Tight Security Bounds for Key-Alternating Ciphers

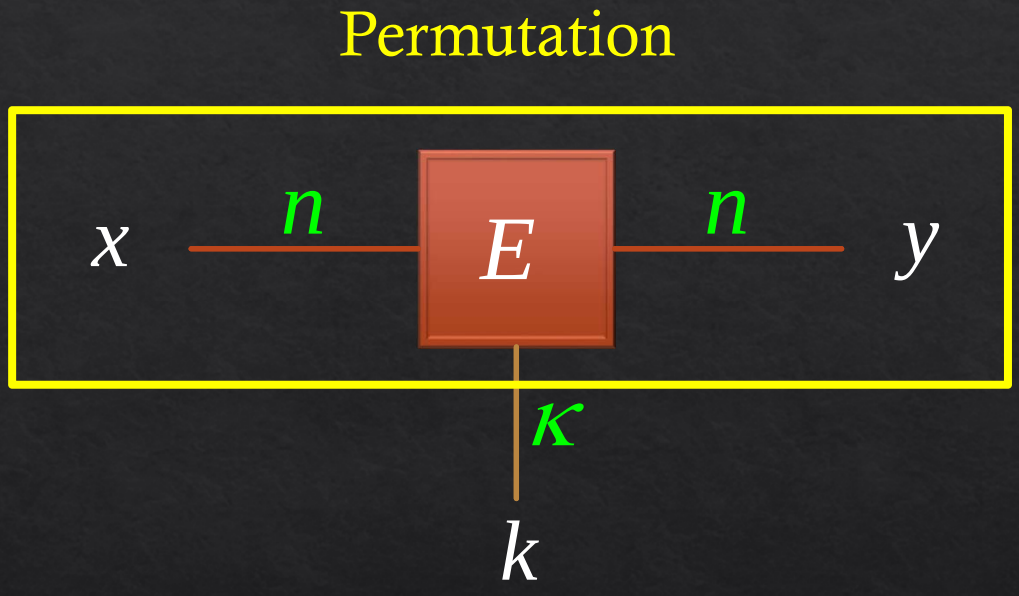
Shan Chen and John Steinberger

Institute for Interdisciplinary Information Sciences

Tsinghua University

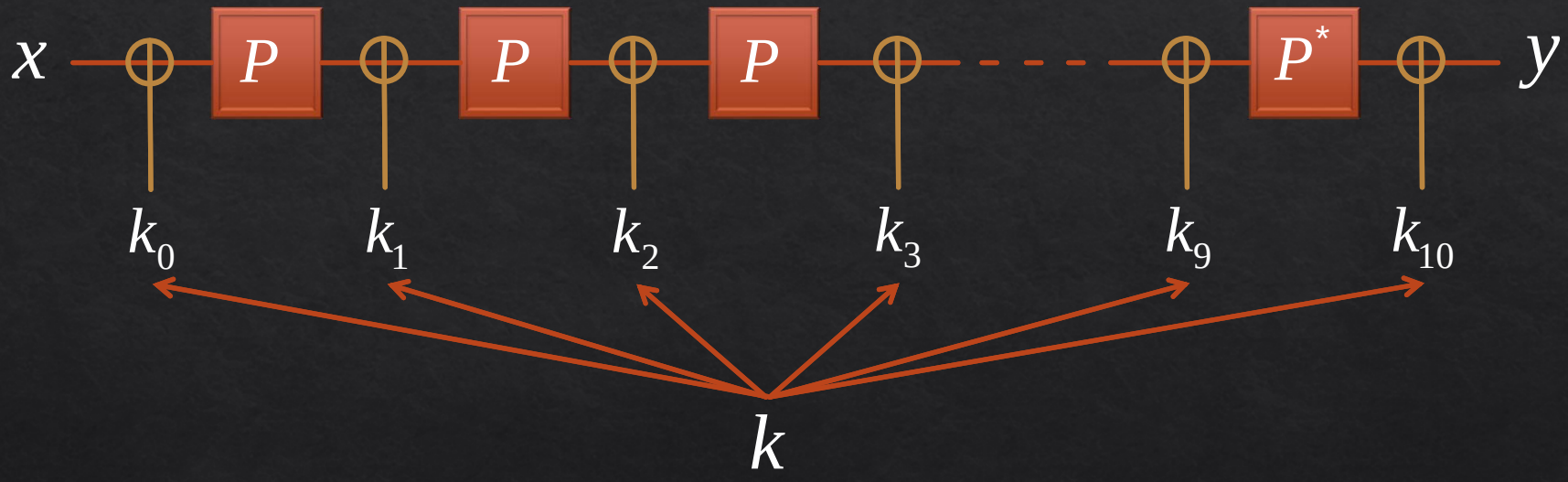
