A Fast and Key-Efficient Reduction from Chosen-Ciphertext to Known-Plaintext Security

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May 24, 2007

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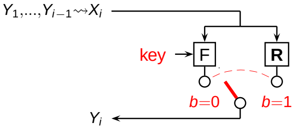

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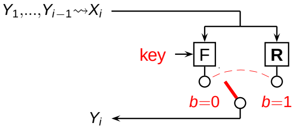

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Goal: weaken assumptions, improve efficiency

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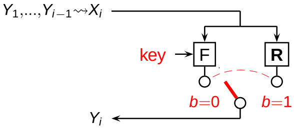

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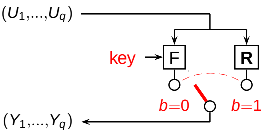

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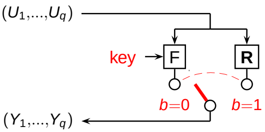

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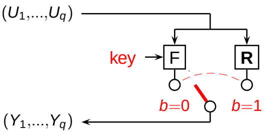

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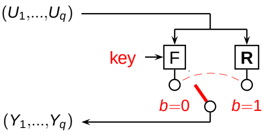

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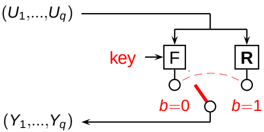

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- be self inverse, i.e., $F_k(F_k(x)) = x$.

KPA vs. CPA: Example 1

I		R	
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

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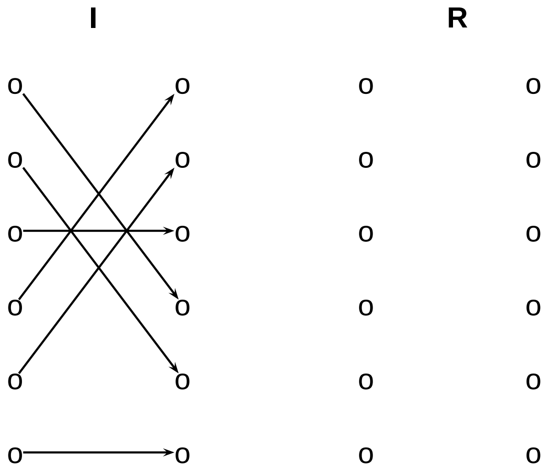

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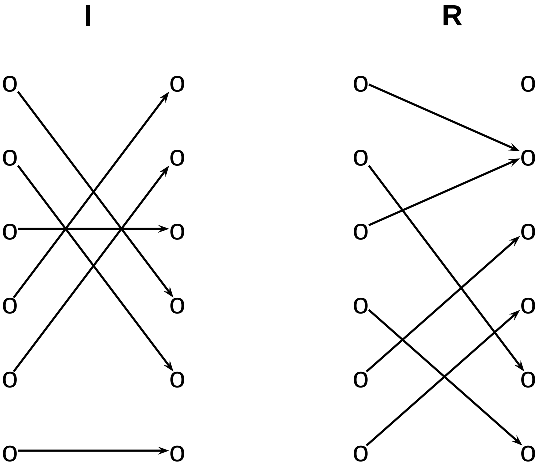

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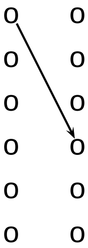

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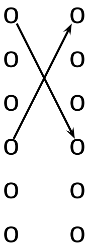

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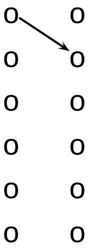

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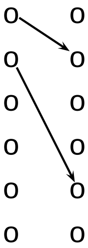

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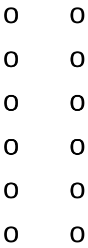

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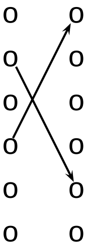

