

a_1 z_1

Our Construction: Main Idea

e.g. k=1, n=2

R OR R



P

V

x_1,x_2

P_Σ → a_1, a_2

com=$\mathrm{Com}_{KN}(a_1, a_2)$

c

w_b

$P_\Sigma(x_b,w_b,a_1, c)$ →

$\mathrm{Sim}_\Sigma(x_{1-b},c)$ →

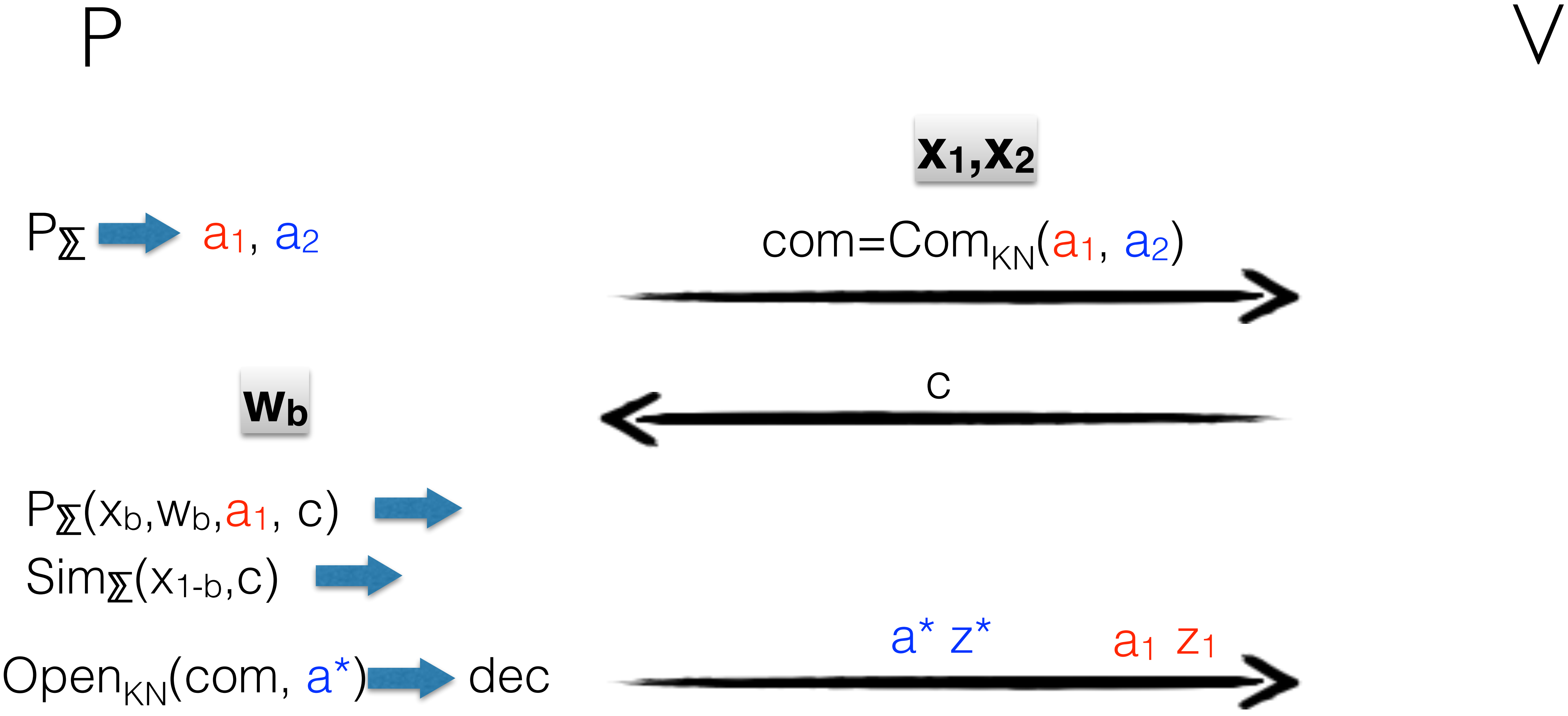
a* z*

a_1 z_1

Our Construction: Main Idea

e.g. k=1, n=2

R OR R



P

V

$\mathbf{x_1,x_2}$

P_Σ → a_1, a_2

com=Com$_{KN}$(a_1, a_2)