China

Block Cipher



AES Cipher

Permutation

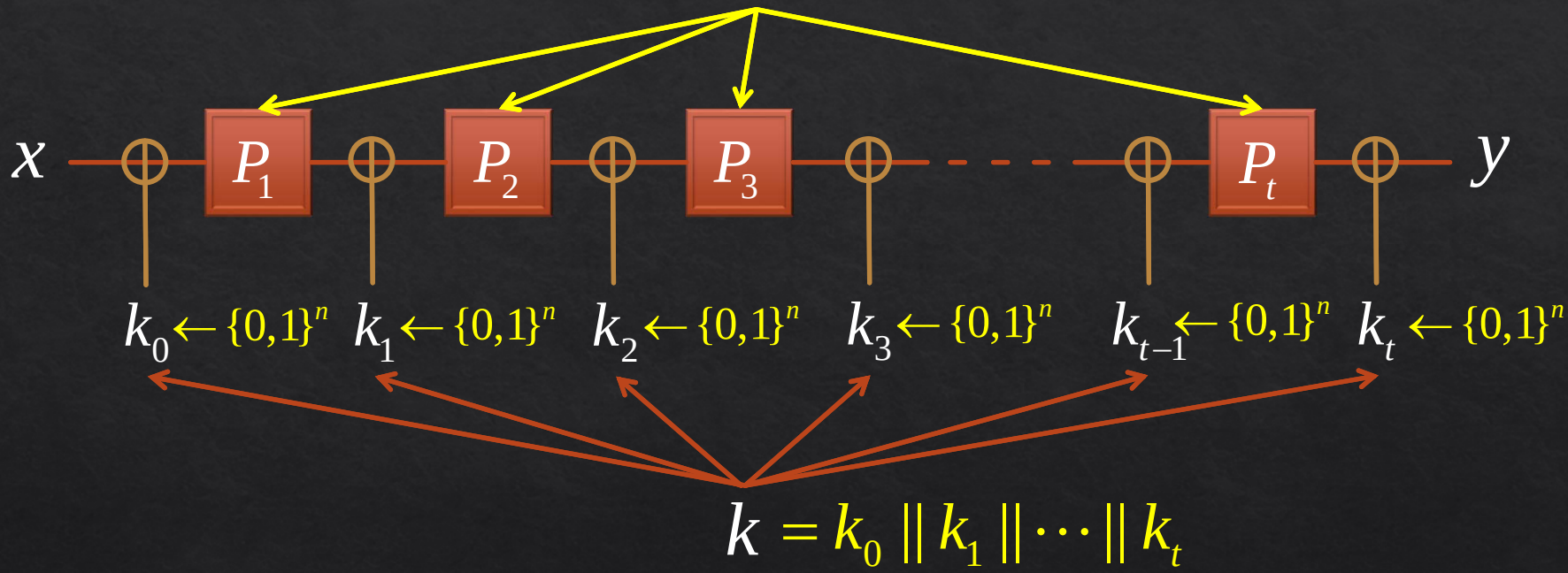


Master Key

$\approx_C$ Random Permutation Q

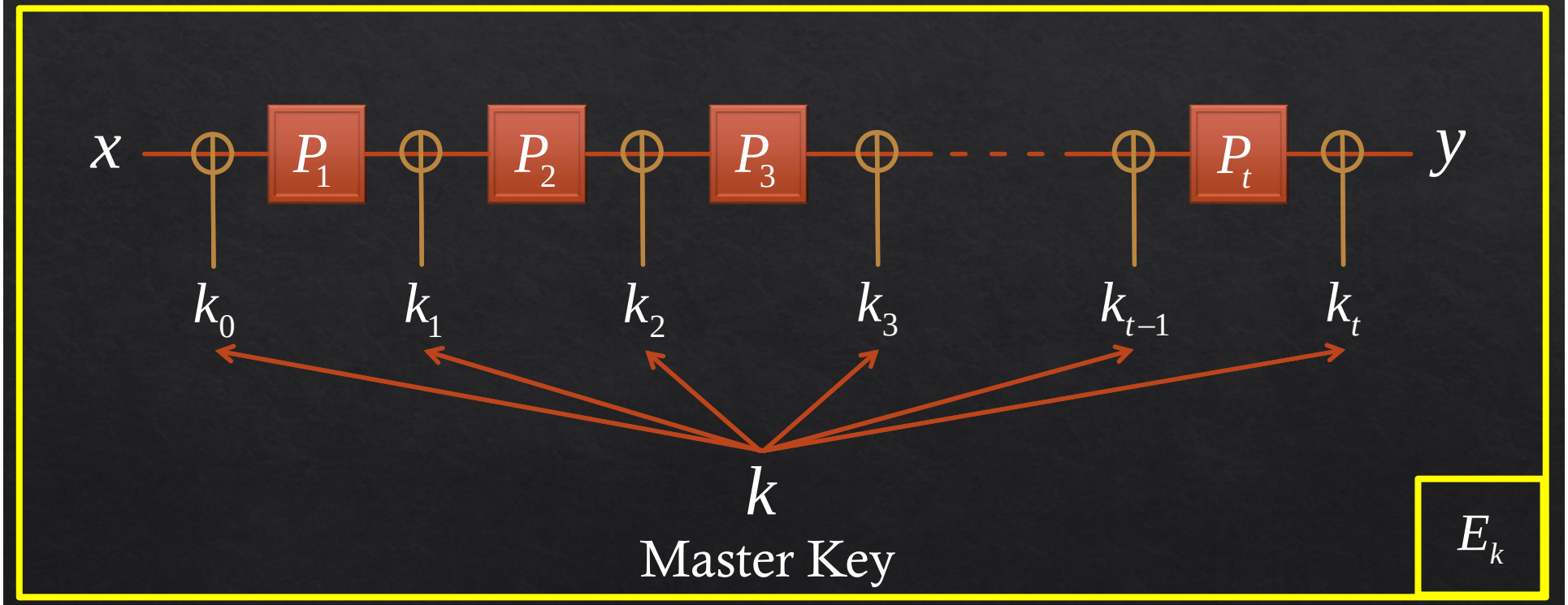
Key-Alternating Ciphers (Ideal Permutation Model)

