Key-Alternating Ciphers in a Provable Setting: Encryption Using a Small Number of Public Permutations

Andrey Bogdanov , Lars R. Knudsen
Gregor Leander , Francois-Xavier Standaert
John Steinberger , Elmar Tischhauser

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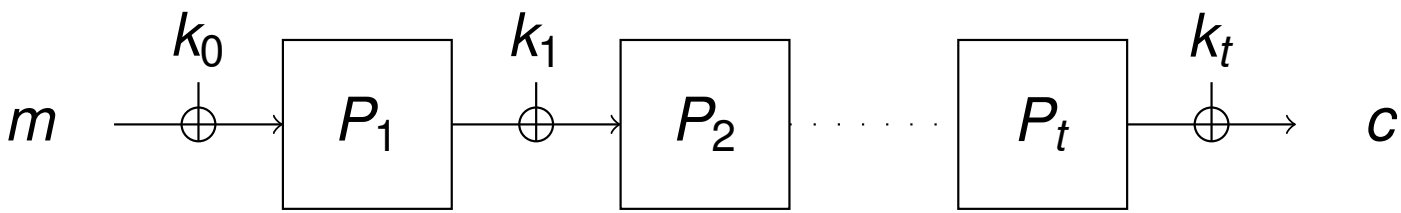
Outline

Outline

The Cipher

The Topic

A key-alternating cipher



Example

Most prominent example: AES

Many others exist.

A natural question

Question

Is this a good way of building block ciphers?

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More precisely:

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Is there a generic way to break all of them?

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More precisely:

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Is there a generic way to break all of them?

Answer

We do not know!

Only one round studied: Even-Mansour '91

A natural question

Answer

We do not know!

Very surprising!

- cf. progress in provable security
- cf. generic group model
- cf. SHA-3 competition

SHA-3

Folklore

We are much more confident with designing block ciphers than with designing hash functions.

Example

DES is still okay, MD4/MD5 and SHA-1: not really

SHA-3

Folklore

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DES is still okay, MD4/MD5 and SHA-1: not really

When it comes to provable security: This is different.

Provable Security

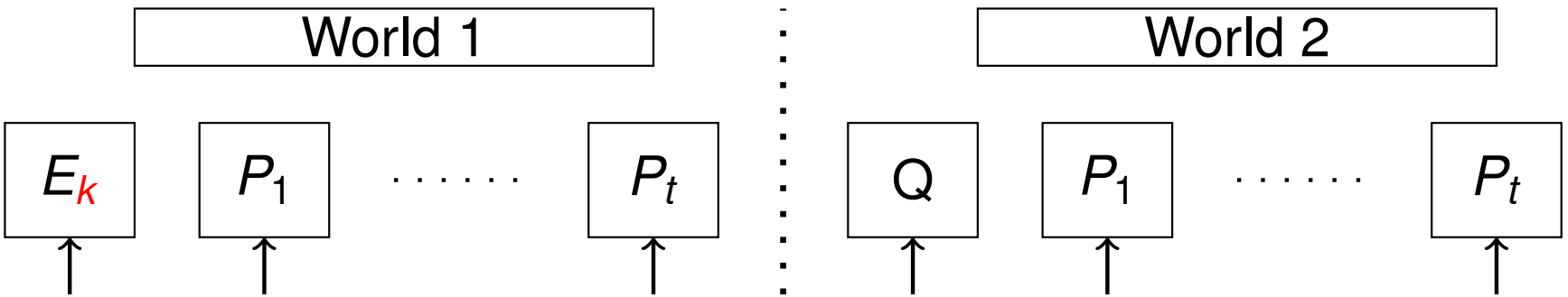
All SHA-3 finalist come with a proof in the idealized model. No AES finalist had one.

Outline

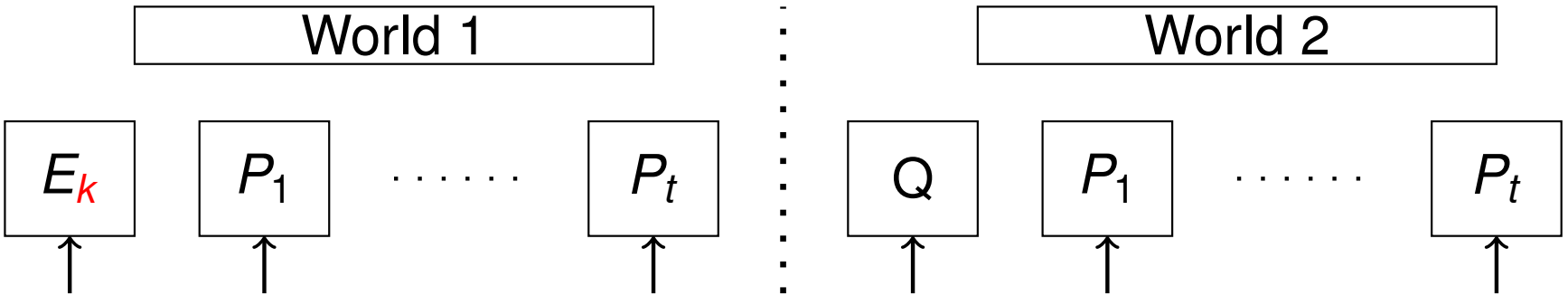
The Setting

The Setting

- Ideal round functions
- Information theoretical adversary
- Two worlds



The Result



Theorem (informal)

