

Dual System Encryption via Doubly Selective Security:

Framework, Fully-secure Functional Encryption for
Regular Languages, and More

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AIST, Japan
@Eurocrypt 2014, Copenhagen

Our Results in **One Slide**

Framework
for fully-secure FE
(with tighter reduction)

Instantiations:

The first fully secure

- FE for regular languages
- ABE with short ciphertext
- unbounded ABE

and more

Our Results in **One Slide**

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focus in this talk

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The first fully secure

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and more

1

Introduction

Functional Encryption Syntax

FE for predicate $R:A \times B \rightarrow \{0,1\}$ or family $\{R_k\}_k$

- $\text{Setup}(k, 1^\lambda) \longrightarrow PK, MSK$
- $\text{Encrypt}(Y, M, PK) \longrightarrow CT$  for ciphertext attribute Y
- $\text{KeyGen}(X, MSK, PK) \longrightarrow SK$  for key attribute X
- $\text{Decrypt}(CT, SK) \longrightarrow M$ if $R(X, Y)=1$

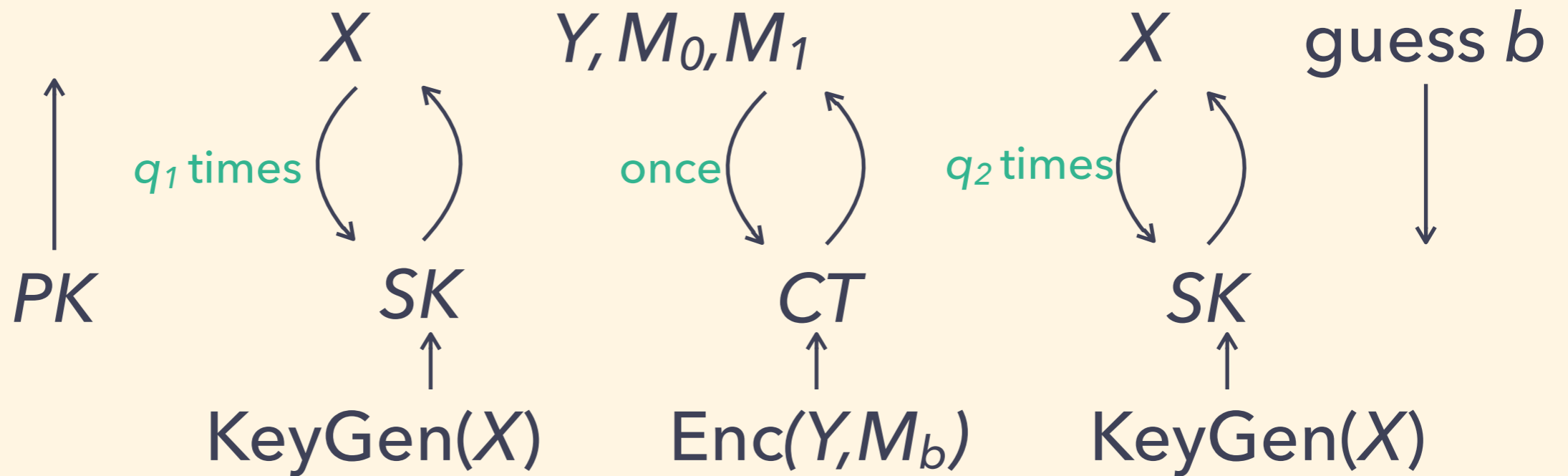
FE here means the class

“Public-index Predicate Encryption” of FE [BSW11].

Definition of Full Security for FE



Adversary



Challenger

Non-triviality condition: $R(X, Y) = 0$

Definition of **Selective Security** for FE



Adversary

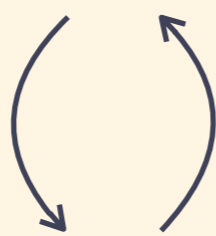
Y



PK

q_1 times

X

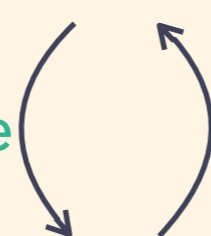


SK

KeyGen(X)

M_0, M_1

once

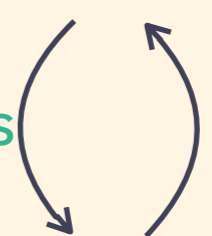


CT

Enc(Y, M_b)

q_2 times

X



SK

KeyGen(X)

guess b



Challenger

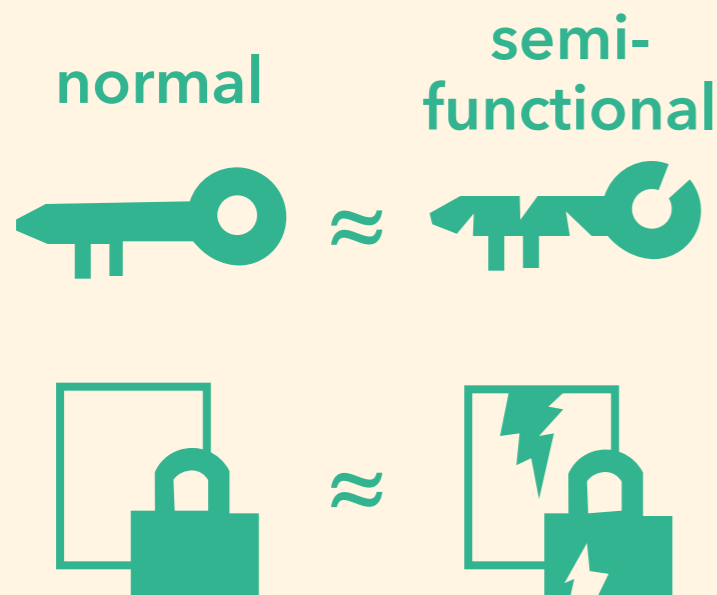
Non-triviality condition: $R(X, Y) = 0$

Approaches for Full Security



Partitioning

- IBE [BB04b, Waters05]
- Seem not to work with richer FE



Dual-System Encryption [Waters09]

- Work also with richer FE:
 - ABE [LOSTW10, OT10, LW12, ...]
 - Inner-product enc [OT12, ...]
 - Spatial encryption [AL10, ...]

Dual System Also Offers Simplicity.

**An original FE
scheme**

Selectively-secure

**Similar scheme but
in composite-order
bilinear group**

**A candidate for
fully-secure scheme**

Boneh-Boyen IBE

(selectively secure)

$$CT = (g^s, g^{s(h_1+h_2ID)}, e(g, g)^{as}M)$$

$$SK = (g^{a+r(h_1+h_2ID')}, g^r)$$

Lewko-Waters IBE

(fully secure)

$$CT = (g_1^s, g_1^{s(h_1+h_2ID)}, e(g_1, g_1)^{as}M)$$

$$SK = (g_1^{a+r(h_1+h_2ID')}g_3^{w_1}, g_1^r g_3^{w_2})$$

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Abstract Selective Secure FE

$$CT = (g_1^{c(s,h)}, e(g_1, g_1)^{as}M)$$

$$SK = g_1^{k(a,r,h)}$$

Abstract Fully Secure FE ?

$$CT = (g_1^{c(s,h)}, e(g_1, g_1)^{as}M)$$

$$SK = g_1^{k(a,r,h)} \cdot g_3^w$$

Apply to any scheme?

Successful Applications

IBE

ABE

Spatial Encryption

Selective

Full

BB04

LW10

GPSW06

LOSTW10

BH08

AL10

Unsuccessful Applications

FE for regular languages

ABE w/ short ciphertexts

Fully-unbounded ABE

Selective

Full

Waters12

?

ALP11

?

RW13

?

**Open
problem!**

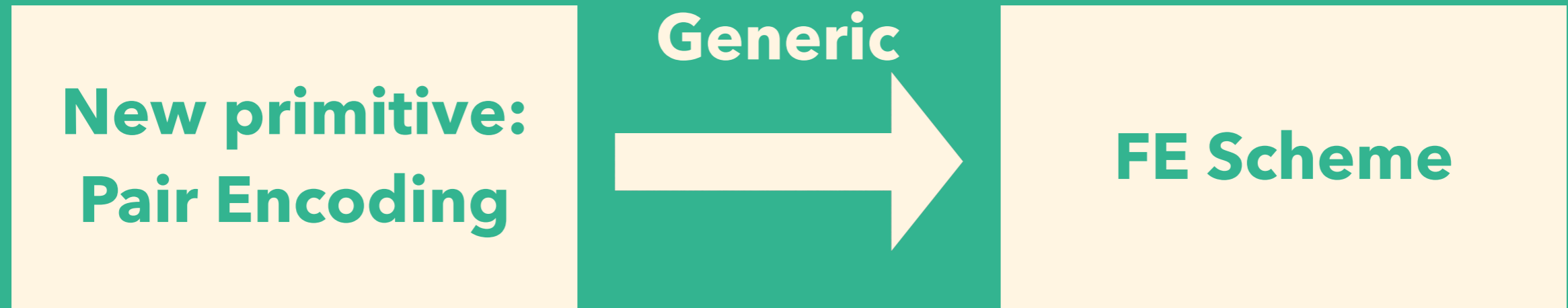
We ask:

Why did “traditional” dual systems fail for some schemes?

How to overcome that barrier?

To *systematically* answer, we
provide a generic framework.

