Sometimes-Recurse Shuffle

Almost-Random Permutations in Logarithmic Expected Time

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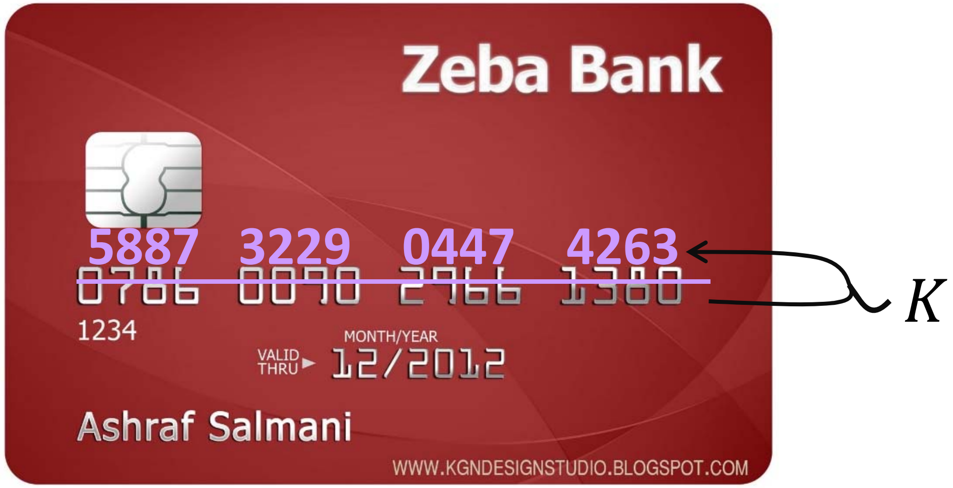
Enciphering a Credit-Card Number

(also called a "PAN")



$$E\colon\ \breve{}\ \times \{0,1,\ldots,9\}^{16} \to \{0,1,\ldots,9\}^{16}$$

Format-Preserving Encryption (FPE): ← named & popularized by **T. Spies**

[NBS FIPS 74: 1981]

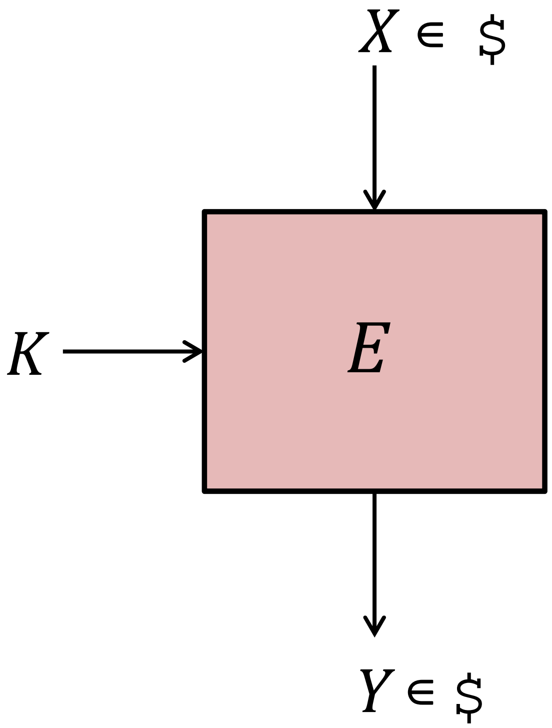
$$E\colon\ \breve{}\ \times \$\, y\!\!\to\ \$$$

FPE $\overset{?}{=}$ Blockcipher

Sort of

$E\colon\ \breve{}\ \times \$\, \text{y}\!\to\ \$$

$E(K,\cdot)$ is a permutation on \$



Assumption: $\$\ = [N] = \{0, \ldots, N-1\}$

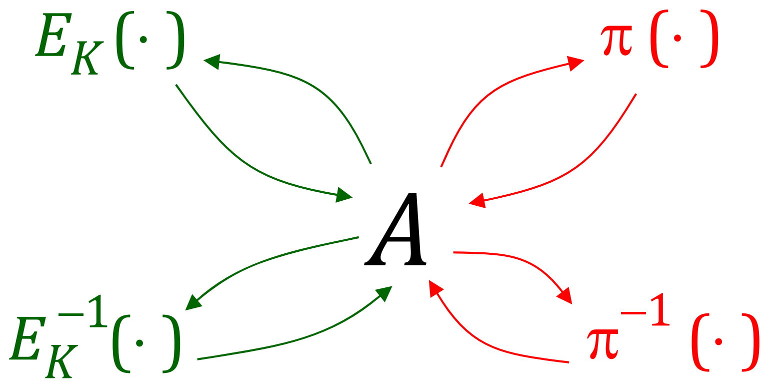
Not *that* limiting – many natural messages spaces can be efficiently put into 1-to-1 correspondence with $[N]$

[Black, Rogaway 2002]

[Bellare, Ristenpart, Rogaway, Stegers 2009]