No adversary can distinguish the two worlds with less than

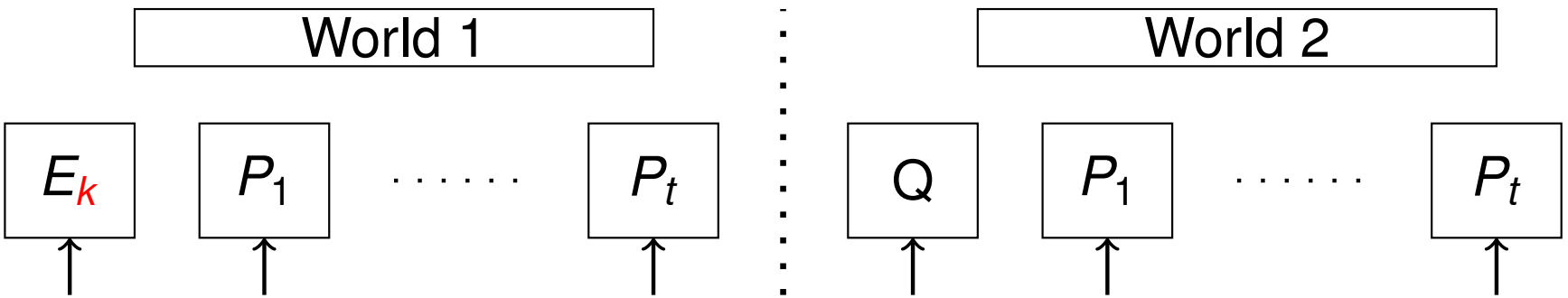
- $2^{n/2}$ *queries for one round (Even-Mansour)*
- $2^{2n/3}$ *queries for more than one round*
- $2^{3n/4}$ *queries for more than two rounds*

Outline

A Proof Outline

Initial Game

- Sample permutations P_i and Q uniform at random
- Choose random keys
- Goal of the adversary A: distinguish the worlds



Lazy Sampling

Start with empty lists for P_i and E.

Upon a query to P_i (or E):

- Select all P_i uniform at random (among all permutations consistent with previous queries).
- Construct E.
- Answer the query accordingly.
- Update lists.

Goal of the adversary A: distinguish the worlds

Clear: Results in the same distribution.

The Hybrid

We change the game *a bit*. Lazy sampling: Start with empty lists for P_i and E.

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Modified Game: Upon a query to P_i (or E):

- Select a random answer y (maintaining P_i (or E) as a permutation)
- Check consistency
- If consistent: Output y and update lists.
- If inconsistent: crash!

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Consistency

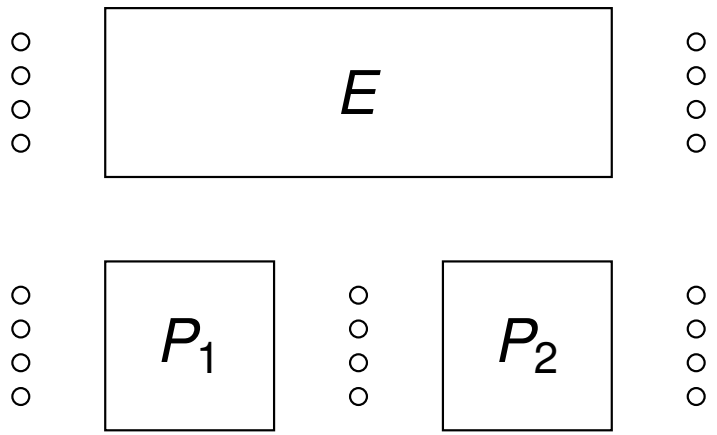
P_i and E_k are consistent iff

$$E_k(x) = P_t(\ldots P_2(P_1(x \oplus k_0) \oplus k_1)\ldots) \oplus k_t$$

The Hybrid: In a Picture

For Simplicity

Only $n = 2$ and zero keys.