Uniformly Random and Independent

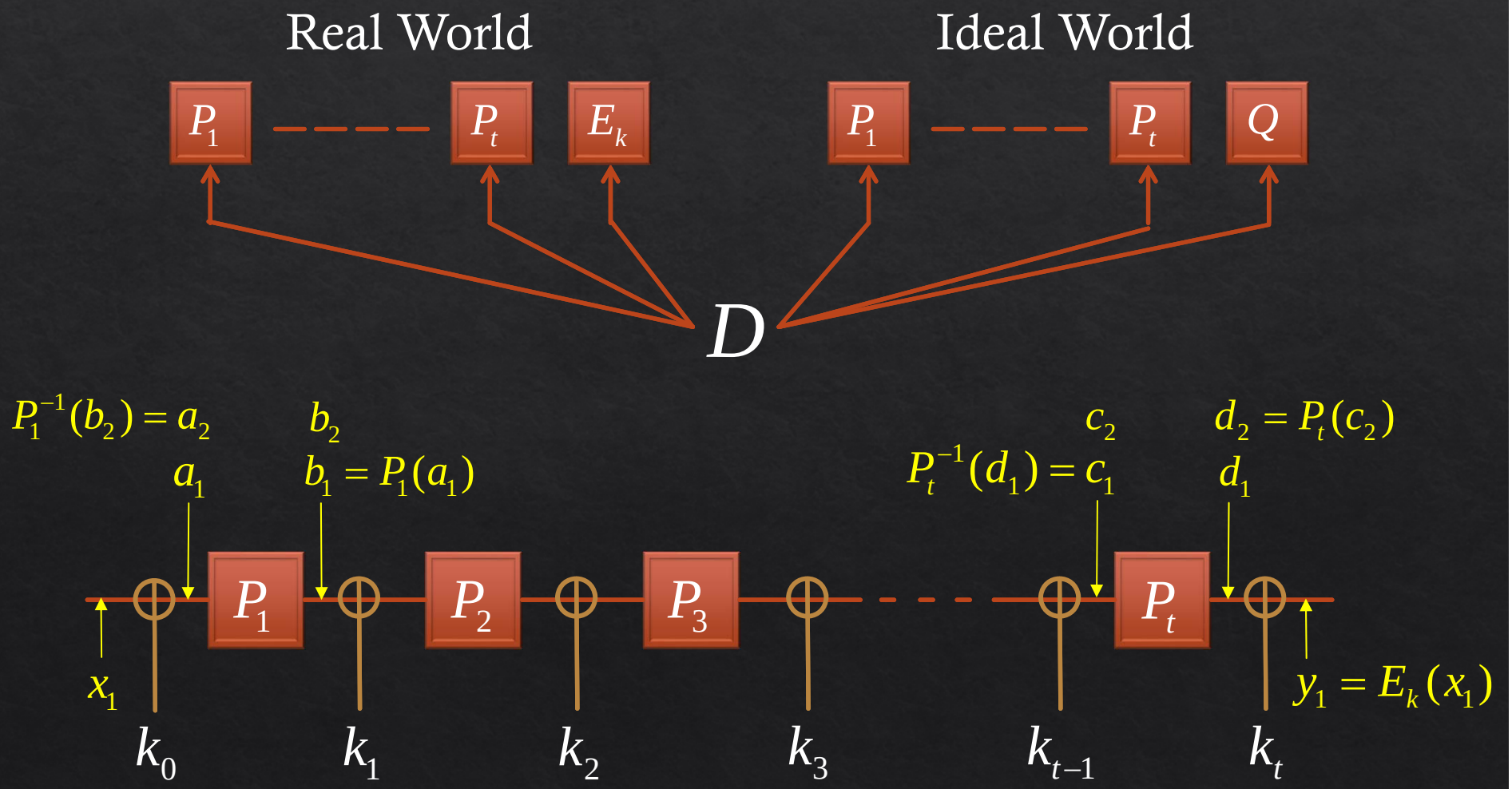


Master Key

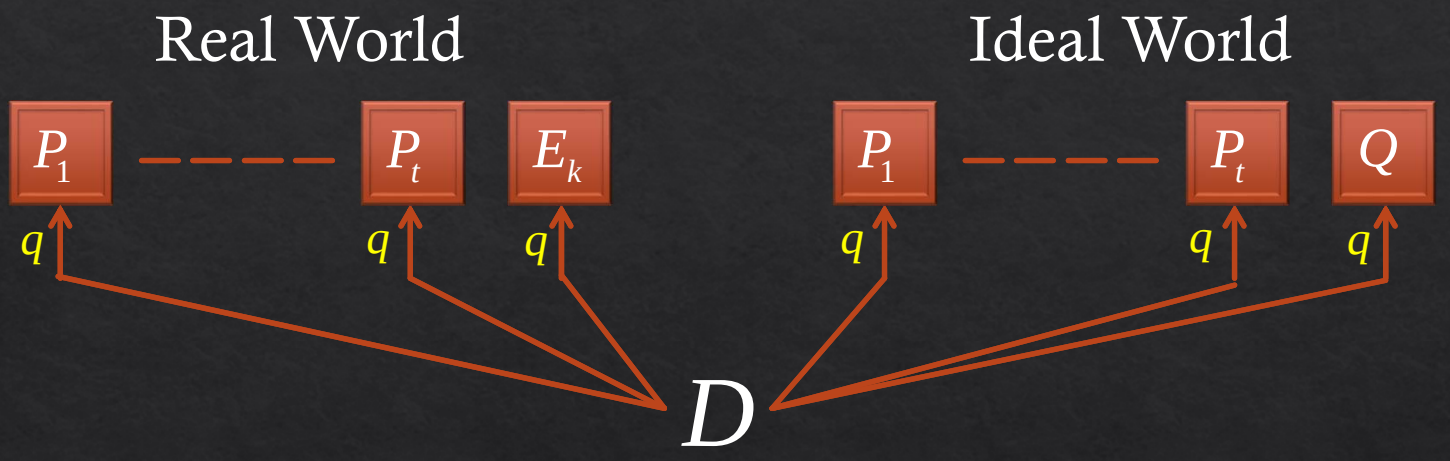
Key-Alternating Ciphers



Indistinguishability Experiment



Indistinguishability Security



$$\mathrm{Adv}(D) := \left|\Pr[D^{P_1,\cdots,P_t,E_k} = 1] - \Pr[D^{P_1,\cdots,P_t,Q} = 1]\right|$$

Previous Work

- Security: $N = 2^n$
  - $t = 1, \Omega(N^{1/2})$ [EM97]
  - $t \geq 2, \Omega(N^{2/3})$ [BKLSST12]
  - $t \geq 3, \Omega(N^{3/4})$ [S12]
  - $\forall t = 2k, \Omega(N^{t/(t+2)})$ [LPS12]
  - $\forall t, \Omega(N^{t/(t+1)})$ [CS14]