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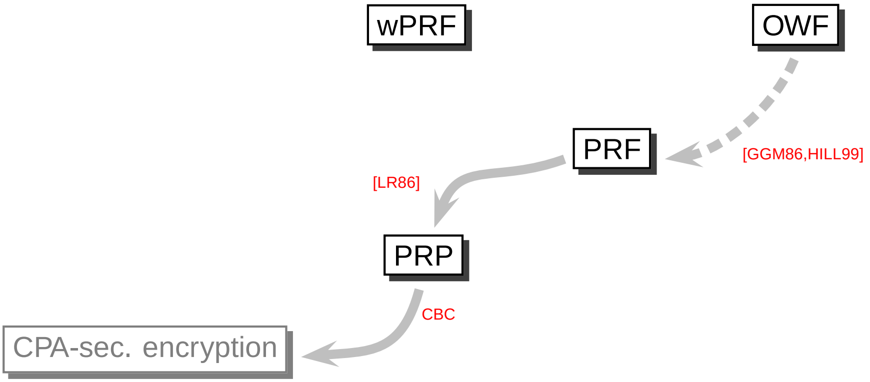

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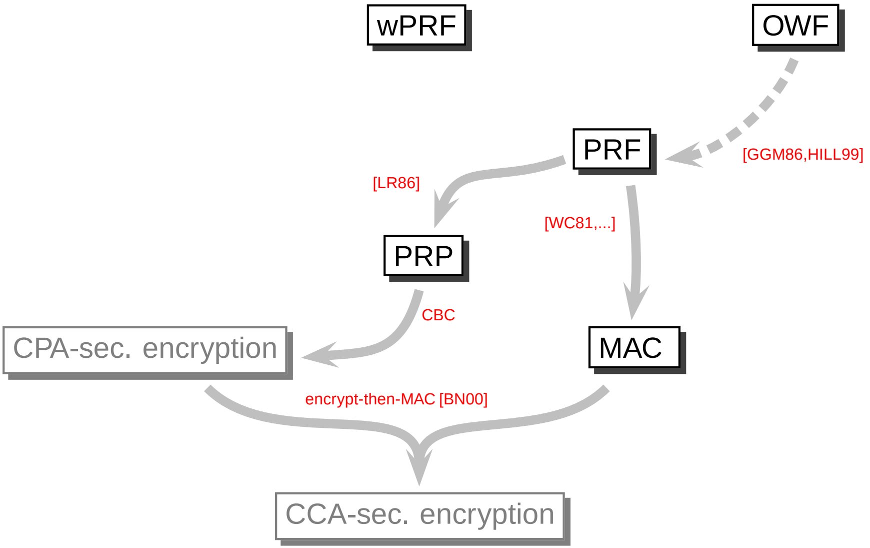
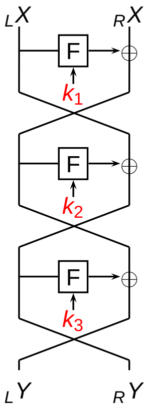


Feistel-Networks?

Feistel-Networks with PRFs do produce a PRP.



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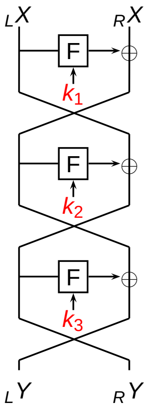

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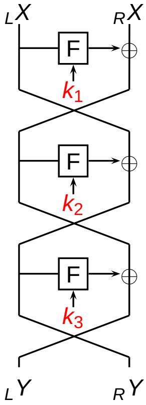
Reason

The wPRF F can have 0 as fixpoint,

$$\mathsf{F}_k(0) = 0 \qquad \text{(for all keys } k\text{)}$$

and hence

$$\psi[\mathsf{F}_{k_1}\mathsf{F}_{k_2}\mathsf{F}_{k_3}](0) = 0.$$



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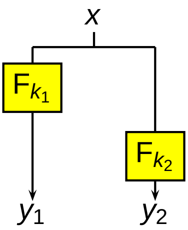

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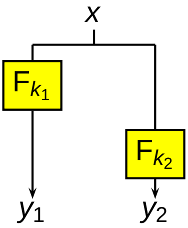