Our Framework Result

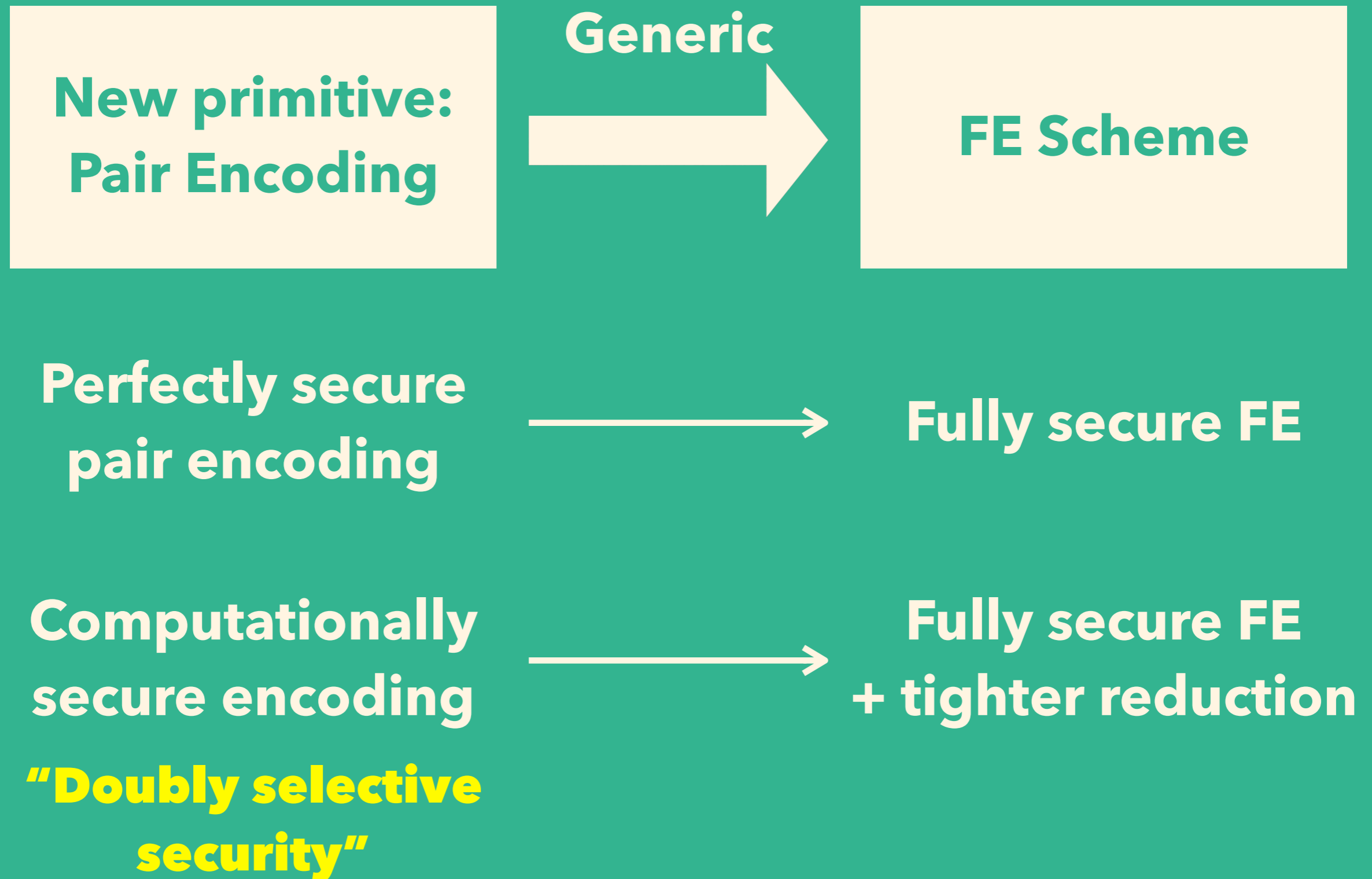


Perfectly secure
pair encoding

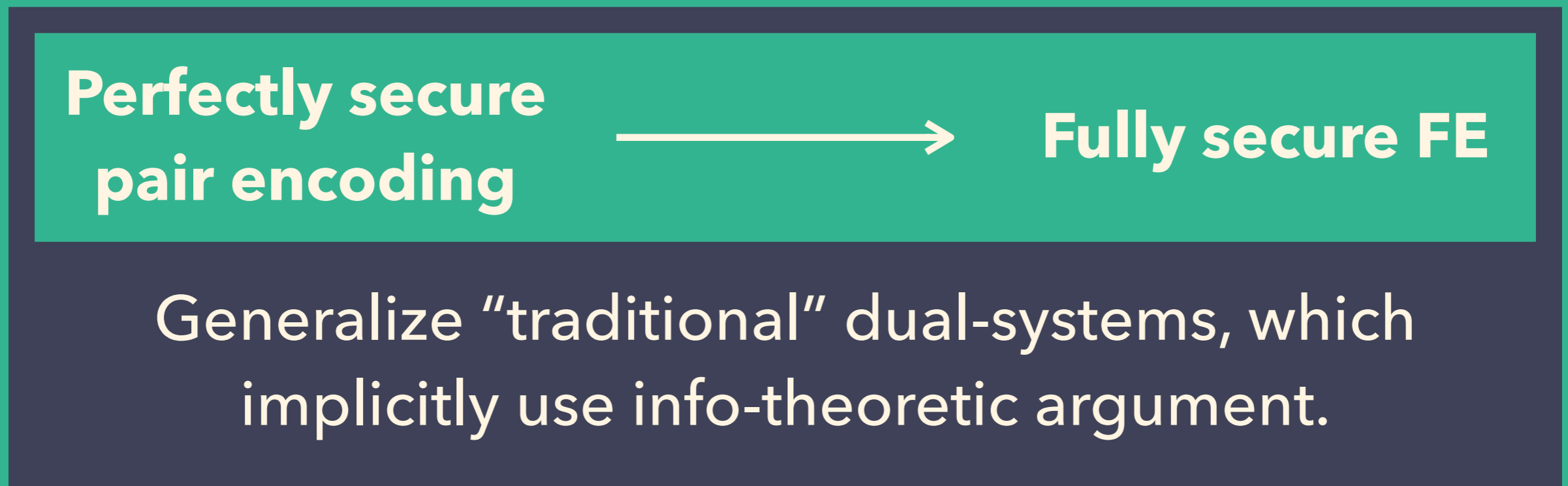
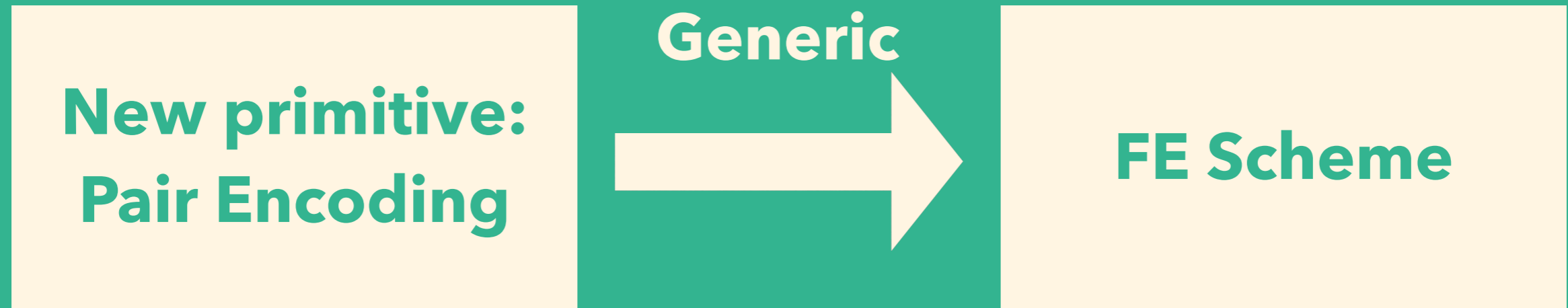
Computationally
secure encoding

**"Doubly selective
security"**

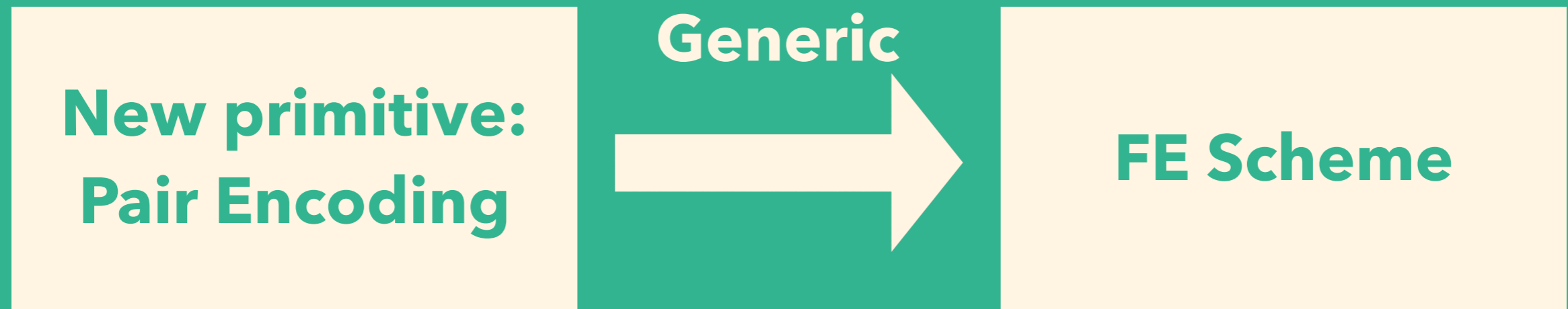
Our Framework Result



Our Framework Result



Our Framework Result



Generalize Lewko-Waters12 ABE
+ New techniques for tighter reduction.

**Computationally
secure encoding**



**Fully secure FE
+ tighter reduction**

A Glance at Pair Encoding

Recall the abstract scheme

$$CT = (g_1^{c(s,h)}, e(g_1, g_1)^{as} M)$$

$$SK = g_1^{k(a,r,h)} \cdot g_3^w$$

Pair encoding consists of $c()$ and $k()$.

Our Answer to Instantiations

	Selective	Fully-secure
FE for regular languages	Waters12	
ABE w/ short ciphertexts	ALP11	
Fully-unbounded ABE	RW13	

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Why traditional dual systems failed:

(Implicit) encodings were not perfect.

Our Answer to Instantiations

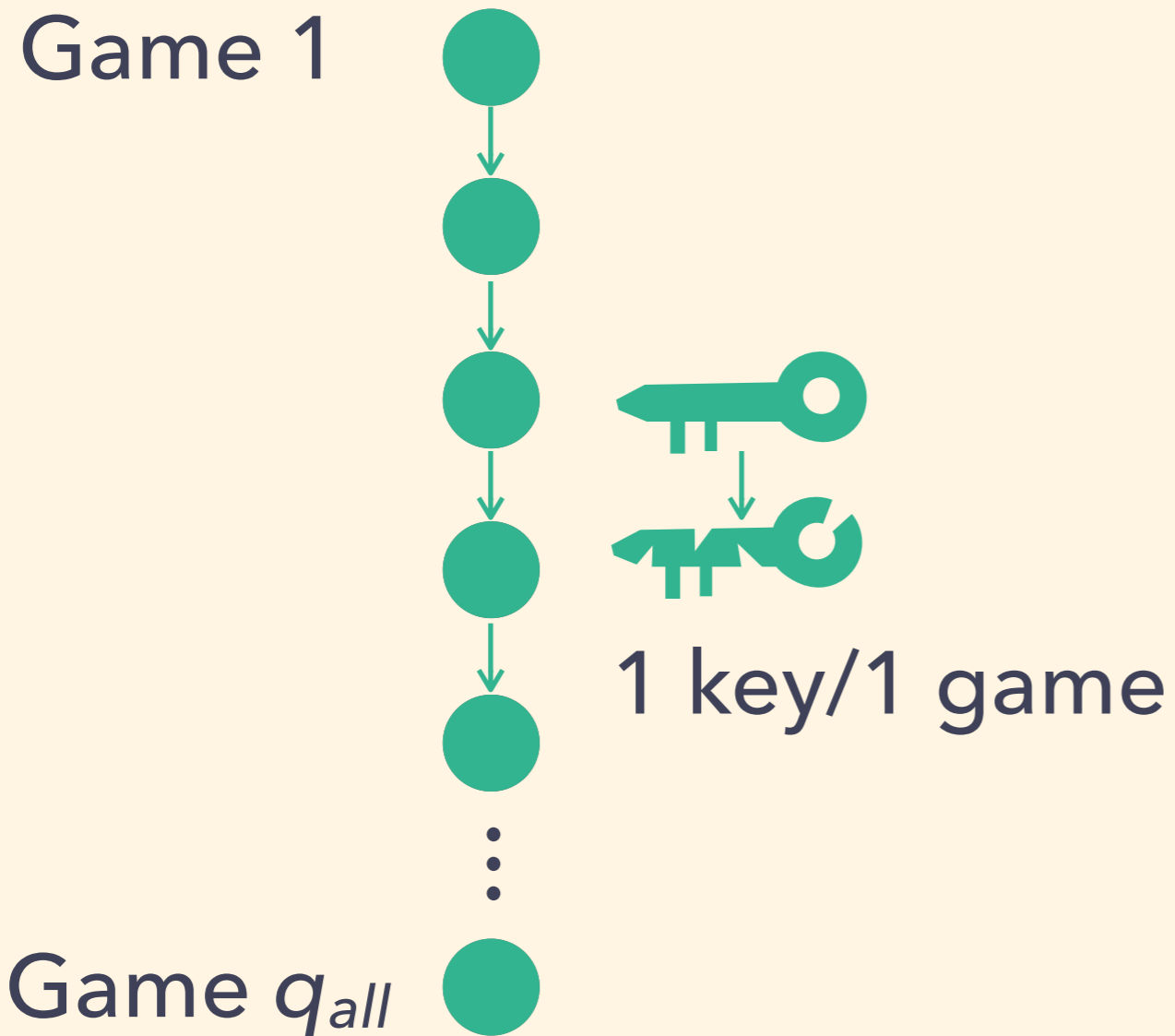
	Selective	Fully-secure
FE for regular languages	Waters12	New!
ABE w/ short ciphertexts	ALP11	New!
Fully-unbounded ABE	RW13	New!
	↓	↑

Why traditional dual systems failed: (Implicit) encodings were not perfect.

How to overcome: Use computationally secure encodings

A Glance at Tighter Reduction

All prior dual-system proofs
(except [Chen-Wee Crypto13])



Reduction = $O(q_{all})$, $q_{all} = q_1 + q_2$

A Glance at Tighter Reduction

All prior dual-system proofs
(except [Chen-Wee Crypto13])

Our new approach

Game 1



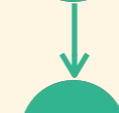
⋮



1 key/1 game

Game q_{all}

1



q_1



one big jump!



...



q_2 keys in 1 game

q_{all}



Reduction = $O(q_{all})$, $q_{all} = q_1 + q_2$

Reduction = $O(q_1)$

Related work on Dual-System Framework

- [Chen-Wee Crypto13]: Dual-system groups
 - Unify prime- and composite-order groups but only to specific predicates (HIBE).
 - Ours unifies for any predicate (but specific to composite-order).
- [Wee TCC14]: Predicate Encoding
 - Independently abstracting perfectly secure encoding.

2 Framework

Pair Encoding: Definition

Pair Encoding for predicate $R=\{R_k\}_k$

$\text{Enc1}(X) \longrightarrow \mathbf{k}(\mathbf{a}, \mathbf{r}, \mathbf{h})$ where $\mathbf{r}=(r_1, \dots, r_m)$

$\text{Enc2}(Y) \longrightarrow \mathbf{c}(\mathbf{s}, \mathbf{h})$ where $\mathbf{s}=(\mathbf{s}, s_1, \dots, s_w)$

Pair Encoding: Definition

Pair Encoding for predicate $R=\{R_k\}_k$

Param(k) \longrightarrow $|\mathbf{h}|$ where $\mathbf{h}=(h_1,\dots,h_m)$

Enc1(X) \longrightarrow $k(\mathbf{a},\mathbf{r},\mathbf{h})$ where $\mathbf{r}=(r_1,\dots,r_m)$

Enc2(Y) \longrightarrow $\mathbf{c}(\mathbf{s},\mathbf{h})$ where $\mathbf{s}=(s,s_1,\dots,s_w)$

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Pair(X,Y) \longrightarrow \mathbf{E}

- Correctness : If $R(X,Y)=1$, $k(\mathbf{a},\mathbf{r},\mathbf{h}) \mathbf{E} \mathbf{c}(\mathbf{s},\mathbf{h})^\top = \mathbf{a}\mathbf{s}$

Pair Encoding: Definition

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Pair(X,Y) \longrightarrow \mathbf{E}

- **Correctness :** If $R(X,Y)=1$, $k(\mathbf{a},\mathbf{r},\mathbf{h}) \mathbf{E} \mathbf{c}(\mathbf{s},\mathbf{h})^\top = \mathbf{as}$
- **Security:** If $R(X,Y)=0$, ...to be defined.