Measuring quality

$E\colon \breve{\ } \times \$ \rightarrow \$$



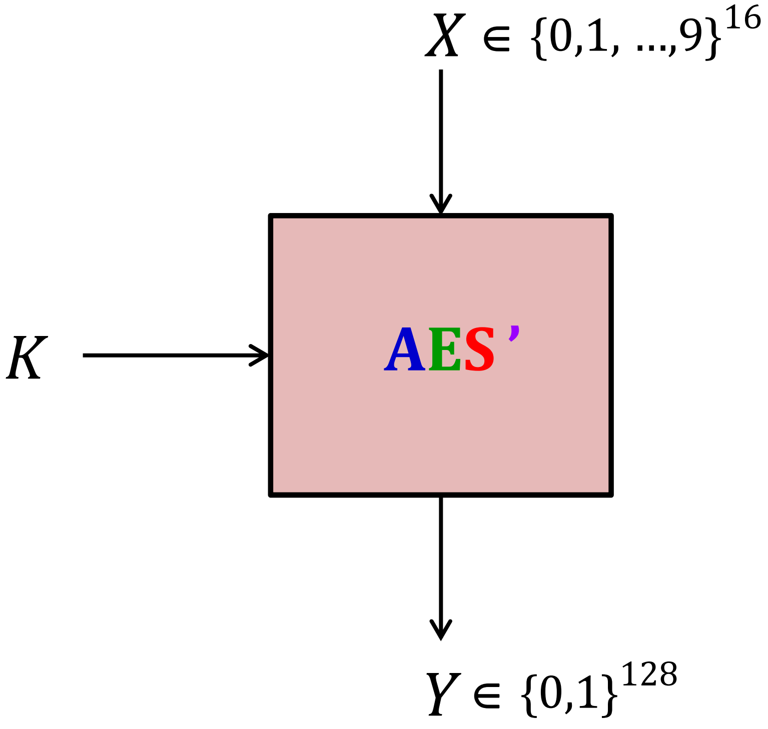
$$\boxed{\mathbf{Adv}_E^{\mathrm{sprp}}(q) = \max_{A \text{ asks } q \text{ queries}} \Pr[A^{E_K \; E_K^{-1}} \rightarrow 1] - \Pr[A^{\pi \; \pi^{-1}} \rightarrow 1]}$$

$$\boxed{\Delta_E(q) = \max_{\substack{A \text{ asks } q \\ \text{nonadaptive queries}}} \Pr[A^{E_K} \rightarrow 1] - \Pr[A^{\pi} \rightarrow 1]}$$

When q=N
these coincide

One approach to FPE

De novo construction



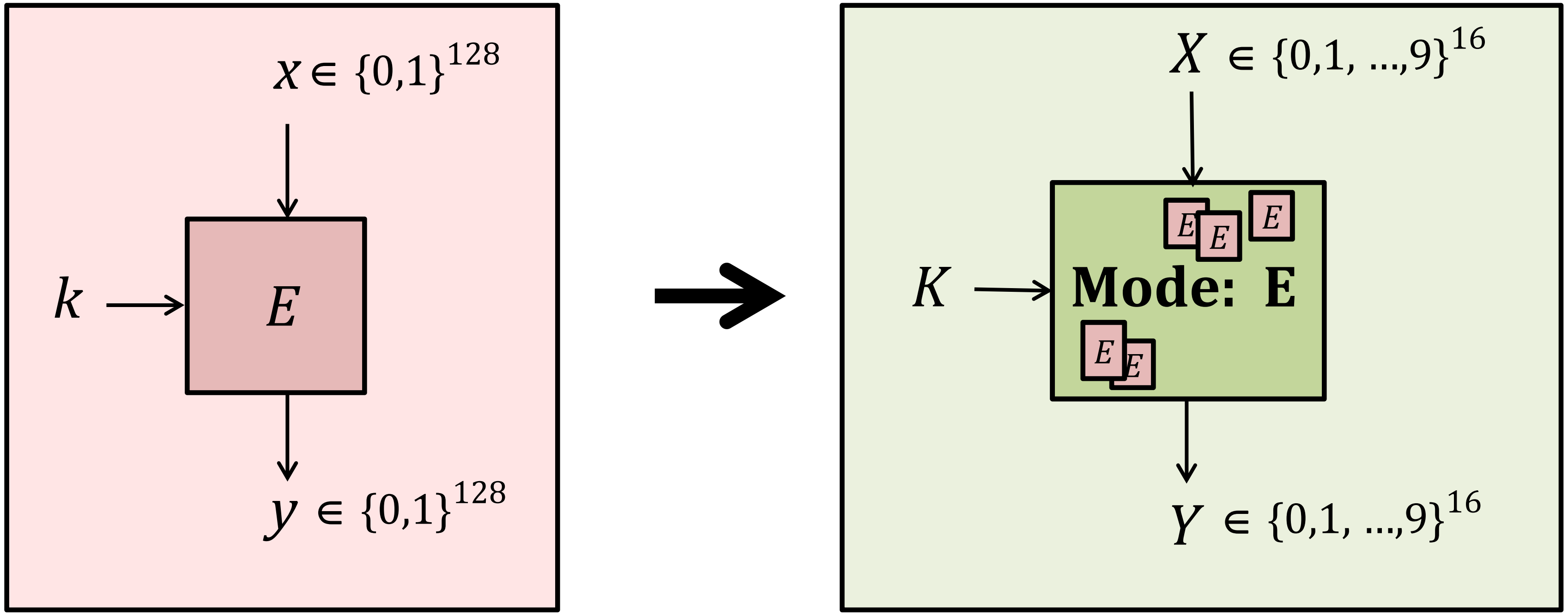
Lots of problems

- Unclear **how** to extend conventional blockciphers to small/unusual domains.
- **Security assurances** earned by existing blockcipher **forfeit**
- **Existing HW** and **SW** not exploitable

There exist designs that allow short binary strings, like **HPC**, but don't go as far as [*N*]

Another approach to FPE

PRF to PRP conversion



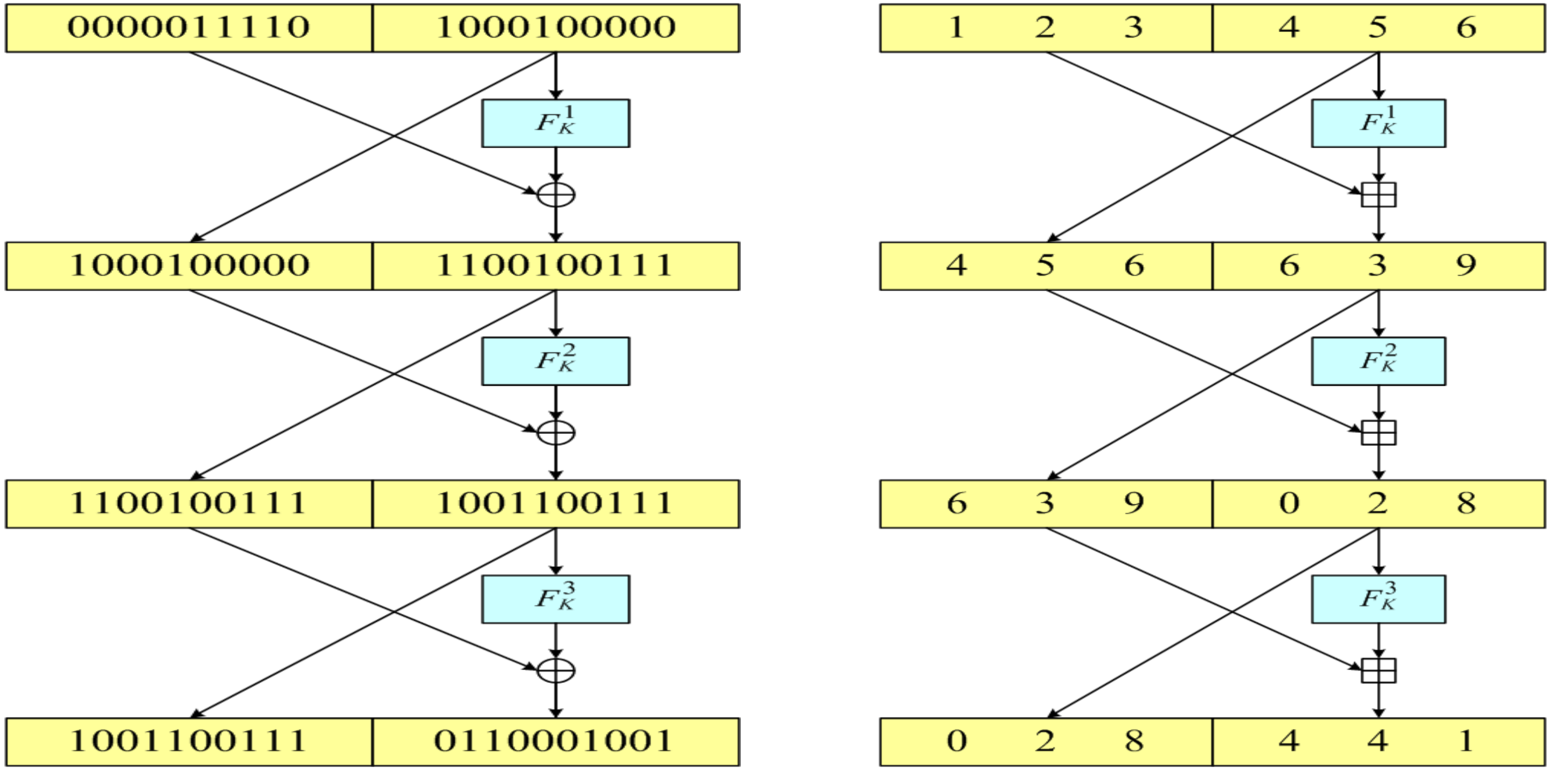
PRP with a domain $\{0,1\}^b$ → **PRP** with a domain $[N]$

PRF with a domain $\{0,1\}^b$ → **PRP** with a domain $[N]$

Random function with domain $\{0,1\}^b$ → Random permutation with domain $[N]$

That’s what Feistel does

Random function with domain $\{0,1\}^b$ → Random permutation with domain $[N]$



Poor concrete security

Luby-Rackoff: proven security to $q \sim N^{1/4}$

Patarin: provable security to $q \sim N^{1/2}$

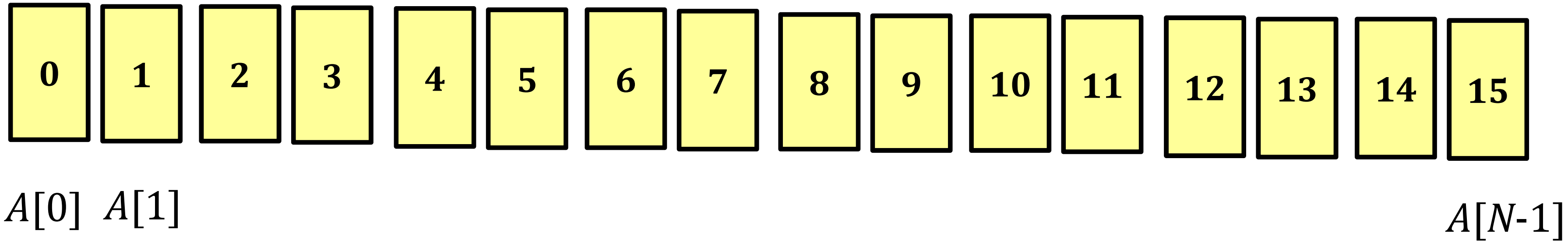
Folklore: inf th attacks to $q \sim N^{1/2}$

Goal: security to $q = N$

full security **[Ristenpart, Yilek 2013]**

Full security <u>is</u> feasible

At least if you spend $\Omega(N)$ time



```
for $j$ from $0$ to $N-1$ do $A[i] \leftarrow i$
for $j$ from $N-1$ downto $1$ do
    $i \twoheadleftarrow [\,j\,]$
    $A[i] \leftrightarrow A[j]$
```

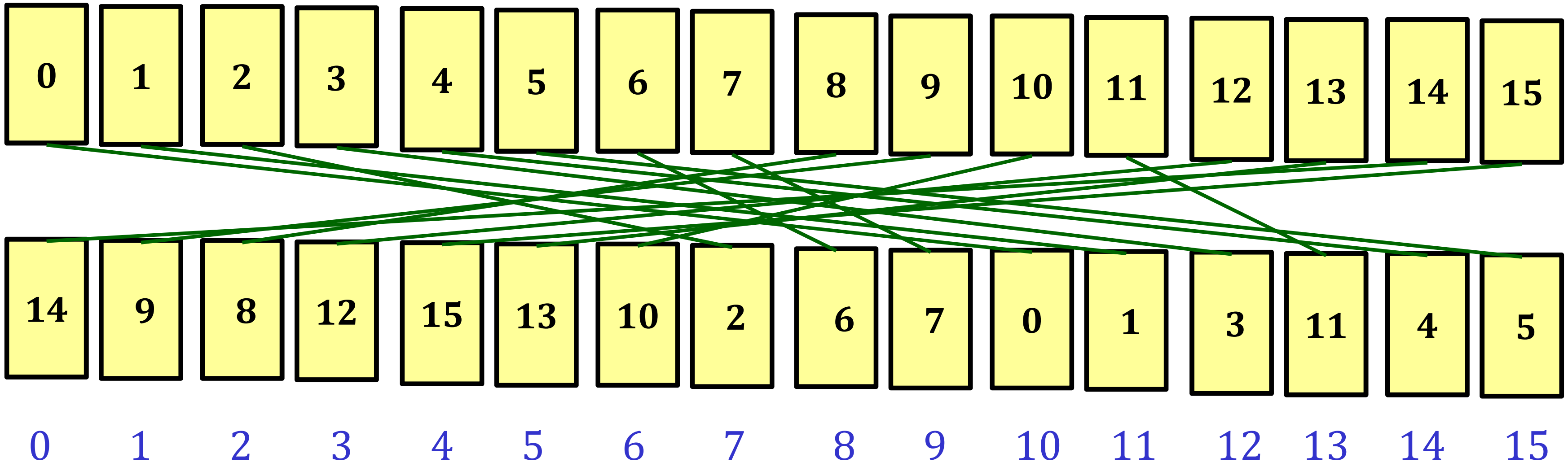
Let key K name these choices: sequence of numbers in $[N]$, $[N-1]$, …,$[3]$, $[2]$, $[1]$

“Knuth Shuffle”

(Fisher-Yates)

The Route Towards Better Methods/Bounds

Enciphering Scheme ↔ Card Shuffle

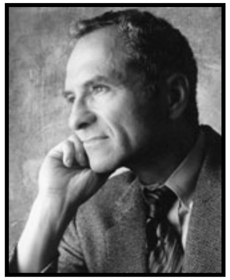


A point in x∈$ ↔ A particular card

A key K∈˘ y ↔ Randomness used to shuffle the cards

Image $E_K(x)$ ↔ Where that card ends up with the given randomness

An **oblivious** shuffle: can follow the path of a card without attending to the other cards. [Naor, 1989]



[Thorp 1973]

