RUHR-UNIVERSITÄT BOCHUM

Optimal Security Proofs for Full-Domain Hash, revisited

Cambridge, EUROCRYPT 2012

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1 Introduction

2 Our results

3 Extensions

4 Conclusions

RSA-Full Domain Hash Signatures

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procedure KeyGen $p, q \in_R \mathbb{P}$, $N = pq$ $e \in_R \mathbb{Z}_{\varphi(N)}$ Pick $H : \{0,1\}^* \to \mathbb{Z}_N$ return $(pk = (N, e, H), sk = (p, q))$	**procedure Sign**(sk, m) return $\sigma = H(m)^{\frac{1}{e}} \mod N$ **procedure Verify**(pk, m, σ) if $\sigma^e \mod N = H(m)$ then return 1 else return 0

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- RSA-FDH signatures are unique.

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- Can RSA-FDH be tightly secure?
- Exactly 10 years ago at EUROCRYPT 2002 in Amsterdam, Coron answered this by showing that a loss of a factor of q_s is optimal

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  - Proof does not hold for small e
- We show a tight proof for small e
  - Proof is to Φ-Hiding , which is stronger than RSA
- We then show some generalizations and extensions.

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Theorem 1 (Coron)

If there is a reduction $\mathcal{R}$ from RSA-FDH to inverting RSA, with security loss less than q_s, then we can efficiently invert RSA.

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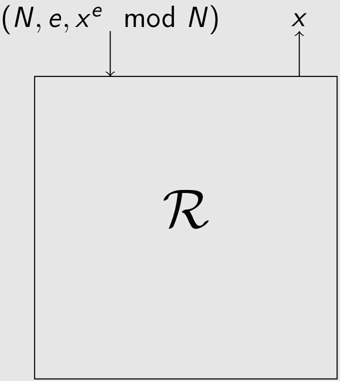

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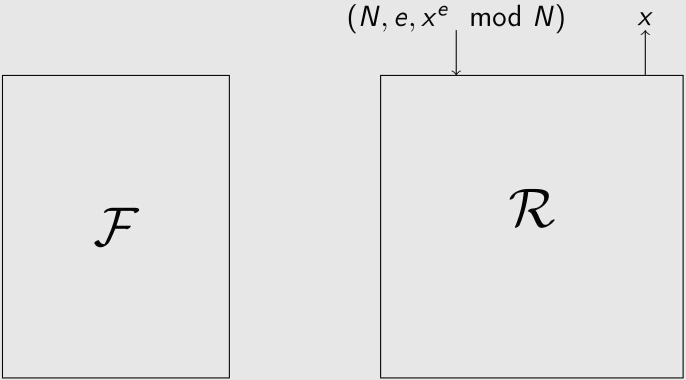

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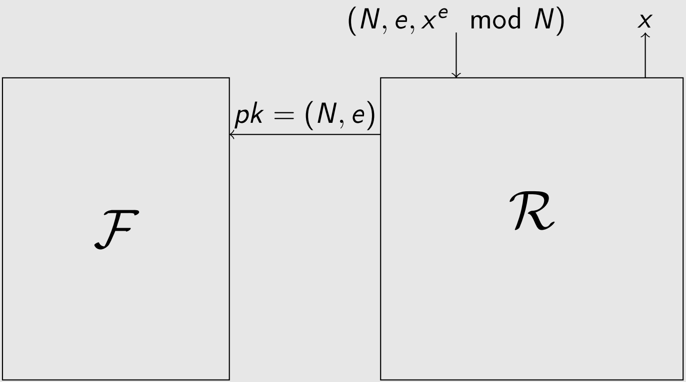

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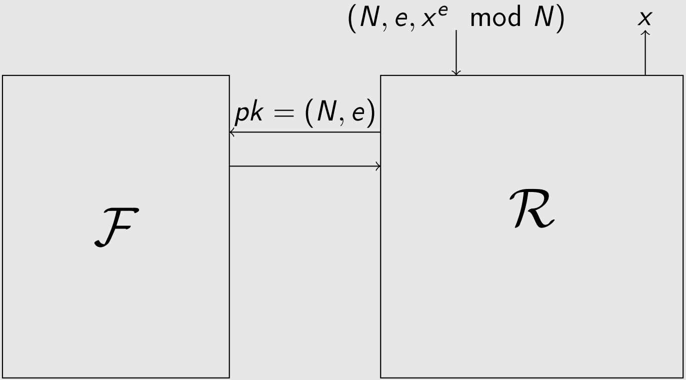

