A Full Characterization of Completeness for Two-party Randomized Function Evaluation

Daniel Kraschewski, Hemanta K. Maji, Manoj Prabhakaran, Amit Sahai

EUROCRYPT 2014

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- results hold with respect to UC as well as standalone security notions

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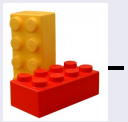
- Who has more  -bricks?  or  ?

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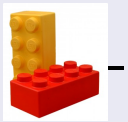
- Who has more  -bricks?  or  ?
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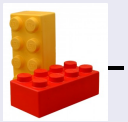
- Who has more  -bricks?  or  ?
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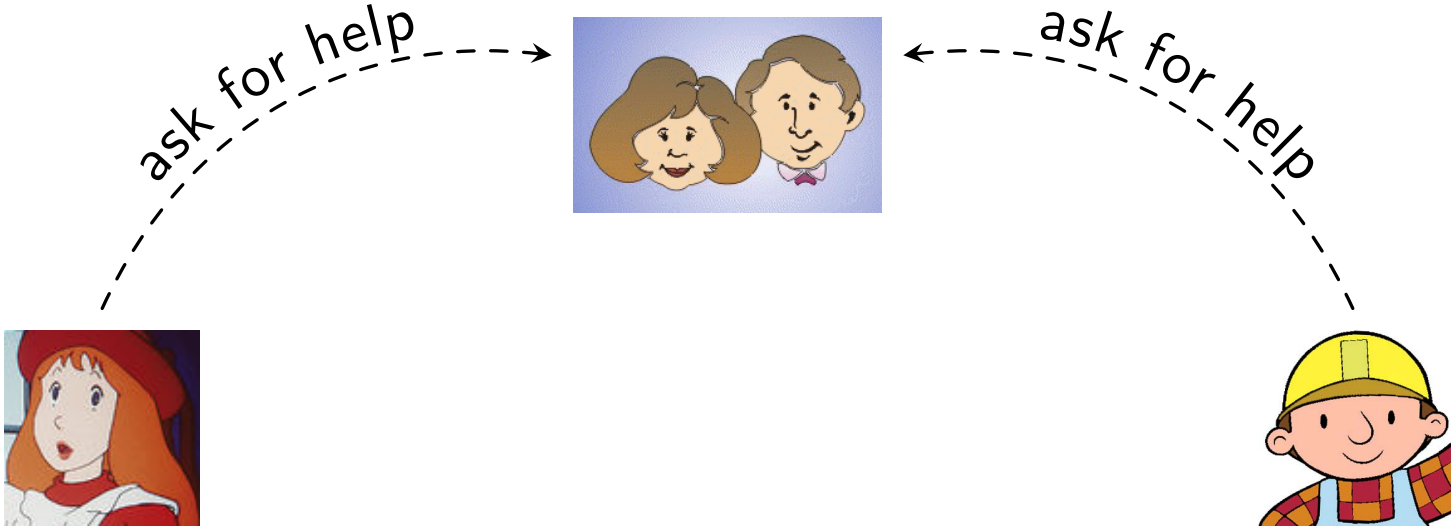

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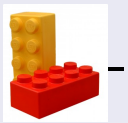
- Who has more  -bricks?  or  ?
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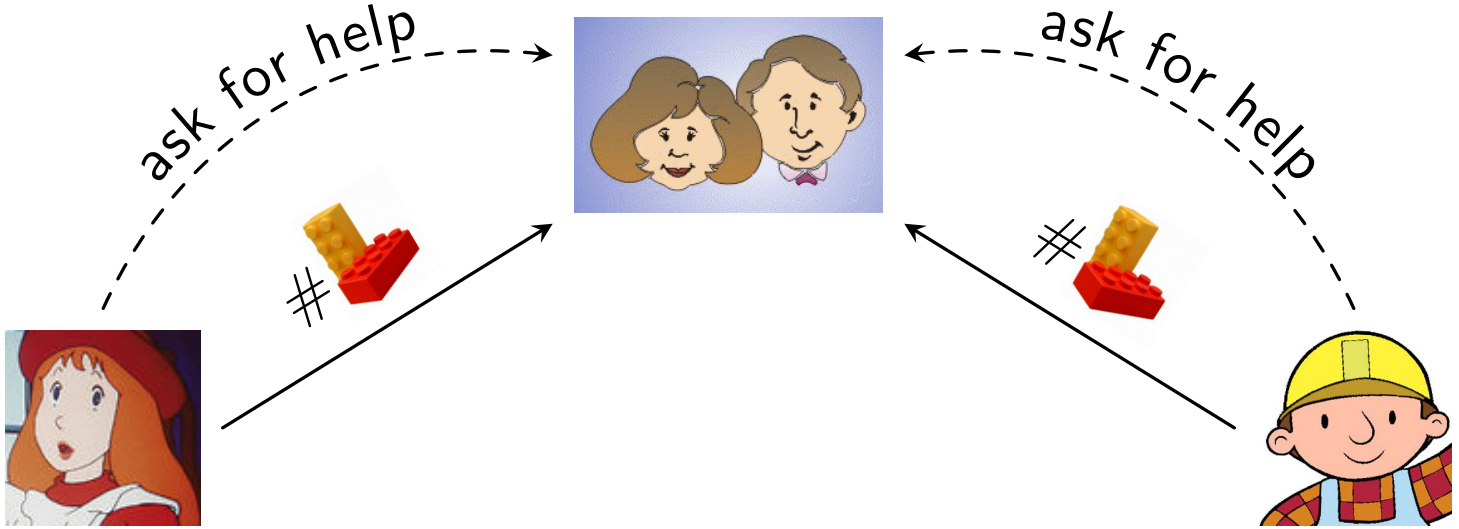

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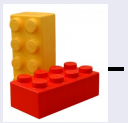
- Who has more  -bricks?  or  ?
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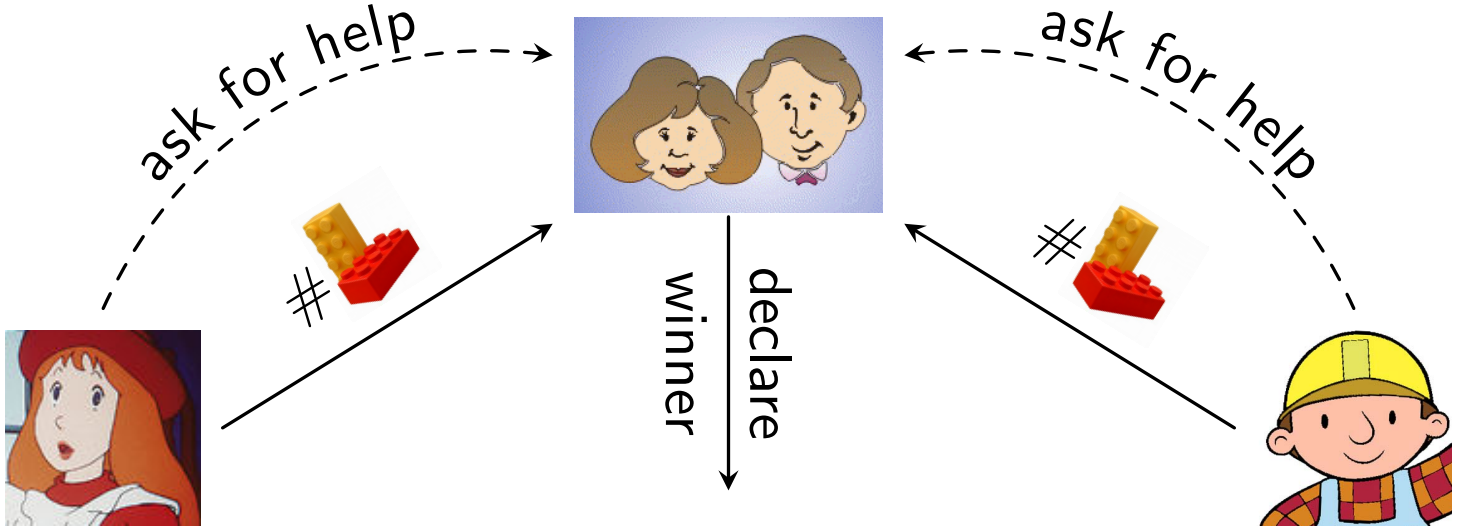

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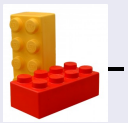
- Who has more  -bricks?  or  ?
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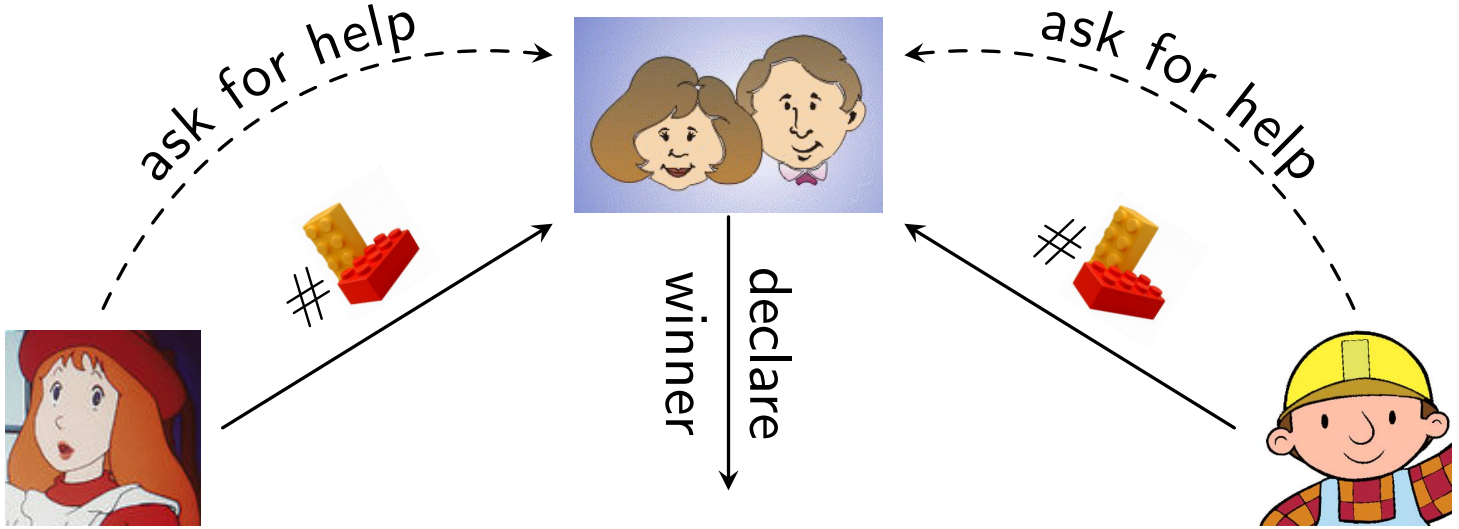

What about less general trusted 3rd parties?