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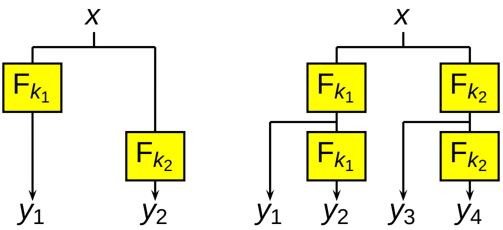

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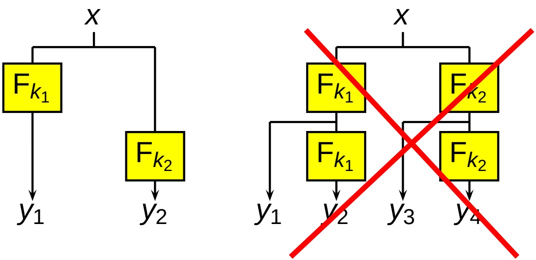

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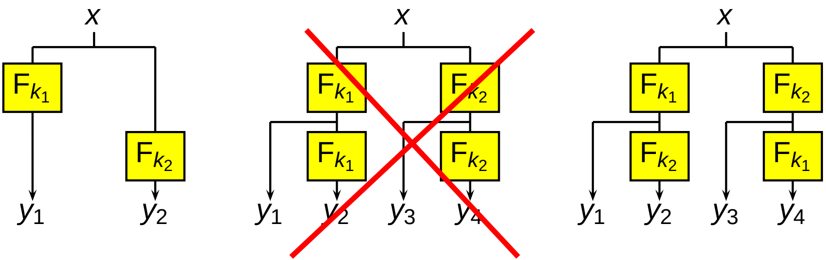

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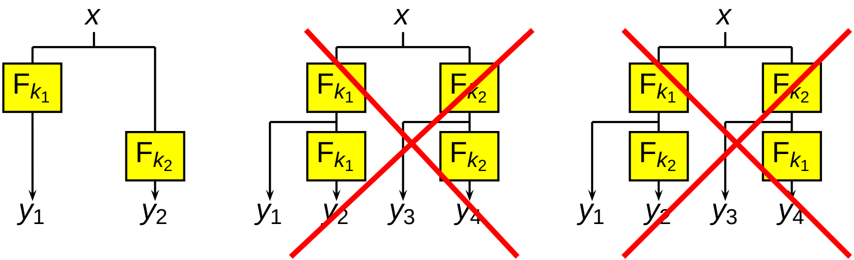

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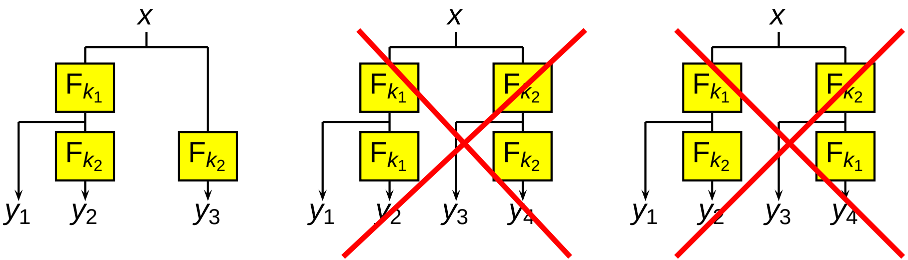

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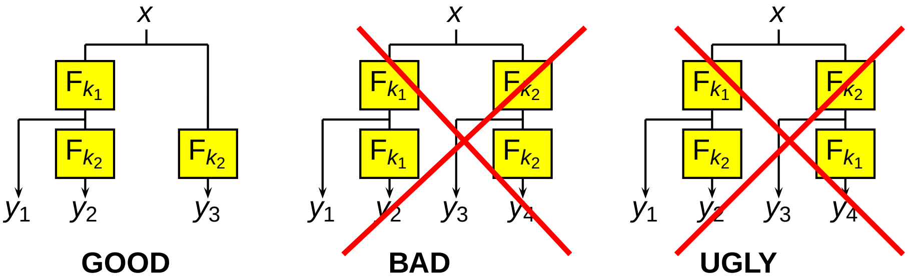