D has to make at least $N^{1/2}$ queries to distinguish the real world from the ideal world with advantage > 0.5

$$\frac{t}{t+2} = \frac{t/2}{t/2+1}$$

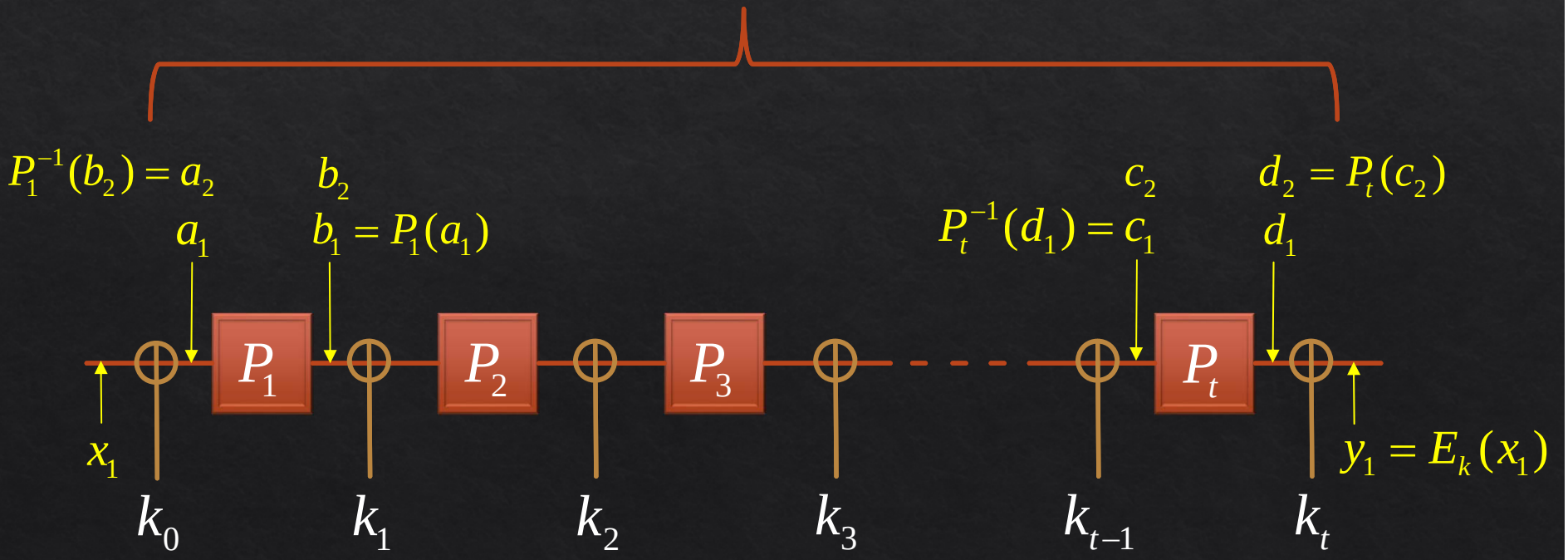
- Attack:
  - $\forall t, O(N^{t/(t+1)})$ [BKLSST12]

$N^{t/(t+1)}$ queries are sufficient to distinguish the real world from the ideal world with advantage > 0.5

Transcripts

Transcript:

$$\tau = \{(a_1, b_1), (a_2, b_2), (c_1, d_1), (c_2, d_2), (x_1, y_1)\}$$



Information-Theoretic Setting

Transcript:

$$\tau = \{(a_1, b_1), (a_2, b_2), (c_1, d_1), (c_2, d_2), (x_1, y_1)\}$$

1. No query direction 2. No query order

We can assume w.l.o.g. that D is deterministic.

$$P_2(a_2) \to b_2 \quad P_1^{-1}(d_1) \to c_1 \quad E_k(x_1) \to y_1 \cdots$$

Statistical Distance of Transcripts

$$\mathrm{Adv}(D) := \left|\Pr[D^{P_1,\cdots,P_t,E_k} = 1] - \Pr[D^{P_1,\cdots,P_t,Q} = 1]\right|$$

$$S = \{\tau \in \mathrm{T} : D(\tau) = 1\}$$

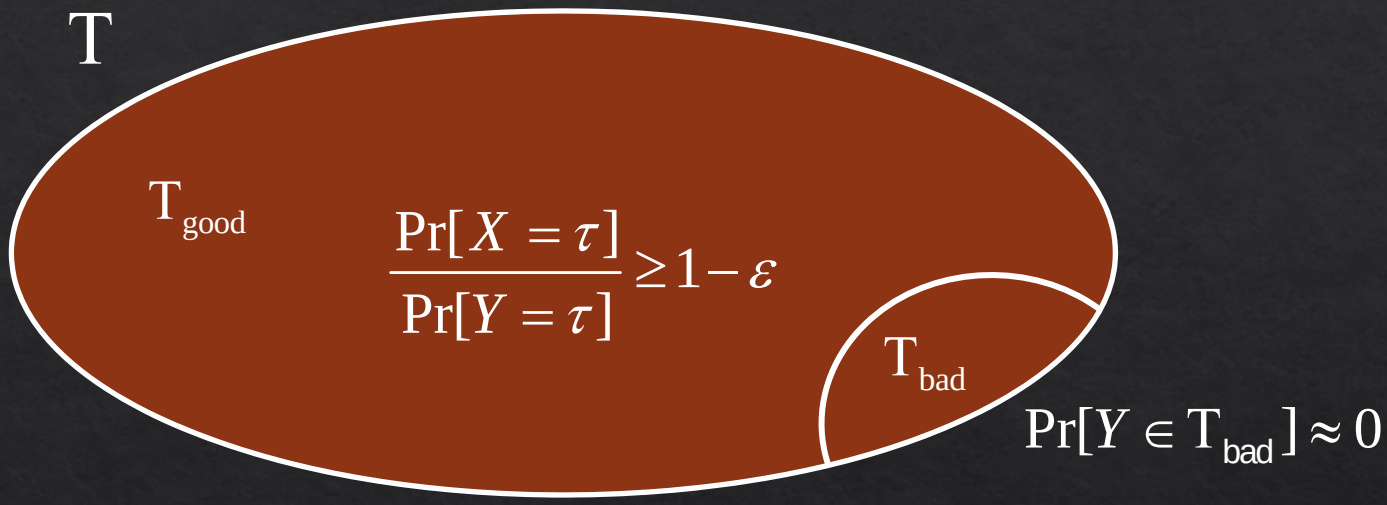
$$= \left|\Pr[\boldsymbol{X} \in S] - \Pr[\boldsymbol{Y} \in S]\right|$$