Simple primitives can be very powerful

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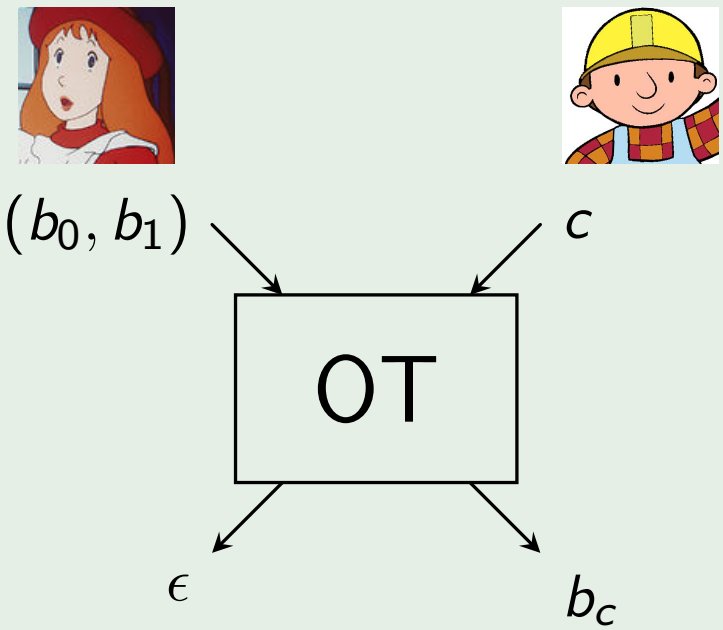
Oblivious Transfer



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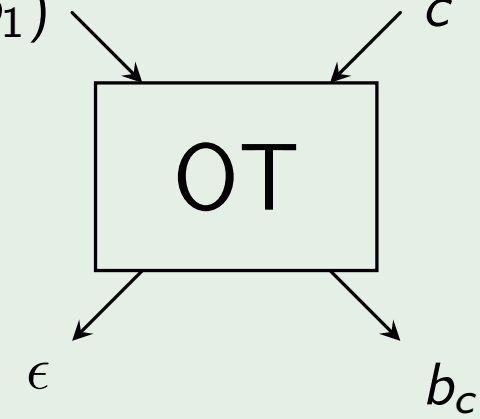

complete (= all-powerful)

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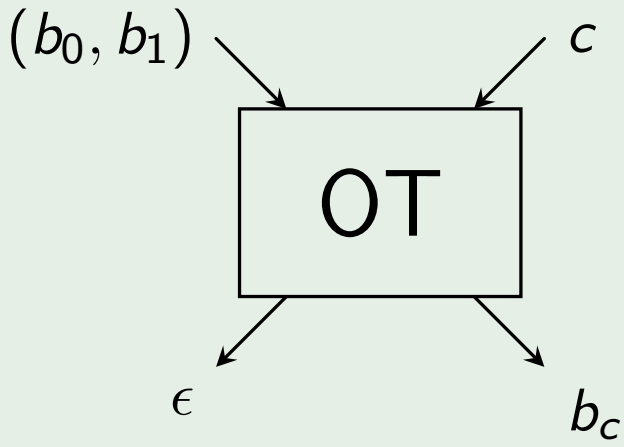

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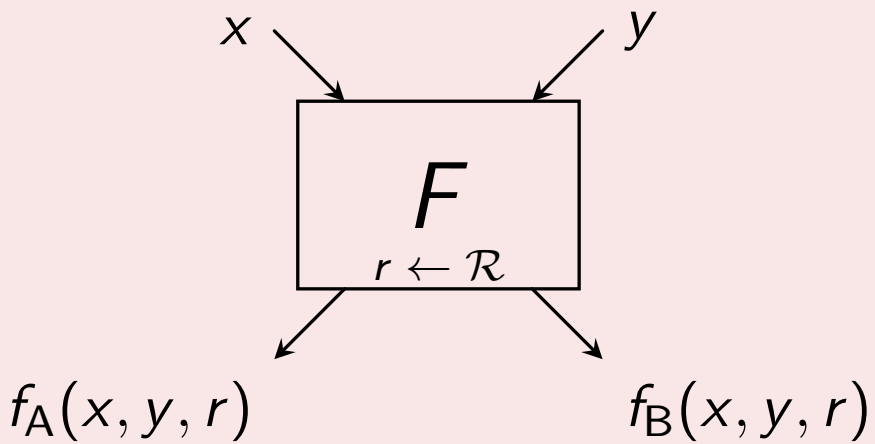
General crypto-gate $F = (f_A, f_B)$

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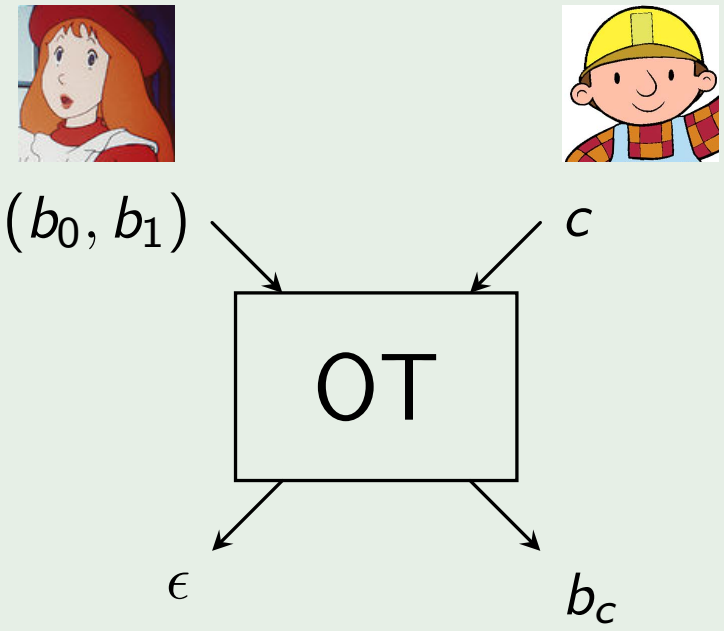

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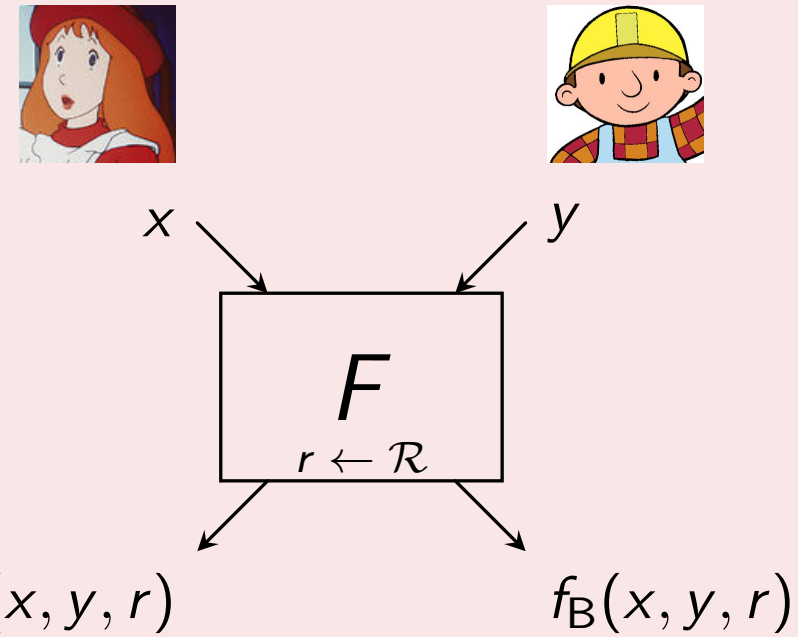

Which ones are complete?

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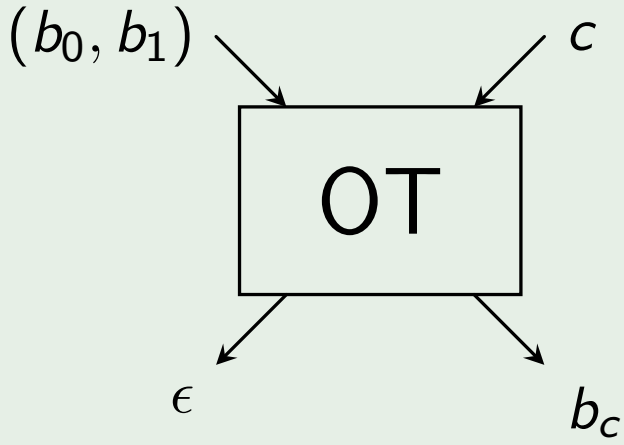

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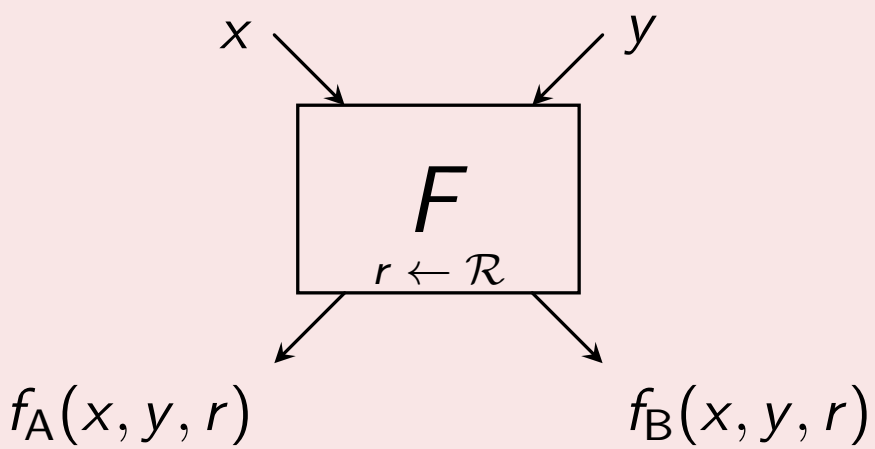

Which ones are complete?

Special cases

symmetric: $f_A = f_B$

asymmetric: $f_A = \epsilon$

Known completeness criteria

		semi-honest	malicious
deterministic			
randomized			

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* except for noisy channels [Crépeau-Kilian-88, Crépeau-Morozov-Wolf-04]

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Main results

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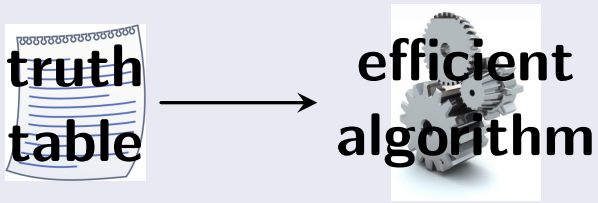
1. 

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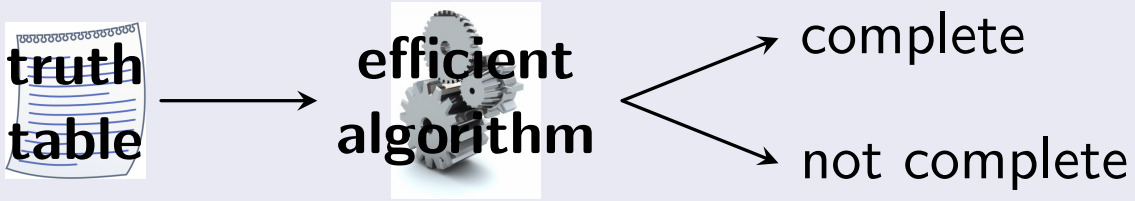
1. 

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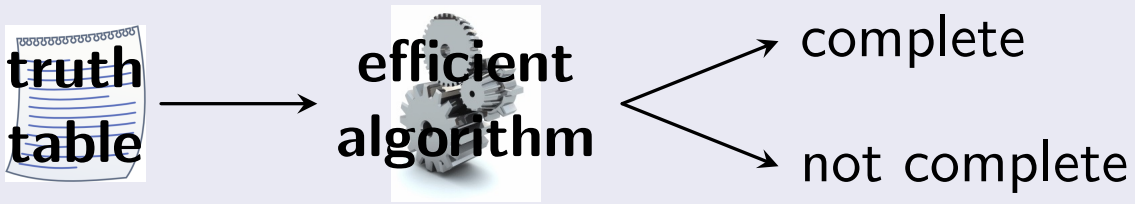
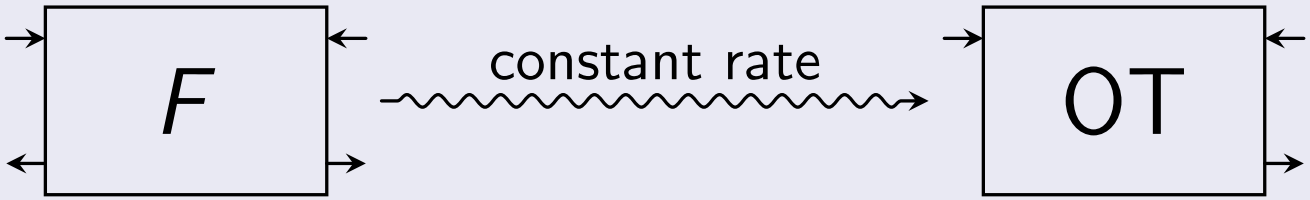
1. 

2. 