Thorp Shuffle

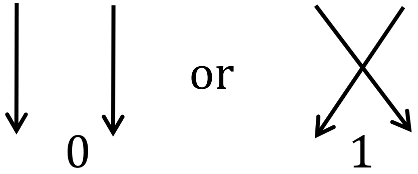
TH$[N, r]$

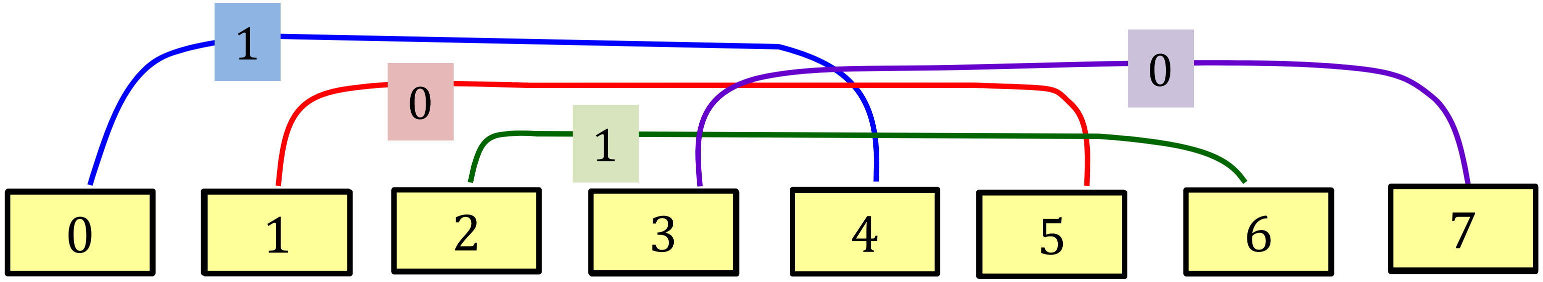
repeat r times

1. Pair cards at posns x and $x + N/2$

2. Flip a coin for each pair

3. The coins indicate if pairs go

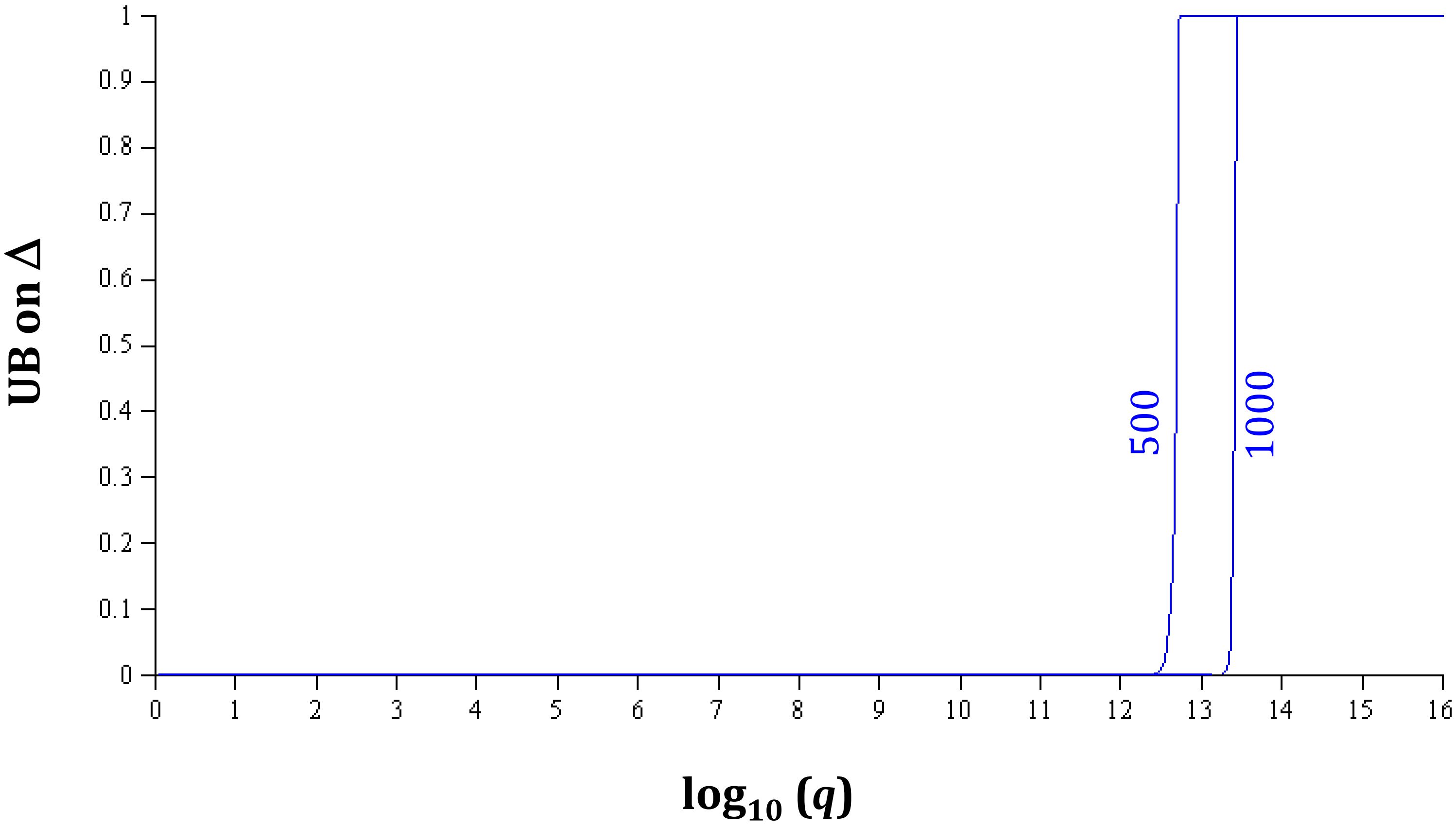




Security of Thorp

TH$[10^{16}, r]$

[Morris, Rogaway, Stegers 2009]
[Hoang, Rogaway 2010]



Swap-or-Not SN[N, r]

[Hoang, Morris, Rogaway 2012]

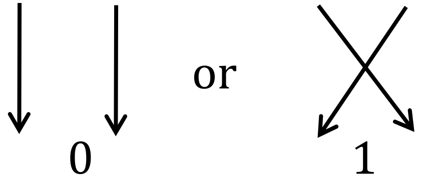
repeat r times

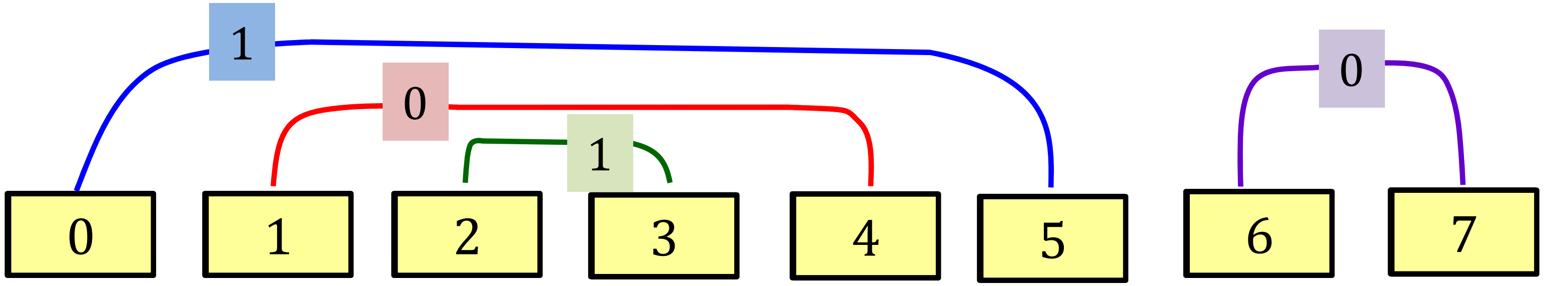
1. $K \twoheadleftarrow [N]$ Eg, $K = 5$ below

2. Pair x and $K{-}x \pmod{N}$

3. Flip a coin for each pair

4. The coins indicate if pairs go





Swap-or-Not SN[N, r]

[Hoang, Morris, Rogaway 2012]

As a **blockcipher**

```
**algorithm** $E_{K_1 \ldots K_R\ F}(x)$    // $x \in \{0, \ldots, N-1\}$
**for** $i \leftarrow 1$ **to** $r$ **do**
        $x' \leftarrow K_i - x$
        $x^* \leftarrow \max(x, x')$
        **if** $F(i, x^*) = 1$ **then** $x \leftarrow x'$
**return** $x$
```