List of queries:

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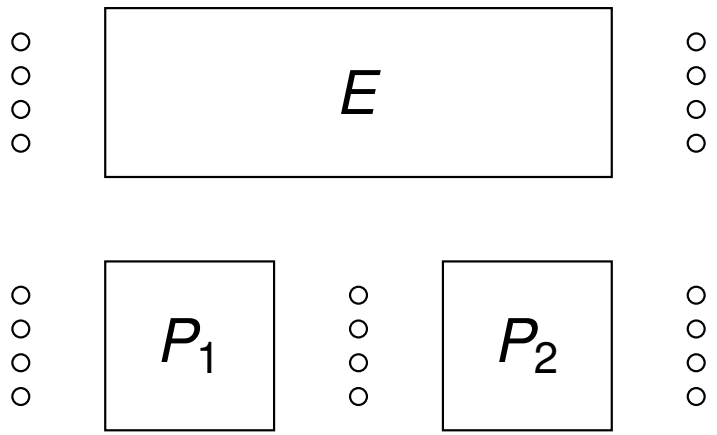

List of queries:

$$P_1(0) =$$

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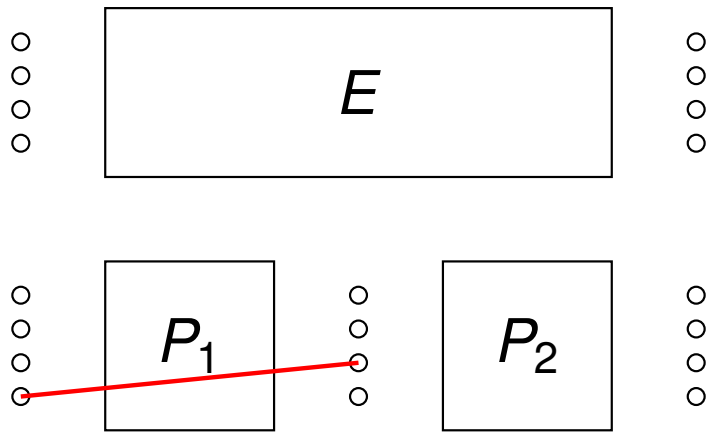

List of queries:

$$P_1(0) = 1$$

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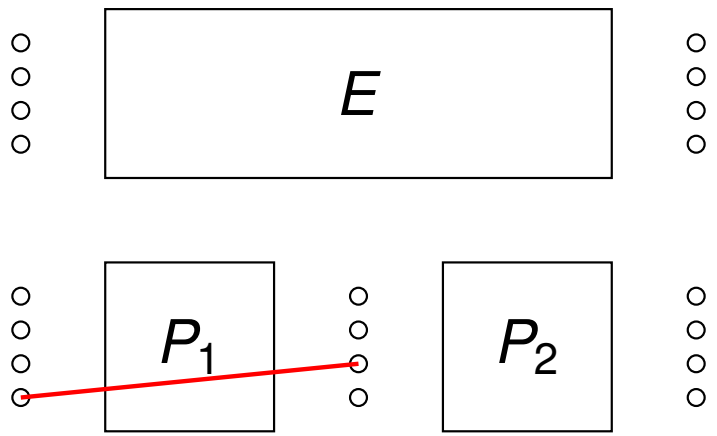

List of queries:

$$P_1(0) = 1 \quad P_2(2) =$$

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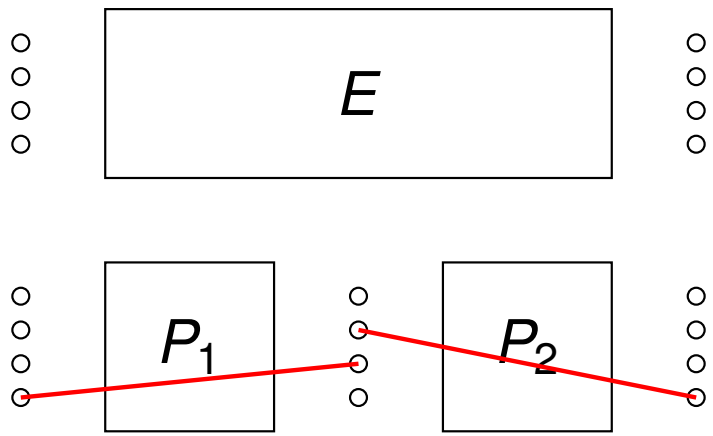

List of queries:

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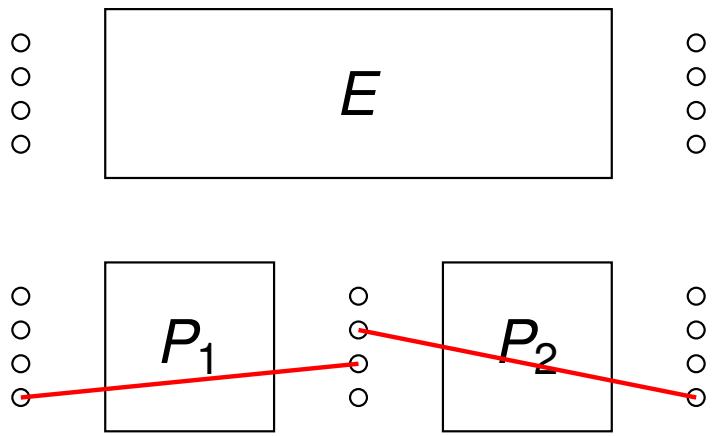