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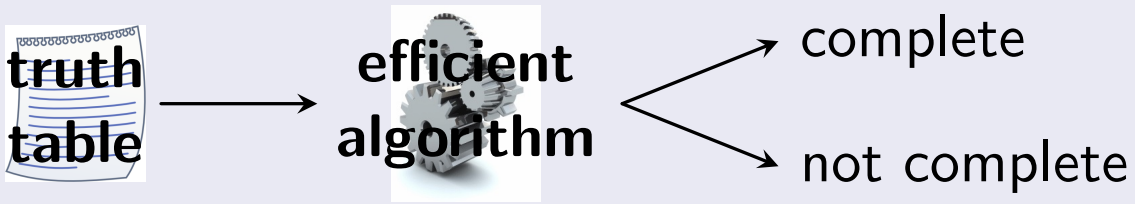
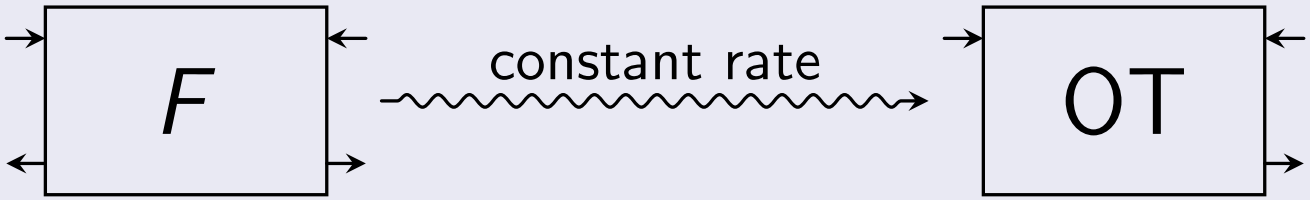
1. 

2. 

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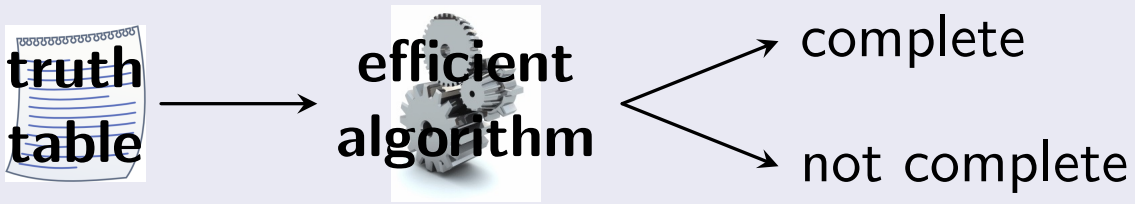
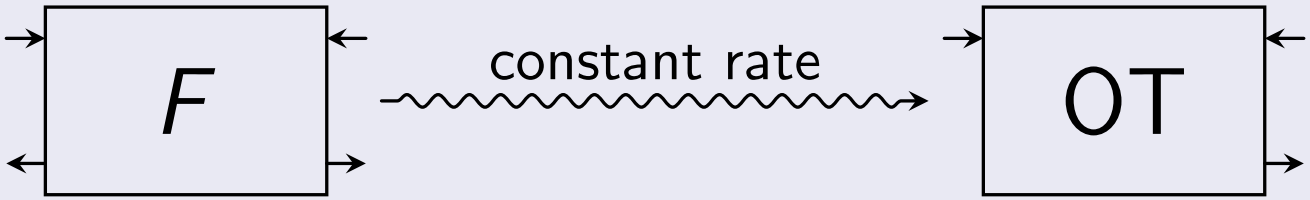
1. 

2. 

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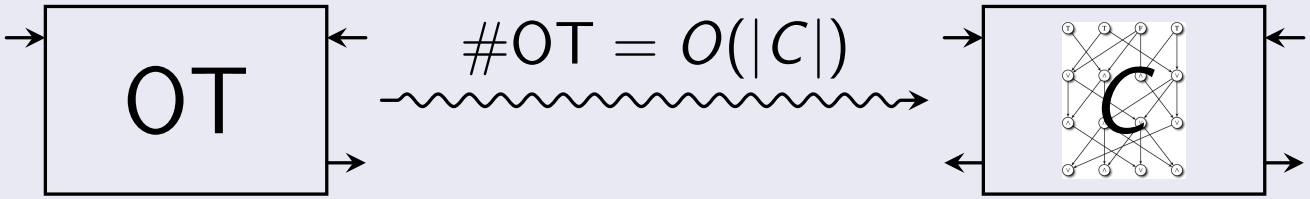

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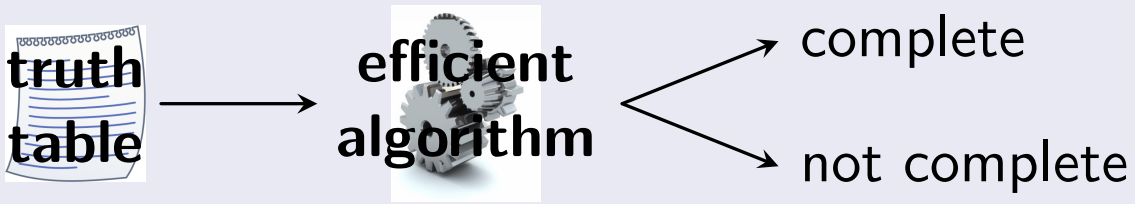
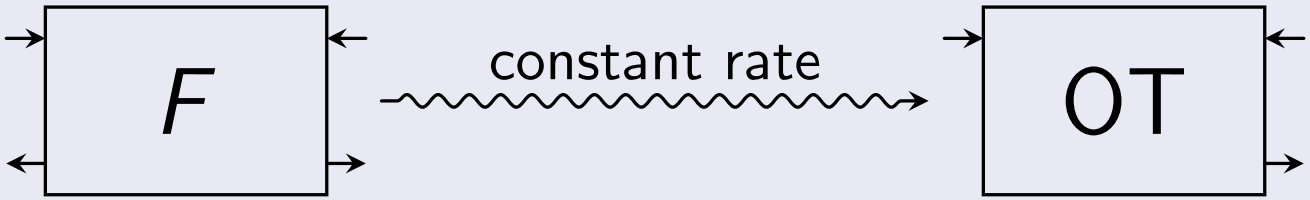
1. 

2. 

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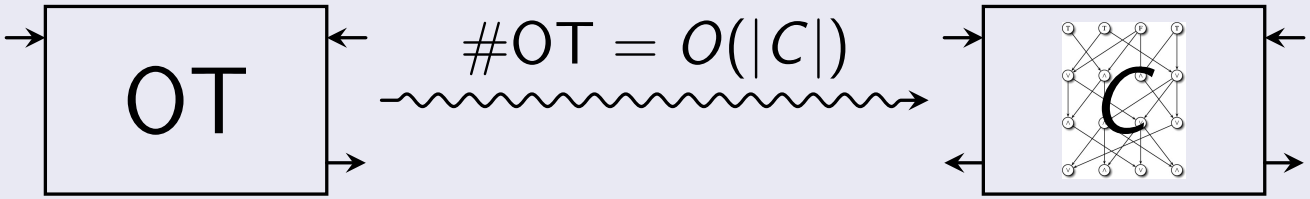

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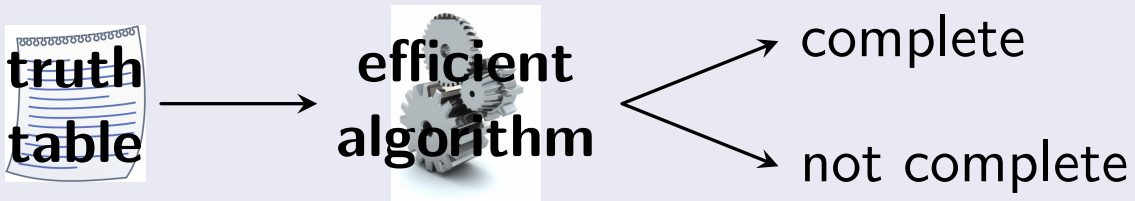
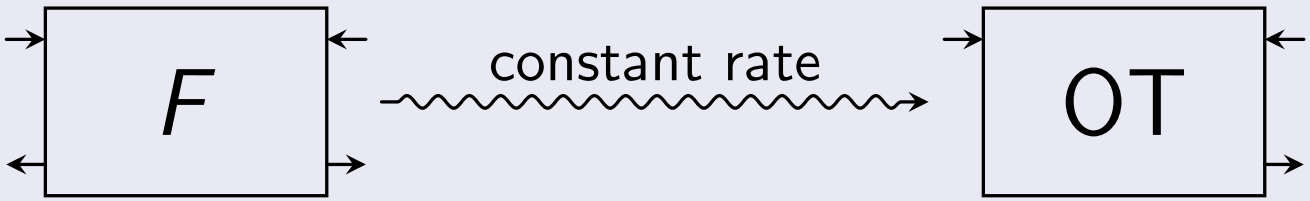
1. 

2. 

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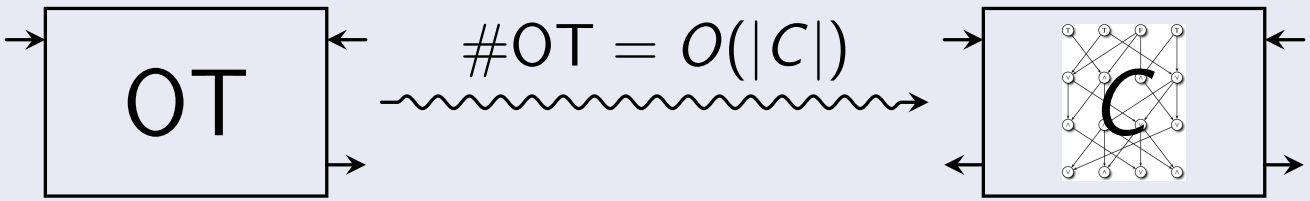

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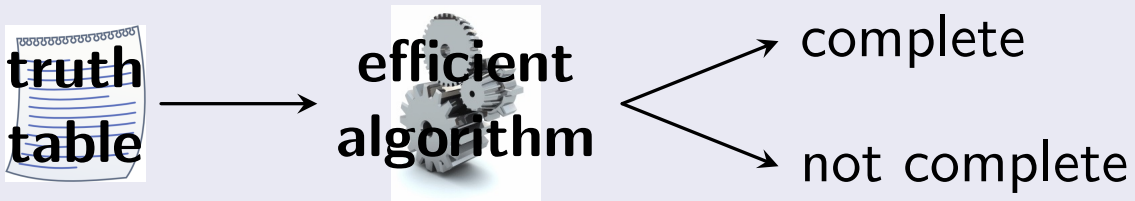
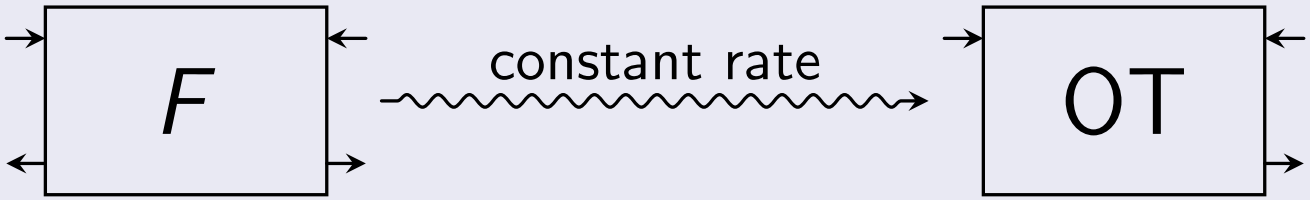
1. 

2. 