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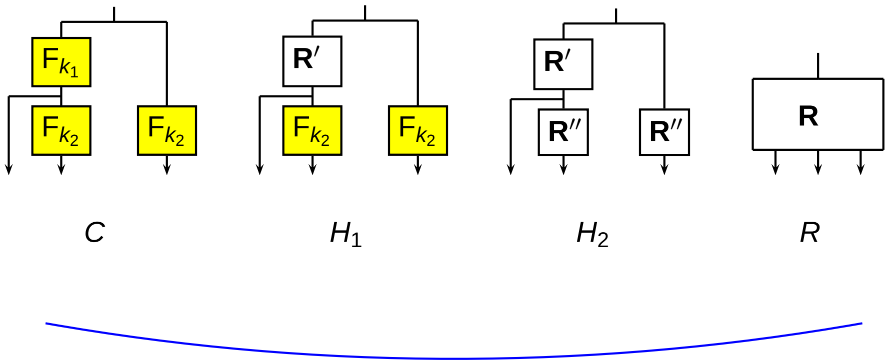

$$\mathbf{Adv}_{t,q}^{\mathrm{KPA}}(C, R) \leq ?$$

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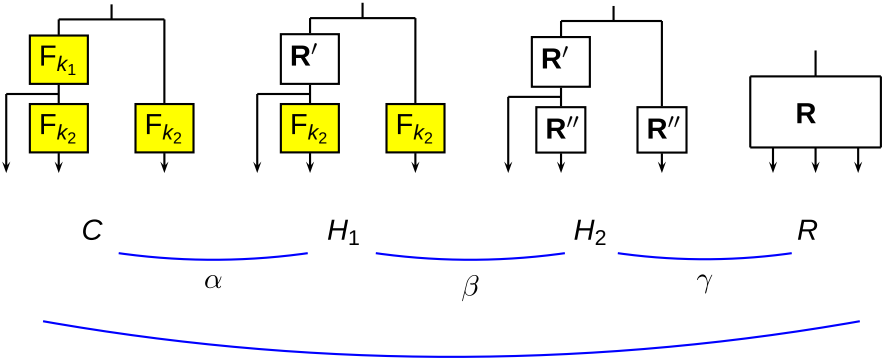

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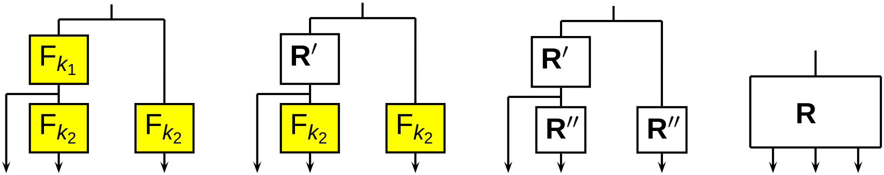

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$\leq \mathbf{Adv}^{\text{KPA}}_{t,q}(\mathsf{F},\mathbf{R}')$ $\leq \mathbf{Adv}^{\text{KPA}}_{t,2q}(\mathsf{F},\mathbf{R}'')+q^2/2^{n+1}$ $\leq q^2/2^{n+1}$

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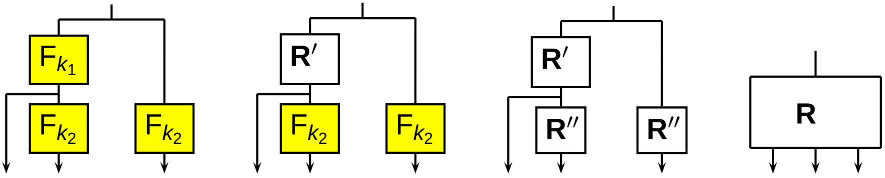