$\mathbf{w_b}$

c

$P_\Sigma(x_b, w_b, a_1, c)$ →

$Sim_\Sigma(x_{1-b}, c)$ →

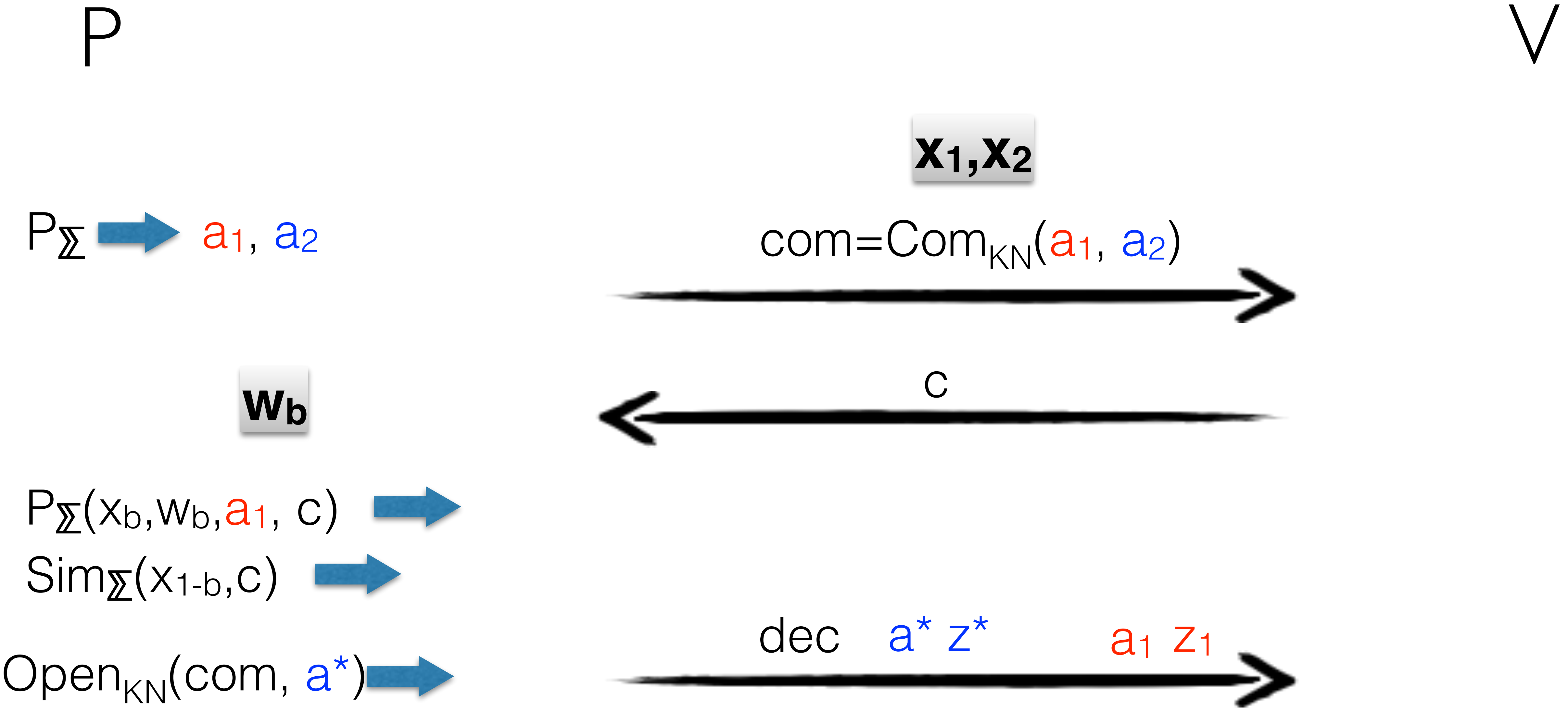
Open$_{KN}$(com, a^*) → dec

a^* z^* a_1 z_1

Our Construction: Main Idea

e.g. k=1, n=2

R OR R



P