List of queries:

$$P_1(0) = 1 \quad P_2(2) = 0 \quad E(0) =$$

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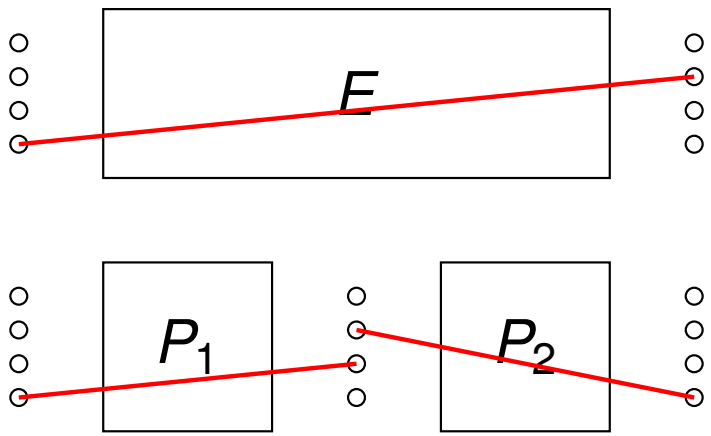

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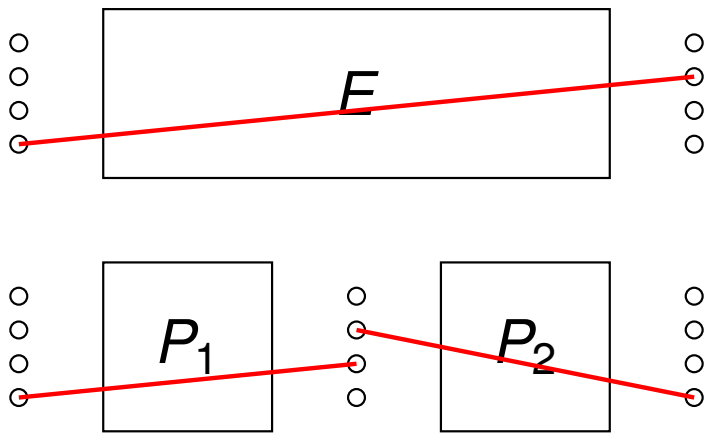

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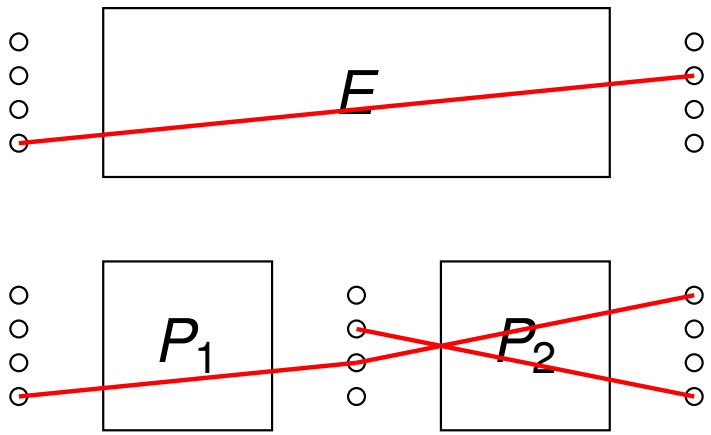

List of queries:

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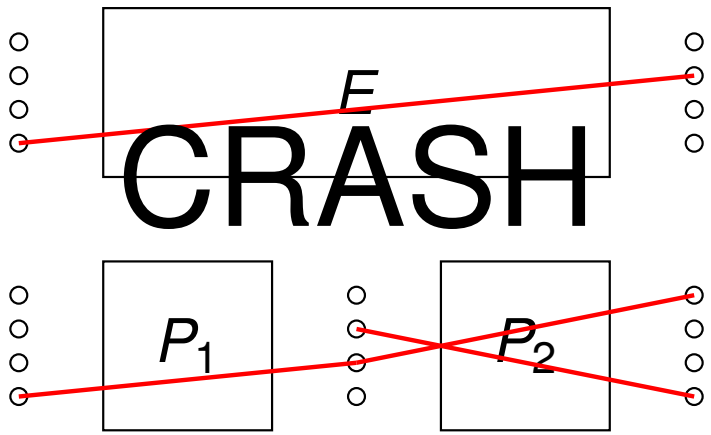

List of queries:

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Almost Done?

