

# Functional Encryption with Bounded Collusions via Multi-Party Computation

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# Functional Encryption

[Sahai-Waters'05, Boneh-Sahai-Waters'11, O'Neill'10...]


mpk



**(M)**

Alice

msk



Bob

**Enc(M)**

**$SK_{F_1}, SK_{F_2}, \dots$**



Charlie

“I can compute  **$F_1(M), F_2(M), \dots$** ”,  
but learn **nothing else**  
about **M**



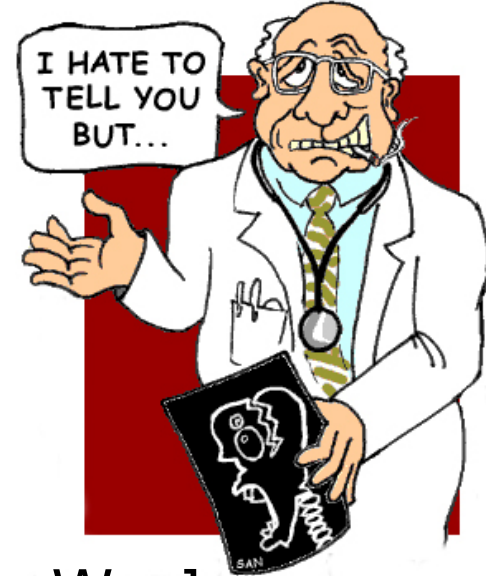
## Big Question:

Can we construct Functional Encryption for all functions?

### State of the art:

- (anonymous) Identity-Based Encryption:  
[S'82,BF'01,BDOP'04,BW'06]
- Attribute-based Encryption: Boolean formulas  
[GPSW'06,LOSTW'10,...]
- Predicate Encryption: inner products [KSW'08,...]

# Our Result 1:



**THEOREM** [Agrawal, Gorbunov, Vaikuntanathan, Wee]

## General Unbounded query FE is IMPOSSIBLE

- Even in a weak non-adaptive simulation def.
- For Pseudo-Random Functions  
(where  $M = \text{seed}$  and  $F_i = \text{PRFinput}$ )
- Generalizes to “incompressible” functions

\* Concurrent, \*incomparable\*, work by Persiano et al.

# Our Result 2:



[G., Vaikuntanathan, Wee]: **q-bounded**  
**Functional Encryption for all functions!**

## Previously:

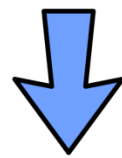
- ◆ **q-bounded IBE**  
[ DKXY'01, CHHIKRSV'07, GLW'12 ]

## Main Motivation:

- ◆ Bounded number of adversaries (having  $SKF_1, SKF_2, \dots, SKF_q$ ) collude to learn “anything else” about message  $M$

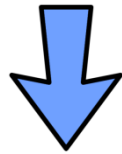
# q-bounded Functional Encryption for Any Function

1-FE for arbitrary circuits [SS'10, Yao'86]



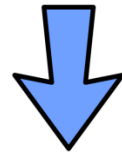
Using MPC [BGW'88]

q-FE for degree-D circuits



FE Bootstrapping Theorem:  
Using Randomized Encodings  
[AIK'05, Yao'86]

q-FE for arbitrary circuits



unbounded-FE for arbitrary circuits

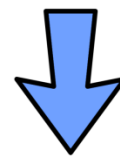


**DONE!**

**DONE!**

# q-bounded Functional Encryption for Any Function

1-FE for arbitrary circuits [SS'10, Yao'86]



Using MPC [BGW'88]

q-FE for degree-D circuits



FE Bootstrapping Theorem:  
Using Randomized Encodings  
[AIK'05, Yao'86]

q-FE for arbitrary circuits



~~unbounded-FE for arbitrary circuits~~

**DONE!**

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