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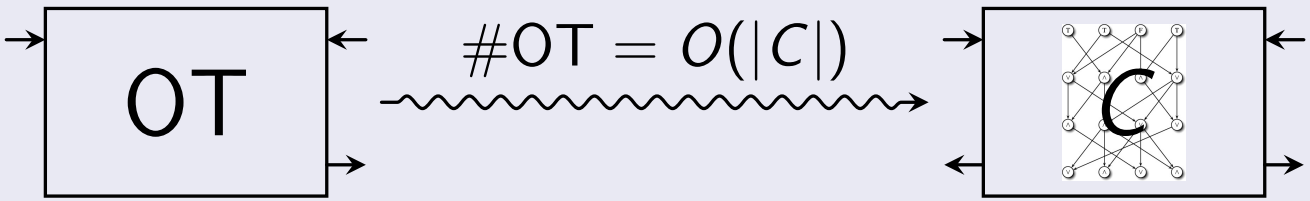

- constant-rate reduction between complete crypto-gates
- robust notion of "crypto-complexity" (independent of underlying gate)
- **new approach for lower bounds?**

Starting point: semi-honest completeness

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Representation of crypto-gates

weighted bipartite graph

- left part: views (x, a) of 
- right part: views (y, b) of 
- edges: $\Pr[a, b \mid x, y]$

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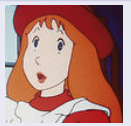
- left part: views (x, a) of 
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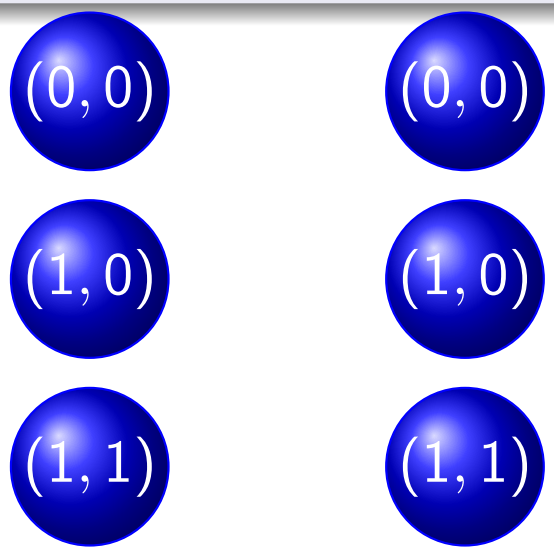

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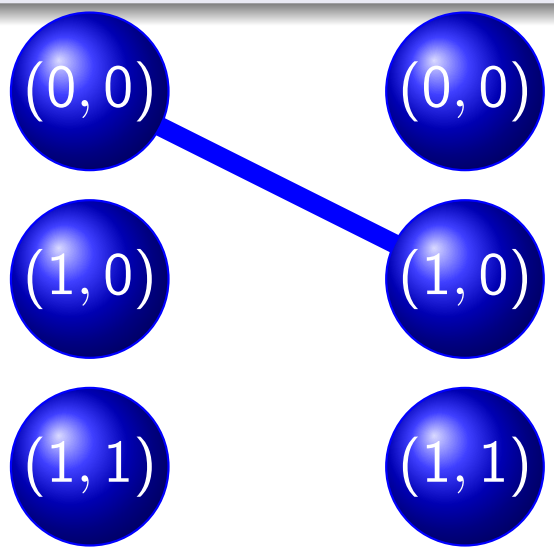

Starting point: semi-honest completeness

Representation of crypto-gates

weighted bipartite graph
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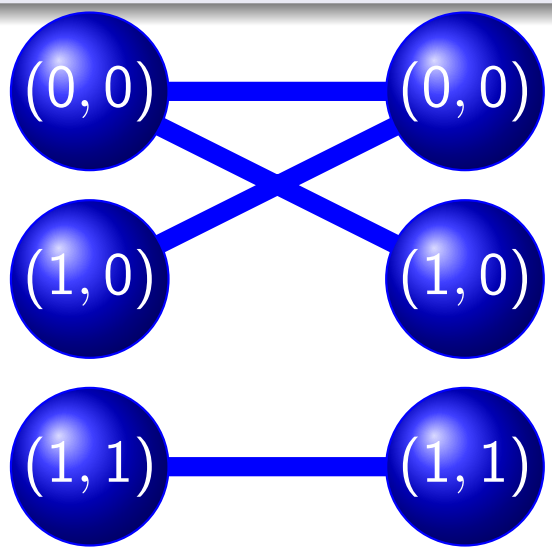

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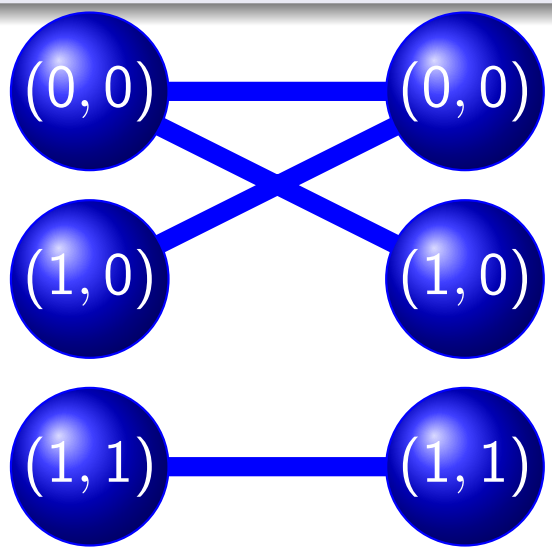

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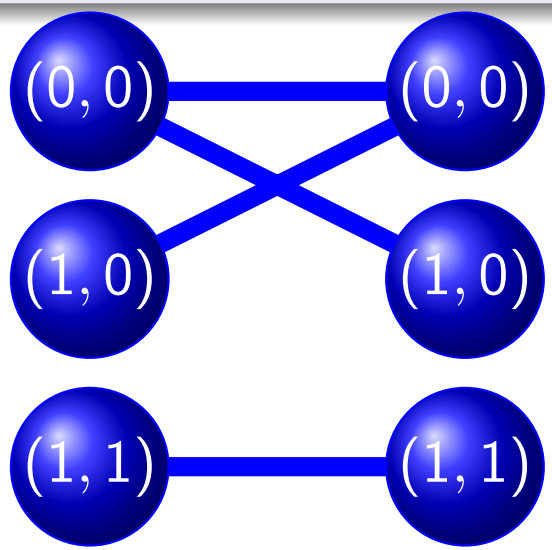

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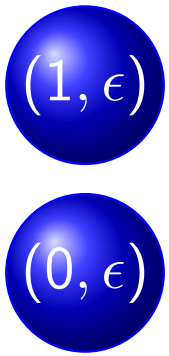

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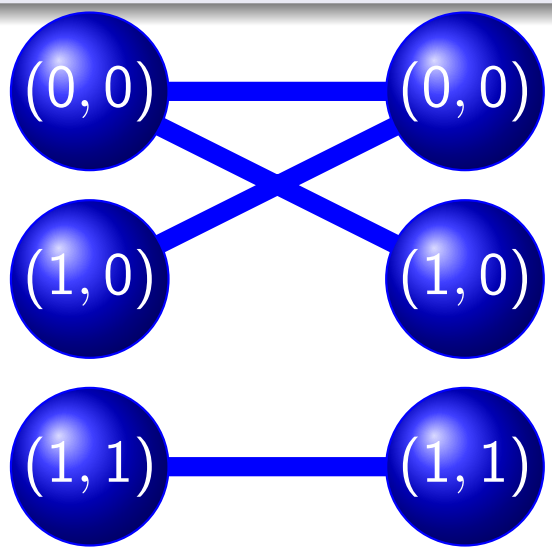

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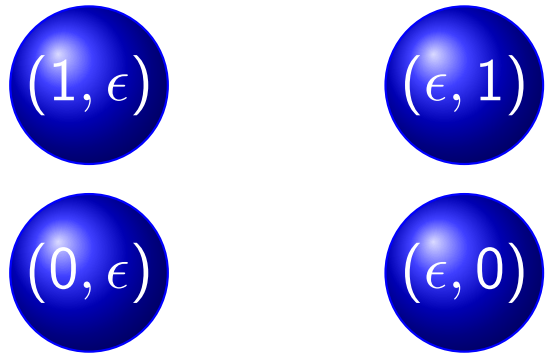

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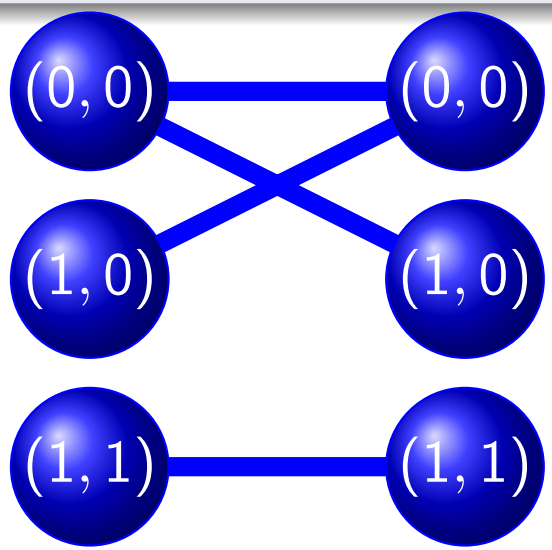

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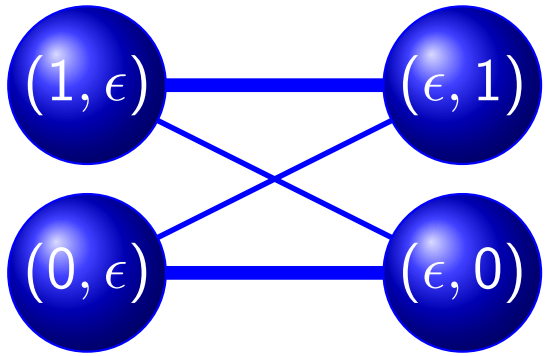

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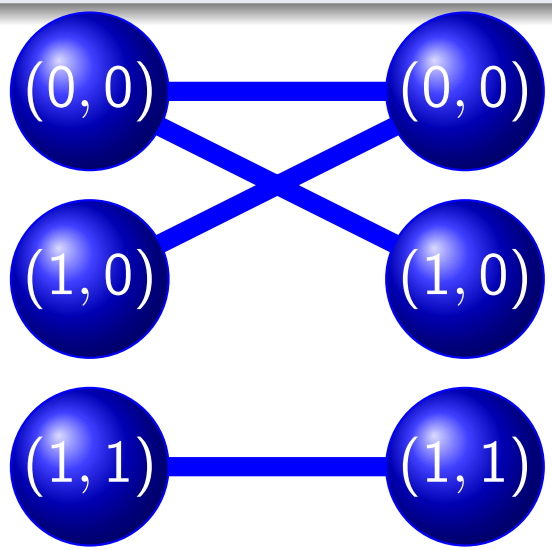

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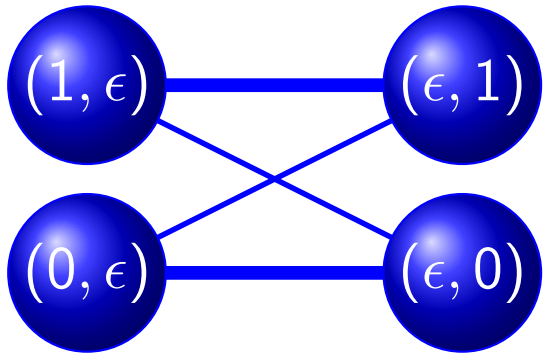

Semi-honest completeness [Maji-Prabhakaran-Rosulek-12]

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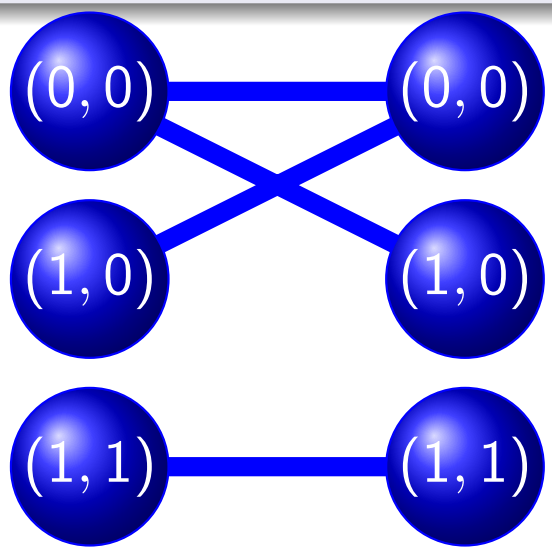

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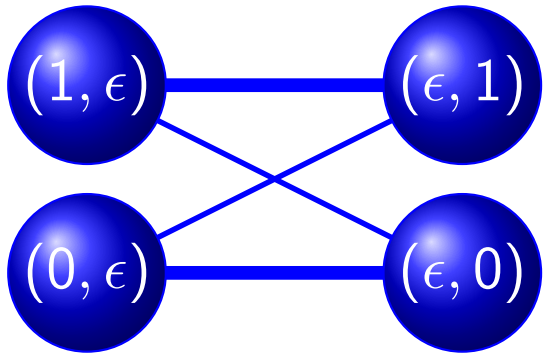