V

x_1,x_2

P_Σ → a_1, a_2

com=$\mathrm{Com}_{KN}(a_1, a_2)$

c

w_b

$P_\Sigma(x_b,w_b,a_1, c)$ →

$\mathrm{Sim}_\Sigma(x_{1-b},c)$ →

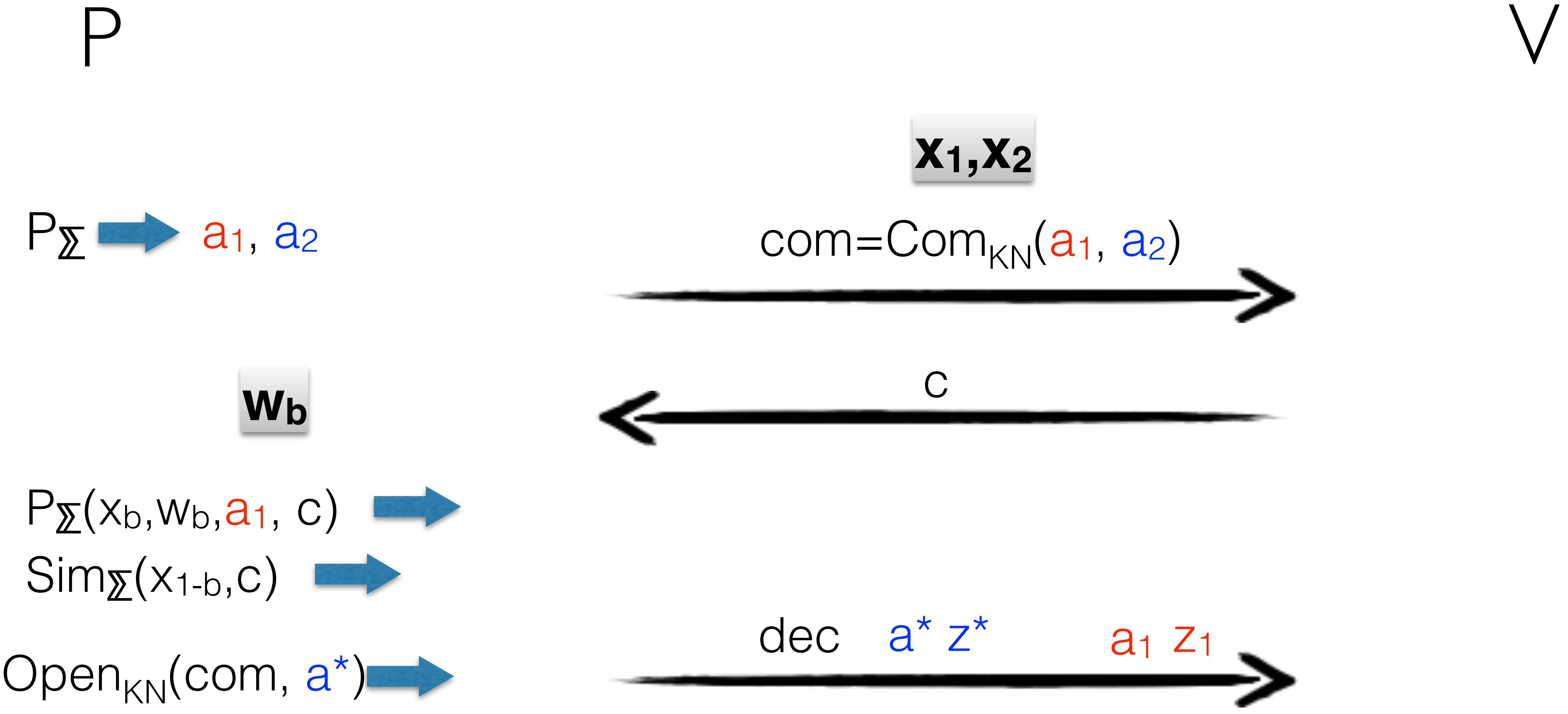
$\mathrm{Open}_{KN}(\mathrm{com}, a^*)$ →