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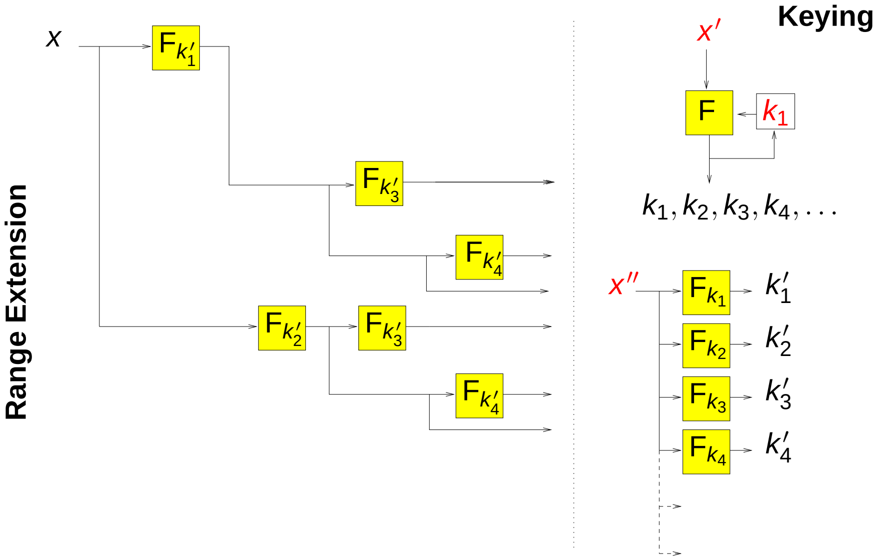
$\Rightarrow$ How can the range of F be extended even more?

Range Extension of wPRFs [DN02]



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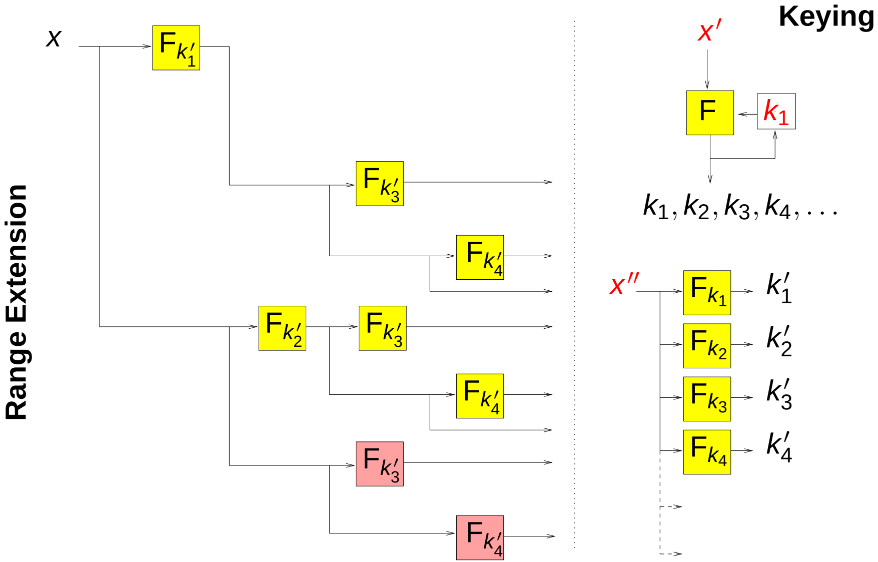

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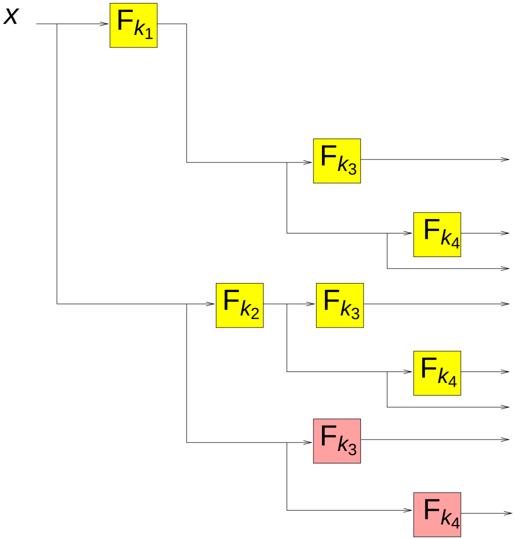

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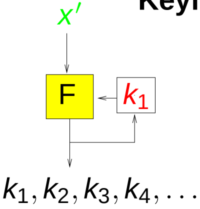

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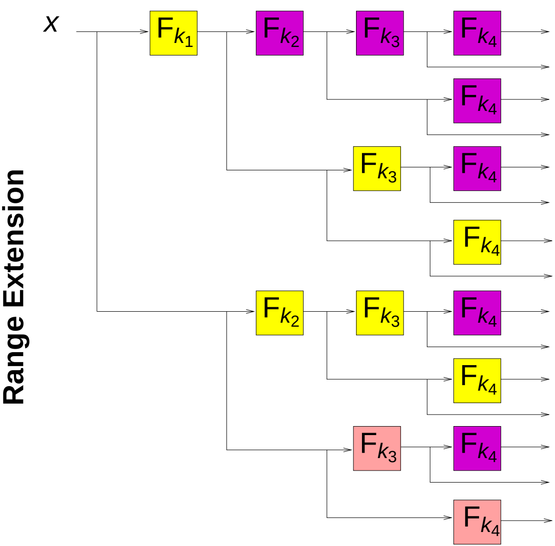

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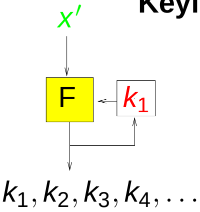

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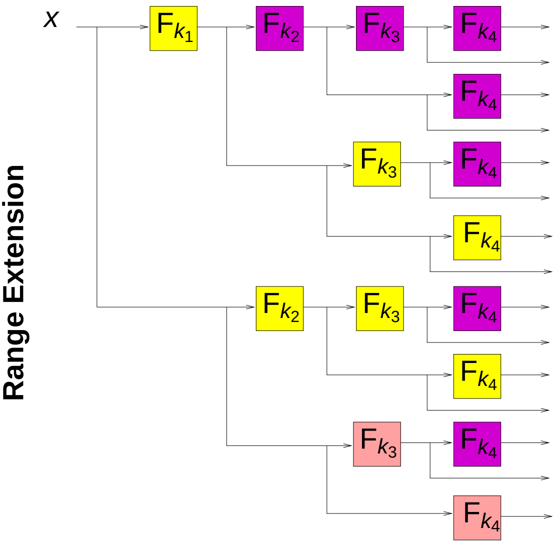
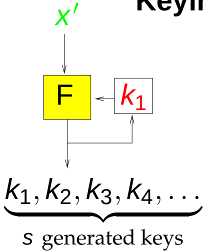


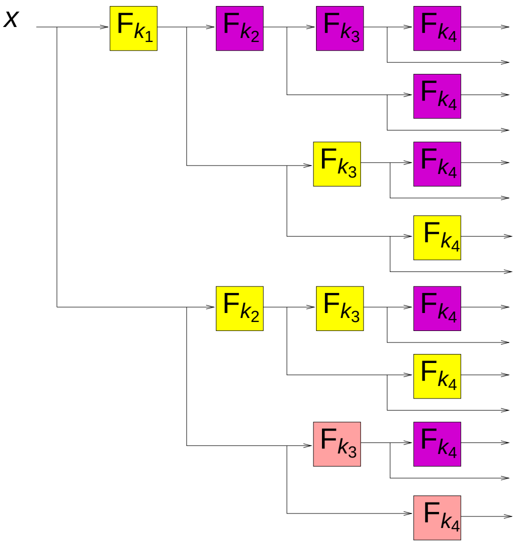
Keying



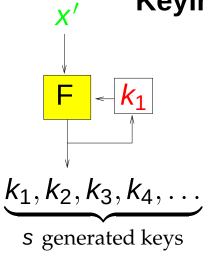
	# keys	out-puts	over-head
DN02	3	1.4^s	$2s$
MT05	3	1.7^s	$2s$
this	**1**	$\mathbf{2}^s$	**s**

Range Extension of wPRFs [this paper]