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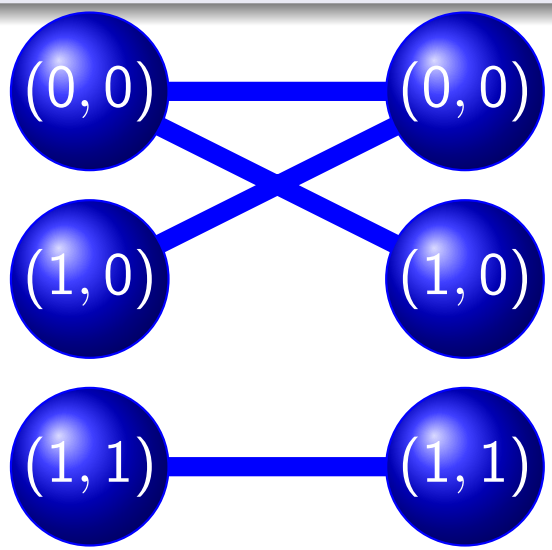

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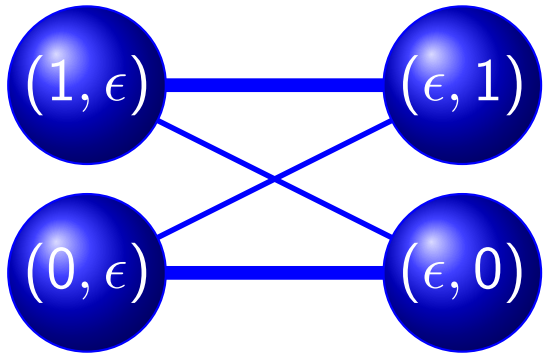

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$\Leftrightarrow$ adjacency matrix has full-rank non-diagonal 2×2-submatrix

Malicious completeness

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Redundancy

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maliciously use only part of the crypto-gate, yet emulate honest behavior

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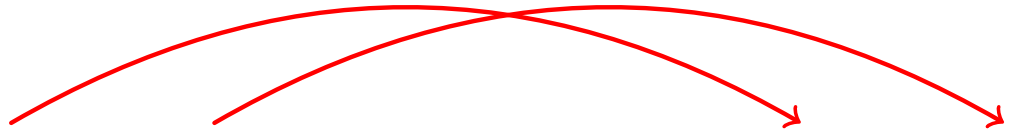

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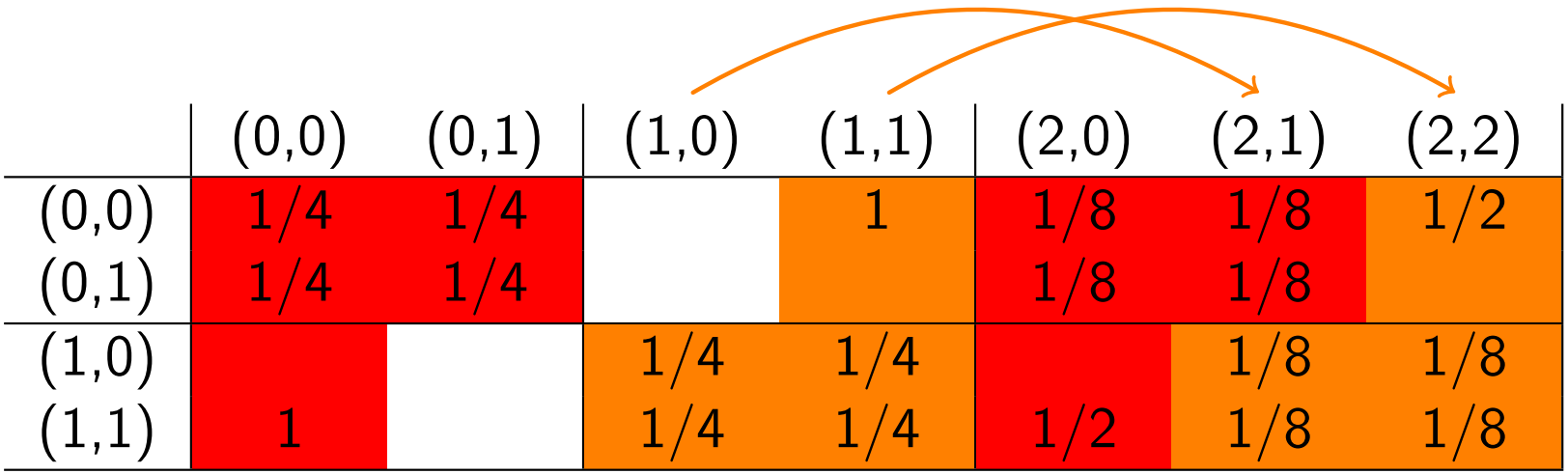

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Efficient characterization of malicious completeness

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Efficient characterization of malicious completeness

1. detect redundancies (use linear programming)

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Complete construction

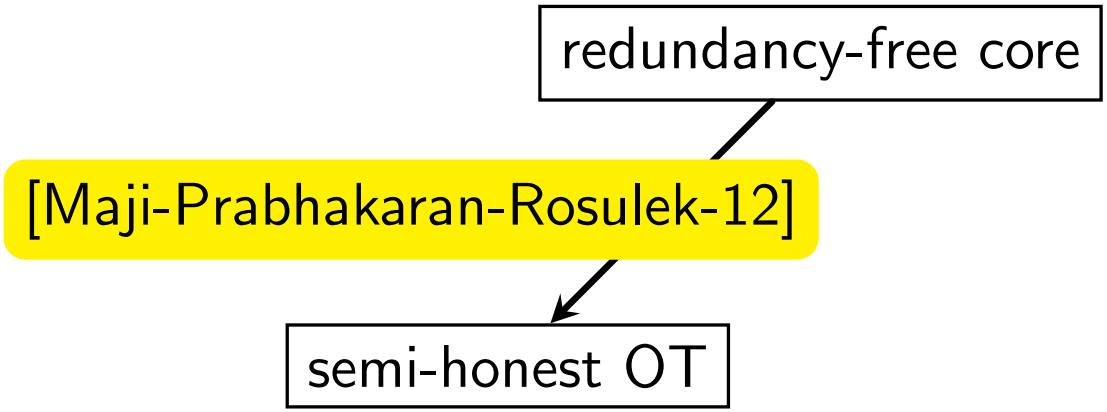
Complete construction

given crypto-gate

Complete construction

redundancy-free core

Complete construction



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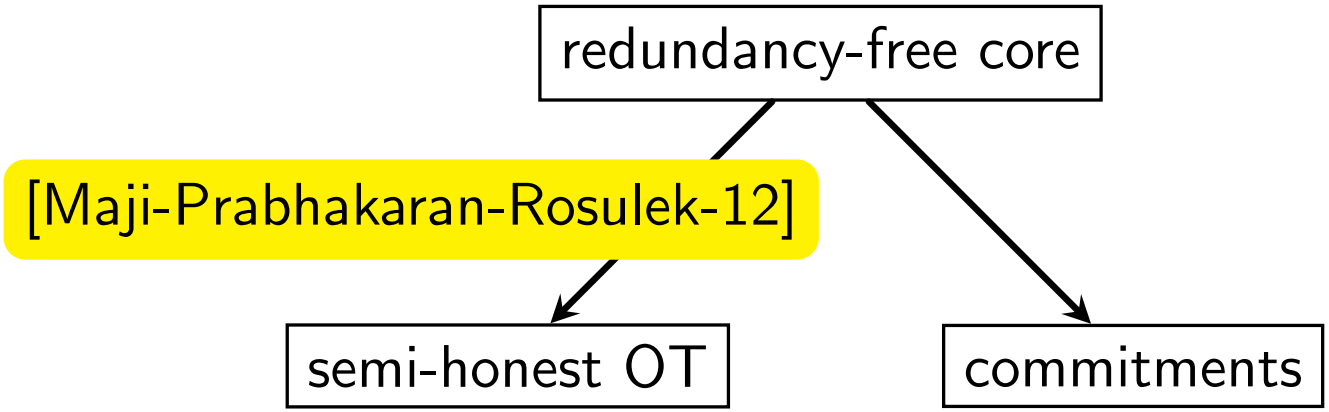

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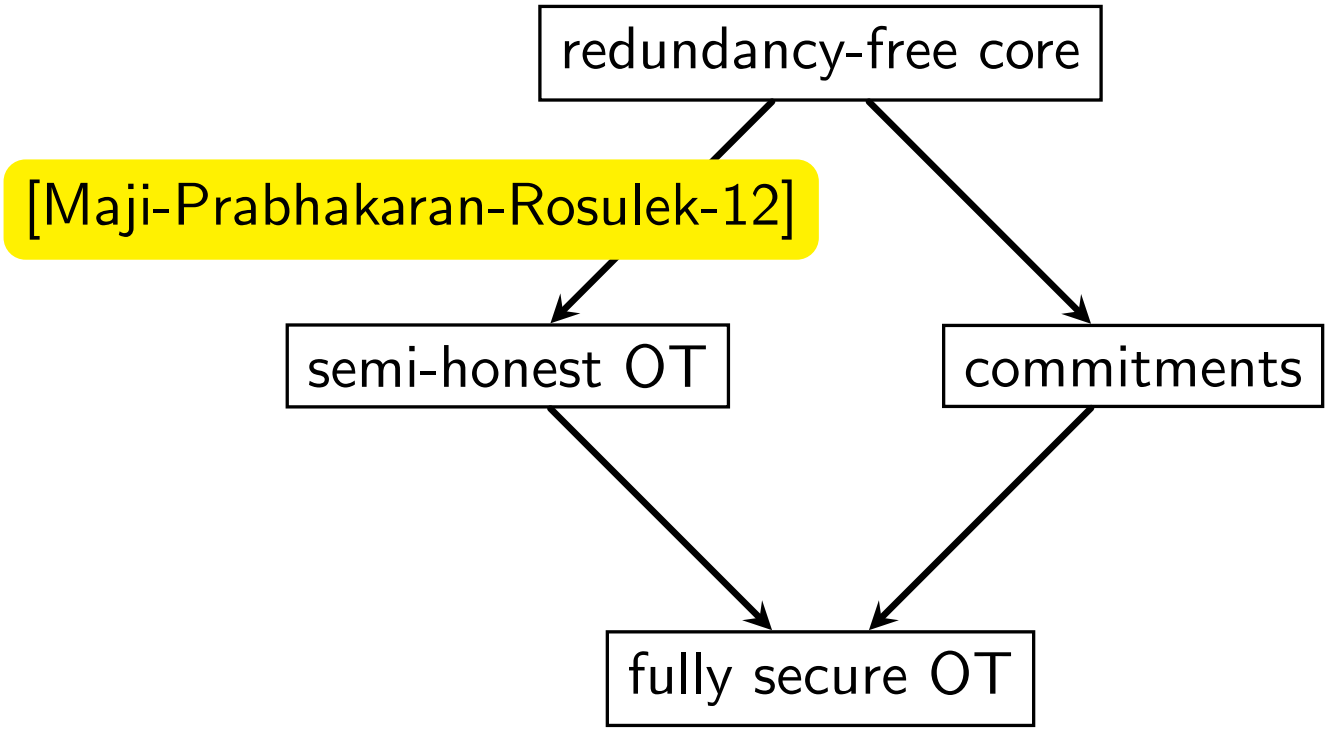