dec a^* z^* a_1 z_1

Our Construction: Main Idea

e.g. k=1, n=2

R OR R



P

V

x_1, x_2

P_Σ → a_1, a_2

com=$\text{Com}_{KN}(a_1, a_2)$

c

w_b

$P_\Sigma(x_b, w_b, a_1, c)$ →

$\text{Sim}_\Sigma(x_{1-b}, c)$ →

$\text{Open}_{KN}(\text{com}, a^*)$ →

dec a^* z^* a_1 z_1

-Is dec a valid opening for a^*
and a_1 w.r.t com?

-Is (a^*, c ,z^*) accepting for ∑?

-Is (a_1, c ,z_1) accepting for ∑?

How to Construct an Efficient (K,N) Trapdoor Commitment

Ingredient 1: DDH

$$(g^{a},g^{b},g^{ab}) \approx (g^{a},g^{b},g^{c})$$

How to Construct an Efficient (K,N) Trapdoor Commitment

Ingredient 1: DDH

$$(g^{a},g^{b},g^{ab}) \approx (g^{a},g^{b},g^{c})$$

↑

DH tuple

How to Construct an Efficient (K,N) Trapdoor Commitment

Ingredient 1: DDH

$$(g^{a},g^{b},g^{ab}) \approx (g^{a},g^{b},g^{c})$$

↑ ↑

DH tuple non-DH tuple

How to Construct an Efficient (K,N) Trapdoor Commitment

Ingredient 2: Instance dependent trapdoor commitment (**IDTC**) from DDH

Given **T**=(g^{a},g^{b},g^{c})

Com(**T**,m) ⇒ dec, com

If T is **NDH** →

Binding Perfect
Hiding Computational

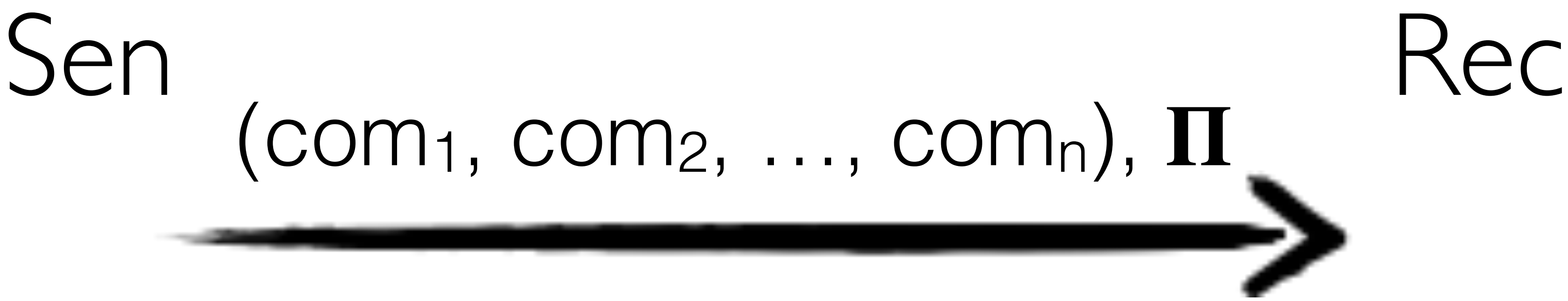
If T is **DH** →

Binding Computational

Equivocal

Constructions of IDTC follow directly from known constructions of Trapdoor Commitments from ∑-Protocols [Dam10, HL10, DN02]