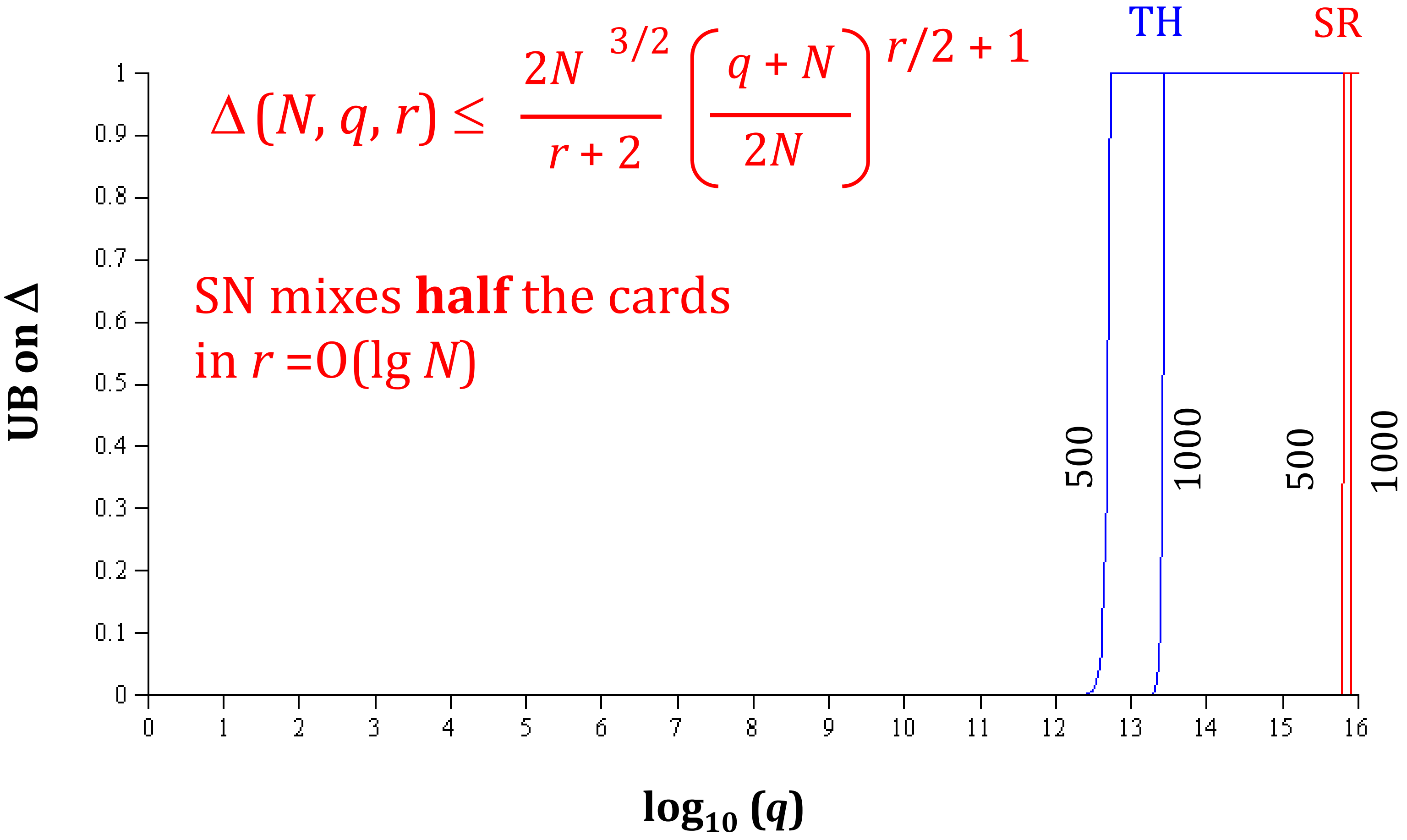
Decryption: Same, with *i* going from *r* downto 1

Bounds for SN apply to SN^{-1}

[Hoang, Morris, Rogaway 2012]

Security of Swap-or-Not

SN$[10^{16}, r]$



$$\Delta(N, q, r) \le \frac{2N^{3/2}}{r+2}\left(\frac{q+N}{2N}\right)^{r/2+1}$$

SN mixes **half** the cards in $r = O(\lg N)$

Bootstrapping an inner shuffle

Icicle & Mix-and-Cut

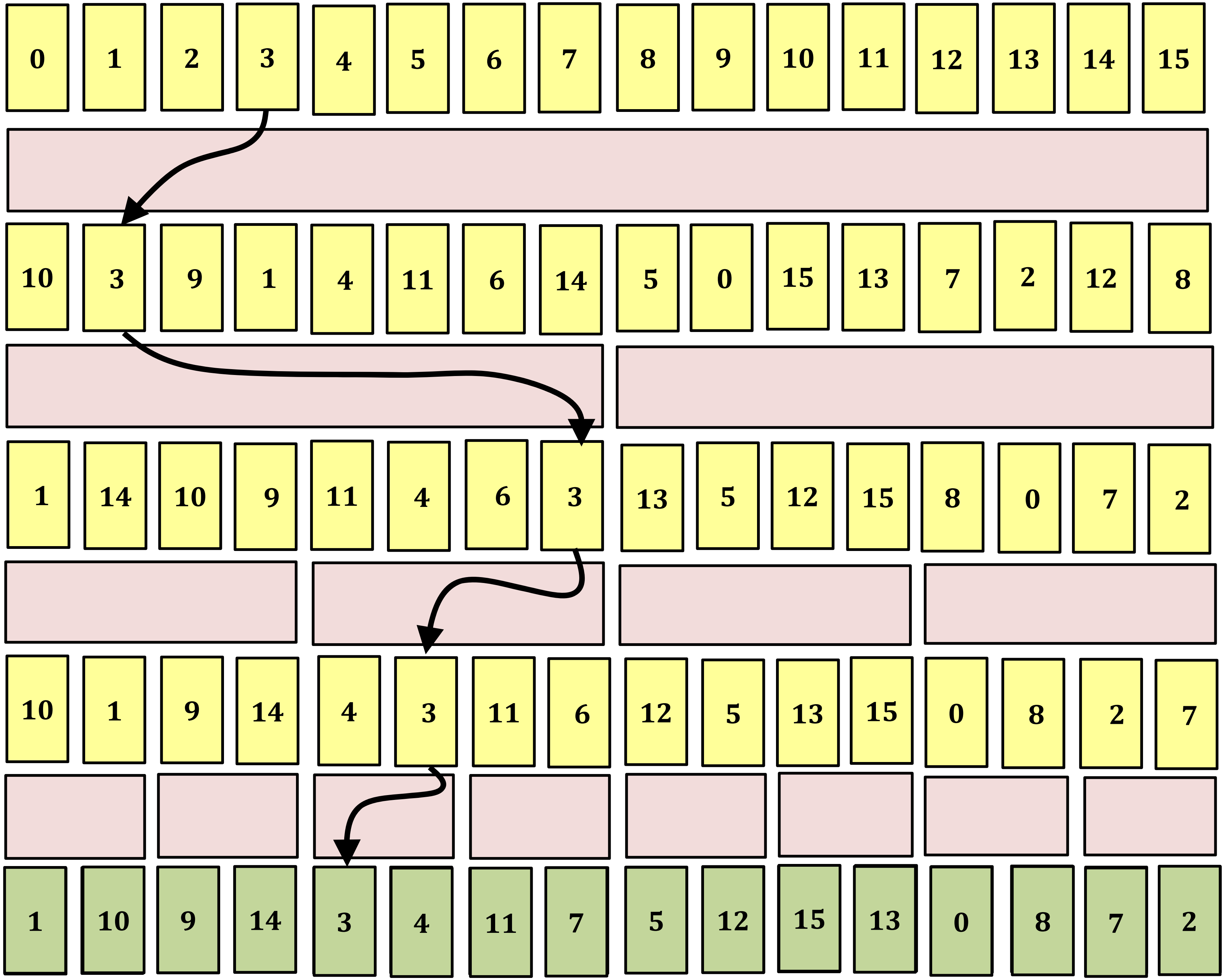
[Ristenpart, Yilek 2013]
following
[Granboulan-Pornin 2007] and
[Czumaj, Kanarek, Kutylowski and 1998]