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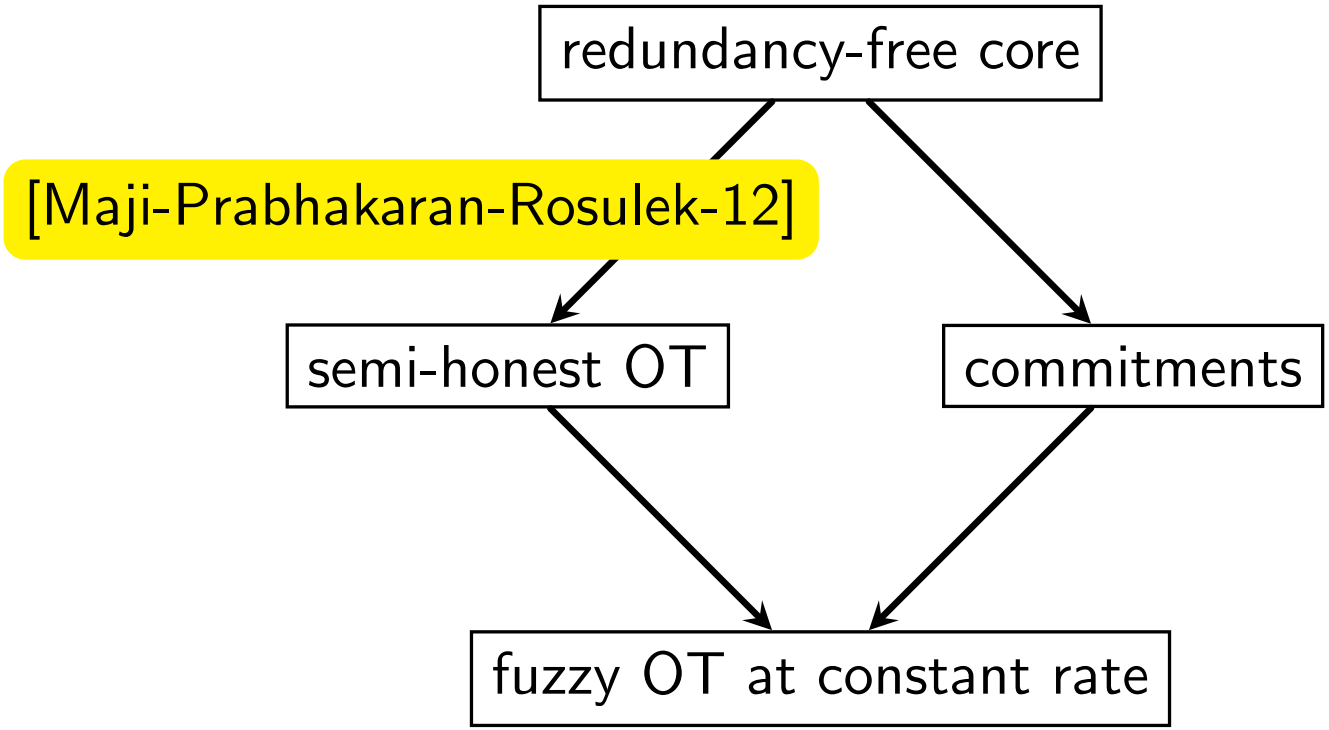

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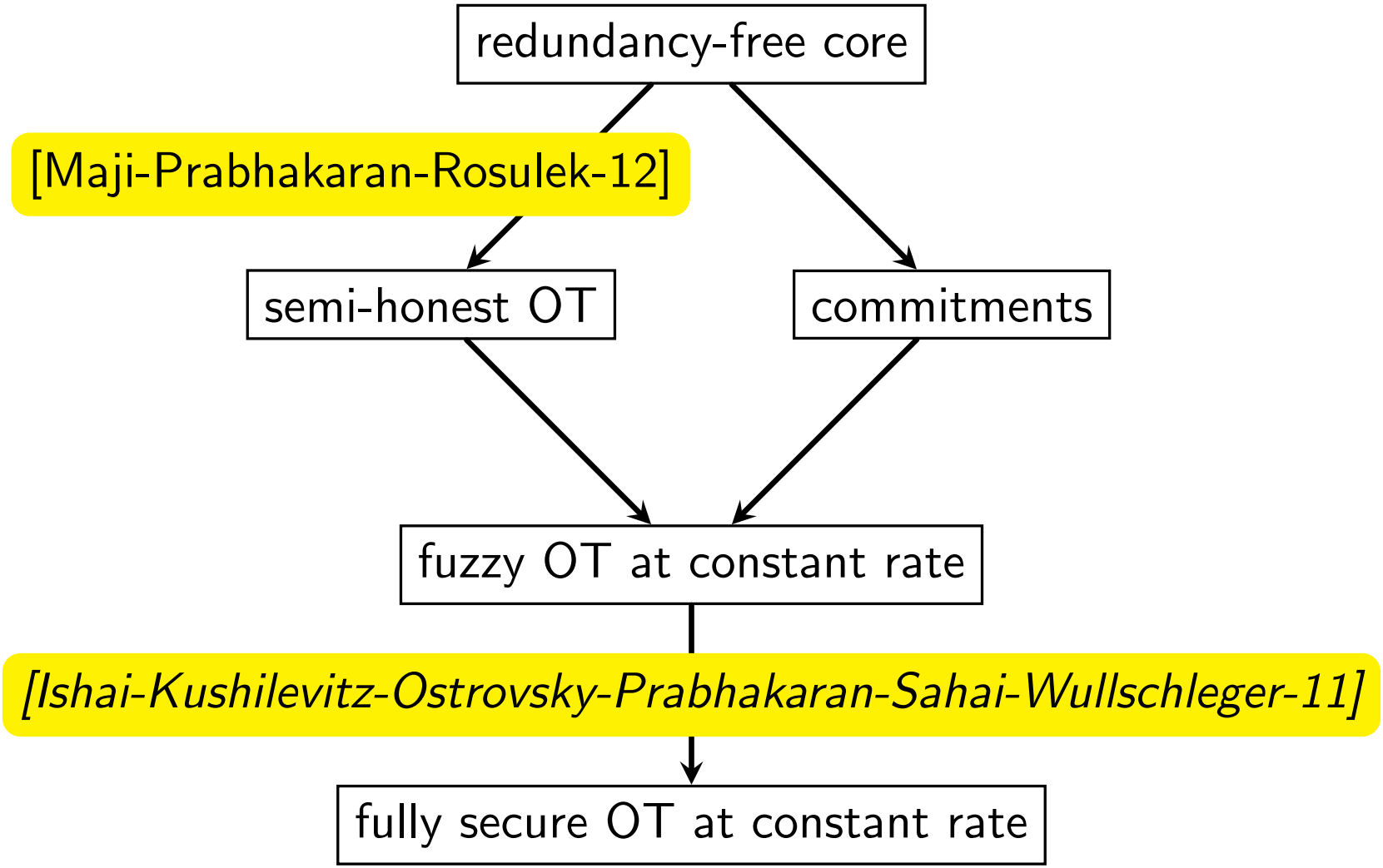

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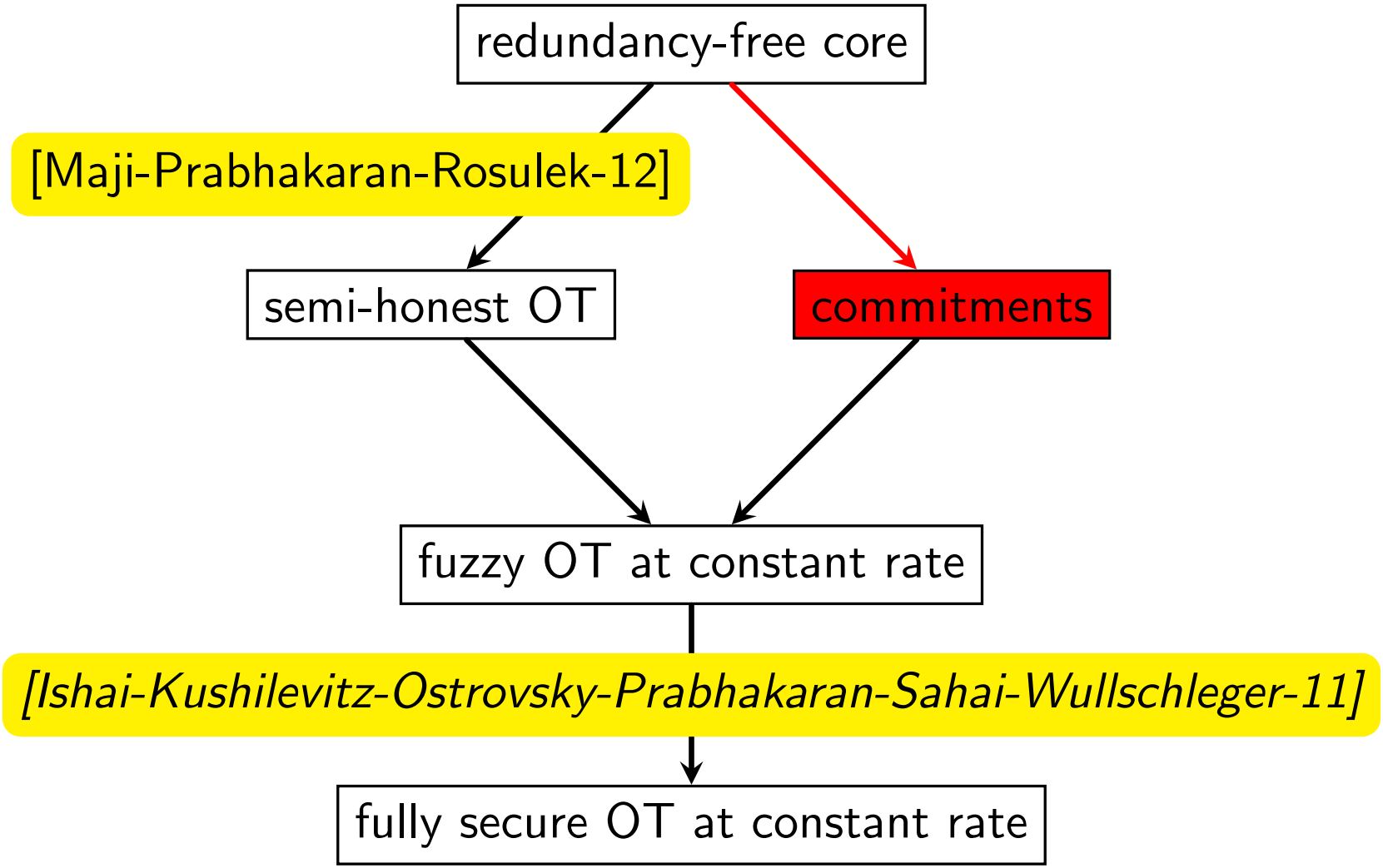

Commitment construction

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use crypto-gate as "channel"

 "sends" (x, a)

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Caveats

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- redundancy-free $\not\Rightarrow$ unfakeable input *distributions*

Technical contributions

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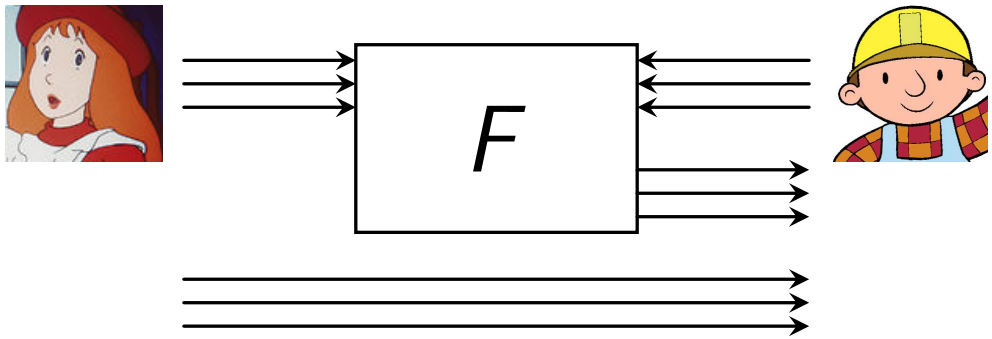
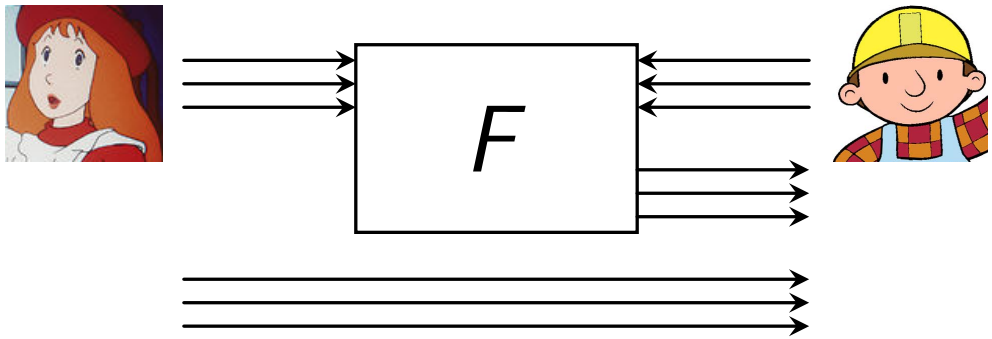