Additional Requirements

Parameter-vanishing

$$\mathbf{k}(a, \mathbf{0}, \mathbf{h}) = \mathbf{k}(a, \mathbf{0}, \mathbf{0})$$

Linearity for \mathbf{k}

$$\mathbf{k}(a_1, \mathbf{r}_1, \mathbf{h}) + \mathbf{k}(a_2, \mathbf{r}_2, \mathbf{h}) = \mathbf{k}(a_1 + a_2, \mathbf{r}_1 + \mathbf{r}_2, \mathbf{0})$$

Linearity for \mathbf{c}

$$\mathbf{c}(\mathbf{s}_1 + \mathbf{s}_2, \mathbf{h}) = \mathbf{c}(\mathbf{s}_1, \mathbf{h}) + \mathbf{c}(\mathbf{s}_2, \mathbf{h})$$

Linearity implies homogeneity: $\mathbf{k}(0, \mathbf{0}, \mathbf{0}) = 0, \mathbf{c}(\mathbf{0}, \mathbf{0}) = 0$

Pair Encoding: Example for IBE

Param $\longrightarrow 2$ That is, $\mathbf{h}=(h_1,h_2)$

Enc1(ID) $\longrightarrow \mathbf{k}(\mathbf{a},r,\mathbf{h}) = (\mathbf{a}+r(h_1+h_2ID), r)$

Enc2(ID') $\longrightarrow \mathbf{c}(\mathbf{s},\mathbf{h}) = (\mathbf{s}, \mathbf{s}(h_1+h_2ID'))$

Pair(ID, ID') $\longrightarrow \mathbf{E} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- **Correctness** If $ID=ID'$

$$(\mathbf{a}+r(h_1+h_2ID), r) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{s} \\ \mathbf{s}(h_1+h_2ID') \end{pmatrix} = \mathbf{as}$$

Composite-order Bilinear Groups

G, G_T of order $N=p_1p_2p_3$

with bilinear map $e: G \times G \rightarrow G_T$

have prime-order subgroups G_1, G_2, G_3

Orthogonality: $e(g_i, g_j)=1$ iff $i \neq j$

Subgroup Decision: Decide if $T \in G_1$ or $T \in G_{12}$

Constructing FE from Pair Encoding

FE for predicate R from Pair encoding for R

Setup $\longrightarrow PK=(g_1, g_1^h, e(g_1, g_1)^a, g_3), MSK=a$

Encrypt(Y, M, PK) $\longrightarrow CT=(g_1^{c(s,h)}, e(g_1, g_1)^{as}M)$ $Enc2(Y)=c(s,h)$

KeyGen(X, MSK) $\longrightarrow SK=g_1^{k(a,r,h)} \cdot R_3$ $Enc1(X)=k(a,r,h)$

Decrypt(CT, SK) $\longrightarrow e(g_1^{k(a,r,h)E}, g_1^{c(s,h)} \cdot R_3)$
 $= e(g_1, g_1)^{as}$ $k(a,r,h) E c(s,h)^T = as$

Security Proof of Our Framework

Semi-functional Ciphertexts/Keys

Can Be Defined in Terms of Pair Encoding Scheme



$$g_1^{c(s,h)}.g_2^{c(0,0)=0}$$



$$g_1^{k(a,r,h)}.g_2^{k(0,0,0)=0}$$



$$g_1^{c(s,h)}.g_2^{c(\hat{s},\hat{h})}$$



$$g_1^{k(a,r,h)}.g_2^{k(0,\hat{r},\hat{h})}$$

$$e(g_1,g_1)^{as} \text{ unmodified}$$



$$g_1^{k(a,r,h)}.g_2^{k(\hat{a},\hat{r},\hat{h})}$$

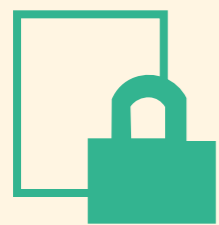


$$g_1^{k(a,r,h)}.g_2^{k(\hat{a},0,0)}$$

Each randomness except "semi-param" \hat{h} is fresh for each.

Semi-functional Ciphertexts/Keys

Can Be Defined in Terms of Pair Encoding Scheme



$$\textcircled{C} \quad g_1^{c(s,h)} \cdot g_2^{c(0,0)=0}$$



$$\textcircled{C} \quad g_1^{c(s,h)} \cdot g_2^{c(\hat{s},\hat{h})}$$

$e(g_1, g_1)^{as}$ unmodified



$$\textcircled{K} \quad g_1^{k(a,r,h)} \cdot g_2^{k(0,0,0)=0}$$



$$\textcircled{K1} \quad g_1^{k(a,r,h)} \cdot g_2^{k(0,\hat{r},\hat{h})}$$



$$\textcircled{K2} \quad g_1^{k(a,r,h)} \cdot g_2^{k(\hat{a},\hat{r},\hat{h})}$$



$$\textcircled{K3} \quad g_1^{k(a,r,h)} \cdot g_2^{k(\hat{a},0,0)}$$

Each randomness except "semi-param" \hat{h} is fresh for each.

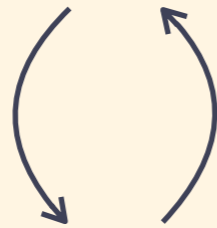
Recall Definition for Full Security



PK



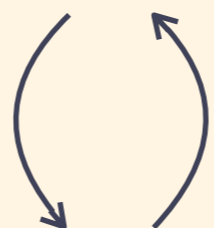
X



SK

$\text{KeyGen}(X)$

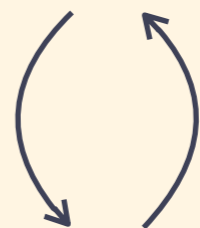
Y, M_0, M_1



CT

$\text{Enc}(Y, M_b)$

X



SK

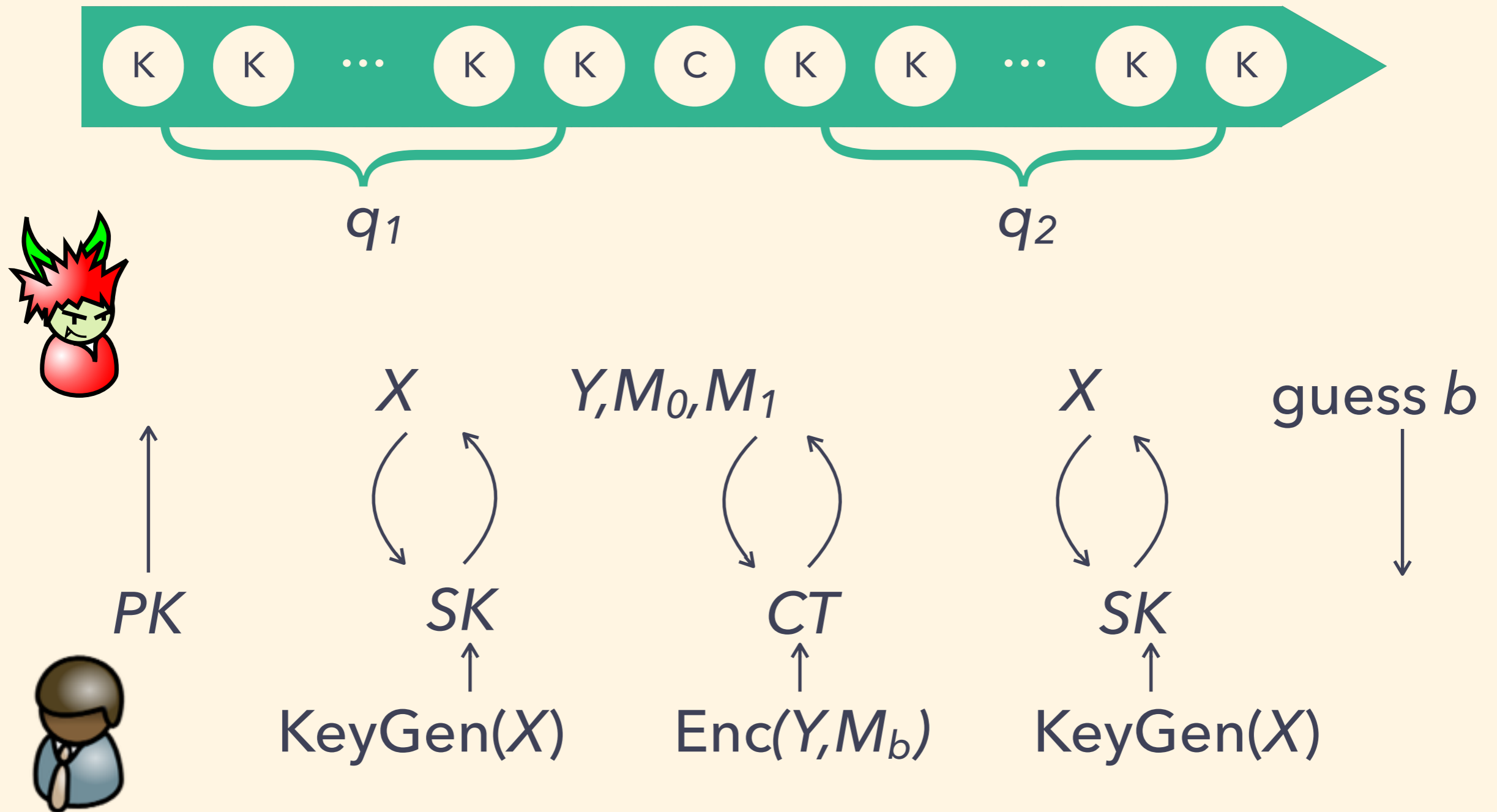
$\text{KeyGen}(X)$

guess b



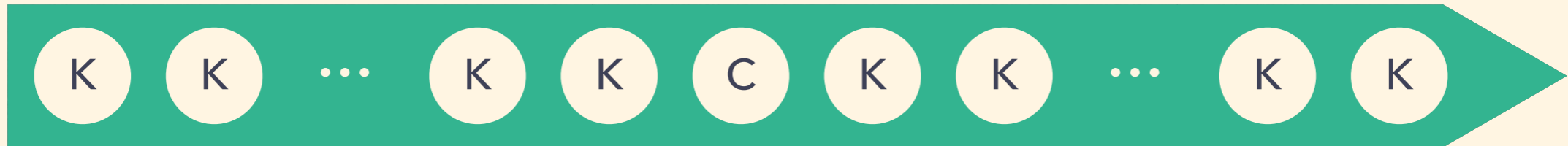
Recall Definition for Full Security

Notation in timeline

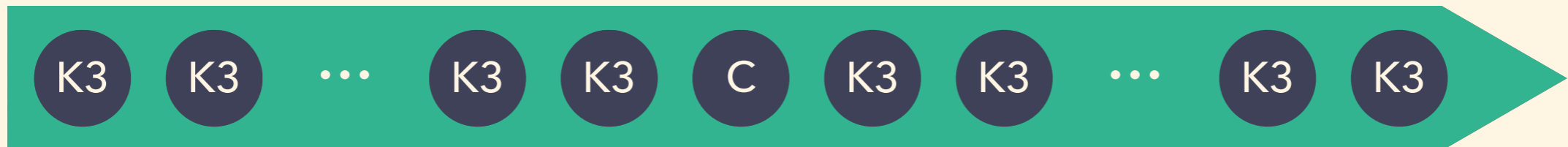


Aim of the Proof

Real game: all normal




Final game: all semi-functional

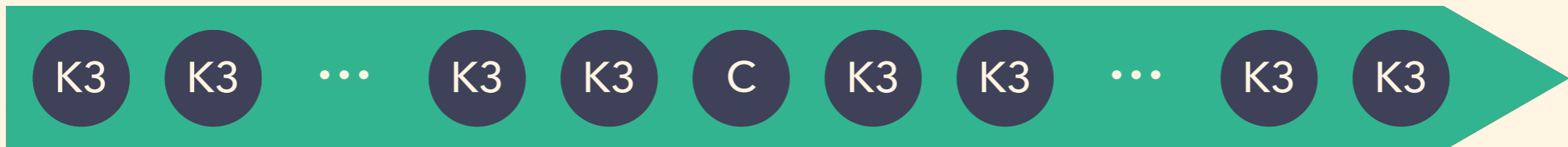


Final Game

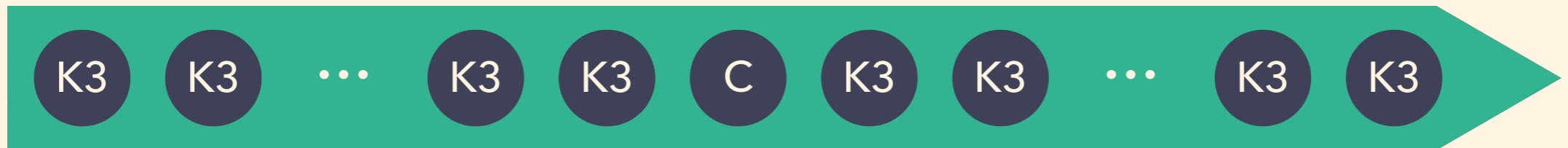
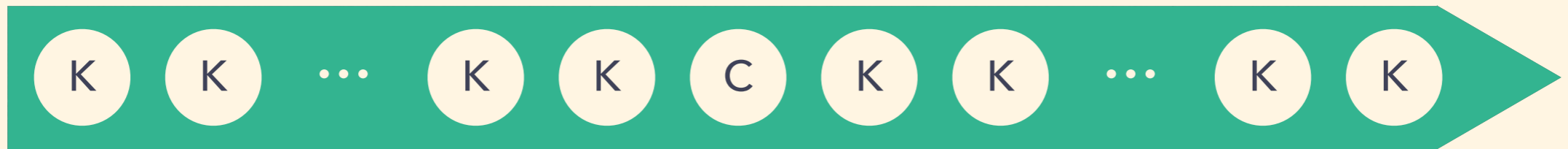
Adversary will have no advantage.