- Apply some inner shuffle.
  It needs to be a pseudorandom separator (PRS):
  the set of elements in the left & right output pile should be near uniform
- Recurse down left & right output piles

Icicle

- Use SN[N, O(lg N)] is a PRS
- Total time: $O(\lg^2 N)$

Mix-and-Cut

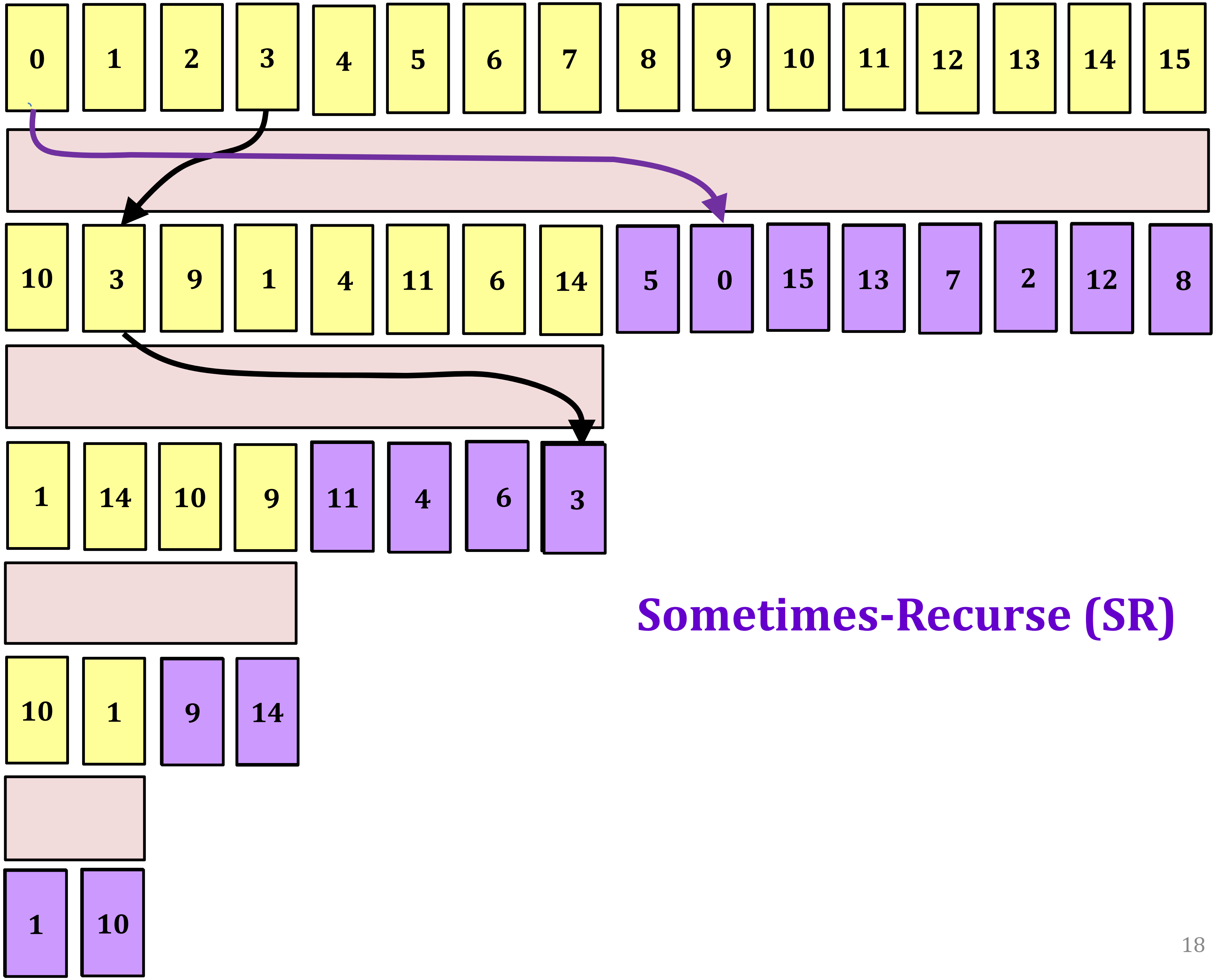

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
10 3 9 1 4 11 6 14 5 0 15 13 7 2 12 8
1 14 10 9 11 4 6 3 13 5 12 15 8 0 7 2
10 1 9 14 4 3 11 6 12 5 13 15 0 8 2 7
1 10 9 14 3 4 11 7 5 12 15 13 0 8 7 2

Sometimes-Recurse (SR) Shuffle

- Apply some inner shuffle.
  It needs to mix **half** the cards (in the inverse shuffle):
- Recurse down **one** of the two output piles – the **left**, say

Anticipated instantiation:

- Use SN[N, O(lg *N*)] as the inner shuffle



Sometimes-Recurse (SR)

SR as a Blockcipher

```
algorithm $E^{N}_{K,F}(x)$      // $x \in \{0, \ldots, N-1\}$
if $N$=1 then return $x$

for $i \leftarrow 1$ to $r_N$ do
        $x' \leftarrow K_i - x$
        $x^* \leftarrow \max(x, x')$                 SN($N$, $r_N$)
        if $F(i, x^*)=1$ then $x \leftarrow x'$

if $x \le N/2$ then return $E^{\lfloor N/2 \rfloor}_{K,F}(x)$
                else return $x$
```

