Multi-Instance Security and its Application to Password-Based Cryptography

Stefano Tessaro

MIT

Joint work with
Mihir Bellare (UC San Diego)
Thomas Ristenpart (Univ. of Wisconsin)

Scenario: File encryption

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Alternative solution:
Password-based cryptography.

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Key-derivation function

KDF

q1w2e3

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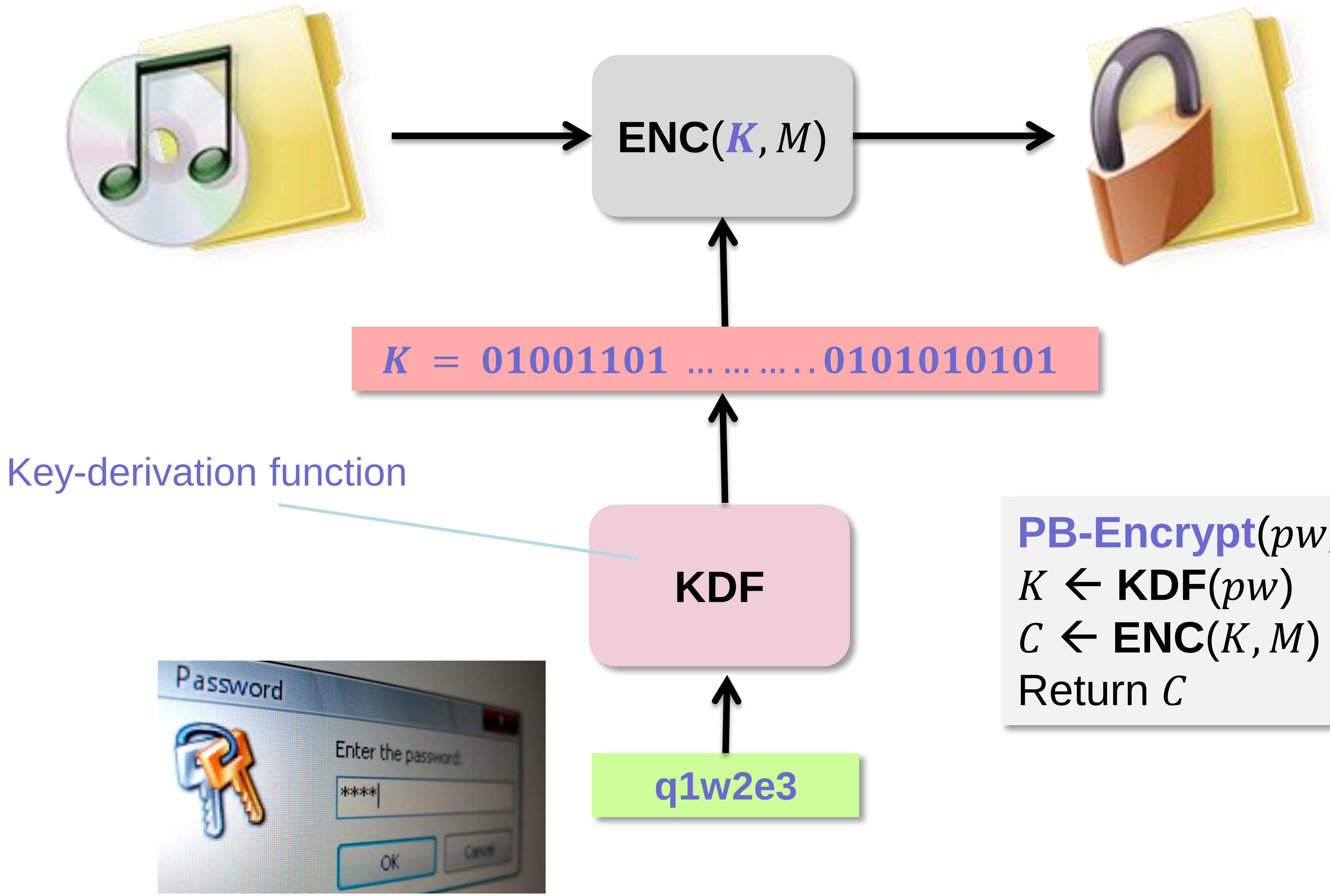