Intuition: decryption contains random $e(g_2, g_2)^{\hat{a}\hat{s}}$

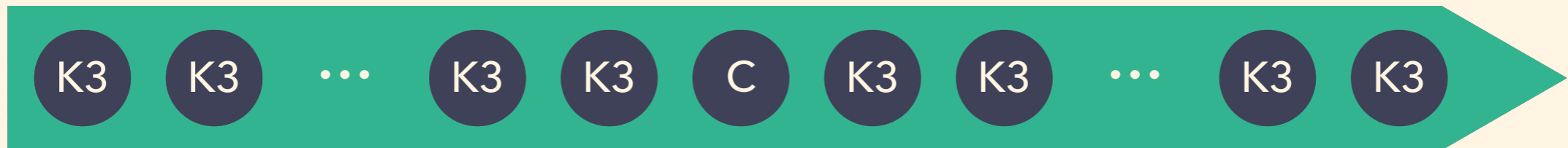
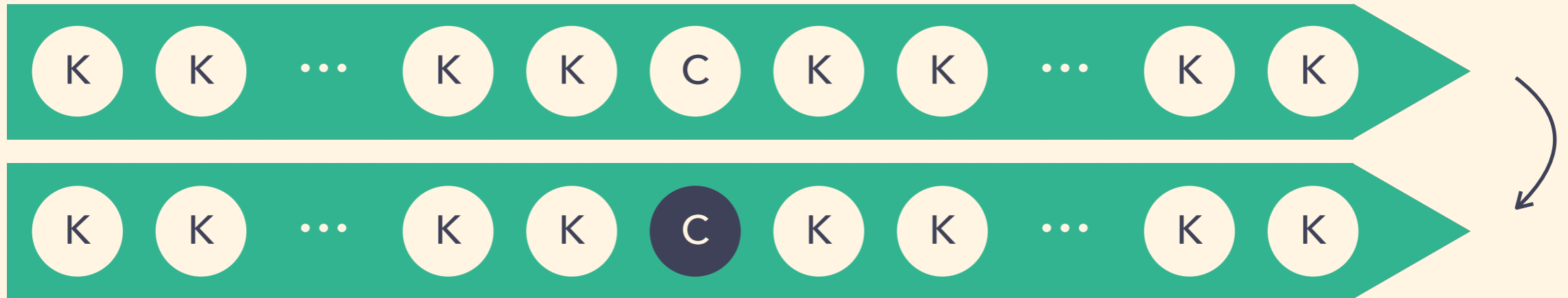
$$\textcircled{C} \quad g_1^{c(s,h)} \cdot \underline{g_2^{c(\hat{s},\hat{h})}} \qquad \textcircled{K3} \quad g_1^{k(a,r,h)} \cdot \underline{g_2^{k(\hat{a},0,0)}}$$