$$\leq \max_{S \subseteq \mathrm{T}} \left|\Pr[X \in S] - \Pr[Y \in S]\right|$$

$$= \Delta(X, Y)$$

$$\mathrm{Adv}(D) \leq \Delta(X, Y)$$

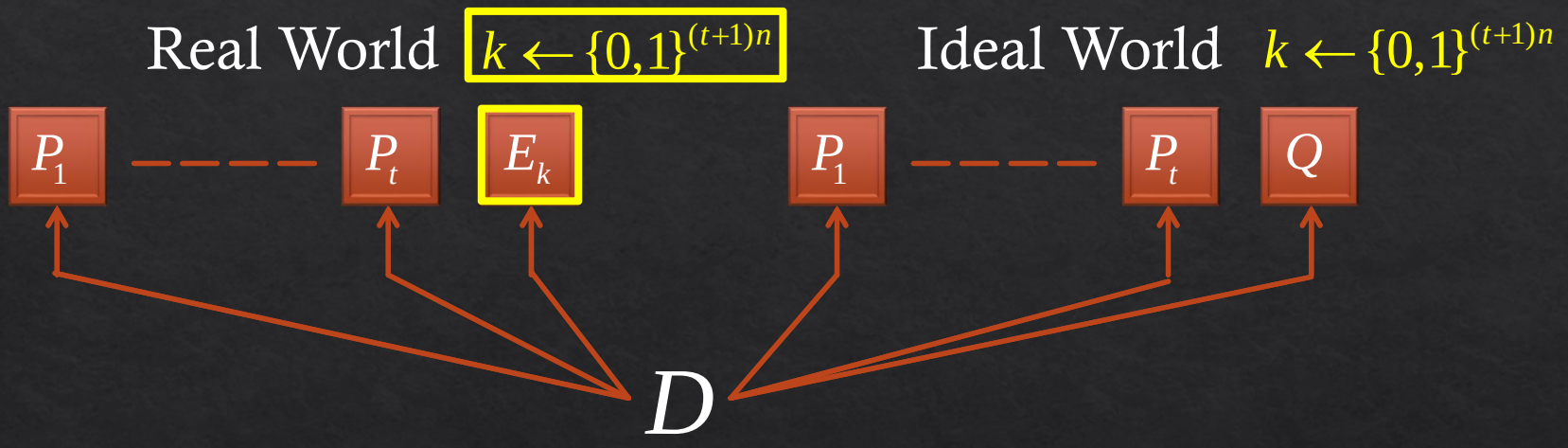
Patarin's H-coefficient Technique [P09]



$$\Delta(X,Y) \leq \varepsilon + \Pr[Y \in T_{bad}]$$

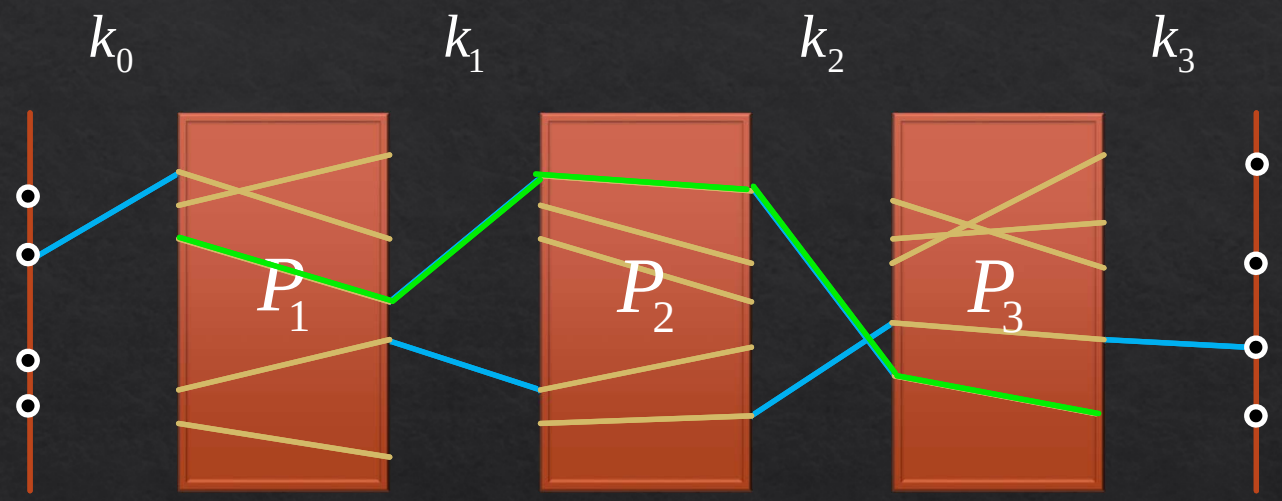
$$\Delta(X,Y) = 1 - \underset{\tau \sim Y}{E}\left[\min\left(1, \frac{\Pr[X=\tau]}{\Pr[Y=\tau]}\right)\right]$$