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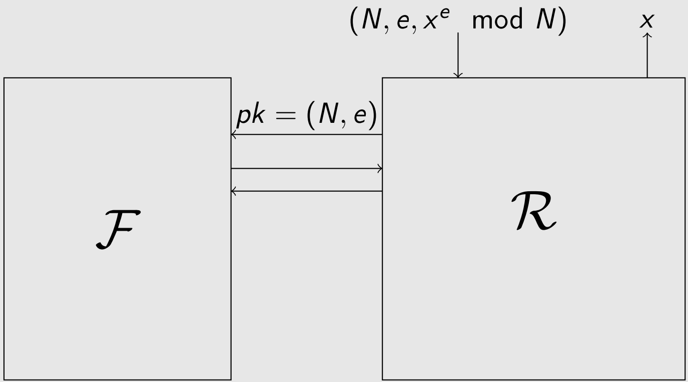

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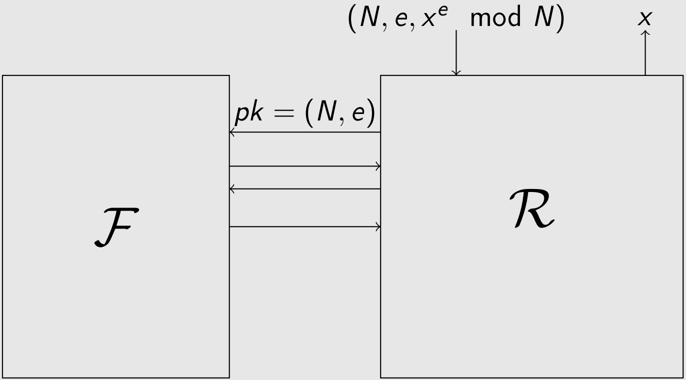

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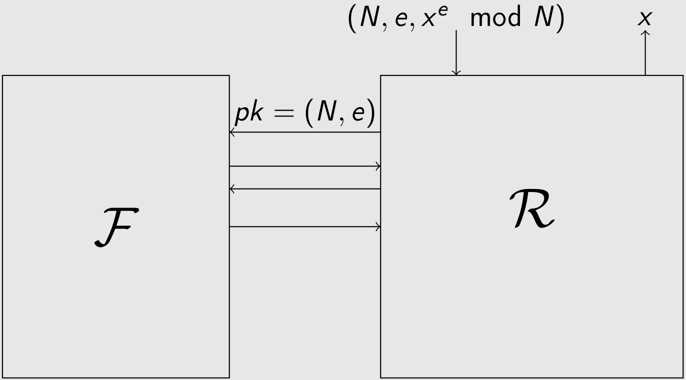

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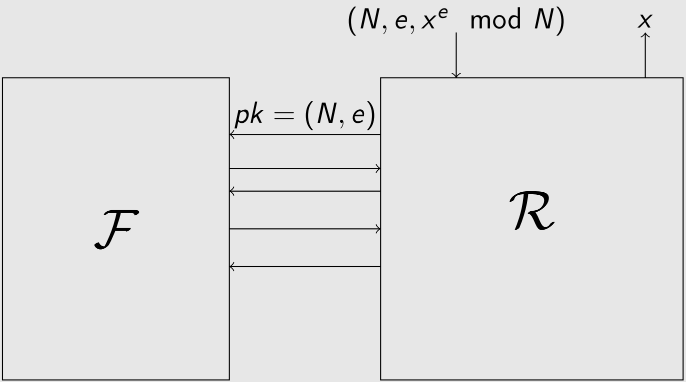

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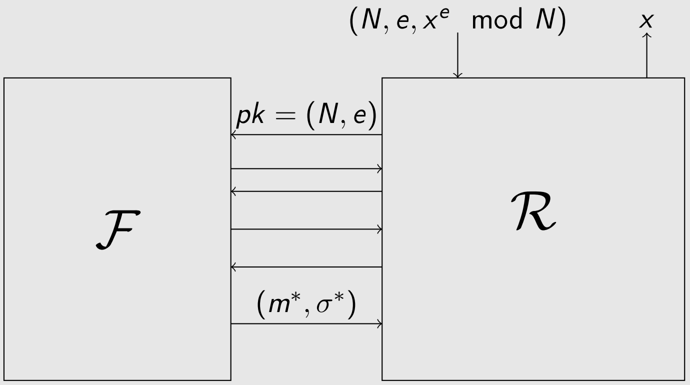