Range Extension

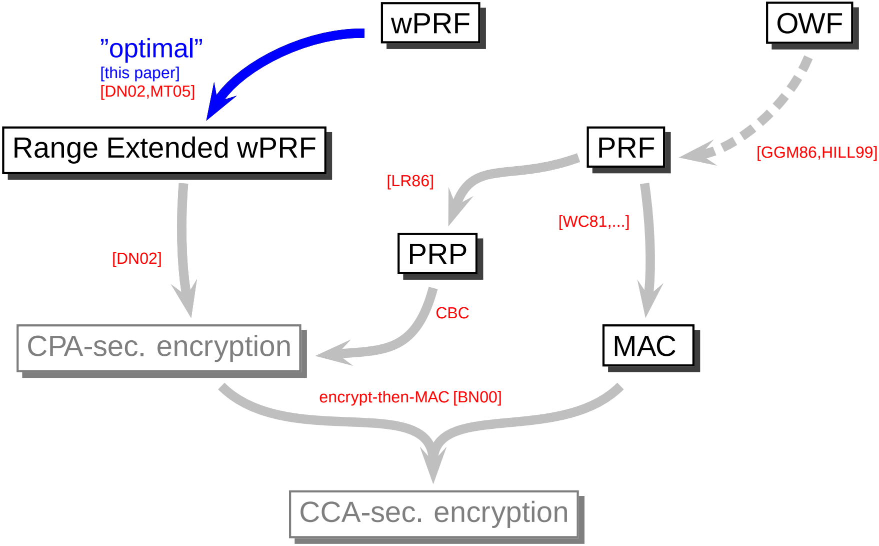


Keying



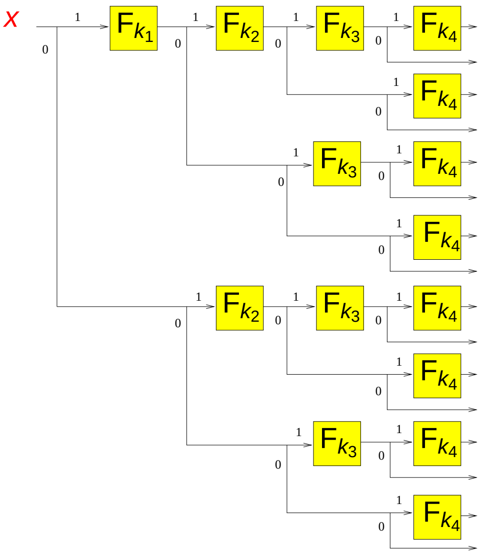
	# keys	out-puts	over-head
DN02	3	1.4^s	$2s$
MT05	3	1.7^s	$2s$
this	**1**	$\mathbf{2}^s$	**s**
better	no, this is “optimal”		

Overview

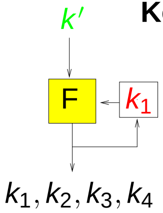

wPRF
OWF
"optimal"
[this paper]
[DN02,MT05]
Range Extended wPRF
PRF
[GGM86,HILL99]
[LR86]
[WC81,...]
PRP
[DN02]
CBC
CPA-sec. encryption
MAC
encrypt-then-MAC [BN00]
CCA-sec. encryption

wPRF ⇒ PRF [this paper]

All Outputs



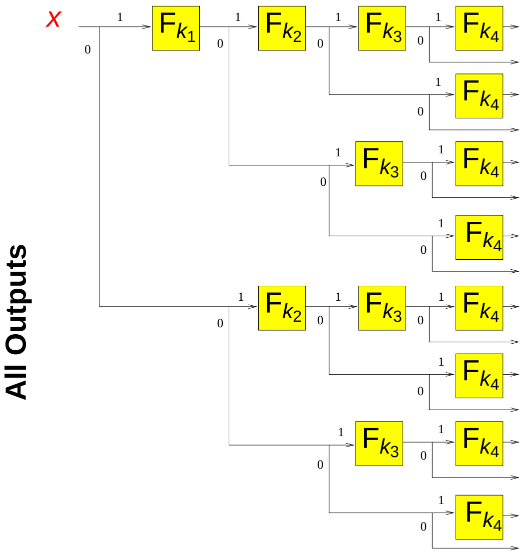
Keying



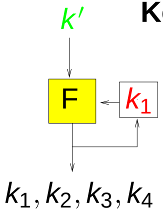
Input: 4-bits $b_1b_2b_3b_4$

Output: block $b_1b_2b_3b_4$

wPRF ⇒ PRF [this paper]