Two Steps To Go

1. Show that one cannot win in the modified game.
2. Show that the modified game is only *slightly* different.

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Step 1: Easy

Sketch on the next slides.

Almost Done?

Two Steps To Go

1. Show that one cannot win in the modified game.
2. Show that the modified game is only *slightly* different.

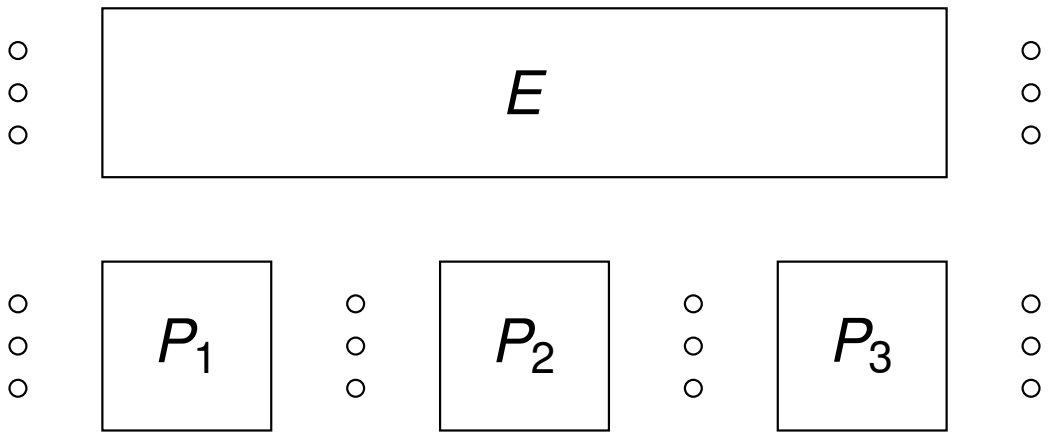
Step 1: Easy

Sketch on the next slides.

Step 2: Not so easy

Quite involved and technical: See paper.

On Step 2: The Modified Game is different



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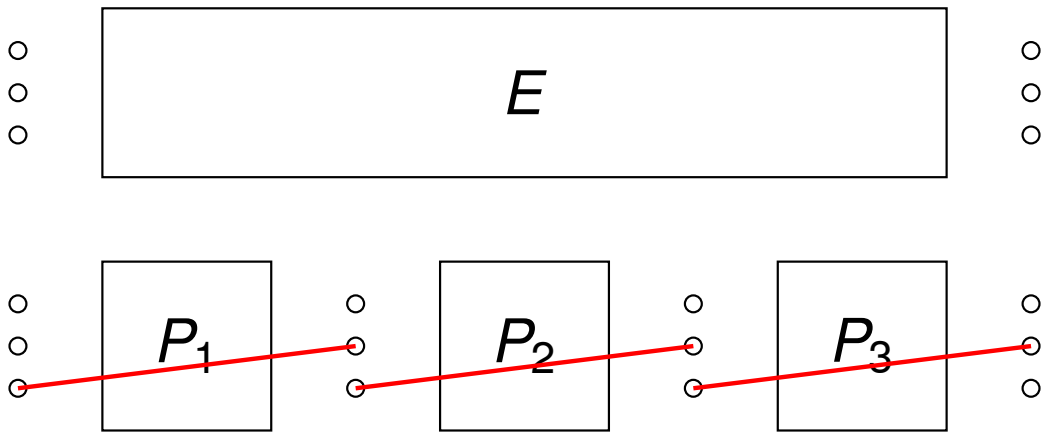

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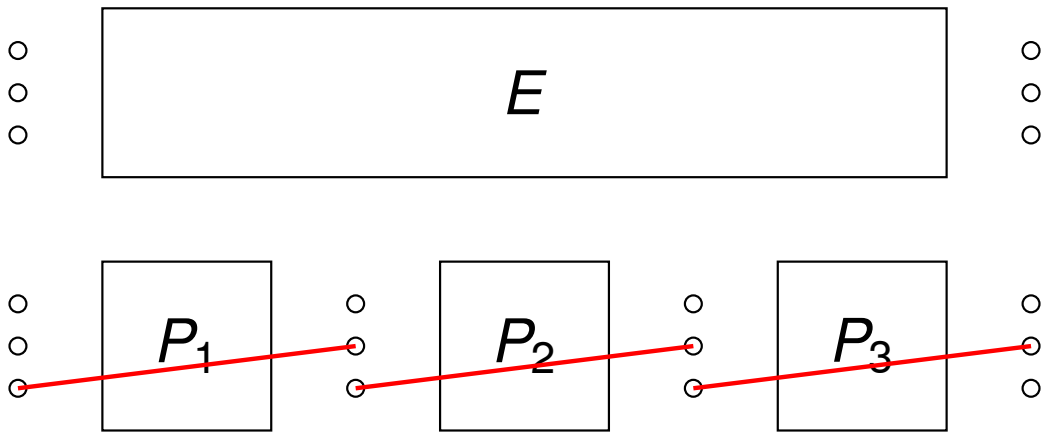