How to Construct an Efficient (K,N) Trapdoor Commitment



Sen

Rec

$(\mathrm{com}_1, \mathrm{com}_2, \ldots, \mathrm{com}_n), \boldsymbol{\Pi}$

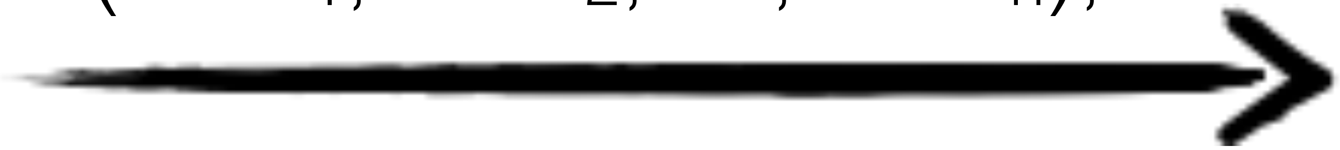
How to Construct an Efficient (K,N) Trapdoor Commitment

At least k are perfectly binding

Sen

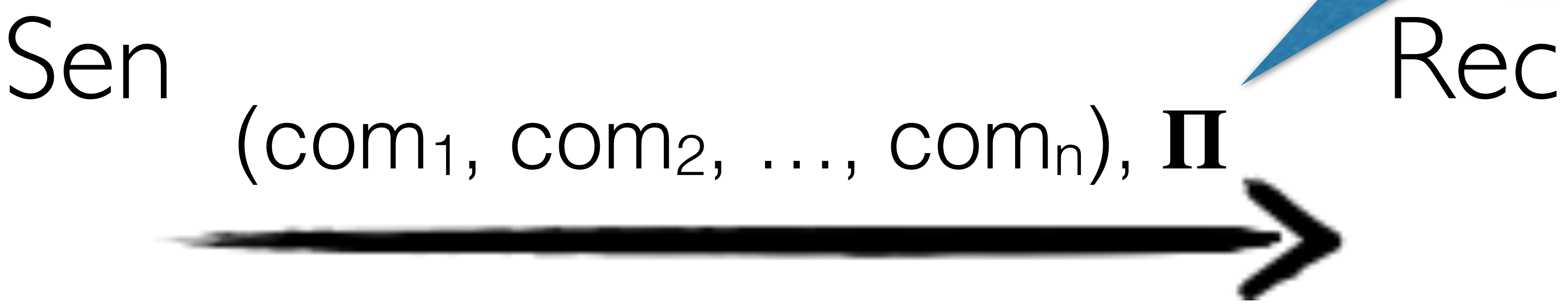
Rec

$(\mathrm{com}_1, \mathrm{com}_2, \ldots, \mathrm{com}_n), \mathbf{\Pi}$



How to Construct an Efficient (K,N) Trapdoor Commitmen

At least k are perfectly binding



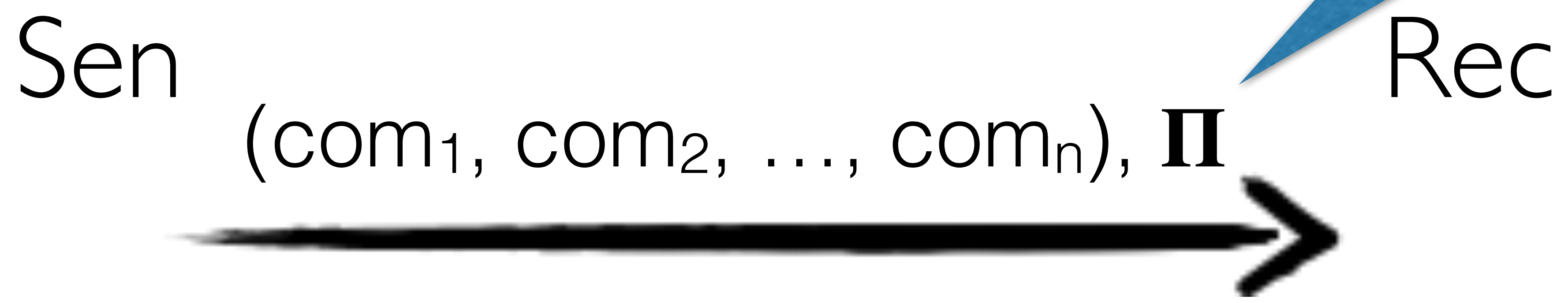
Sen

Rec

$(\mathrm{com}_1, \mathrm{com}_2, \ldots, \mathrm{com}_n), \mathbf{\Pi}$