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Problem: Weak passwords are unavoidable

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Security on NBCNEWS.com

Breach shows even experts choose bad passwords

Easy-to-guess passwords such as '123456' are all too common

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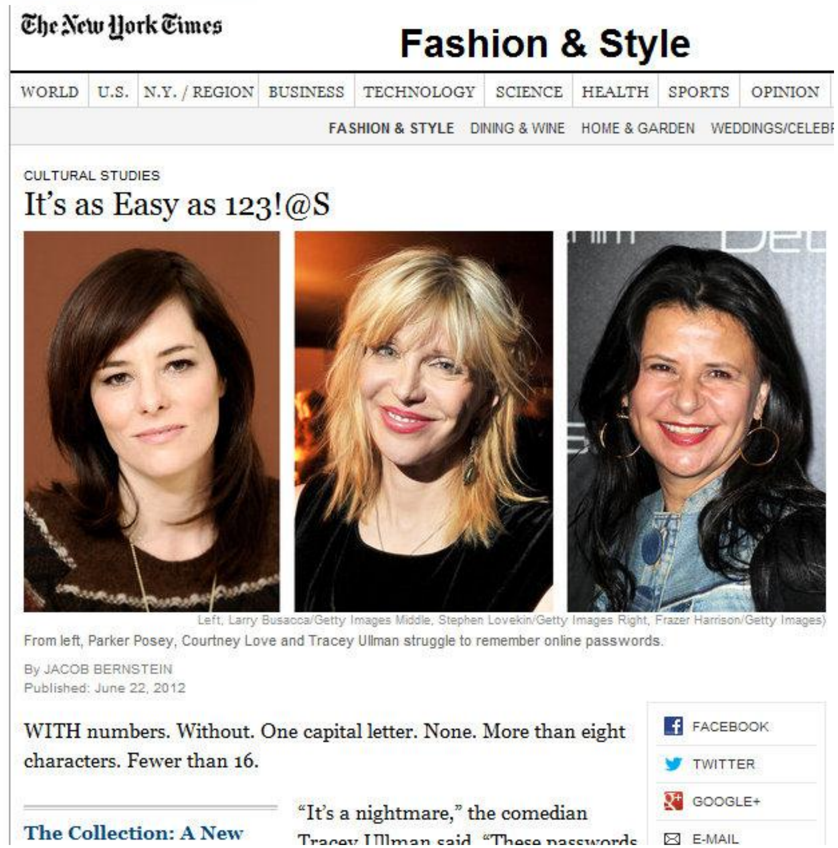
Anonymous' massive year-end attack on the global-security consulting fi
even top-tier executives at the world's largest corporations don't have a cl
importance of a strong password.

Enter a password

abc35

Password Strength:



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It's as Easy as 123!@S

Left, Larry Busacca/Getty Images Middle, Stephen Lovekin/Getty Images Right, Frazer Harrison/Getty Images)

From left, Parker Posey, Courtney Love and Tracey Ullman struggle to remember online passwords.

By JACOB BERNSTEIN
Published: June 22, 2012

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GOOGLE+
E-MAIL

WITH numbers. Without. One capital letter. None. More than eight characters. Fewer than 16.

The Collection: A New

"It's a nightmare," the comedian
Tracey Ullman said. "These passwords

Problem: Weak passwords are unavoidable

Mitigating dictionary attacks via iteration

$\text{KDF} = \text{H}^c$

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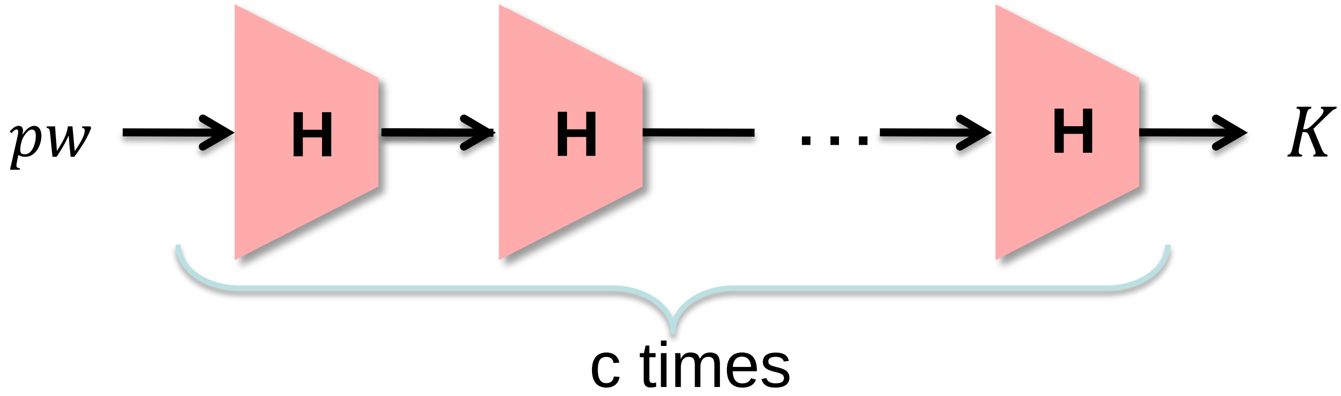
KDF = H^c



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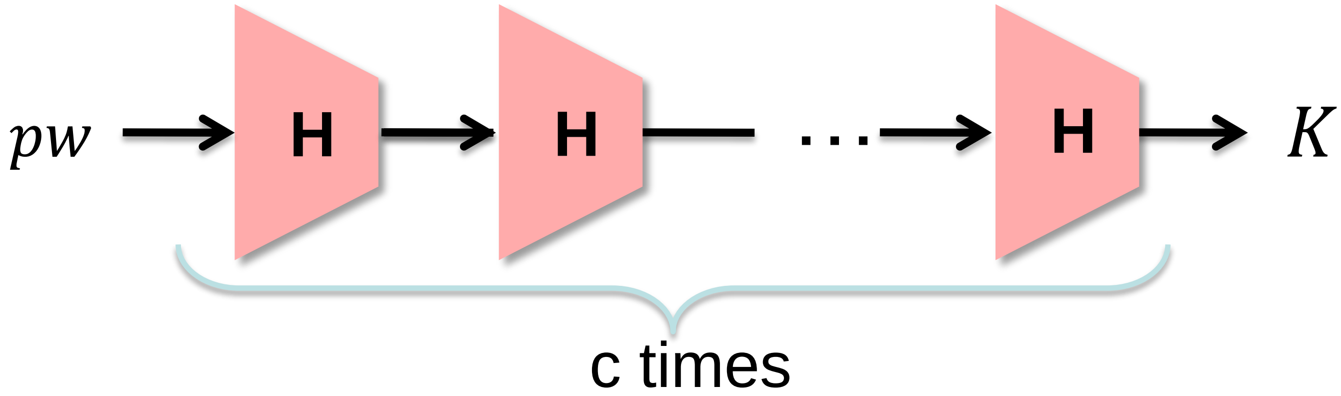

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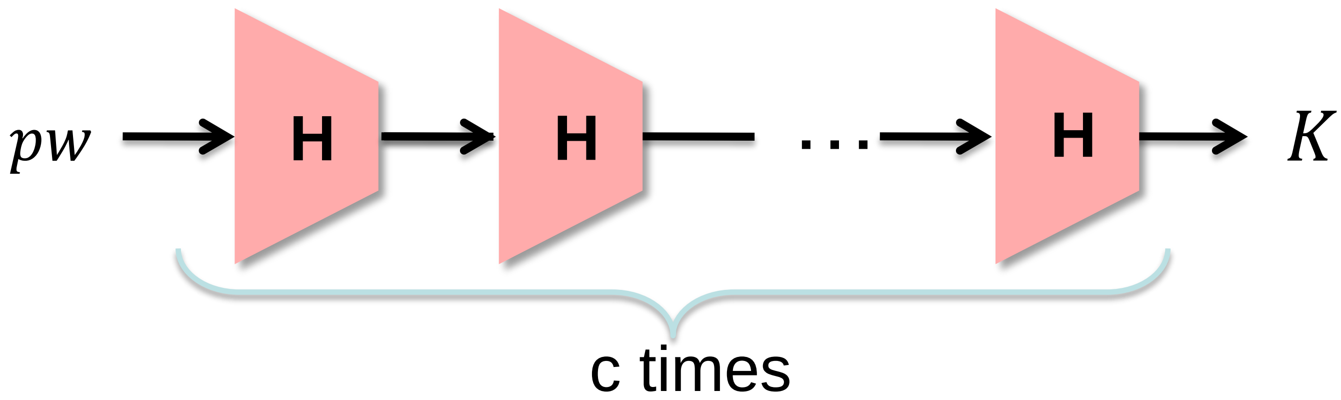