$$E(0) = ?$$

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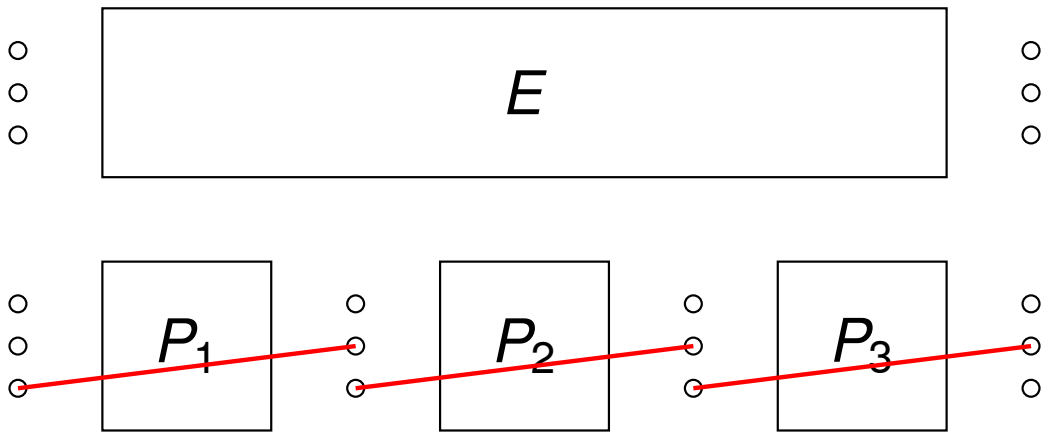

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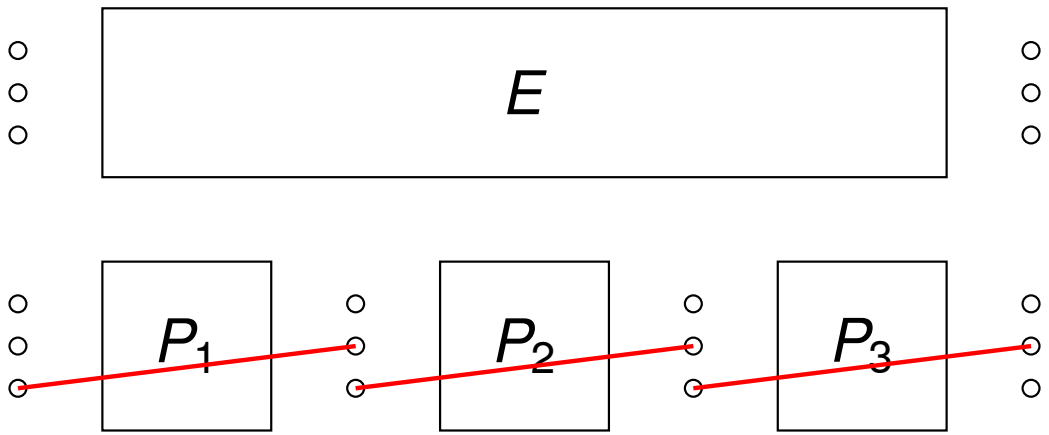

$E(0) = ?$

- Modified Game:
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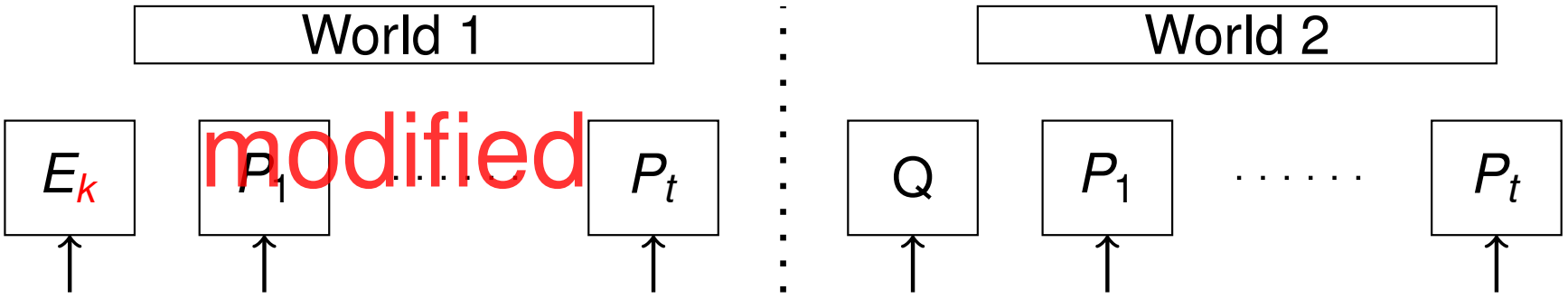
- Original Game: 8 possibilities

