How to Efficiently and Simultaneously Compute the Probabilities of All Iterative Characteristics

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Differential Cryptanalysis

- Introduced by Biham and Shamir [BS90].
- Studies the development of differences through the encryption function.
- Based on differential characteristics $(\Omega_P \rightarrow \Omega_1 \rightarrow \Omega_2 \rightarrow \cdots \rightarrow \Omega_C)$
- Easy to compute differential transitions through linear (affine) operations.
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- S-boxes, require to start looking at probabilities ...

Wait for tomorrow's IACR distinguished lecture by Eli!

Difference Distribution Tables

- The difference distribution table of an S-box counts how many pairs with a given input difference lead to a given output difference.
- It is an essential part of identifying high-probability transitions for constructing the differential characteristics, later used in the attack.
- (Sometimes, the table also stores the actual pairs, and not only their number).

The Difference Distribution Tables of DES $S1$

Input XOR	Output XOR															
	0_x	1_x	2_x	3_x	4_x	5_x	6_x	7_x	8_x	9_x	A_x	B_x	C_x	D_x	E_x	F_x
0_x	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1_x	0	0	0	6	0	2	4	4	0	10	12	4	10	6	2	4
2_x	0	0	0	8	0	4	4	4	0	6	8	6	12	6	4	2
3_x	14	4	2	2	10	6	4	2	6	4	4	0	2	2	2	0
4_x	0	0	0	6	0	10	10	6	0	4	6	4	2	8	6	2
5_x	4	8	6	2	2	4	4	2	0	4	4	0	12	2	4	6
6_x	0	4	2	4	8	2	6	2	8	4	4	2	4	2	0	12
7_x	2	4	10	4	0	4	8	4	2	4	8	2	2	2	4	4
8_x	0	0	0	12	0	8	8	4	0	6	2	8	8	2	2	4
9_x	10	2	4	0	2	4	6	0	2	2	8	0	10	0	2	12
A_x	0	8	6	2	2	8	6	0	6	4	6	0	4	0	2	10
B_x	2	4	0	10	2	2	4	0	2	6	2	6	6	4	2	12
C_x	0	0	0	8	0	6	6	0	0	6	6	4	6	6	14	2
D_x	6	6	4	8	4	8	2	6	0	6	4	6	0	2	0	2
E_x	0	4	8	8	6	6	4	0	6	6	4	0	0	4	0	8
F_x	2	0	2	4	4	6	4	2	4	8	2	2	2	6	8	8
								⋮								
$3E_x$	4	8	2	2	2	4	4	14	4	2	0	2	0	8	4	4
$3F_x$	4	8	4	2	4	0	2	4	4	2	4	8	8	6	2	2

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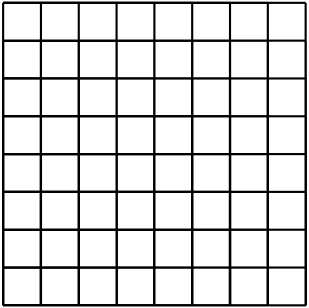
- In other words, iterative differences.
- These differences are extremely important in block cipher cryptanalysis (wait for Eli's talk, read Eli's & Adi's book, or just ask a friendly cryptanalyst).
- Also very useful for the cryptanalysis of hash functions.

Constructing the Full DDT

- For an n-bit to n-bit S-box $S[\cdot]$, one can run the following algorithm:
  1. Set $DDT[i][j] \leftarrow 0$ for all i, j
  2. For all x
     - For all y: $DDT[x \oplus y][S[x] \oplus S[y]] + +$
- Time: $O(2^{2n})$, Memory: $O(2^n)$



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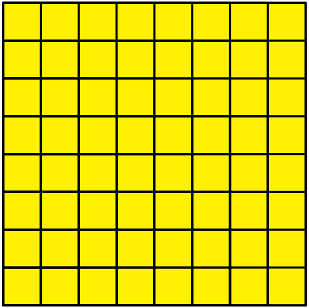

Constructing the Row/Column of the DDT

- For an n-bit to n-bit S-box $S[\cdot]$, and an input difference Δ_{IN} one can run the following algorithm:
  1. For all j's set $DDT[\Delta_{IN}][j] \leftarrow 0$
  2. For all x:
     - $DDT[\Delta_{IN}][S[x] \oplus S[x \oplus \Delta_{IN}] + +$
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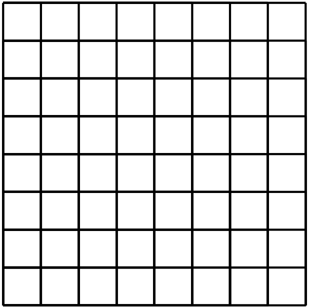