Reveal the Key



D is given the key for free AFTER making all of its queries

Definition of Bad Transcripts



$$\tau \in \mathrm{T}_{\mathrm{bad}} \Leftrightarrow \exists l, \#(p_l)_\tau > C \cdot \underset{\tau \sim Y}{E}[\#(p_l)]$$

Example: $\underset{\tau \sim Y}{E}[\#(p_3)] = \frac{q^3}{N^2}$ $\#(p_3)_\tau > C\frac{q^3}{N^2}$

Markov Inequality $\Rightarrow \Pr[Y \in \mathrm{T}_{\mathrm{bad}}] = O(t^2)\frac{1}{C} \approx 0$

Lower Bounding the Probability Ratio for Good Transcripts (Major Challenge)

$$\tau = \left\{ \begin{array}{cccc} \tau_1 & \tau_2 & \tau_t & \tau_0 \\ (u_1^1, v_1^1) & (u_1^2, v_1^2) & \cdots & (u_1^t, v_1^t) & (x_1, y_1) \\ (u_2^1, v_2^1) & (u_2^2, v_2^2) & & (u_2^t, v_2^t) & (x_2, y_2) \\ \vdots & \vdots & & \vdots & \vdots \\ (u_q^1, v_q^1) & (u_q^2, v_q^2) & \cdots & (u_q^t, v_q^t) & (x_q, y_q) \\ P_1 & P_2 & P_t & E_k / Q \end{array} \right\} \cup \{k^*\}$$

$$\frac{\Pr[X = \tau]}{\Pr[Y = \tau]} \geq 1 - \varepsilon$$

$$\frac{\Pr[X = \tau]}{\Pr[Y = \tau]} = \frac{\Pr[E_k \triangleright \tau_0, P_1 \triangleright \tau_1, \cdots, P_t \triangleright \tau_t, k = k^*]}{\Pr[Q \triangleright \tau_0, P_1 \triangleright \tau_1, \cdots, P_t \triangleright \tau_t, k = k^*]}$$

$$= \frac{\Pr[E_k \triangleright \tau_0 \mid Q \triangleright \tau_0] \Pr[P_1 \triangleright \tau_1, \cdots, P_t \triangleright \tau_t, k = k^*]}{\Pr[Q \triangleright \tau_0 \mid Q \triangleright \tau_0] \Pr[P_1 \triangleright \tau_1, \cdots, P_t \triangleright \tau_t, k = k^*]}$$

Lower Bounding the Probability Ratio for Good Transcripts (Major Challenge)

$$\frac{\Pr[X=\tau]}{\Pr[Y=\tau]}=\frac{\Pr[E_k \rhd \tau_0 \mid G]}{\Pr[Q \rhd \tau_0 \mid G]} \geq 1-\varepsilon \qquad G \Leftrightarrow P_1 \rhd \tau_1, \cdots, P_t \rhd \tau_t, k=k^*$$

$$\tau_0=\{(x_1,y_1),(x_2,y_2),\cdots,(x_q,y_q)\}$$

Ideal World

$$\Pr[Q \rhd \tau_0 \mid G]=\Pr[Q \rhd \tau_0]=\Pr[x_1 \to y_1, x_2 \to y_2, \cdots, x_q \to y_q]$$

$$=\Pr[x_1 \to y_1] \times \Pr[x_2 \to y_2 \mid x_1 \to y_1] \times \vdots \times \Pr[x_q \to y_q \mid x_i \to y_i, i<q]$$

$$=\frac{1}{N}\cdot\frac{1}{N-1}\cdots\frac{1}{N-q+1}$$

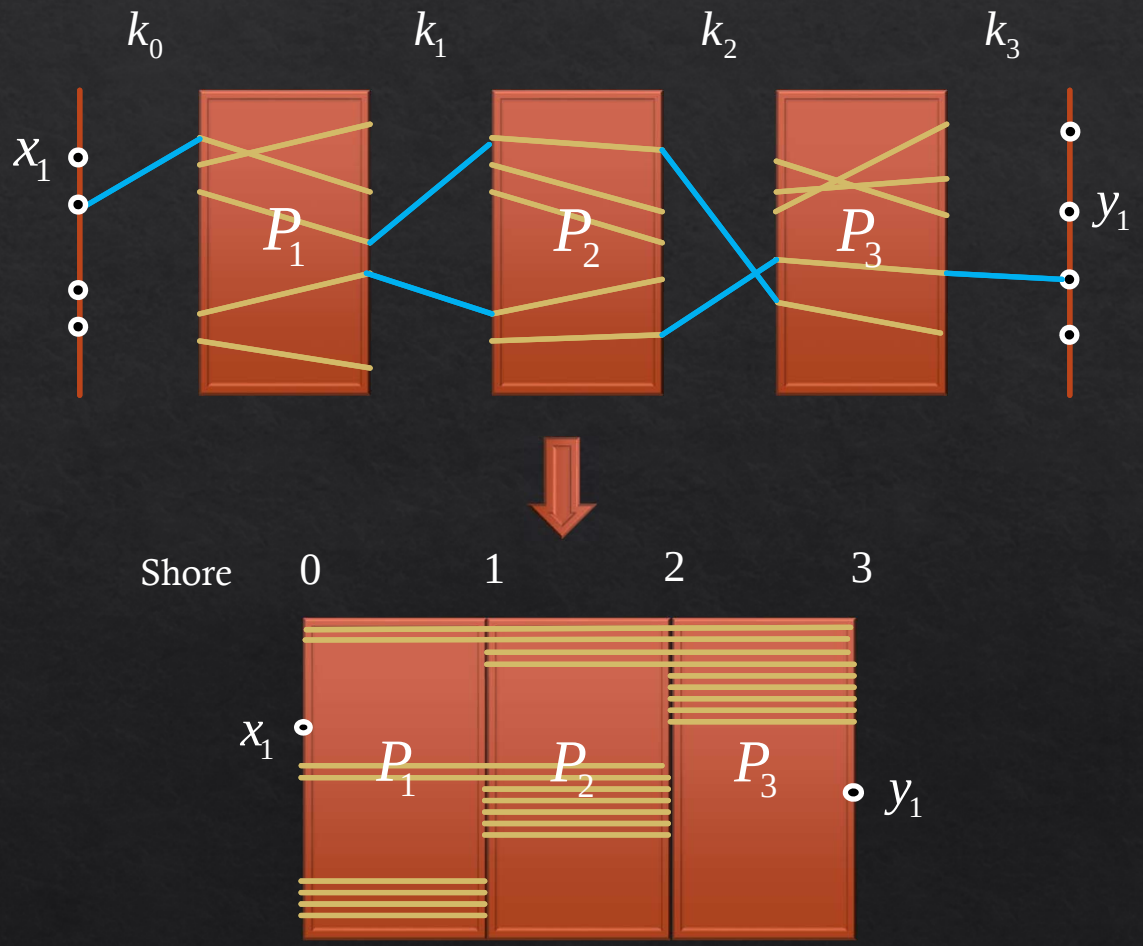
$$=1/N \quad =1/(N-1) \quad =1/(N-q+1)$$