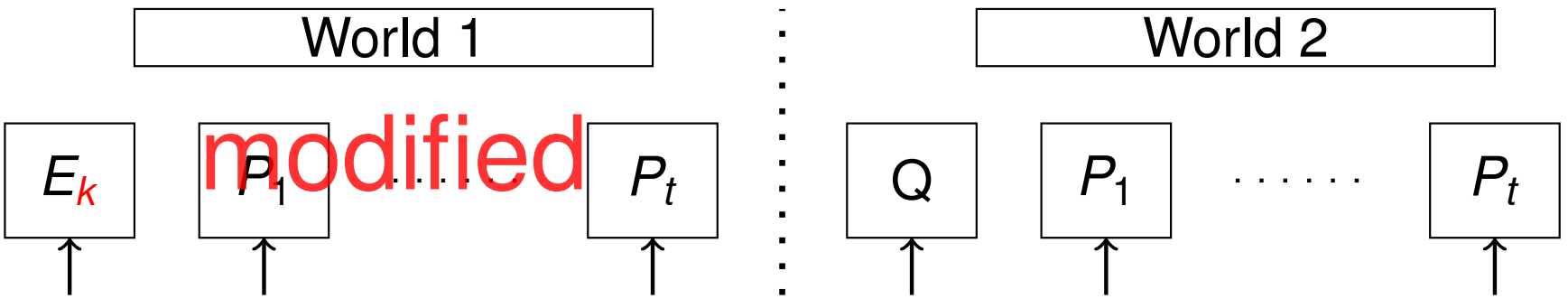
Step 1



First observation

As long as the oracle does not crash, both worlds are the same.

Step 1



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What is the probability for a crash?

Step 1, continued

Question

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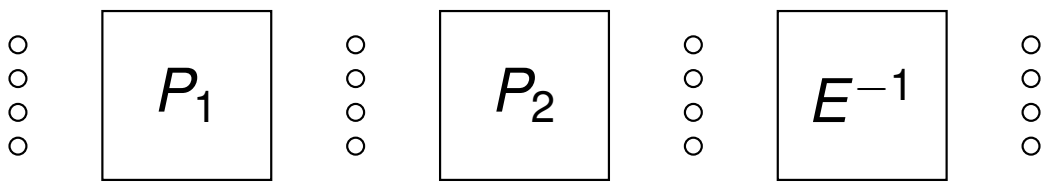

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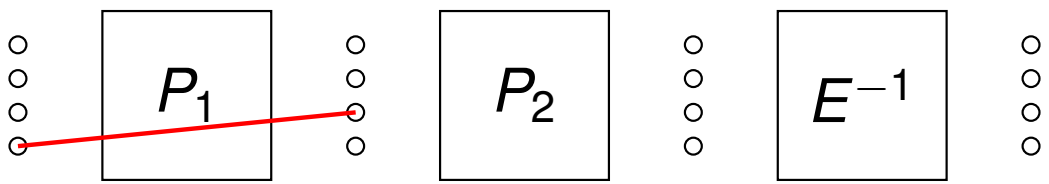

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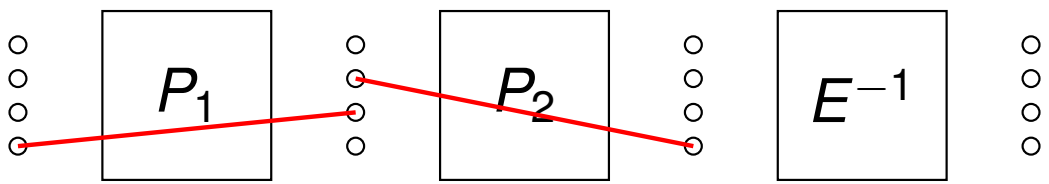

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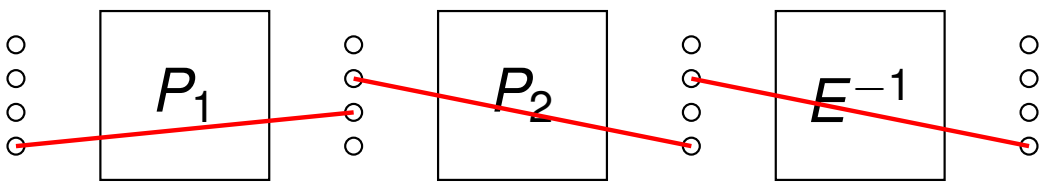