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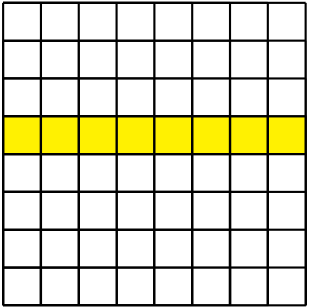
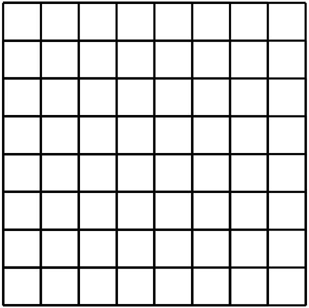


Constructing the Row/Column of the DDT

- For an n-bit to n-bit S-box $S[\cdot]$, and an input difference Δ_{OUT} one can run the following algorithm:
  1. For all j's set $DDT[j][\Delta_{OUT}] \leftarrow 0$
  2. For all x:
     - $DDT[S^{-1}[x] \oplus S^{-1}[x \oplus \Delta_{OUT}]][\Delta_{OUT}] + +$
- Time: $O(2^n)$, Memory: $O(2^n)$



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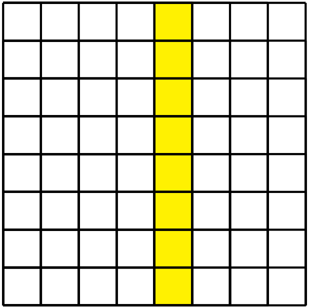
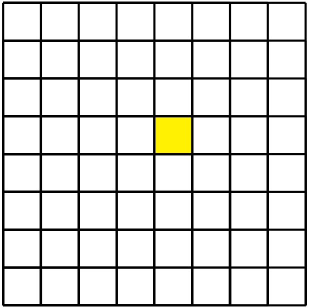


Constructing an Entry of the DDT

- For an n-bit to n-bit S-box $S[\cdot]$, and an input/output difference pair $(\Delta_{IN}, \Delta_{OUT})$:
  1. Set $DDT[\Delta_{IN}][\Delta_{OUT}] \leftarrow 0$
  2. For all x:
     - If $S[x] \oplus S[x \oplus \Delta_{IN}] = \Delta_{OUT}]$ $DDT[\Delta_{IN}][\Delta_{OUT}]++$



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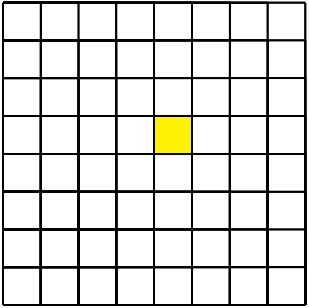
  2. For all x:
     - If $S[x] \oplus S[x \oplus \Delta_{IN}] = \Delta_{OUT}]$ $DDT[\Delta_{IN}][\Delta_{OUT}]++$
- Time: $O(2^n)$, Memory: $O(1)$



How to Construct the Diagonal

1. Construct the entire DDT ($O(2^{2n})$ time and $O(2^n)$ memory).
2. Construct only the diagonal's entries ($O(2^{2n})$ time and $O(n)$ memory).

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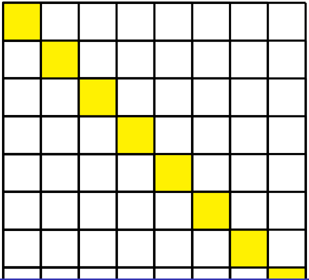
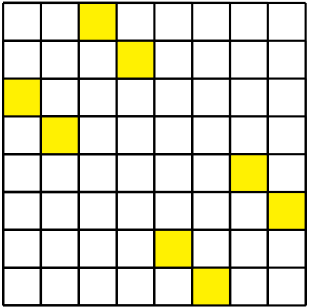
- Or $i \oplus S[i] = j \oplus S[j] \ldots$
- By listing all $x \oplus S[x]$, and looking for collisions in these values, we find all iterative differences. In $O(2^n)$ time and memory!
- Which is exactly **the same time complexity as computing just $DDT[1][1]$!**



Shifted Diagonal Entries

- Similarly, one can compute the entries of the form $(\Delta, \Delta \oplus v)$ for a fixed v.
- Just look for values of $x \oplus S[x]$ which are shifted by v.



Summary

- Presented a new methodology to efficiently construct the diagonal of DDTs.
- Same time complexity for a diagonal as for an entry!
- Useful for 32-bit structures (Super S-box of AES, lightweight crypto).

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- Presented a new methodology to efficiently construct the diagonal of DDTs.
- Same time complexity for a diagonal as for an entry!
- Useful for 32-bit structures (Super S-box of AES, lightweight crypto).
- Also works with additive differences (use $x + S[x]$).
- And allows finding the actual pairs as well.

Questions?

$E\nu\chi\alpha\rho\iota\sigma\tau\omega$!

Thank you for your attention!