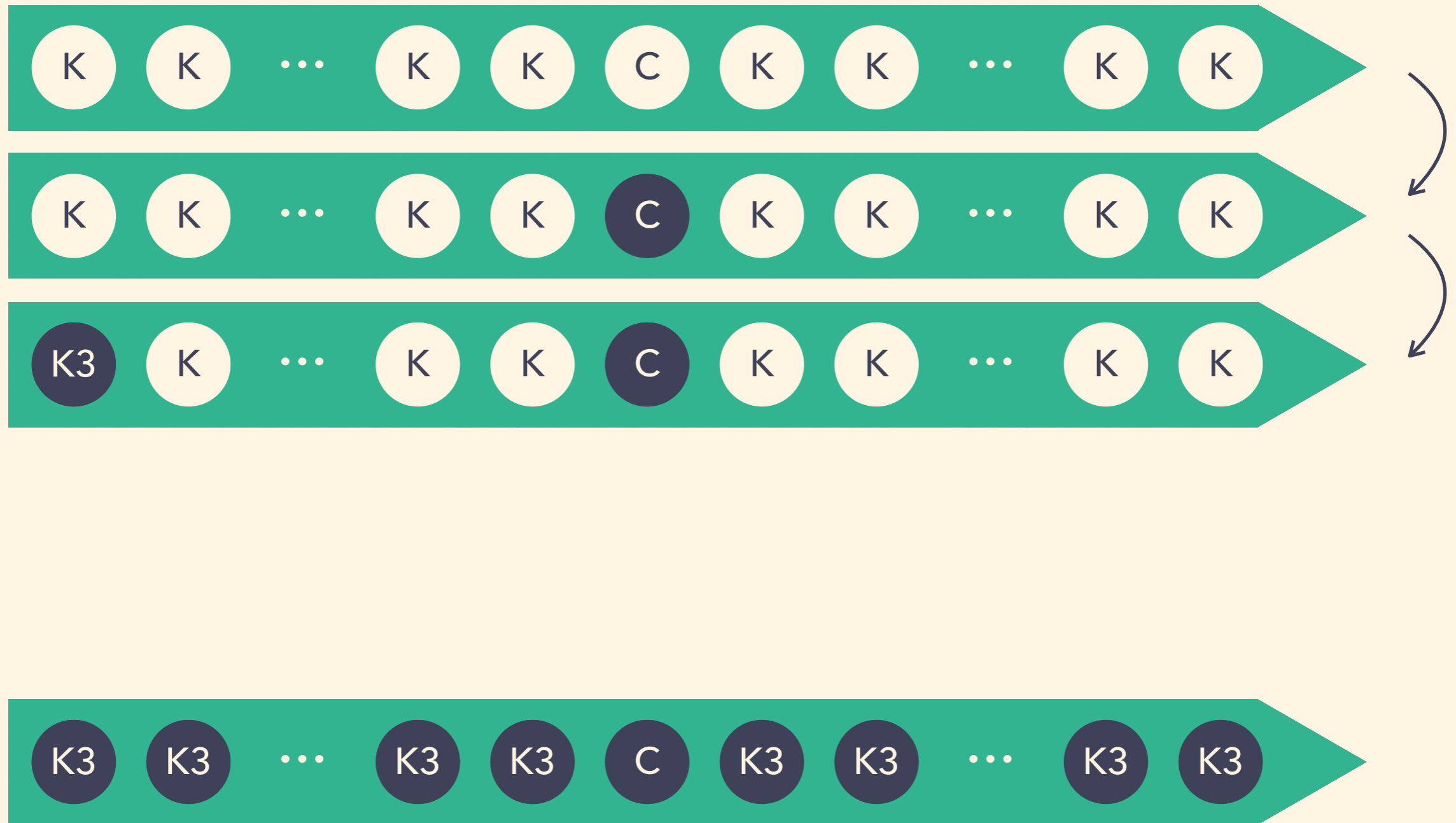
Game Sequence in the Proof



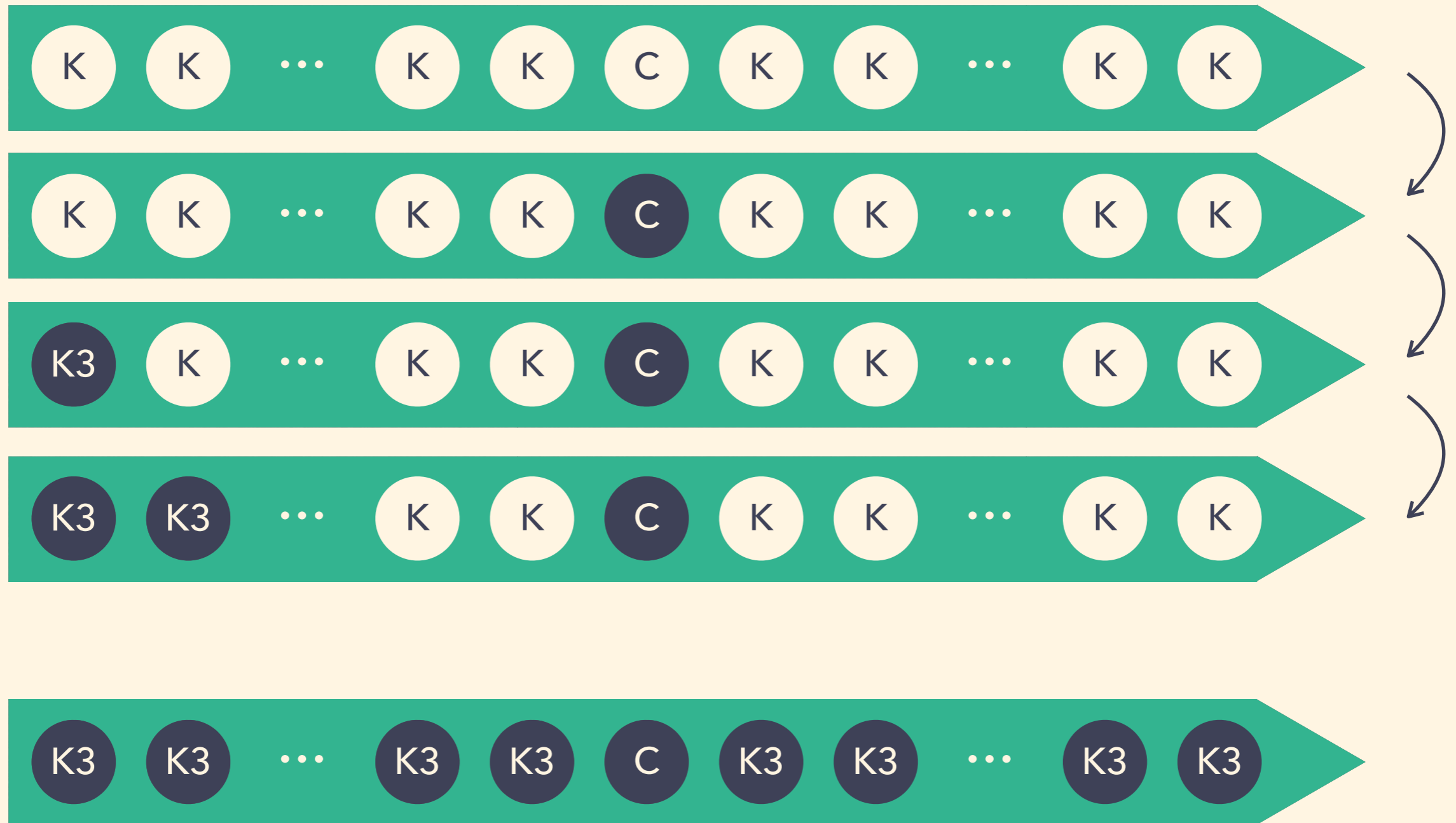
Game Sequence in the Proof



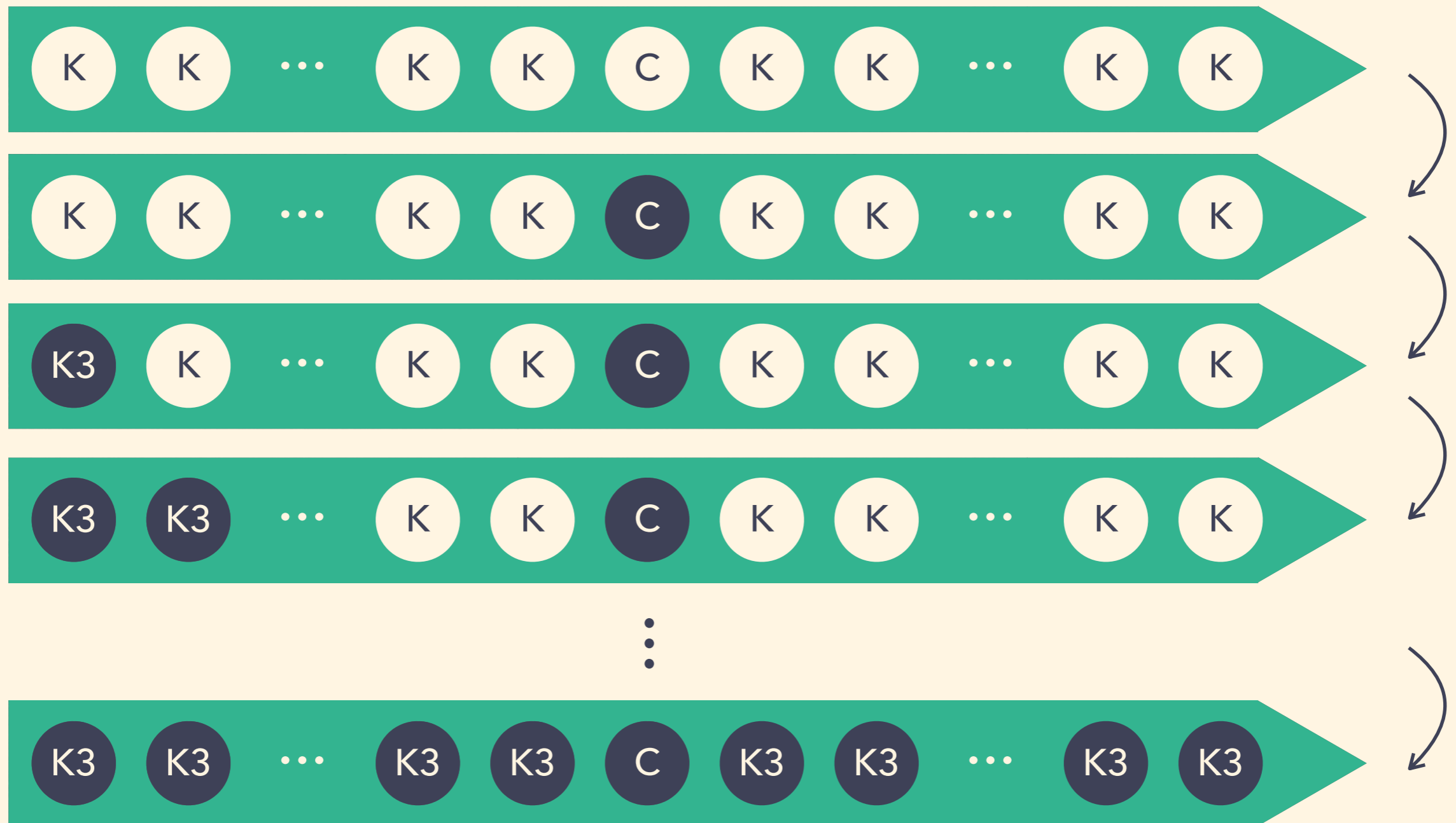
Game Sequence in the Proof



Game Sequence in the Proof

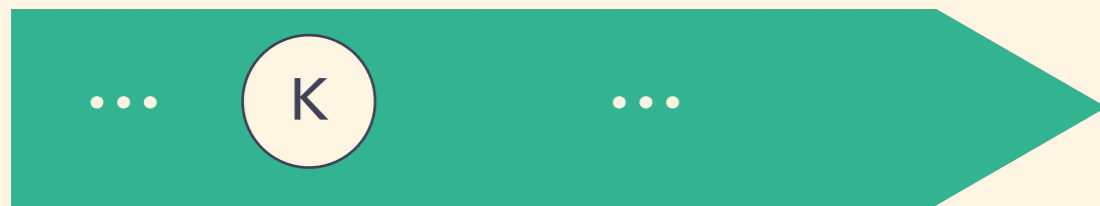


Game Sequence in the Proof

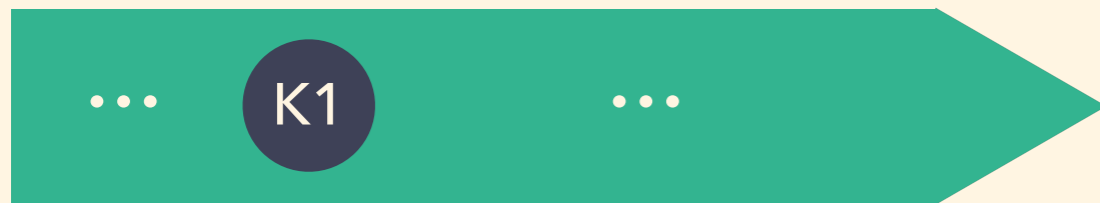


Game Subsequence

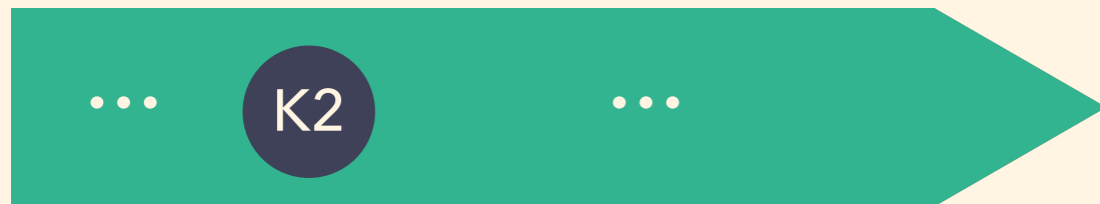
Indistinguishability based on



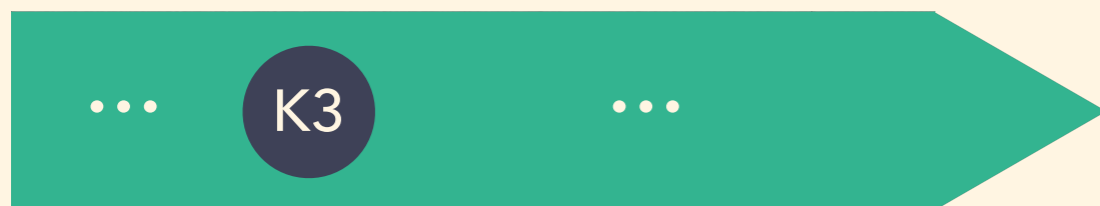
Subgroup Decision



Security of encoding

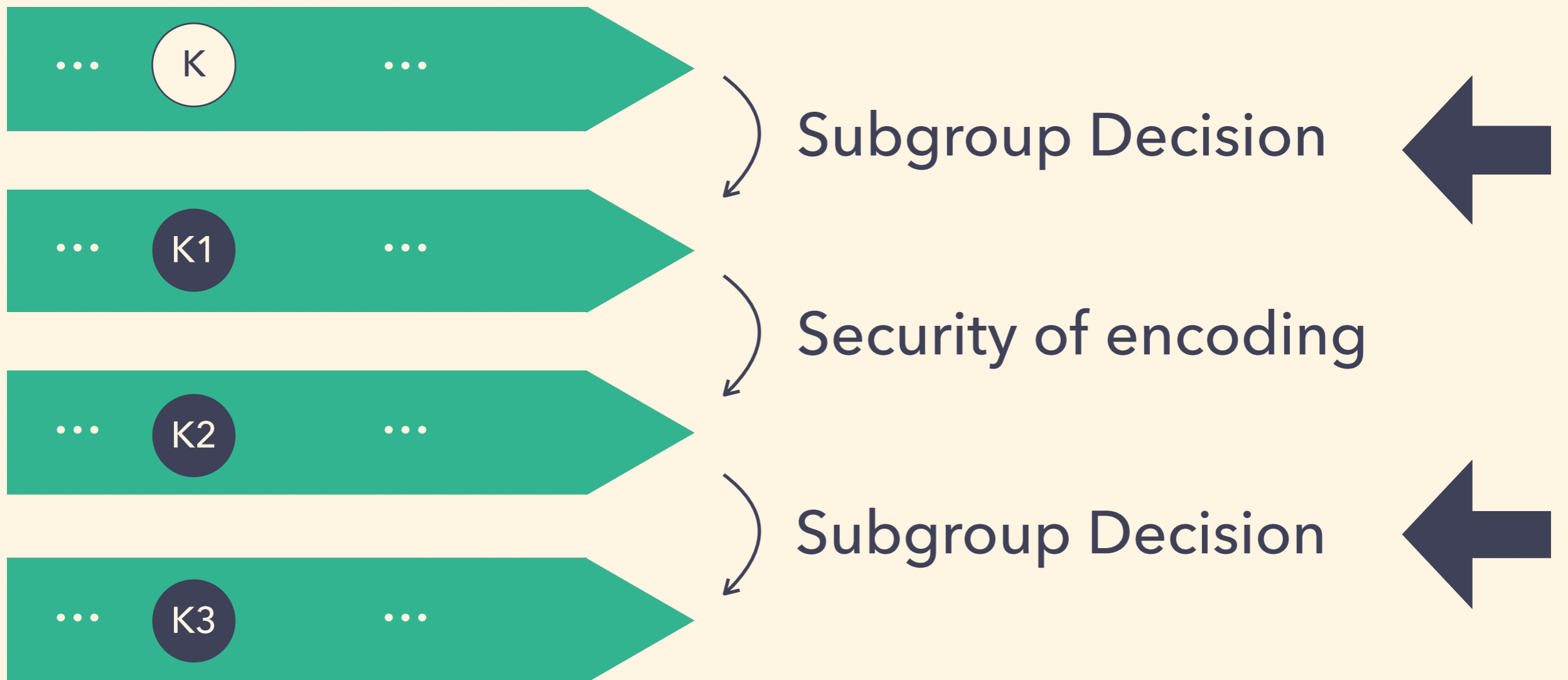


Subgroup Decision



Game Subsequence

Indistinguishability based on

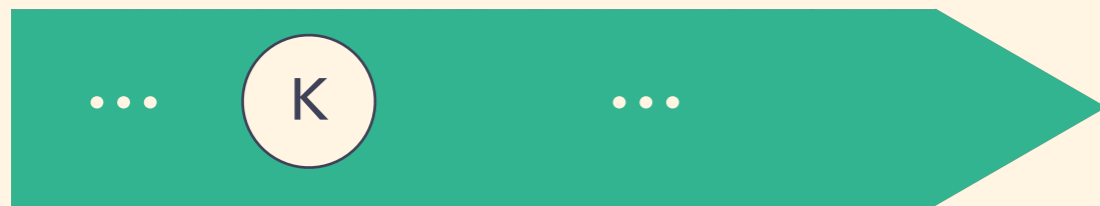


Intuition: These two do not depend on encoding.

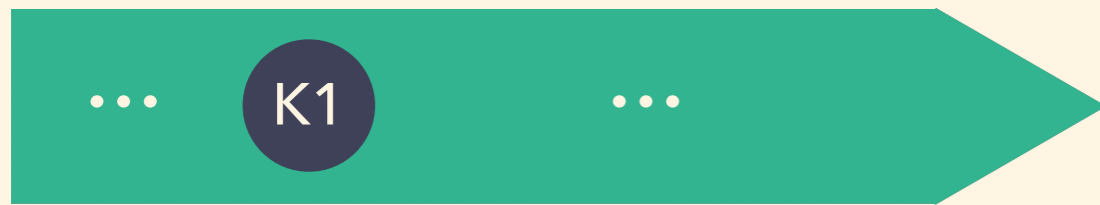
Use linearity, param-vanishing of \mathbf{k} and orthogonality of G .

Game Subsequence

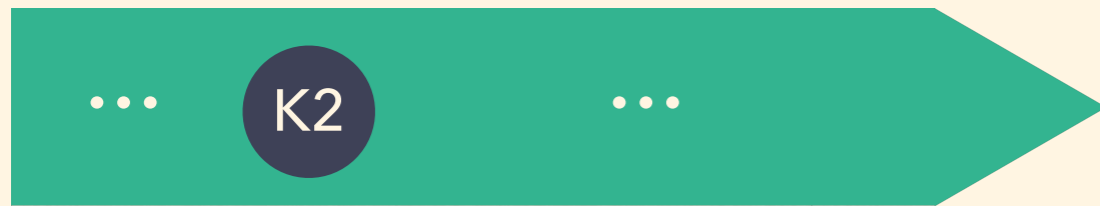
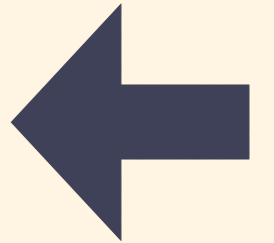
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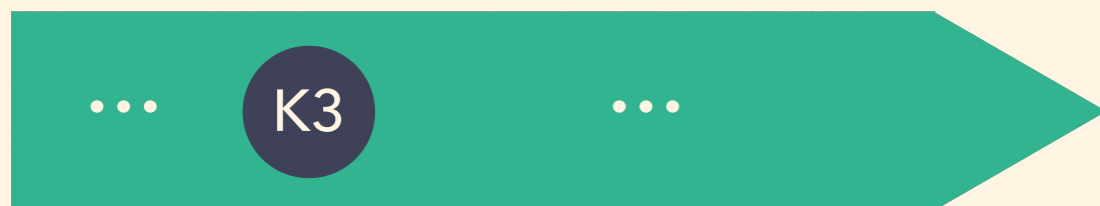
Subgroup Decision



Security of encoding



Subgroup Decision



The 2nd Transition

K1 $g_1^{k(a,r,h)} \cdot g_2^{k(0,\hat{r},\hat{h})}$

K2 $g_1^{k(a,r,h)} \cdot g_2^{k(\hat{a},\hat{r},\hat{h})}$

Security of encoding
(to be defined)

The 2nd Transition

K1 $g_1^{k(a,r,h)} \cdot g_2^{k(0,\hat{r},\hat{h})}$

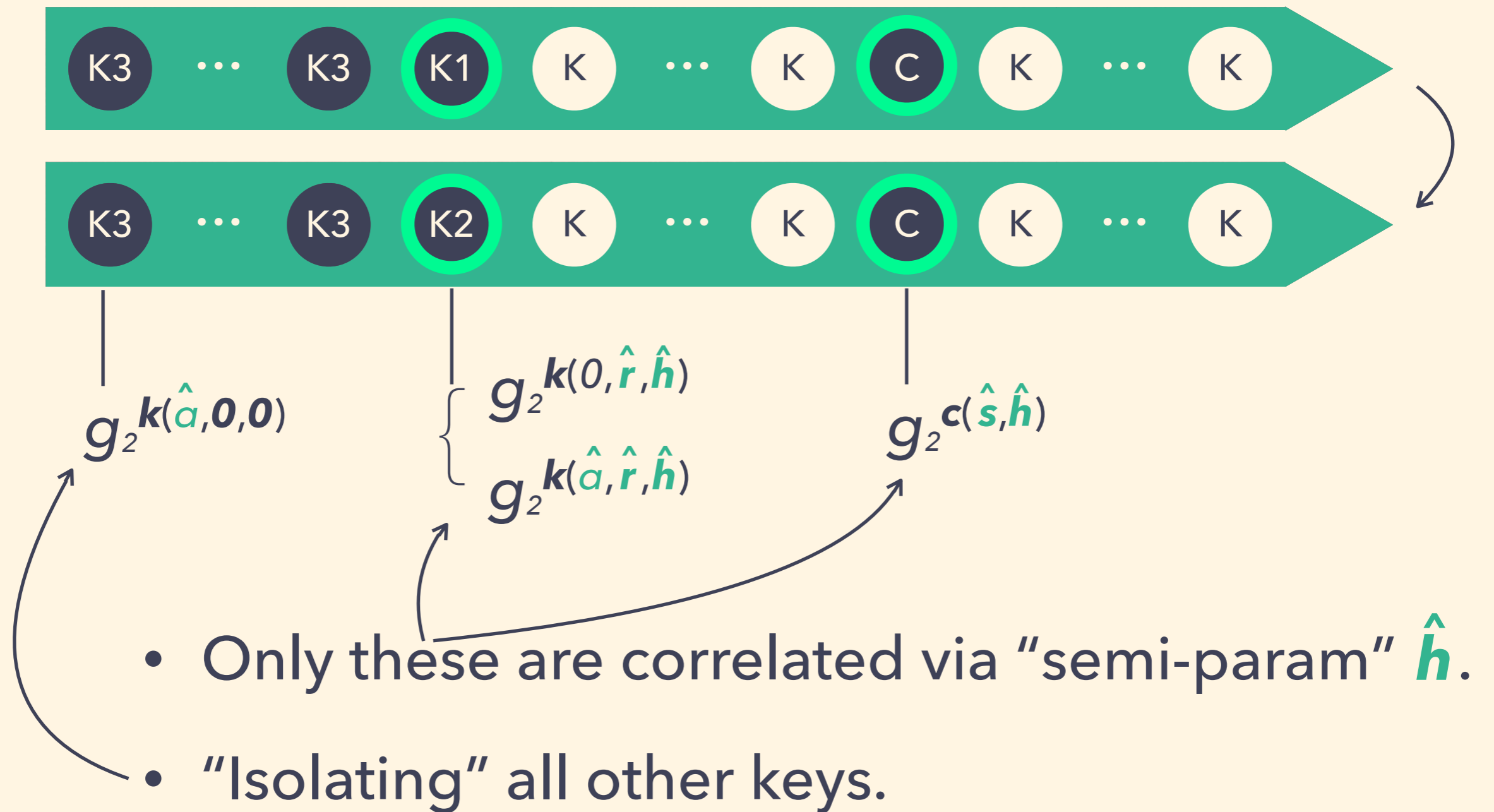
K2 $g_1^{k(a,r,h)} \cdot g_2^{k(\hat{a},\hat{r},\hat{h})}$

Security of encoding
(to be defined)

Idea: just define security of encoding to be exactly the indistinguishability of these two games!

The 2nd Transition

In More Details



Defining Security of Encoding

Computationally secure encoding



Cannot
distinguish

$$\left\{ \begin{array}{l} g_2^{k(0, \hat{r}, \hat{h})} \\ g_2^{k(\hat{a}, \hat{r}, \hat{h})} \end{array} \right. \quad g_2^{c(\hat{s}, \hat{h})}$$

Defining Security of Encoding

Perfectly secure encoding

Identical
(info-theoretic) $\left\{ \begin{array}{l} k(0, \hat{r}, \hat{h}) \\ k(a, \hat{r}, \hat{h}) \end{array} \right.$ $c(\hat{s}, \hat{h})$

Computationally secure encoding



Cannot
distinguish

$\left\{ \begin{array}{l} g_2^{k(0, \hat{r}, \hat{h})} \\ g_2^{k(a, \hat{r}, \hat{h})} \end{array} \right.$ $g_2^{c(\hat{s}, \hat{h})}$

Defining Security of Encoding

Perfectly secure encoding

Identical
(info-theoretic)

$$\left\{ \begin{array}{l} k(0, \hat{r}, \hat{h}) \\ k(a, \hat{r}, \hat{h}) \end{array} \right. \quad c(\hat{s}, \hat{h})$$

Computationally secure encoding



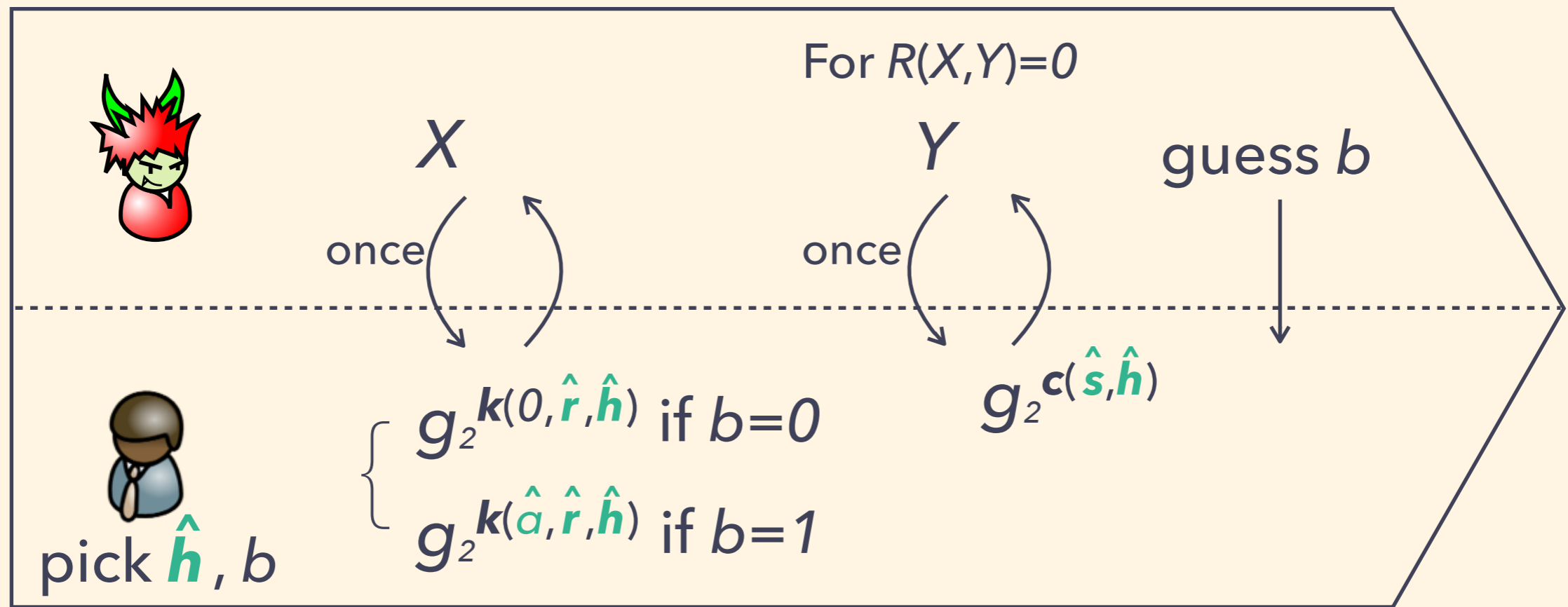
Cannot
distinguish

$$\left\{ \begin{array}{l} g_2^{k(0, \hat{r}, \hat{h})} \\ g_2^{k(\hat{a}, \hat{r}, \hat{h})} \end{array} \right. \quad g_2^{c(\hat{s}, \hat{h})}$$

1st flavor: k before c

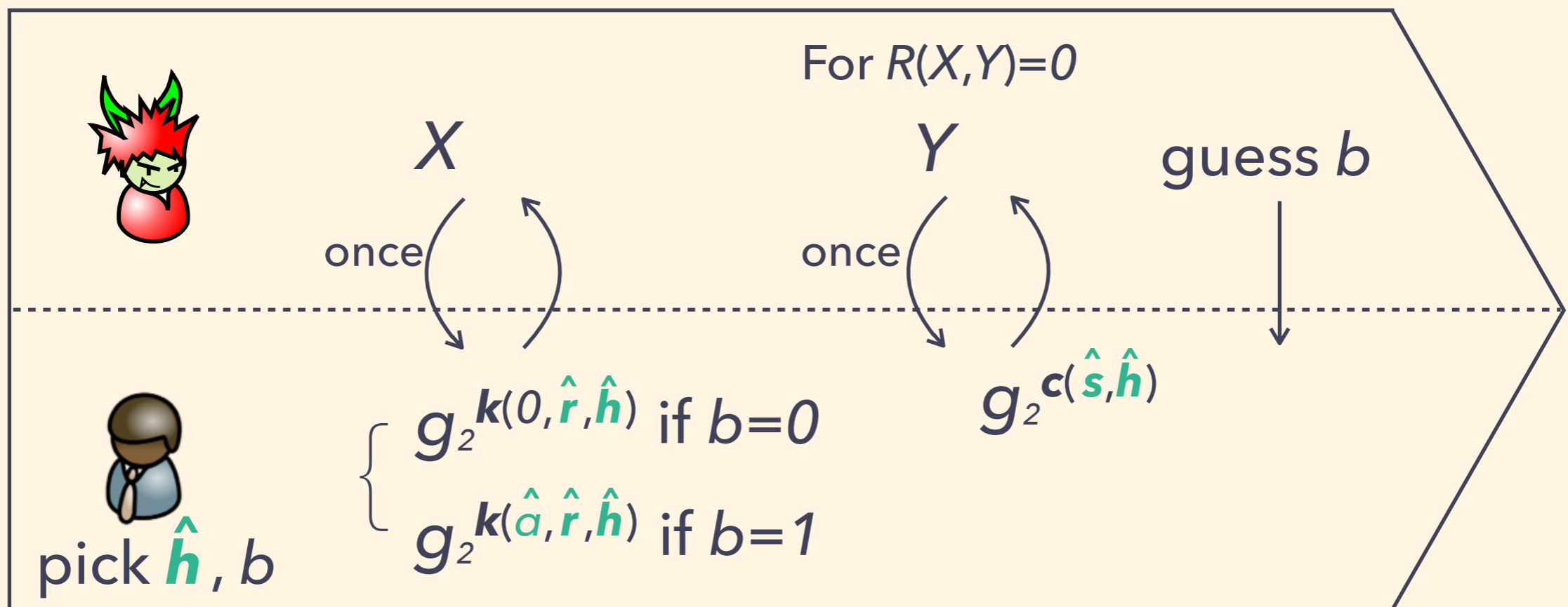
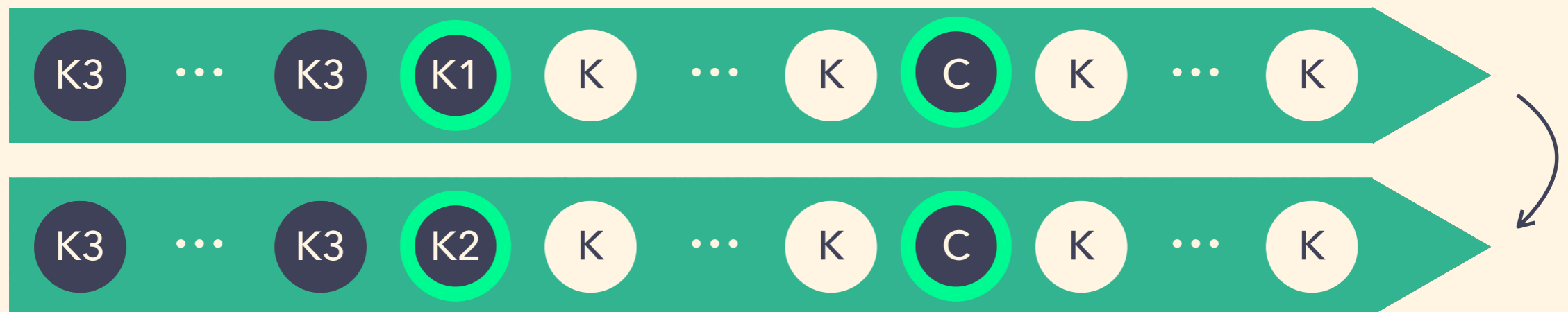
2nd flavor: c before k

Computationally Security 1: k before c

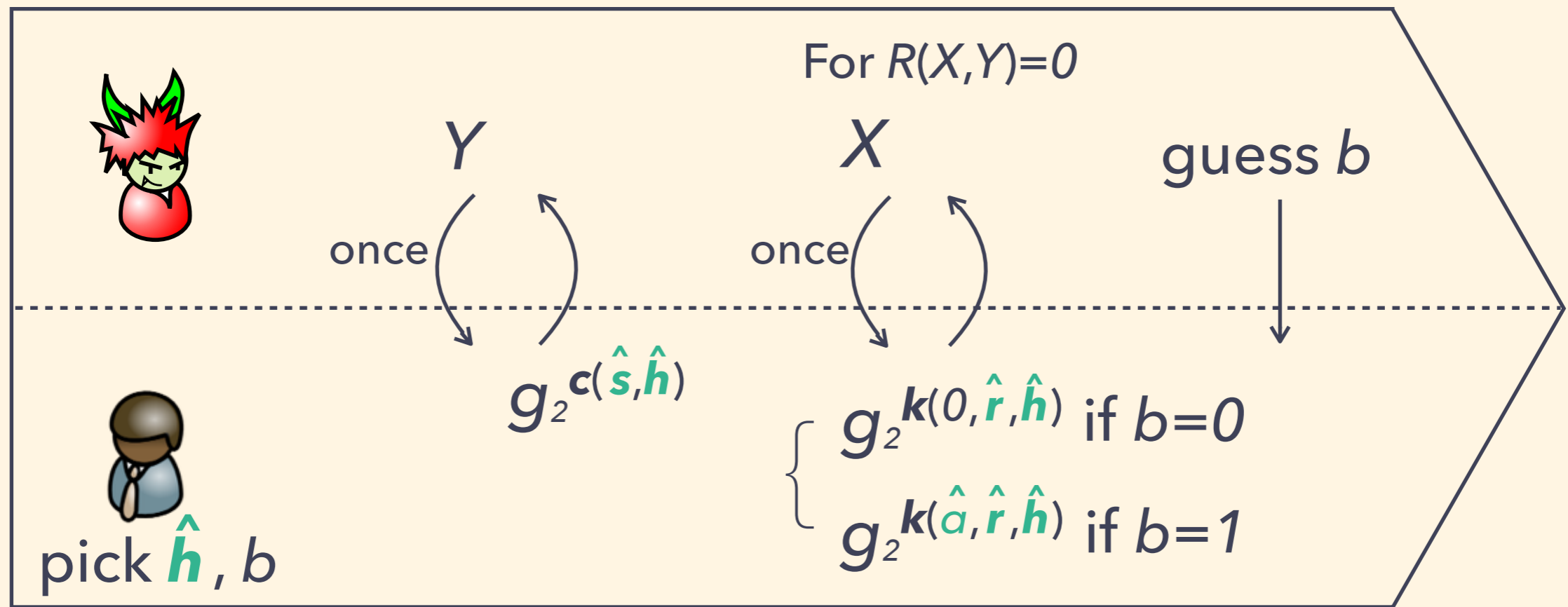


Computationally Security 1: *k* before *c*

For Transitions of *Pre-challenge Keys*

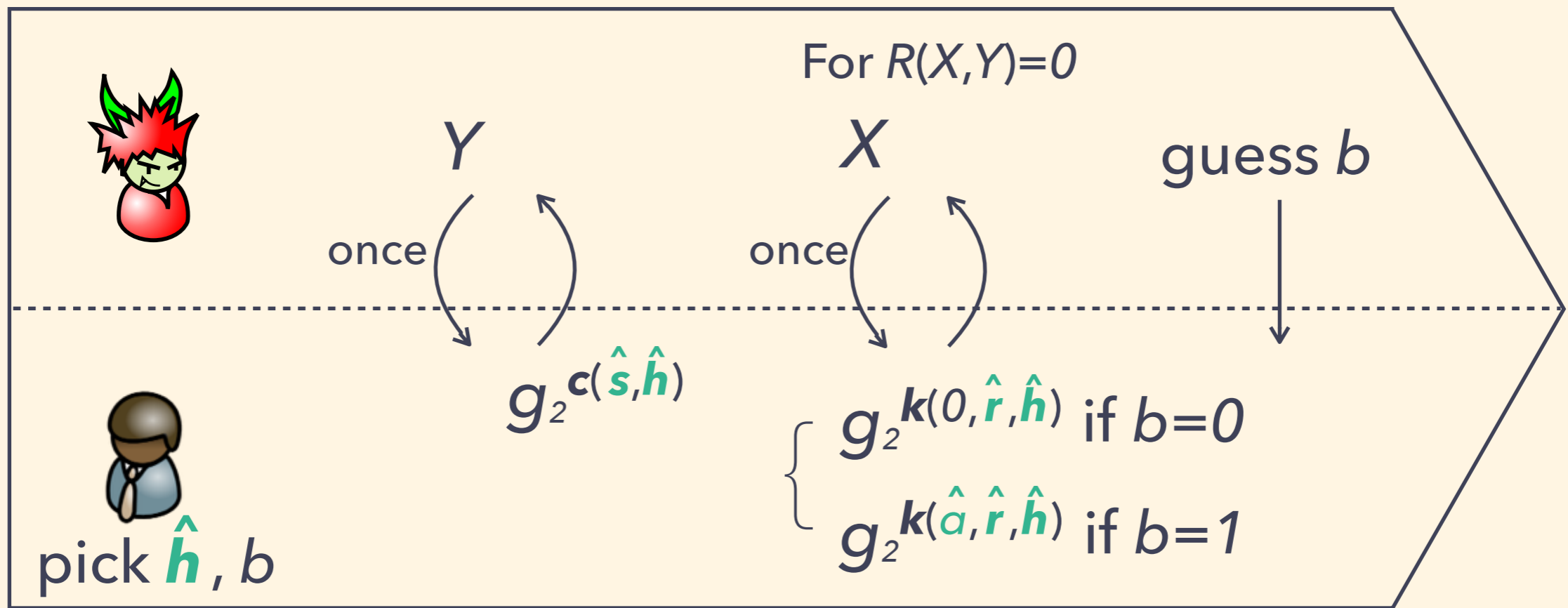
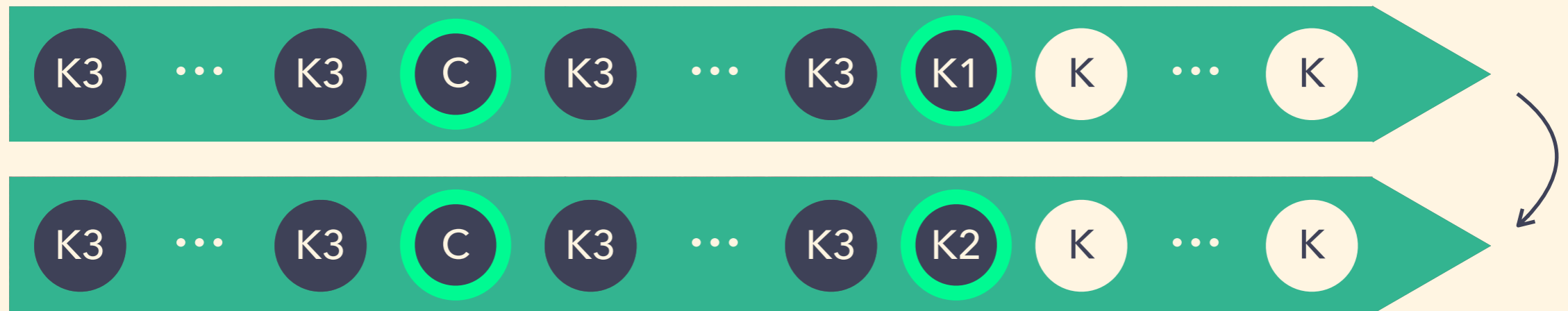


Computationally Security 2: c before k

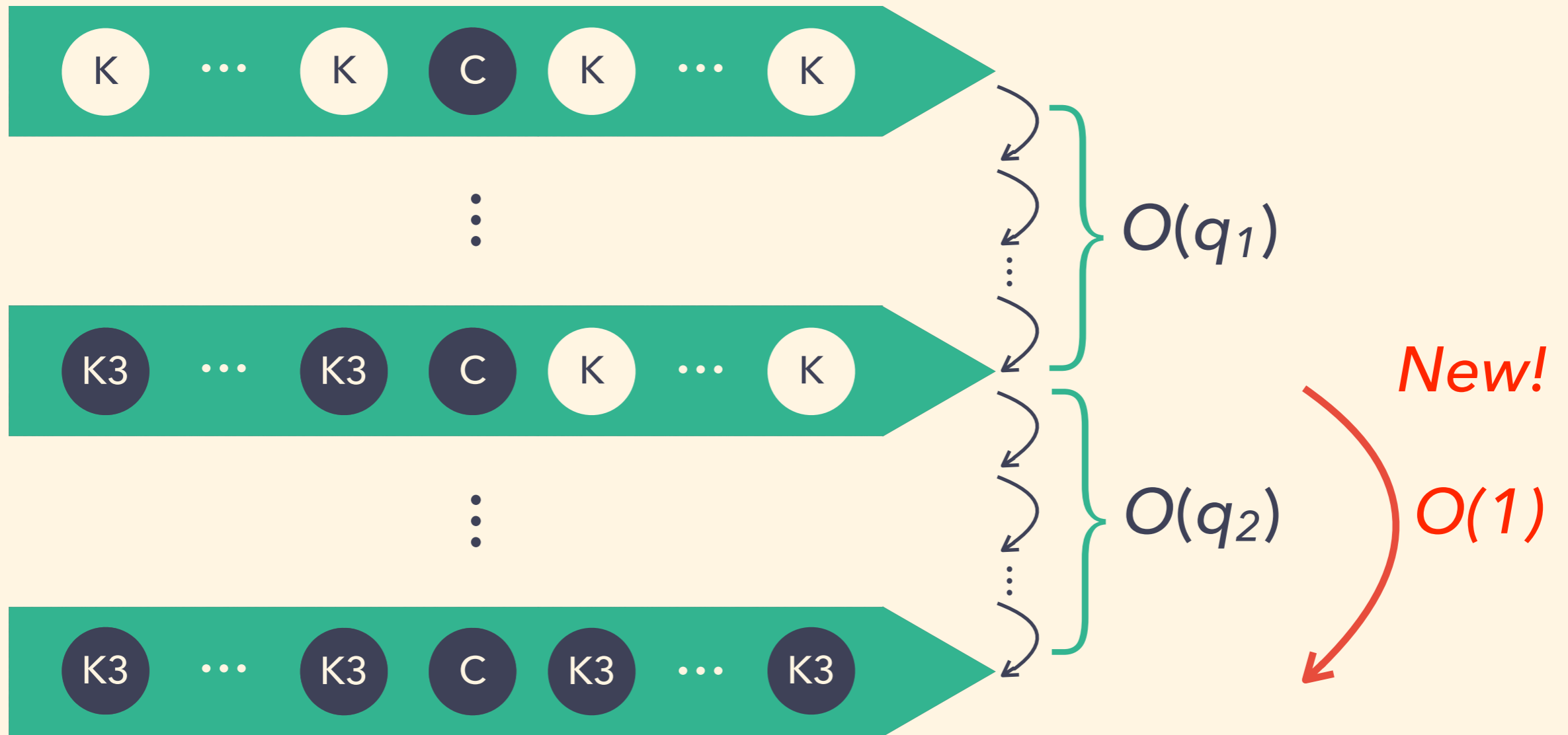


Computationally Security 2: c before k

For Transitions of *Post-challenge Keys*

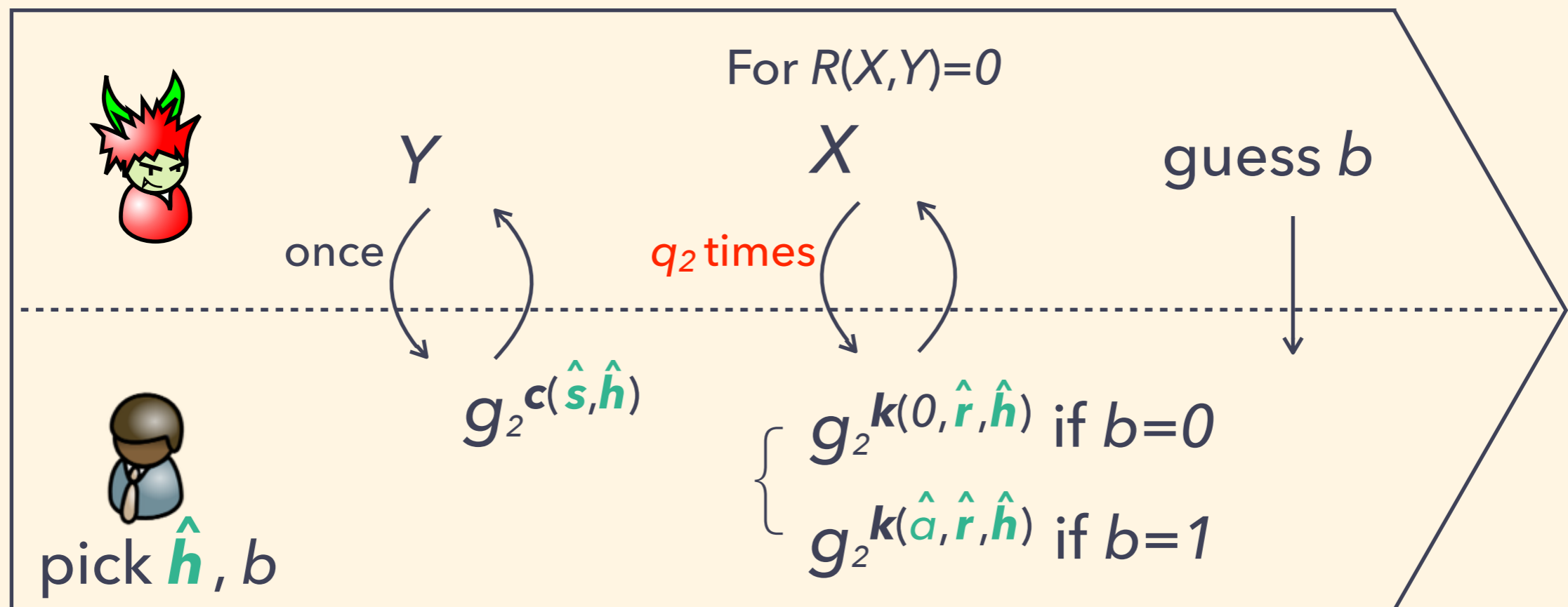


Tighter Security Proof

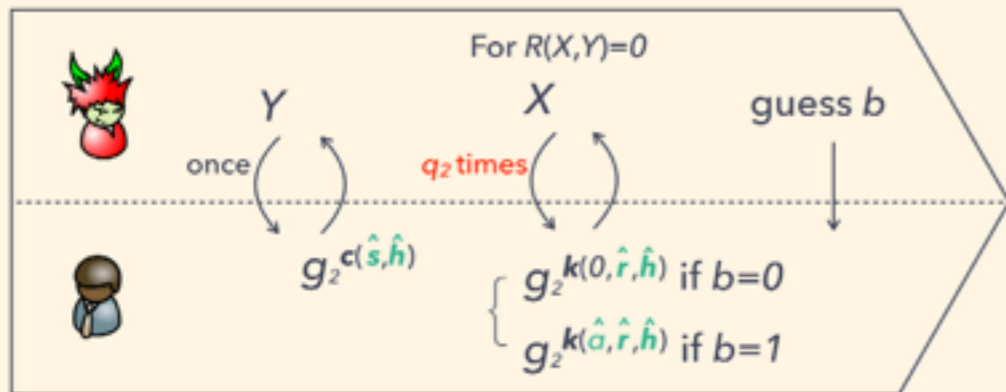


Refining Computationally Security 2

For Transitions of *Post-challenge Keys*

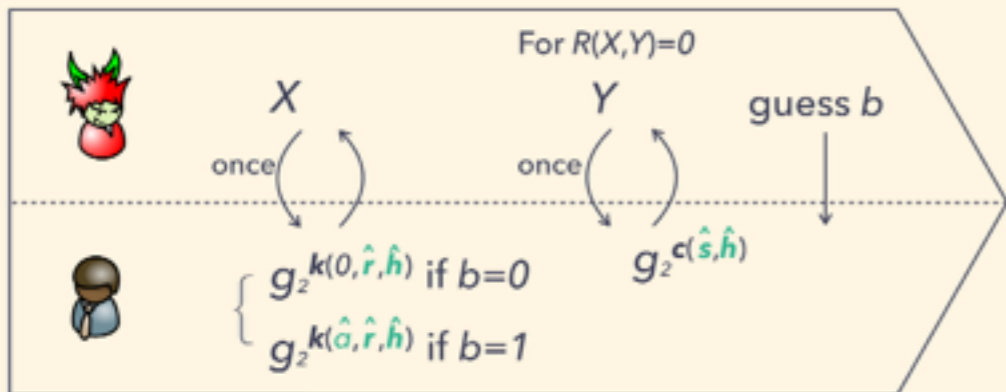


“Doubly Selective Security”