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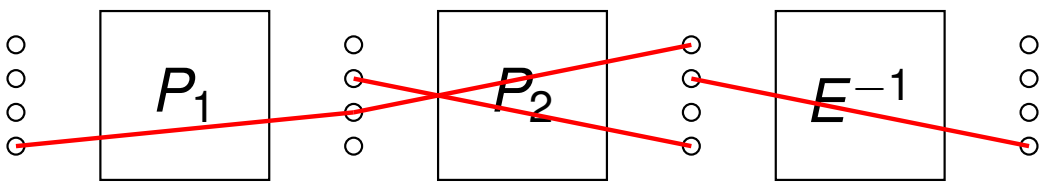

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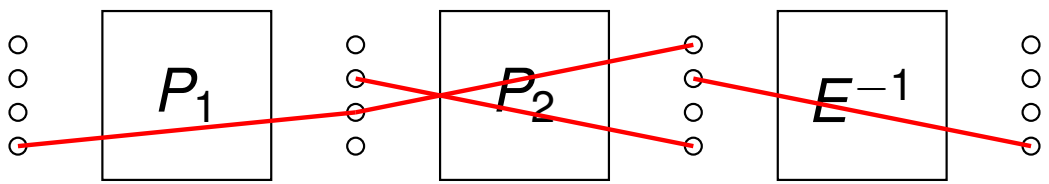

A Crash

A sequence of queries, connected in all but one positions.

Step 1, continued

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Informal

$$q \approx 2^{\frac{t}{t+1}n}$$

The precise statement

Notation:

- n block size
- q number of queries

Theorem

Let $N = 2^n$ and let $q = N^{\frac{t}{t+1}}/Z$ for some $Z \geq 1$. Then, for any $t \geq 1$, and assuming $q < N/100$, we have

$$\mathbf{Adv}^{\mathrm{PRP}}_{E,N,t}(q) \leq \frac{4.3q^3t}{N^2} + \frac{t+1}{Z^t}.$$

For $t \geq 2$ this implies $q \approx 2^{2n/3}$.

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Interpreting The Result

Theorem (informal)

With idealized permutations as round function, the key-alternating cipher is secure.

What does this mean? For a concrete cipher?

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Generic Attacks

If you want to break the cipher, you have to use special properties of the permutations.

For a Concrete Instance

It does not mean anything.

Any practical instance is, well practical. Thus not ideal.

Outline

Further Results

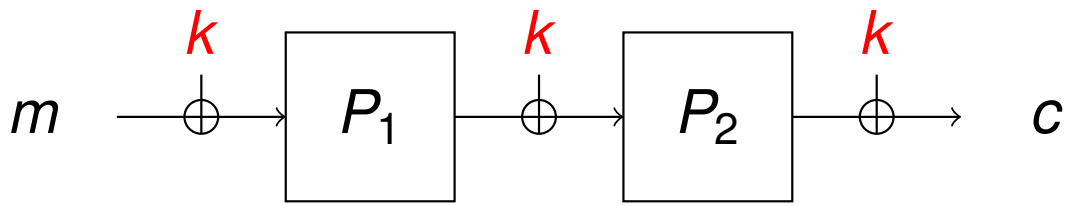
More in the paper:

- Study the expected resistance against linear cryptanalysis
- A concrete proposal using AES

A Concrete Proposal or How It Started

A Proposal

A block cipher secure against related-key attacks?



($P_{1,2}$ is AES with fixed key.)

Intuition

If P_1 and P_2 are complicated enough than related-key attacks do not work.

Future Work

This work leaves many questions open:

- Improve the bound
- Get closer to actual constructions, e.g.
  - identical round keys
  - identical round functions
- New Constructions

Future Work: Improve The Bound

Conjecture

The actual lower bound is

$$q \approx 2^{t/(t+1)n}$$

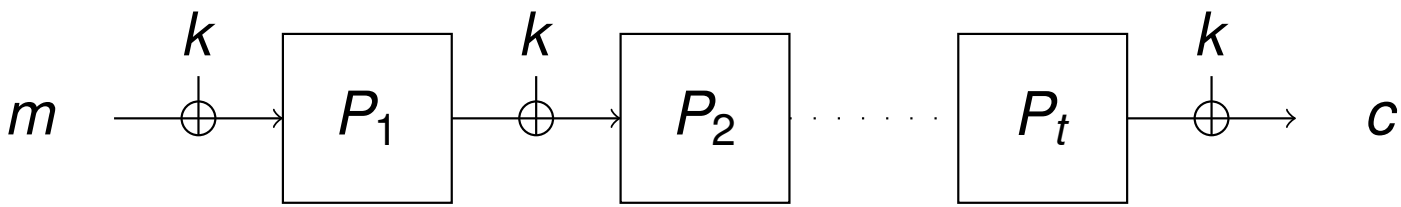
- This is actually the upper bound.
- Already improved to $2^{3/4n}$ for $t \geq 3$
- Challenging step: A bound that improves with t.

The End

Thanks!

Future Work: Closer to actual constructions I

Identical round keys:



Future Work: Closer to actual constructions II

Identical round functions: