Cryptographic and Physical Zero-Knowledge Proof Systems for Solutions of Sudoku Puzzles

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Sudoku

Can be generalized to an n×n grid, where $n=k^2$.

Here, k=3, n=9.

The question

- How to convince someone that you solved a Sudoku puzzle, without revealing the solution.

- In other words, prove that
  - There is a solution to the puzzle
  - You know the solution
  - But do this without revealing the solution.

- In other words: ZK proofs of knowledge for Sudoku.

Related Work [NNR]

- Where is Waldo?

ZK proofs for Sudoku

- Sudoku is in NP (in fact, it is NP Complete)
  - So why bother designing special proofs?

- Direct ZK proofs for Sudoku are preferable:
  - Efficiency
  - Practicality: Implementable without the aid of computers
  - Understandability (by non-experts!): Ensure that participants have intuitive understanding of the proof.
- Our Results
  - Cryptographic solutions: two machines exchange messages. Security based on computational assumptions.
  - Physical solutions: Implementable by humans without involving computers.

A demo of a Physical ZK protocol for Sudoku

		8		1		7	6	
	3							1
			6		4			2
3			1					
	4	5	7		2	1	9	
					6			5
2			9		5			
4							2	
	9	1		6		4		

Daily SuDoku: Tue 1-May-2007 very hard

Analysis

- Completeness: perfect.

- Soundness: If Prover doesn't know a solution, then it cannot answer at least one of the verifier's 3 choices.
  – Cheating probability ("Soundness error"): 2/3 *(might be too high)*

- Zero-knowledge:
  – Obviously you didn't learn anything from what you saw.
  – This property can be defined and analyzed.
- Knowledge extractor:
  – Can be defined and analyzed (not hard).

We describe several physical protocols

- The protocols use playing cards, or scratch-off cards.

- Possible criteria:
  - Number of cards
  - Number of shuffles
  - Soundness error

	# of cards	shuffles	soundness error
Protocol 1: “one card per cell”	n^2	n	2/3
Protocol 2: “all packets”	$3n^2$	3n	1/9
Protocol 3: “aggregate packets”	$3n^2$	c-1	1/9+8/(9c)
Protocol 4: “triplicate”	n (3n) special cards	3n	0

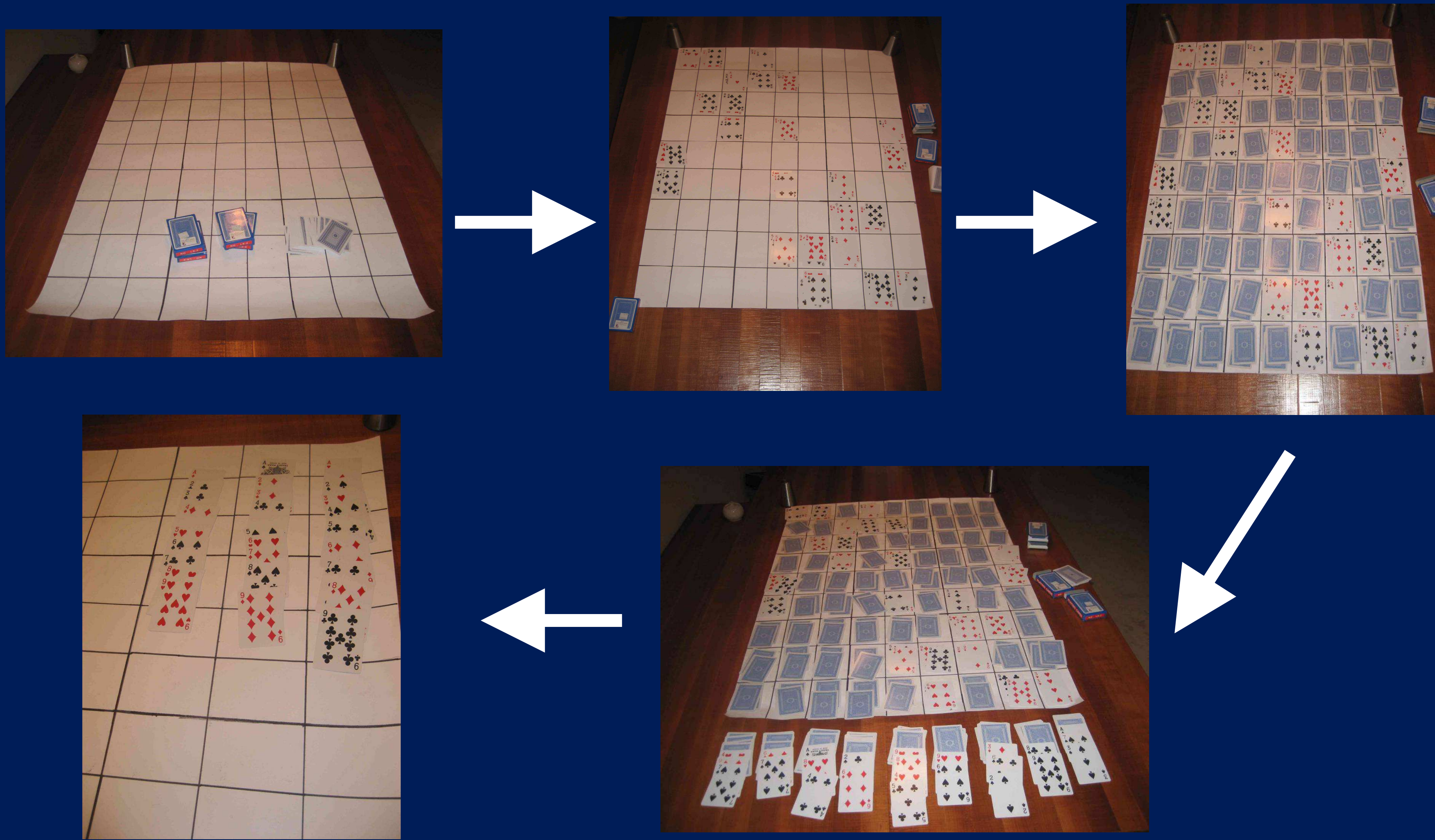
We describe several physical protocols

- The protocols use playing cards, or scratch-off cards.

- Possible criteria:
  - Number of cards
  - Number of shuffles
  - Soundness error

	# of cards	shuffles	soundness error
Protocol 1: “one card per cell”	n^2 **81**	n 9	2/3
Protocol 2: “all packets”	$3n^2$ 243	3n 27	**1/9**
Protocol 3: “aggregate packets”	$3n^2$ 243	**c-1** **3**	1/9+8/(9c)
Protocol 4: “triplicate”	n special cards	3n 27	0

Protocol 2 (using decks of cards)
http://www.wisdom.weizmann.ac.il/~naor/PAPERS/SUDOKU_DEMO



Discussion

- A good way to explain zero-knowledge for kids?

- Open problems:
  - Protocols over the mail?
    - [MN] showed how to implement commitments from scratch-off cards.
    - However, an amplification stage requires many repetitions
    - Not easy for humans

  - Non-interactive proofs (by the puzzle creator) [Berson]

  - Other puzzles?