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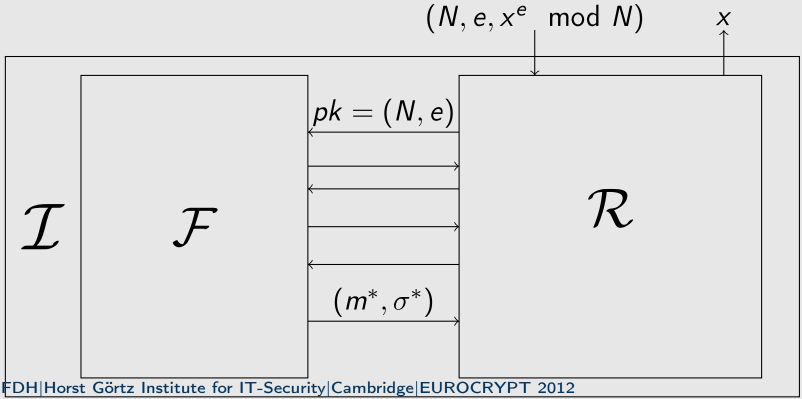

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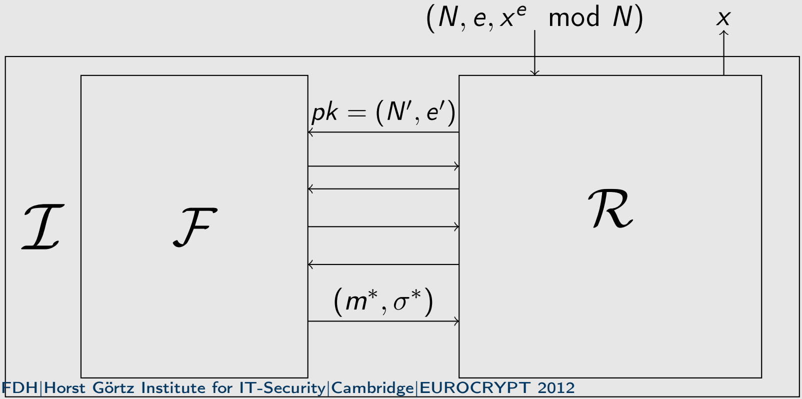

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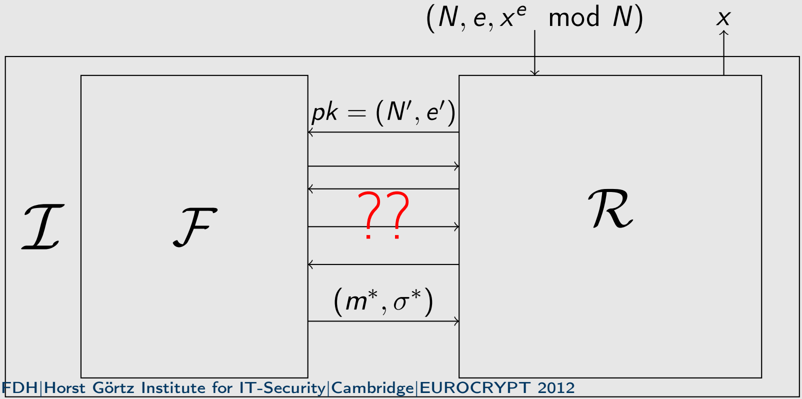

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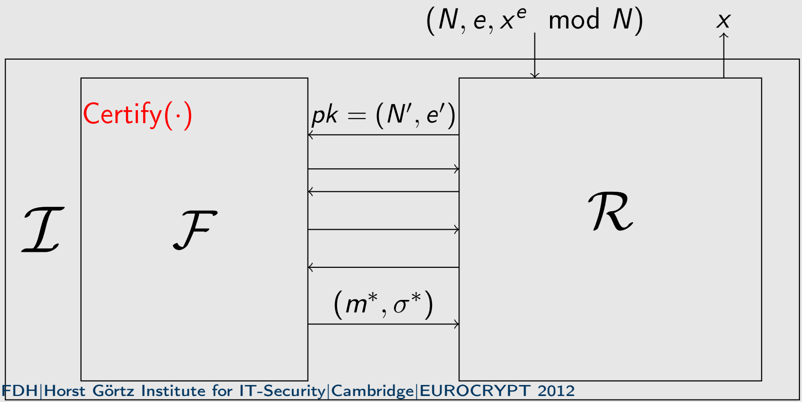
If there is a reduction $\mathcal{R}$ from RSA-FDH to inverting certified RSA, with security loss less than q_s, then we can efficiently invert RSA.