Real World

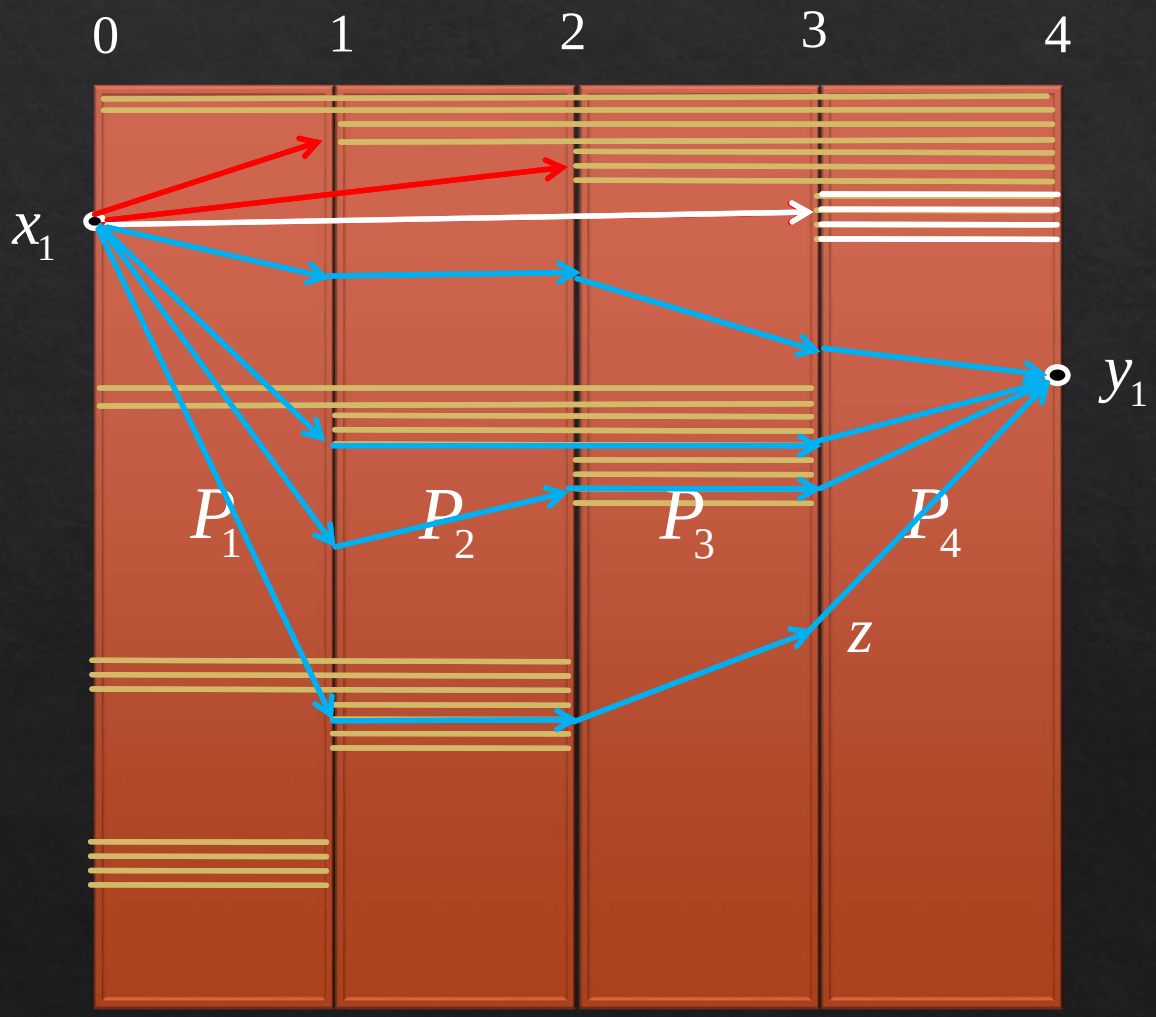
$$\Pr[E_k \rhd \tau_0 \mid G]$$

$$=\Pr[x_1 \to y_1 \mid G] \times \Pr[x_2 \to y_2 \mid x_1 \to y_1, G] \times \vdots \times \Pr[x_q \to y_q \mid x_i \to y_i, i<q, G]$$

Lower Bounding the Probability Ratio for Good Transcripts (Major Challenge)