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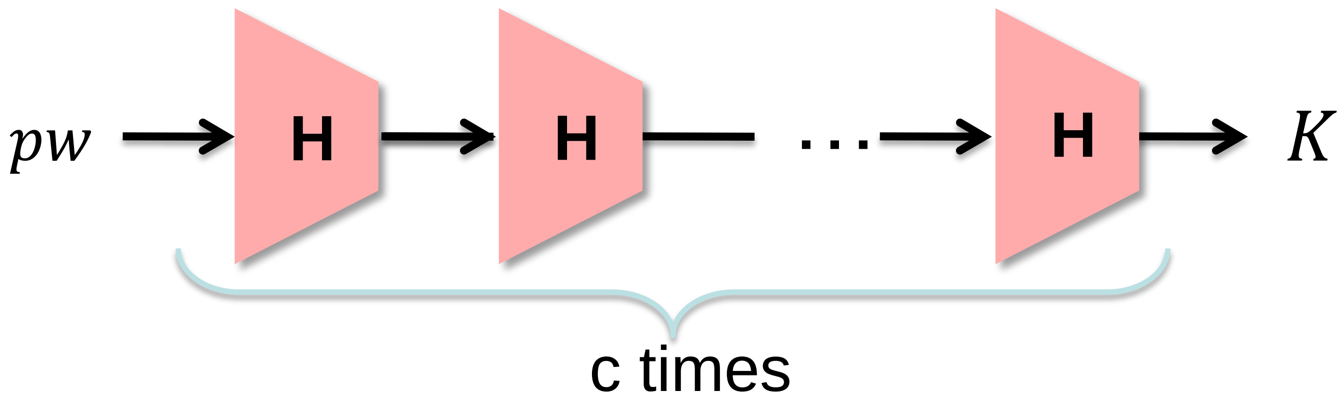

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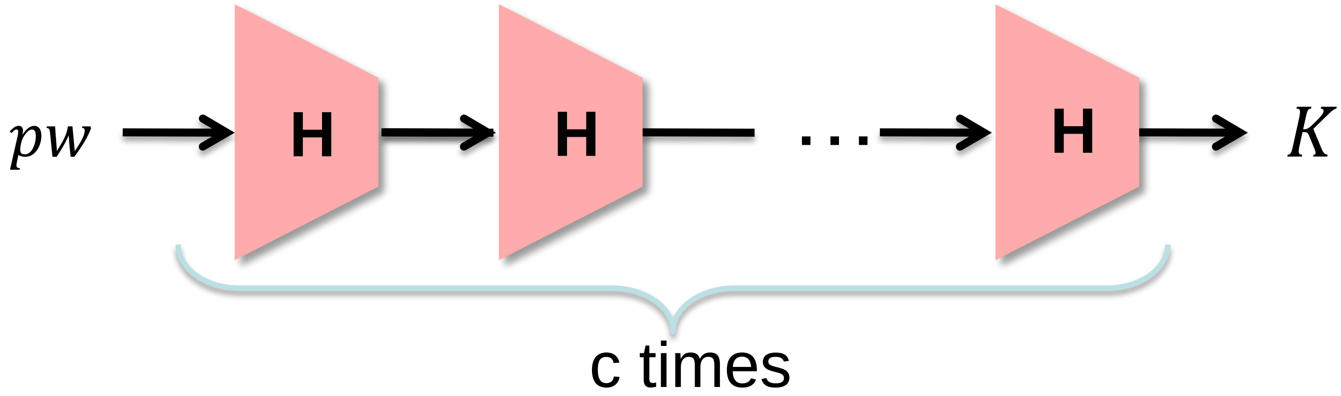