Keying



Input: 4-bits $b_1 b_2 b_3 b_4$

Output: block $b_1 b_2 b_3 b_4$

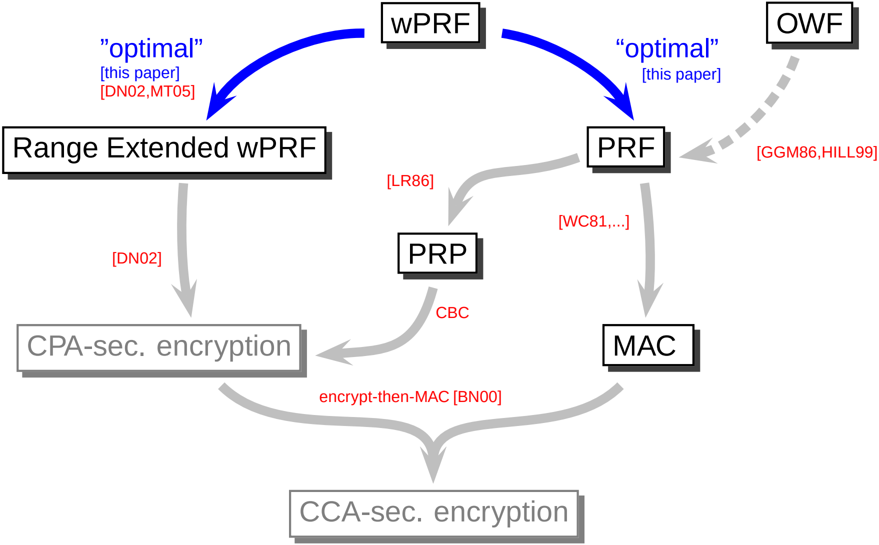
DDH-based wPRF: $x \mapsto x^k$

$\Rightarrow$

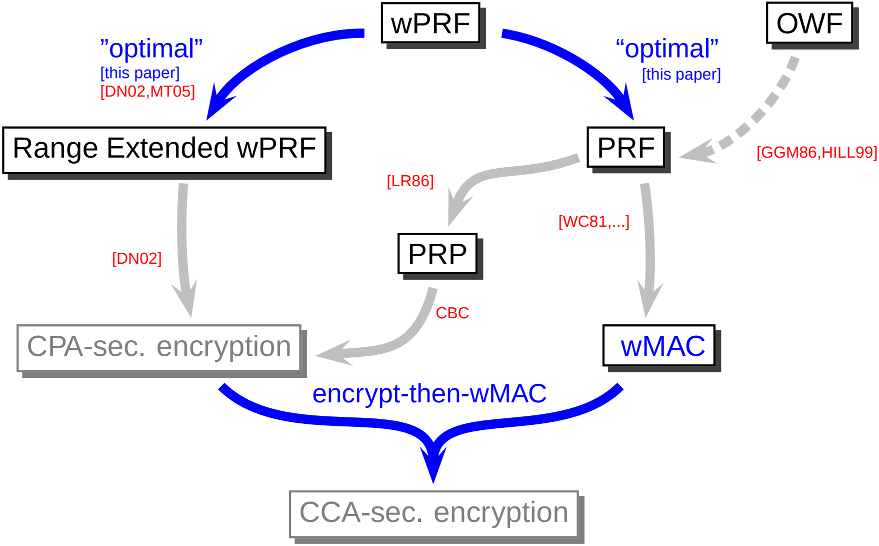
DDH-based PRF [NaoRei97]:

$(b_1, b_2, b_3, b_4) \mapsto x^{\prod_{b_i=1} k_i}$

Conclusions



Conclusions



Conclusions