1) Select $T_1, T_2, \ldots, T_n$

How to Construct an Efficient (K,N) Trapdoor Commitment



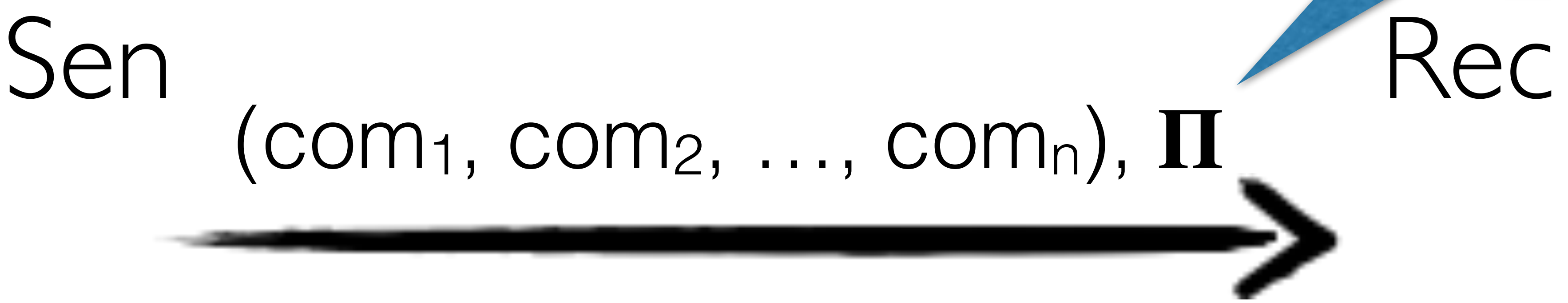
At least k are perfectly binding

1) Select T_1, T_2, …, T_n

2) Run Com($\mathbf{T_i}$,m_i) ⇒ (dec$_i$,com$_i$) **for** i=1,…,n and send (com$_1$, com$_2$, …, com$_n$)

How to Construct an Efficient (K,N) Trapdoor Commitment

At least k are perfectly binding



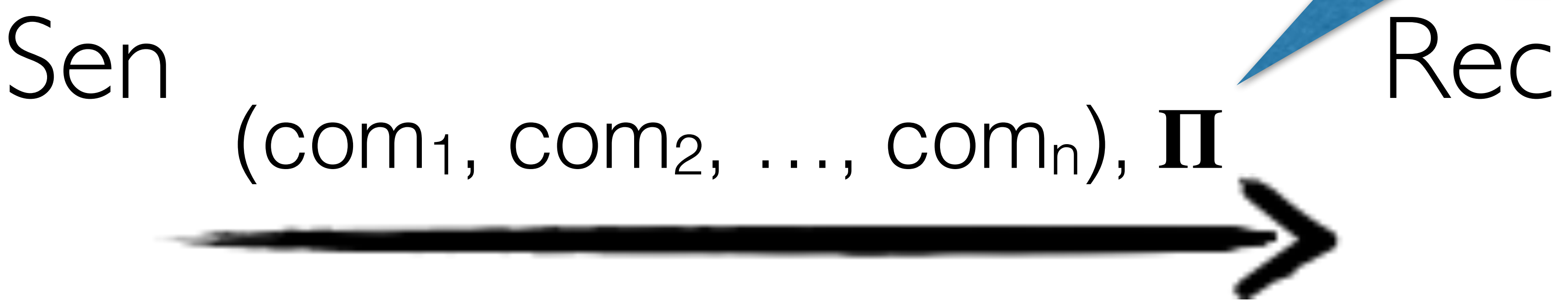
1) Select $T_1, T_2, \ldots, T_n$

2) Run $Com(\mathbf{T_i}, m_i) \Rightarrow (dec_i, com_i)$ **for** $i=1,\ldots,n$ and send $(com_1, com_2, \ldots, com_n)$