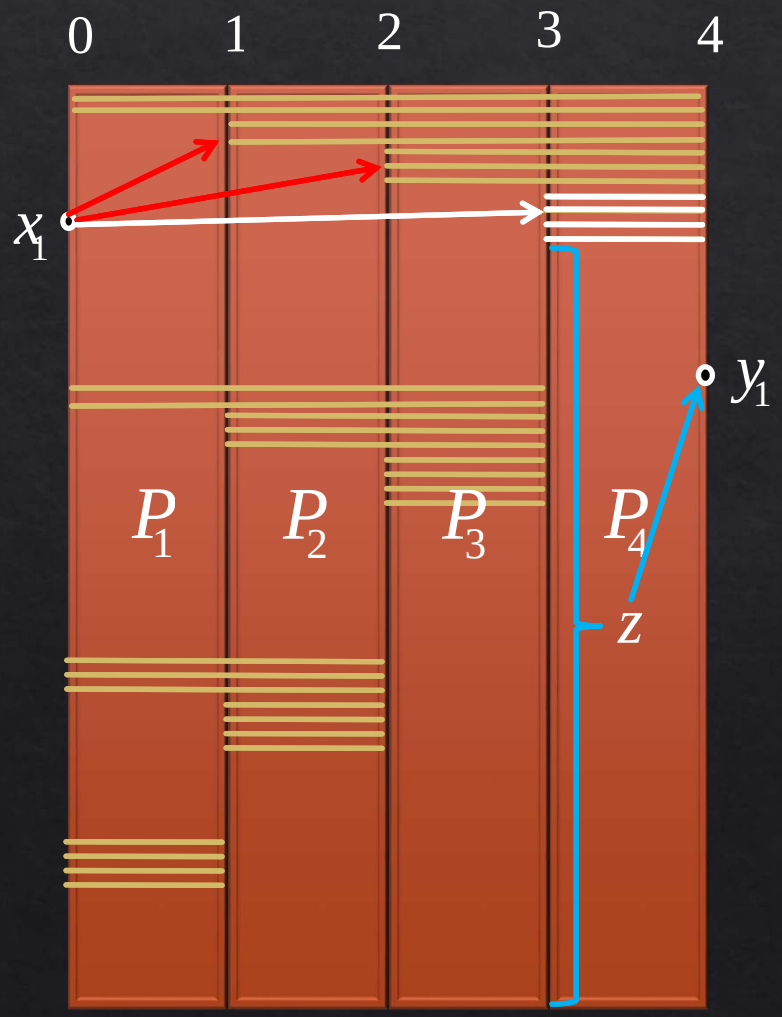


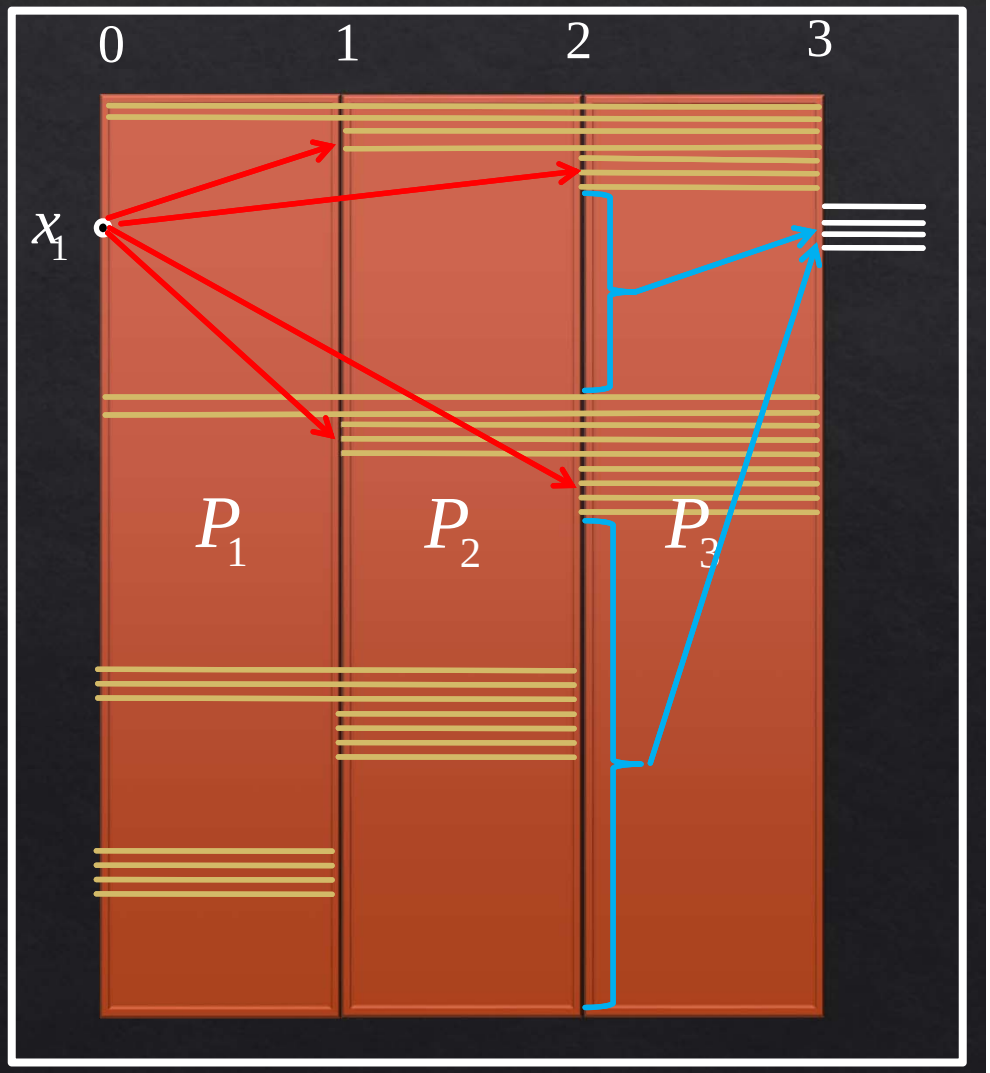
Lower Bounding the Probability Ratio for Good Transcripts (Major Challenge)



Q: What is the probability of z being free?

Lower Bounding the Probability Ratio for Good Transcripts (Major Challenge)





The End

Thanks & Any Questions?