With appropriate r_N
full security in O(lg N) **expected** rounds

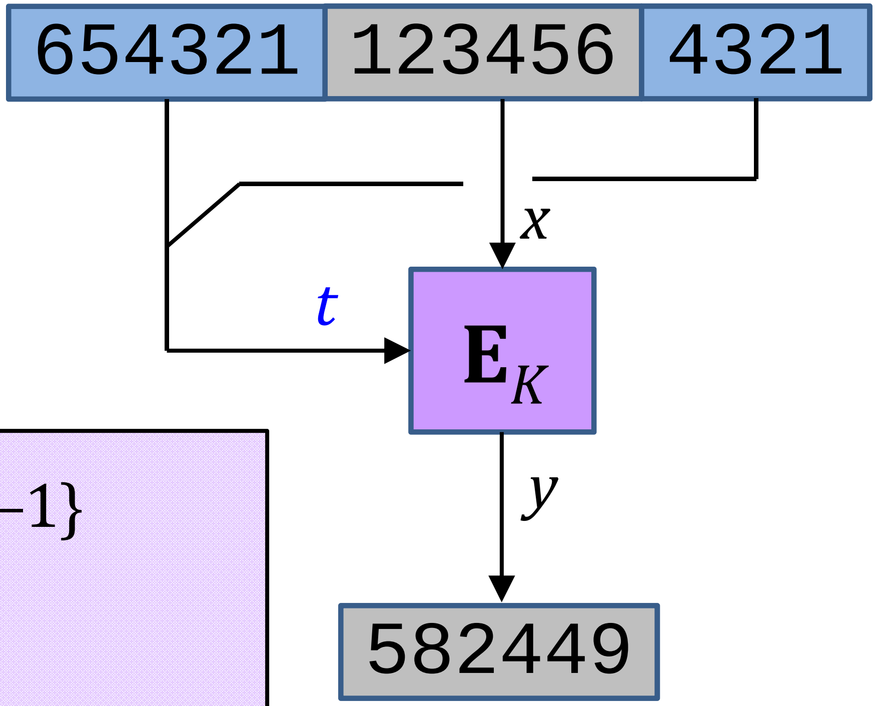
Number of rounds

error to ε = 10^{-10}

Plaintext	Best	Expected	Worst	r_N
6-digits	289	**563**	4411	fixed
	272	**544**	5168	splits ε
16-digits	531	**1048**	18239	fixed
About 80k cycles, 25 μsec	507	**1014**	26365	splits ε
30-digits	869	**1723**	51453	fixed
	840	**1680**	83160	splits ε

Supporting Tweaks

[Liskov, Rivest, Wagner 2002]



```
algorithm $E^{N}_{K,F}(x)$      // $x \in \{0, \ldots, N-1\}$
if $N$=1 then return $x$

for $i \leftarrow 1$ to $r_N$ do
        $x' \leftarrow K_i - x$
        $x^* \leftarrow \max(x, x')$
        if $F(i, x^*, t)$=1 then $x \leftarrow x'$

if $x \le N/2$ then return $E^{\lfloor N/2 \rfloor}_{K,F}(x)$
          else return $x$
```

Doesn't need to depend on t

Choosing the split

Not necessary to choose |Left| = $\lfloor N/2 \rfloor$

Plaintext	**Expected**	$\lvert\text{Left}\rvert$	$\boldsymbol{r_N}$
16-digits	~~**1014**~~ 1010	$\lfloor 0.52\ N/2 \rfloor$	splits ε

Makes little difference

A Potential Concern

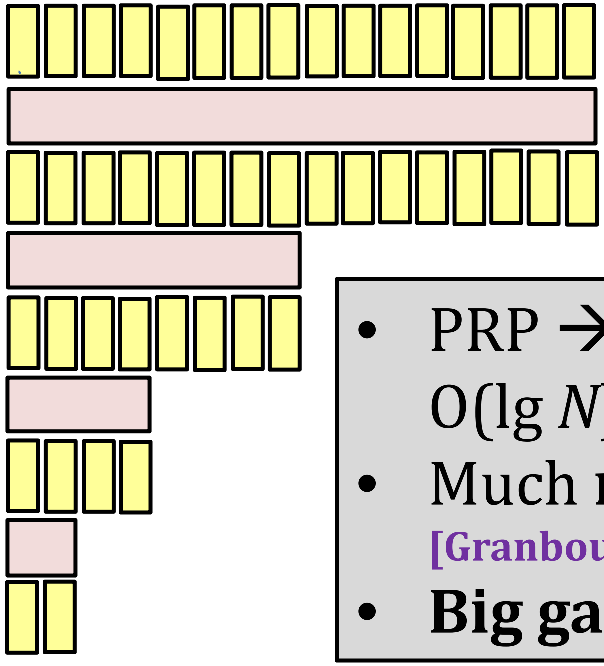
Leaking the runtime

```
algorithm $E^{N}_{K,F}(x)$    // $x \in \{0, \ldots, N-1\}$
if $N$=1 then return $x$

for $i \leftarrow 1$ to $r_N$ do
        $x' \leftarrow K_i - x$
        $x^* \leftarrow \max(x, x')$                  } $\mathrm{SN}(N, r_N)$
        if $F(i, x^*)$=1 then $x \leftarrow x'$

if $x \leq N/2$ then return $E^{\lfloor N/2 \rfloor}_{K,F}(x)$
                 else return $x$
```

Not an issue – the number of repetitions used to encipher x is already revealed by the ciphertext y



Summary

- PRP → PRP, for any [N], with **full security**, in O(lg N) **expected** time
- Much **more efficient** than prior work
  [Granboulan-Pornin 2007], [Stefanov-Shi 2012], [Ristenpart, Yilek 2012]
- **Big gap** to practice remains: r =10 *vs.* r =1000

Open

- O(lg N) **worst-case** time?