3) Prove with $\mathbf{\Pi}$ that at least **k** of the n tuples $T_1, T_2, \ldots, T_n$ are **non-DH** (prove with [CDS94])

How to Construct an Efficient (K,N) Trapdoor Commitment

At least k are perfectly binding



1) Select $T_1, T_2, \ldots, T_n$

2) Run $Com(\mathbf{T_i}, m_i) \Rightarrow (dec_i, com_i)$ **for** $i=1,\ldots,n$ and send $(com_1, com_2, \ldots, com_n)$

3) Prove with Π that at least **k** of the n tuples $T_1, T_2, \ldots, T_n$ are **non-DH** (prove with [CDS94])

Prove with **Π** that at least **k** of the n tuples $T_1, T_2, \ldots, T_n$ are **non-DH**

e.g. k=1, n=2

$\mathbf{T_1}=(g^{a_1},g^{b_1},g^{c_1})$

$\mathbf{T_2}=(g^{a_2},g^{b_2},g^{c_2})$

Prove with **Π** that at least **k** of the n tuples $T_1, T_2, \ldots, T_n$ are **non-DH**

e.g. k=1, n=2

$$\mathbf{T_1}=(g^{a_1},g^{b_1},g^{c_1}) \qquad \mathbf{T_1}'=(g^{a_1},g^{b_1},g^{a_1 \cdot b_1})$$

→

$$\mathbf{T_2}=(g^{a_2},g^{b_2},g^{c_2}) \qquad \mathbf{T_2}'=(g^{a_2},g^{b_2},g^{c_3})$$

Prove with Π that at least **k** of the n tuples $T_1, T_2, \ldots, T_n$ are **non-DH**

e.g. k=1, n=2

$\mathbf{T_1}=(g^{a_1},g^{b_1},g^{c_1})$

$\mathbf{T_2}=(g^{a_2},g^{b_2},g^{c_2})$

→

$\mathbf{T_1}'=(g^{a_1},g^{b_1},g^{a_1 \cdot b_1})$

$\mathbf{T_2}'=(g^{a_2},g^{b_2},g^{c_3})$

P ↔ V

Π^{CDS94}: "**T_1**' is DH **OR** **T_2**' is DH"

Prove with Π that at least **k** of the n tuples $T_1, T_2, \ldots, T_n$ are **non-DH**

e.g. k=1, n=2

$$\mathbf{T_1}=(g^{a_1},g^{b_1},g^{c_1}) \qquad\qquad \mathbf{T_1}'=(g^{a_1},g^{b_1},g^{a_1 \bullet b_1})$$

$$\Rightarrow$$

$$\mathbf{T_2}=(g^{a_2},g^{b_2},g^{c_2}) \qquad\qquad \mathbf{T_2}'=(g^{a_2},g^{b_2},g^{c_3})$$

P $\qquad$ Π^{CDS94}: "$\mathbf{T_1}$' is DH **OR** $\mathbf{T_2}$' is DH" $\qquad$ V

$$\longleftrightarrow$$

V accepts ⇔ One out of

T_1, **T_2** is a non-DH tuple

More Results of Our Work

- Our previous construction works for any (**k**,**n**)
- In the paper you can also find a construction that works for different NP-relations (e.g. R_{Dlog} or R_{DH}) (This construction is non-trivial ad uses as a sub-protocol the construction showed before)
- We give also a compiler that transform a ∑-Protocol (belonging to the class described in [Cra96, Mau15, CD98]) in an **Adaptive-Input PoK**
- **Open problem**
  - Is it possible to extend adaptive PoK to a larger class of ∑-Protocols?

thanks