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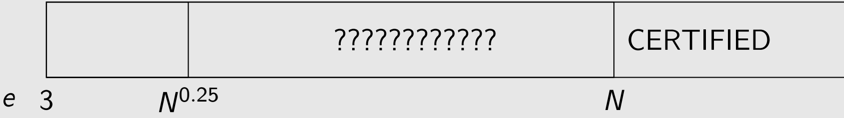

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§ In particular for RSA, the lossy function is e-to-1.

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- Φ-Hiding was introduced in 1999 by Cachin, Micali and Stadler [CMS99].
- Φ-Hiding states that given N and a prime $e < N^{0.25}$ it is hard to distinguish $e|\varphi(N)$ and $gcd(e, \varphi(N)) = 1$.

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If Φ-Hiding is (t', ε')-hard, then RSA-FDH is $(q_h, q_s, t, \varepsilon)$-secure, for any q_h, q_s, with $t \approx t', \varepsilon \approx 2\varepsilon'$.

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- A forgery (m^*, σ^*) gives a collision in the RSA function with probability $1 - \frac{1}{e}$, allowing us to factor or break Φ-Hiding .
- The final security loss is approximately $2 = O(1)$.

Implications of our results

- If we assume that solving Φ-Hiding is equivalent to inverting RSA, then:

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- We can extend our main theorem to any certified trapdoor permutation

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If TDP is (t', ε')-lossy, then TDP-FDH is $(q_h, q_s, t, \varepsilon)$-secure, for any q_h, q_s, with $t \approx t', \varepsilon \approx 2\varepsilon'$.

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- We can show impossibility for any hard problem Π and any certified unique signature scheme Σ.

Theorem 4

If there is a reduction $\mathcal{R}$ from Σ to solving Π, with security loss less than q_s, then we can efficiently solve Π.

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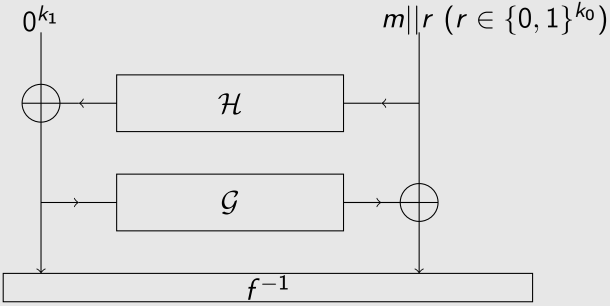

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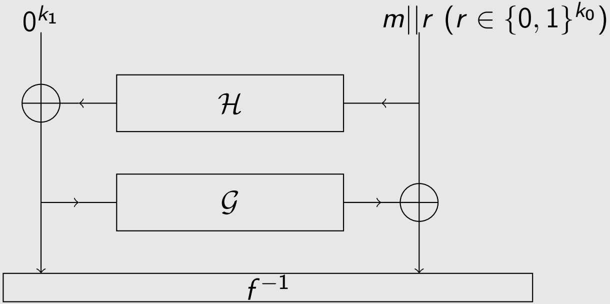
Theorem 5

If Φ-Hiding is (t', ε')-hard, then RSA-PSS-R is $(q_h, q_s, t, \varepsilon)$-secure, for any q_h, q_s, with $t \approx t', \varepsilon \approx 2 \cdot \varepsilon' + \frac{(q_h+q_s)^2}{2^{k_1}}$



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Security Proof	Randomness	Padding	Total overhead
Bellare-Rogaway [BR96]	160	160	320
Coron [Cor02]	30	160	190
This work	0	160	160

Table: Total overhead using RSA-PSS-R for 80 bit security.

Extensions: PSS with message recovery

- When using signatures with recovery we want to minimize bandwidth.
- Signer needs only to send the "enhanced signature".
- Verify is replaced by Recover, which outputs message or $\perp$.
- We use a measure called overhead, which is the difference in size between the message and the "enhanced signature".

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- PSS-R comparable to BLS signatures.

1 Introduction

2 Our results

3 Extensions

4 Conclusions

Conclusion

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Conclusion

- Revisited and corrected Coron's proof.
- Tight security proof for RSA-FDH with small exponents.
- Extensions to TDP and other problems.
- Extensions to PSS and PSS-R.

Many thanks for your attention!

QUESTIONS?