Open problems

- non-interactive completeness $\sim$ Decomposable Randomized Encodings



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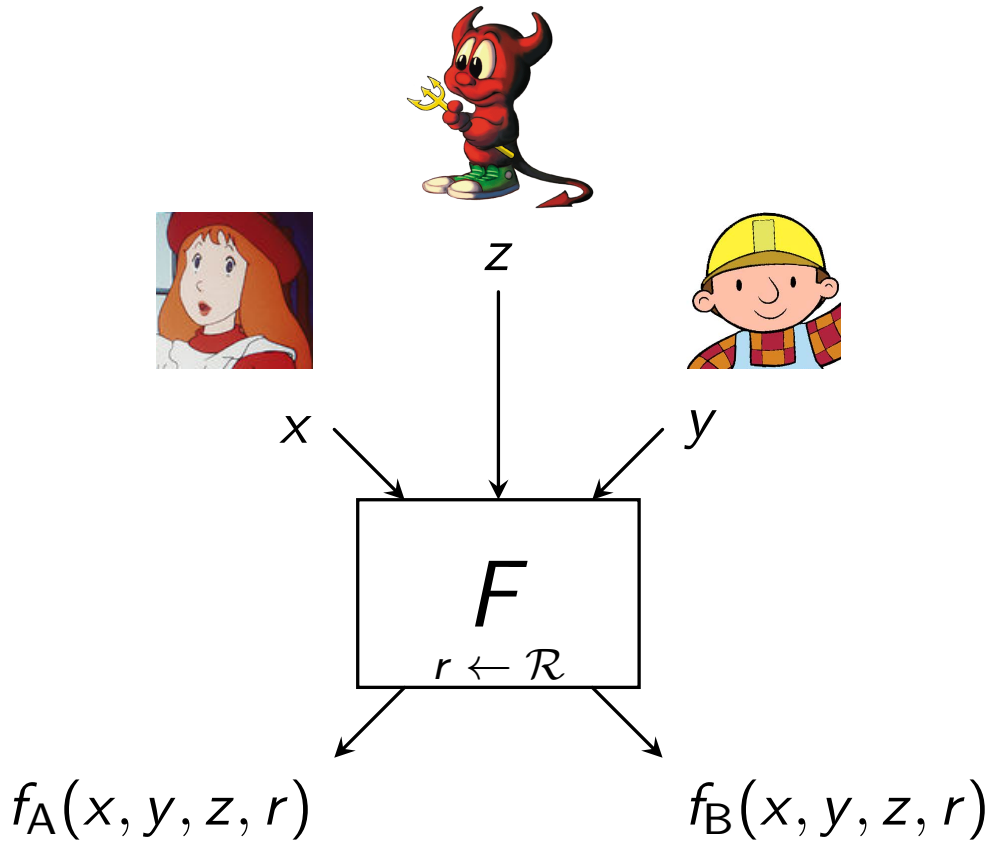

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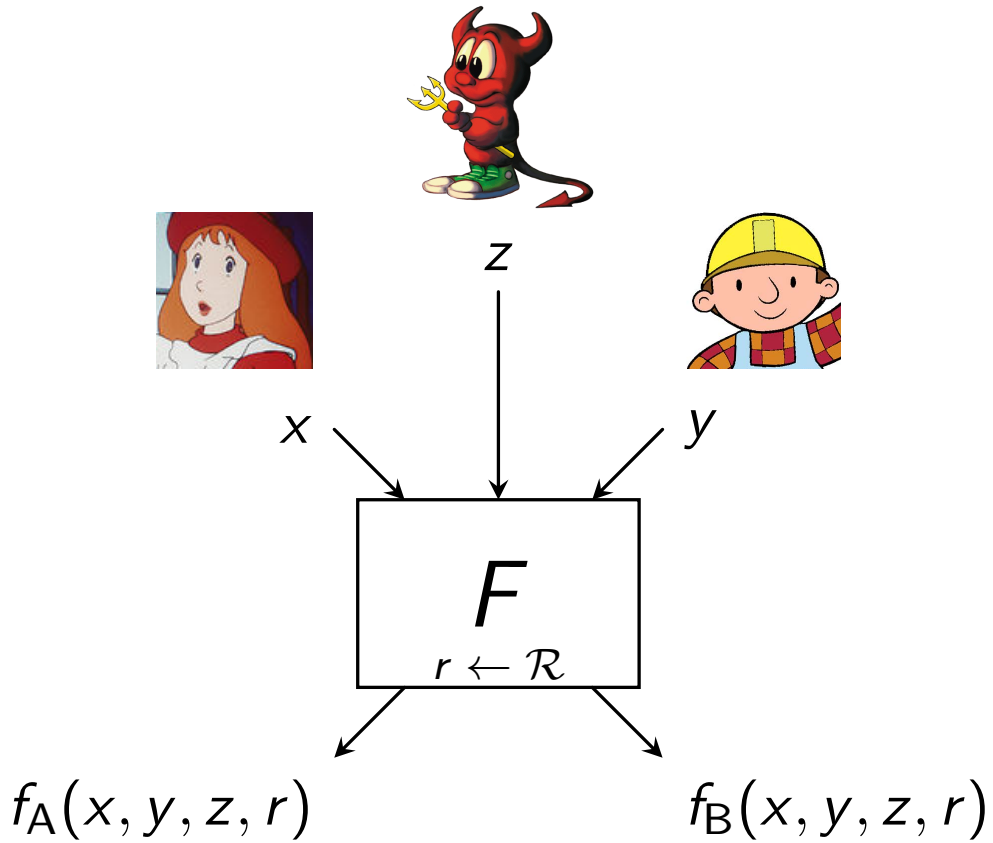

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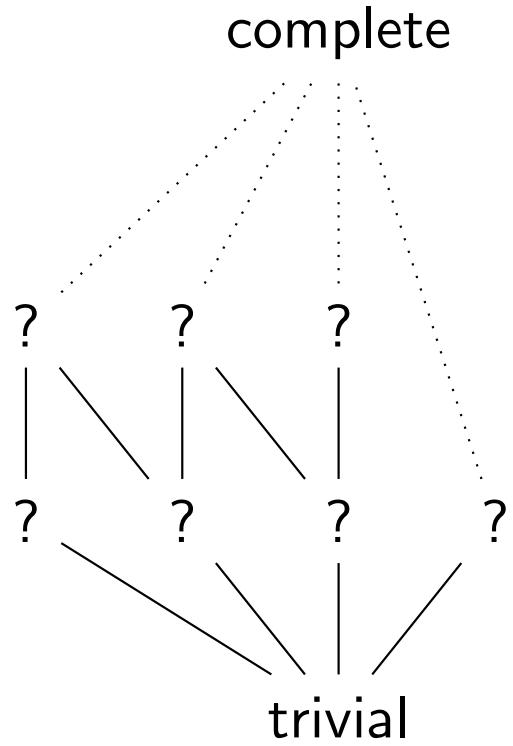

Open problems

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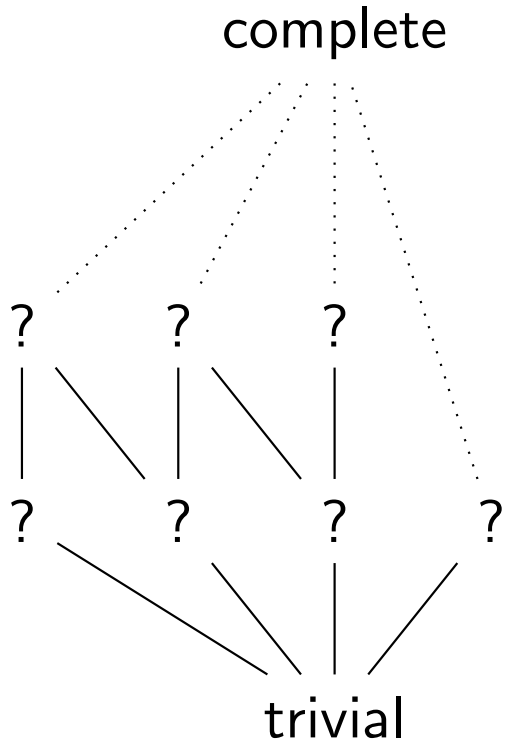

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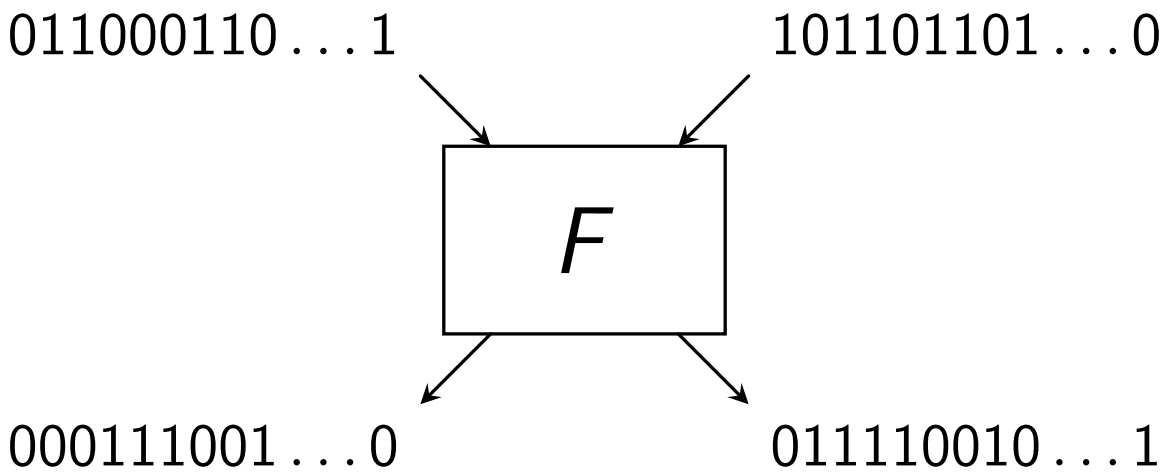

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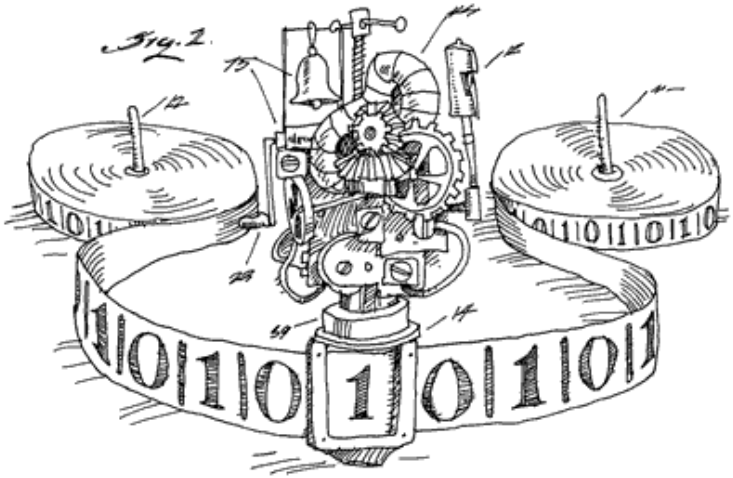
- computationally bounded adversaries



Open problems

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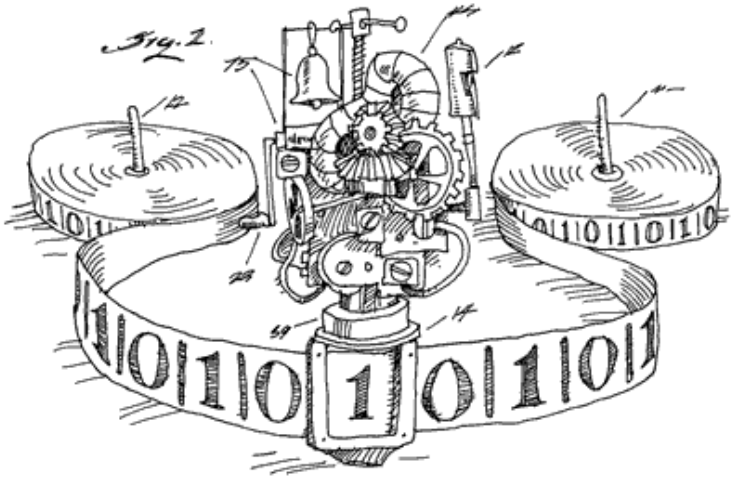

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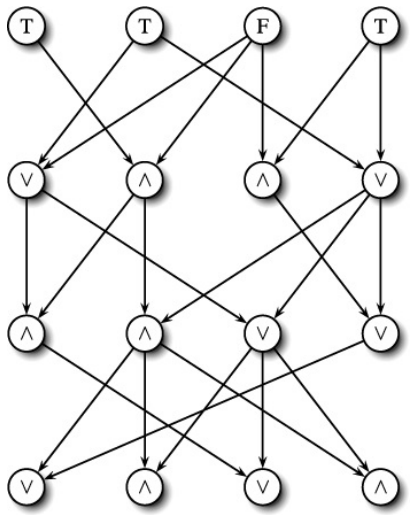
- computationally bounded adversaries (non-black-box reductions)
- lower (crypto-)complexity bounds



Thank you!

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Work supported by NSF grants 07-47027 and 12-28856.

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What's complicated about it?

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cannot use uniform distribution

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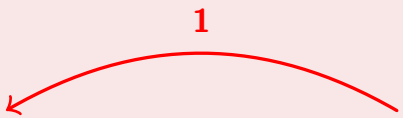

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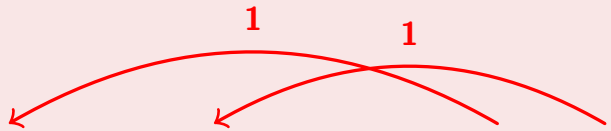

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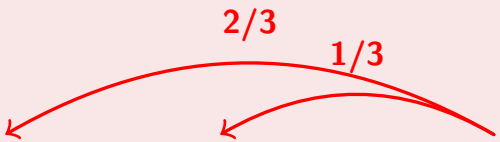

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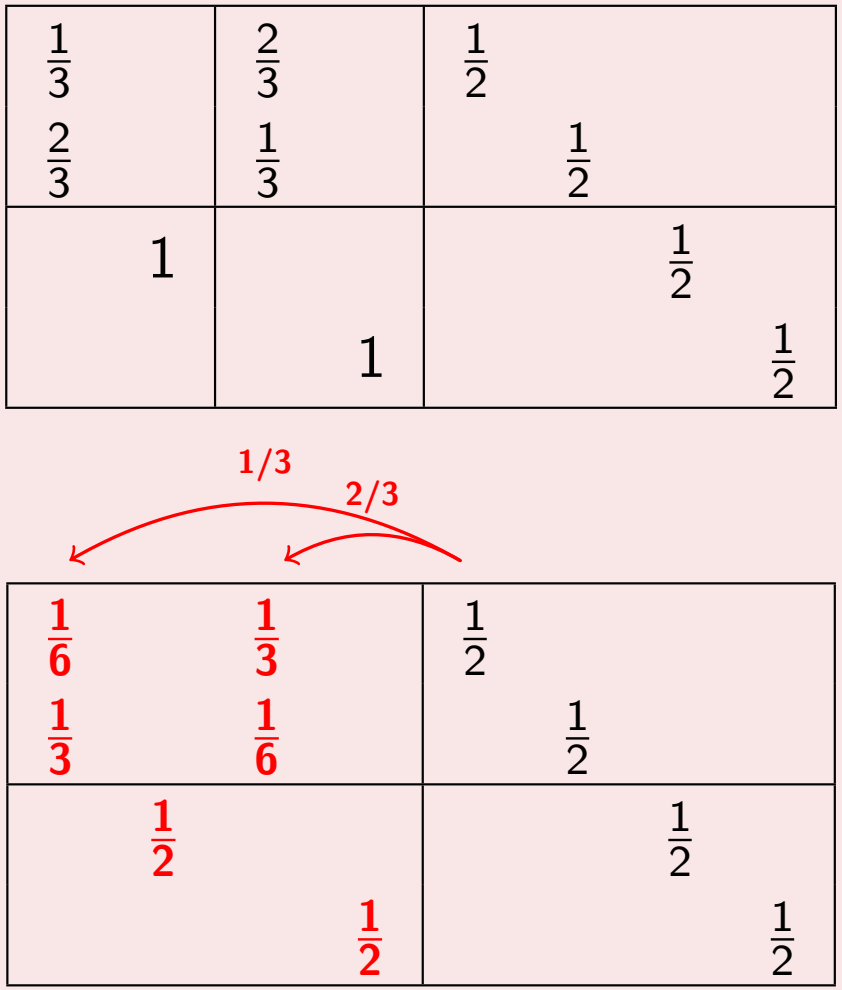

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cannot neglect inputs

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cannot neglect inputs

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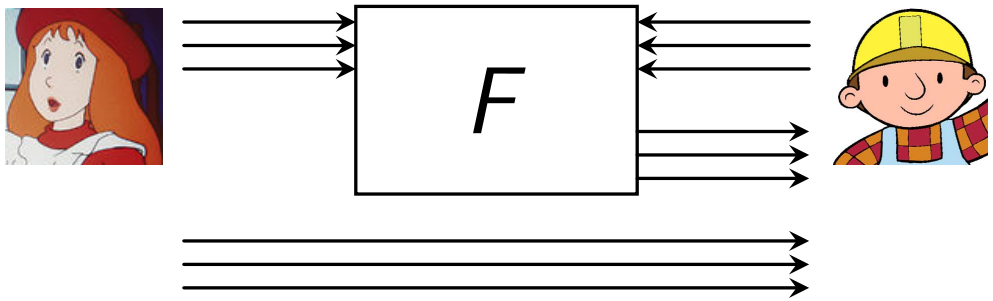
Open Questions & Related Fields

Non-interactive completeness



related to Decomposable Randomized Encodings

what we know

- string-OT from bit-OT
  [Brassard-Crépeau-Santha-96]
- NC^1-NISC from OT, general NISC from OT+PRG
  [Ishai-Kushilevitz-Ostrovsky-Prabhakaran-Sahai-11]

open questions

- general information-theoretic NISC from OT?

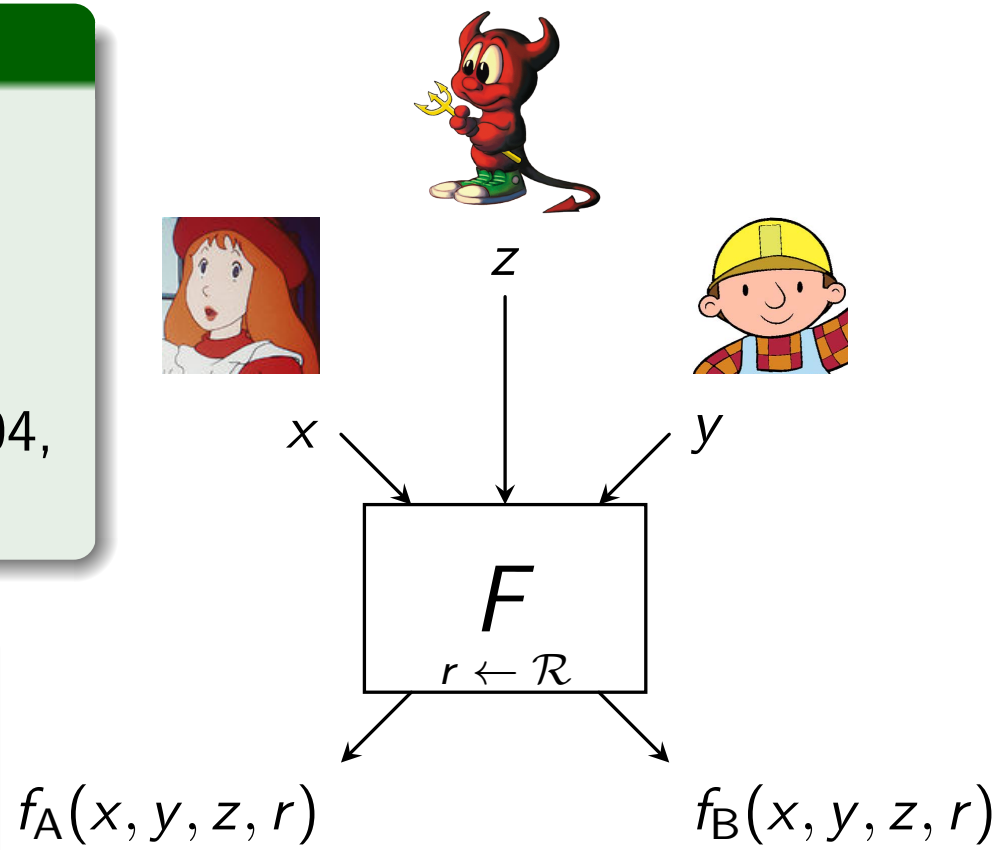
Leaky & unfair primitives

what we know

- completeness criteria for unfair noisy channels [Crépeau-Kilian-88, Damgård-Kilian-Salvail-99, Damgård-Fehr-Morozov-Salvail-04, Wullschleger-09]

open questions

- more complex crypto-gates?
- deterministic crypto-gates?



related to Combiners and Extractors

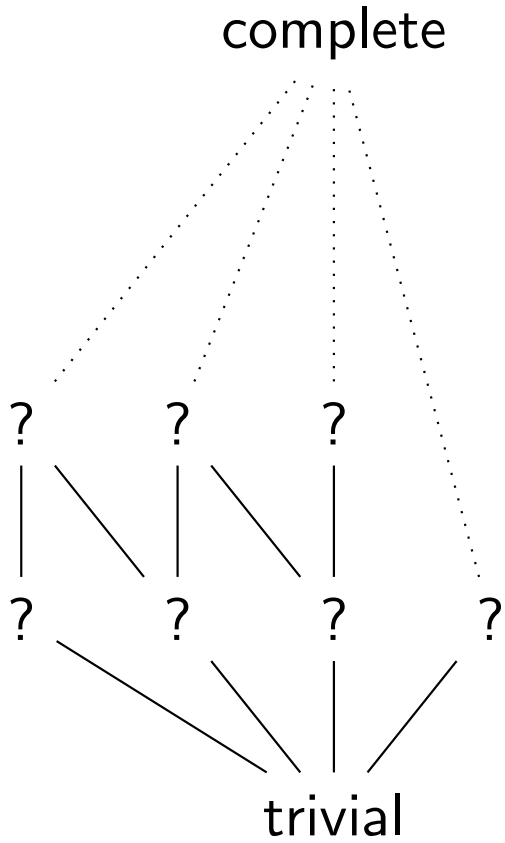
Non-complete crypto-gates

what we know

- classification of trivial crypto-gates [Kushilevitz-92, Beimel-Malkin-Micali-99, Künzler-MüllerQuade-Raub-09, Maji-Prabhakaran-Rosulek-09]
- examples for infinite hierarchy [Kilian-Kushilevitz-Micali-Ostrovsky-00, Maji-Prabhakaran-Rosulek-09]
- Non-complete crypto-gates are symmetric!

open questions

- concrete equivalence classes?
- constant-rate vs arbitrary (efficient) reduction?



related to Black-Box Separations

More than $O(1)$-size

this work

- $O(1)$-size $\rightsquigarrow$ efficient protocol for negligible error
- $O(2^k)$-size $\rightsquigarrow$ exponential complexity for negligible error?

what we know

- highly structured examples (e.g., string-OT, OPE)
- black-box reductions for oracle functionalities, e.g., IC and RO [Luby-Rackoff-88, Coron-Patarin-Seurin-08, Holenstein-Künzler-Tessaro-11, Baecher-Brzuska-Mittelbach-13]
- Random Oracle $\equiv$ Commitments [Mahmoody-Maji-Prabhakaran-12]

open questions

- completeness criteria for oracles?
- good definition for interesting crypto-gates with infinite number of possible inputs?

Computationally bounded adversaries

what we know

- An asymmetric F is complete, iff for some x_0, x_1 it is infeasible to reduce $f(x_1, \cdot)$ to $f(x_0, \cdot)$ [Harnik-Naor-Reingold-Rosen-04].
- Assuming a computational semi-honest OT protocol, (almost) every 2-party functionality is either trivial or complete [Maji-Prabhakaran-Rosulek-10, Rosulek-12].
- In the semi-honest model, any constant round protocol for a non-trivial $O(1)$-size function can be turned into an OT protocol [Lindell-Omri-Zarosim-12].
- black-box separations between OT, key-agreement, CRHF, OWF [Impagliazzo-Rudich-89, Simon-98, Gertner-Kannan-Malkin-Reingold-Viswanathan-00, Gertner-Malkin-Reingold-01]

open questions

- non-black-box reduction of OT to one-way functions?