$Y \rightarrow \text{program } \hat{h} \rightarrow X$

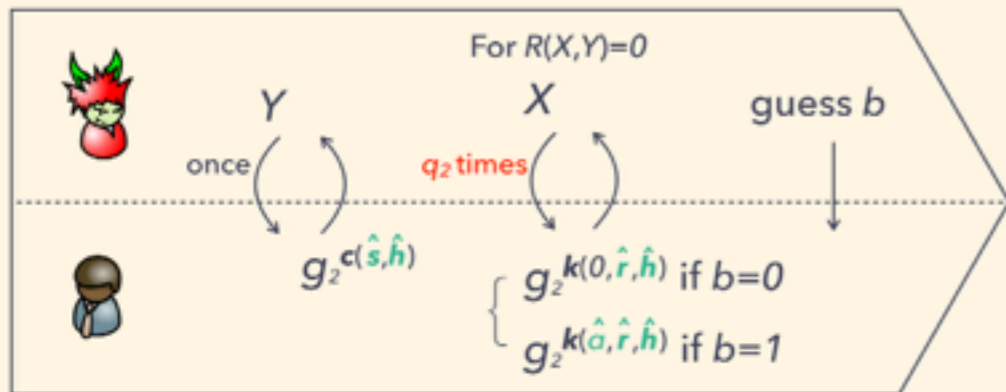
The 2nd notion is called **Selective** Master-key Hiding since the order of queries mimics selective security of FE but in semi-functional space.



$X \rightarrow \text{program } \hat{h} \rightarrow Y$

The 1st notion is called **Co-Selective** Master-key Hiding since the order of queries mimics co-selective security of FE but in semi-functional space.

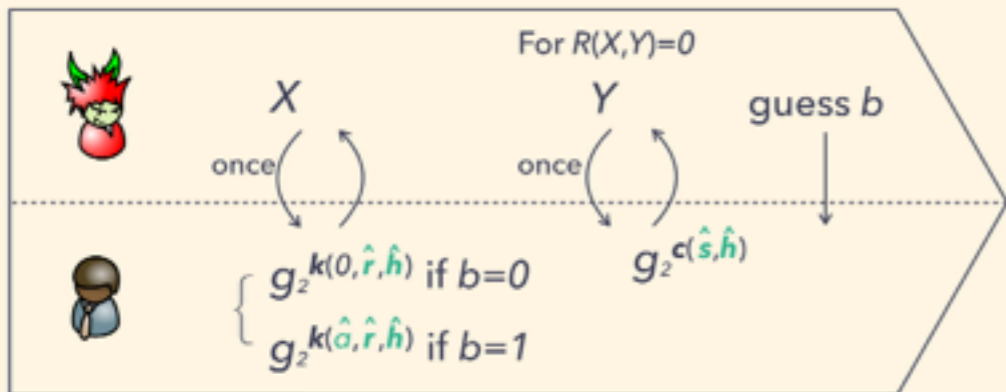
"Doubly Selective Security"



$Y \rightarrow \text{program } \hat{h} \rightarrow X$

The 2nd notion is called **Selective** Master-key Hiding

Can borrow proof techniques for selective security of FE



$X \rightarrow \text{program } \hat{h} \rightarrow Y$

The 1st notion is called **Co-Selective** Master-key Hiding

Can borrow proof techniques for co-selective security of FE or selective security of its dual!

3

Instantiations

Fully Secure IBE

Lewko-Waters IBE $h=(h_1,h_2)$ **Our new IBE** $h=(h_1,h_2,h_3)$

$$k=(a+r(h_1+h_2ID), r)$$

$$k=(a+r_1(h_1+h_2ID)+r_2h_3, r_1, r_2)$$

$$c=(s, s(h_1+h_2ID'))$$

$$c=(s, s(h_1+h_2ID'), sh_3)$$

- Encoding is perfect.
 - $f(x)=h_1+h_2x$ is pair-wise independent
- Full security of IBE: $O(q_{all})$ to Subgroup Decision

- Encoding is perfect.
- Encoding is also selective under 3-party DH.
- Full security of IBE: $O(q_1)$ to Subgroup Decision plus $O(1)$ to 3-party DH

Fully Secure IBE

	Public key	Ciphertext , key	Reduction	Assumption
Waters 05	$O(n)$	$O(1)$	$O(nq_{all})$	DBDH
Gentry 06	$O(1)$	$O(1)$	$O(1)$	q_{all} -ABDHE
Waters 09	$O(1)$	$O(1)$	$O(q_{all})$	DBDH, DLIN
Lewko-Waters 10	$O(1)$	$O(1)$	$O(q_{all})$	subgroup
Chen-Wee 13	$O(n)$	$O(1)$	$O(n)$	DLIN
Our IBE	$O(1)$	$O(1)$	$O(q_1)$	3DH, subgroup

n = ID length

FE for Regular Languages

	Security	Reduction	Assumption
Waters 12	selective	$O(1)$	Q-type
Our FE for regular languages	full	$O(q_1)$	Q-type, subgroup

FE for Regular Languages

Selective security of our encoding

- Borrow techniques from selective security of Waters'.
- Hence, use a similar "Q-type" assumption to Waters'.
 - Q is ciphertext attribute size of *one query*.
 - Q is *not* the number of queries (q_1, q_2).

Co-selective security of our encoding

- New techniques, new Q-type assumption.

KP-ABE with Short Ciphertext

	Ciphertext size	Key size	Security	Reduction	Assumption
A.-Libert-Panafieu 11	$O(1)$	$O(tk)$	selective	$O(1)$	Q-type
Takashima 14	$O(1)$	$O(tk)$	selective	$O(q_{all})$	DLIN
Our ABE w/ short ciphertext	$O(1)$	$O(tk)$	full	$O(q_1)$	Q-type, subgroup

t = max attribute set in ciphertext, k = policy size for key

Unbounded KP-ABE

	Large universe ?	Unbounded attribute repetition?	Security	Reduction	Assumption
Lewko-Waters 11	yes	yes	selective	$O(q_{all})$	subgroup
Lewko-Waters 12	no	yes	full	$O(q_{all})$	Q-type, subgroup
Okamoto-Takashima 12b	yes	no	full	$O(q_{all})$	DLIN
Rouselakis-Waters 13	yes	yes	selective	$O(1)$	Q-type
Our unbounded ABE	yes	yes	full	$O(q_1)$	Q-type, subgroup

More Results

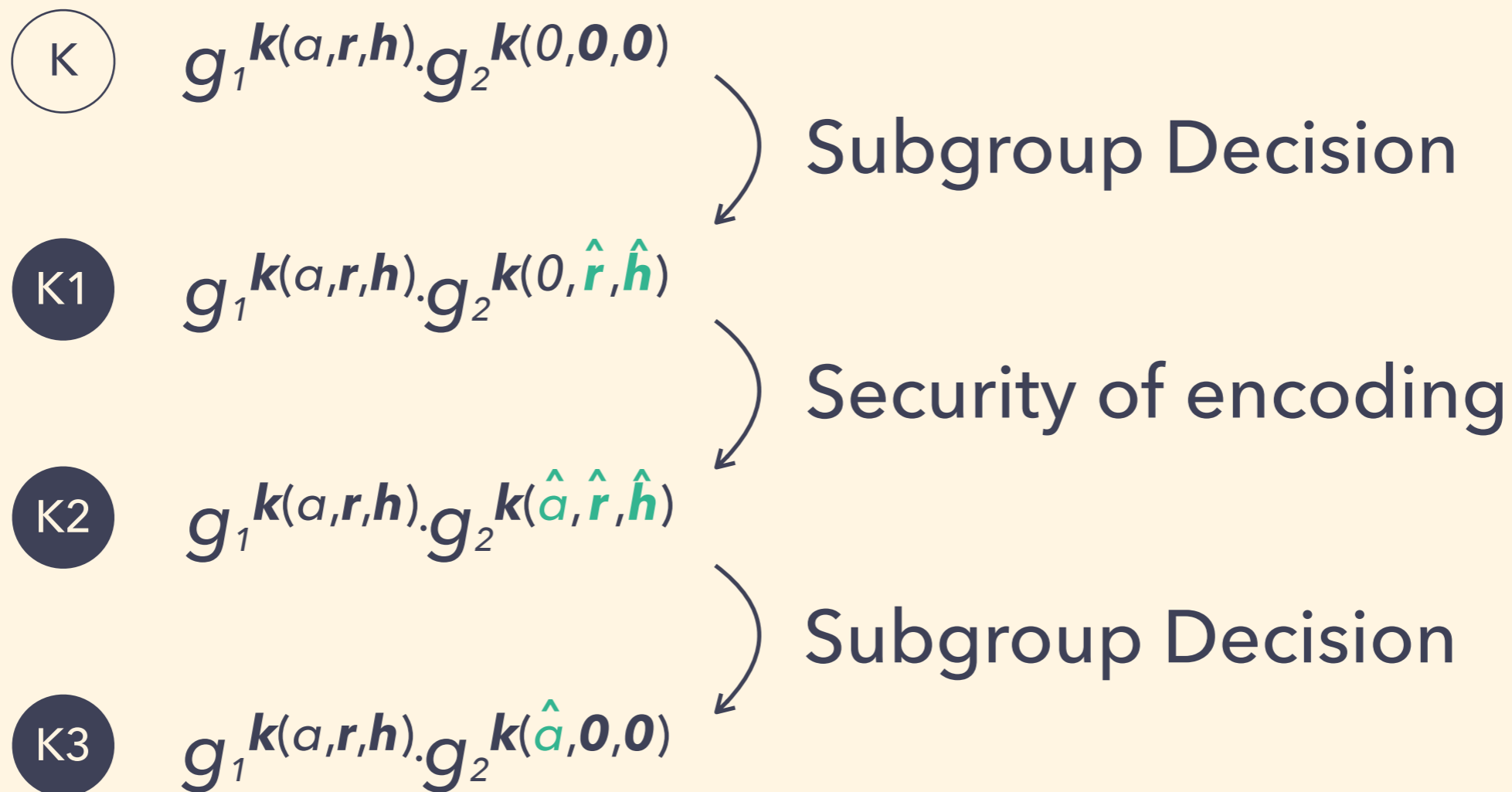
- Generic dual scheme conversion for perfectly-secure encoding.
 - Convert key-policy to ciphertext-policy (& vice versa)
- Fully-secure dual (ciphertext-policy) FE for regular languages.
- Unification of schemes based on dual systems and some improvements.

Take-Home Ideas

- Our framework can be considered as a method for boosting doubly selectively security (of encoding) to fully security (of FE).
- Why does it matters? Proving double selective security of encoding can use techniques from proving classical selective security.

Thank you

Recall the Definitions of Semi-Keys



Proof for the 1st Transition

(K) $g_1^{k(a,r,h)} \cdot g_2^{k(0,0,0)}$

(K1) $g_1^{k(a,r,h)} \cdot g_2^{k(0,\hat{r},\hat{h})}$

Subgroup Decision

Subgroup Decision problem: Decide if $T \in G_1$ or $T \in G_{12}$

Proof for the 1st Transition

Simulated by

$$\textcircled{K} \quad g_1^{k(a,r,h)} \cdot g_2^{k(0,0,0)}$$

$$\textcircled{K1} \quad g_1^{k(a,r,h)} \cdot g_2^{k(0,\hat{r},\hat{h})}$$

$$g_1^{k(a,r,h')} \cdot (g_1^{t_1})^{k(0,r',h')}$$

$$g_1^{k(a,r,h')} \cdot (g_1^{t_1} g_2^{t_2})^{k(0,r',h')}$$

Subgroup Decision problem: Decide if $T \in G_1$ or $T \in G_{12}$

Proof for the 1st Transition

Simulated by

$$\begin{array}{ccc}
 \textcircled{K} & g_1^{k(a,r,h)} \cdot g_2^{k(0,0,0)} & g_1^{k(a,r,h')} \cdot (g_1^{t_1})^{k(0,r',h')} \\
 & \searrow & \swarrow \textcircled{T} \\
 \textcircled{K1} & g_1^{k(a,r,h)} \cdot g_2^{k(0,\hat{r},\hat{h})} & g_1^{k(a,r,h')} \cdot (g_1^{t_1} g_2^{t_2})^{k(0,r',h')}
 \end{array}$$

Subgroup Decision problem: Decide if $T \in G_1$ or $T \in G_{12}$

Simulation is OK due to linearity, param-vanishing of k and "parameter-hiding" of G :

$h = h' \pmod{p_1}$ and $\hat{h} = h' \pmod{p_2}$ are independent.

Proof for the 3rd Transition is similar

K2 $g_1 k(a,r,h).g_2 k(\hat{a},\hat{r},\hat{h})$

K3 $g_1 k(a,r,h).g_2 k(\hat{a},0,0)$



Simulated by

$$g_1 k(a,r,h').(g_1^{t_1} g_2^{t_2}) k(a',r',h')$$

$$g_1 k(a,r,h').(g_1^{t_1}) k(a',r',h')$$

