## **Concurrent Signatures**

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A concurrent signature scheme is a signature scheme where:

- Users can initially exchange non-binding signatures that are somehow linked, and
- All signatures can concurrently be converted to full binding signatures.
- Either no signatures are binding, or all signatures are.

Ambiguous signatures:

- Could have been produced by either of two parties,
- Can convince other party but no-one else,
- E.g. Two party ring signatures, designated verifier signatures.

Keystone:

- Seed for a *keystone fix*,
- Release of keystone removes ambiguity.

Suppose entities A and B wish to exchange signatures on messages  $M_A$  and  $M_B$  respectively.

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The pairs  $\langle \sigma_A, k \rangle$  and  $\langle \sigma_B, k \rangle$  are called *concurrent signatures*.

Fair exchange of signatures allow mutually distrustful parties to exchange signatures in a *fair* way.

*Fair* means: Either each party obtains the other's signature, or neither party does.

Two main approaches to fair exchange of signatures:

- 1. Timed release of signatures,
- 2. Solutions involving a trusted third party.

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- A cannot withhold the keystone because she needs it to obtain a service from *B*. (E.g. Computer Depot)
- A cannot keep B's signature private in the long term (E.g. Credit card system).
- A single third party C will verify both signatures (E.g. Politicians and press)

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We use the ring signature scheme of Abe et al. [AOS02] (an adaptation of Schnorr signature scheme [S91]).

We call our seed a *keystone*, and use a *cryptographic hash function* to create the keystone fix from the keystone.

A concurrent signature scheme is a digital signature scheme comprised of the following algorithms:

**SETUP:** On input a security parameter *l*, outputs the public parameters, a function KGEN, and the participants' public and private keys.

ASIGN: A probabilistic algorithm that produces an ambiguous signature on a message M.

**AVERIFY:** An algorithm which verifies an ambiguous signature.

VERIFY: An algorithm which takes a keystone and an ambiguous signature, verifies the ambiguous signature, and tests whether the keystone is valid.

**SETUP:** On input a security parameter *l*:

- Select two large primes p and q s.t. q|p-1.
- Select an element  $g \in \mathbb{Z}_p^*$  of order q.
- Set the message and keystone spaces  $\mathcal{M}, \mathcal{K}=\{0,1\}^*$ , the signature and keystone fix spaces  $\mathcal{S}, \mathcal{F} \equiv \mathbb{Z}_q$ .
- Select two cryptographic hash functions  $H_1, H_2 : \{0, 1\}^* \to \mathbb{Z}_q$ and set KGEN= $H_1$ .
- Select private keys  $x_i \in_R \mathbb{Z}_q$  and set the public keys  $X_i = g^{x_i} \mod p$ .

**ASIGN:** On input  $\langle X_i, X_j, x_i, h_2, M \rangle$ , pick random  $t \in \mathbb{Z}_q$  and compute:

$$h = H_2(g^t X_j^{h_2} \mod p || M),$$

$$h_1 = h - h_2 \mod q$$
,  $s = t - h_1 x_i \mod q$ .

Output  $\sigma = \langle s, h_1, h_2 \rangle$ .

**AVERIFY:** On input  $\langle \sigma, X_i, X_j, M \rangle$  where  $\sigma = \langle s, h_1, h_2 \rangle$ , verify the equation

$$h_1 + h_2 = H_2(g^s X_i^{h_1} X_j^{h_2} \mod p \mid\mid M) \mod q$$

Output accept or reject.

**VERIFY:** On input  $\langle k, S \rangle$  where  $k \in \mathcal{K}$ ,  $S = \langle \sigma, X_i, X_j, M \rangle$ , check if KGEN(k)=  $h_2$ . If not, output reject, otherwise run AVERIFY(S). Security is defined via the following notions:

**Correctness:** AVERIFY accepts signatures produced by ASIGN.

**Unforgeability:** The adversary should not be able to create (ambiguous) signatures without the appropriate private key.

- **Ambiguity:** The adversary should not be able to distinguish which of two possible signers created an ambiguous signature.
- **Fairness:** If two ambiguous signatures use the same keystone fix f, then a keystone will either convert both signatures into full signatures, or neither.

We define existential unforgeability of a concurrent signature scheme under a chosen message attack using the following game between an adversary E and a challenger C.

Setup: *C* runs SETUP for a security parameter *l*. *E* is given public parameters and the public keys  $\{X_i\}$ . *C* retains the private keys  $\{x_i\}$ .

Queries: E can make the following queries to C: KGen Queries, KReveal Queries, ASign Queries, and Private Key Extract Queries.

**Output:** Finally *E* outputs a tuple  $\sigma = \langle s, h_1, f \rangle$  with public keys  $X_c, X_d$ , and message  $M \in \mathcal{M}$ . The adversary wins if AVERIFY( $\langle s, h_1, f \rangle, X_c, X_d, M$ )= accept, and if either:

- 1. No ASign query on either  $\langle X_c, X_d, f, M \rangle$  or  $\langle X_d, X_c, h_1, M \rangle$  was made by *E*, and no Private Key Extract query was made on either  $X_c$  or  $X_d$ .
- 2. No ASign query on  $\langle X_c, X_i, f, M \rangle$  was made for any  $X_i \neq X_c, X_i \in \mathcal{U}$ , no Private Key Extract query on  $X_c$  was made, and either f was a previous output from a KGen query or E produces a keystone k such that f = KGEN(k).

**Definition:** A concurrent signature scheme is *existentially unforgeable under a chosen message attack* if the probability of success of any polynomially bounded adversary in the above game is negligible. **Theorem:** Our concrete concurrent signature scheme is existentially unforgeable under a chosen message attack in the random oracle model, assuming the hardness of the discrete logarithm problem.

Security results are also proved for the ambiguity and fairness properties of the concrete scheme.

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- To investigate methods whereby the revelation of the keystone does not depend entirely on the initial signer.

- Introduced the notion of concurrent signatures and compared it to previous work,
- Discussed applications for concurrent signatures,
- Presented a concrete concurrent signature scheme,
- Related the security of the concrete scheme to the hardness of the discrete logarithm problem in an appropriate security model.

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