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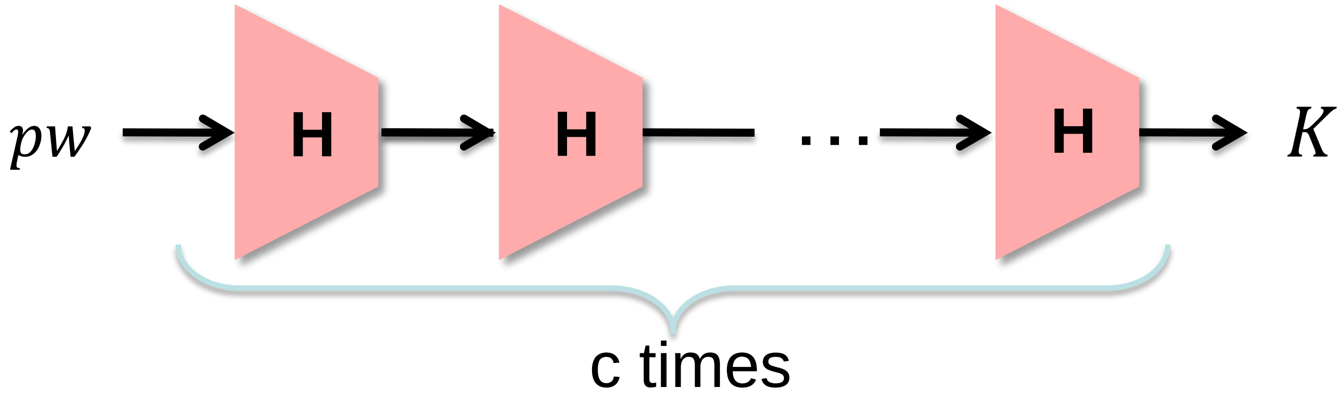

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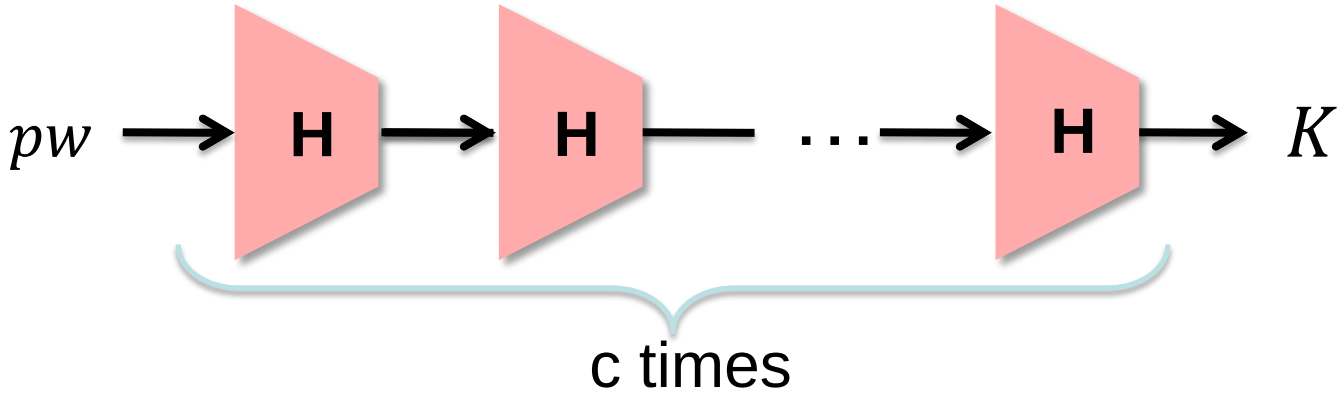

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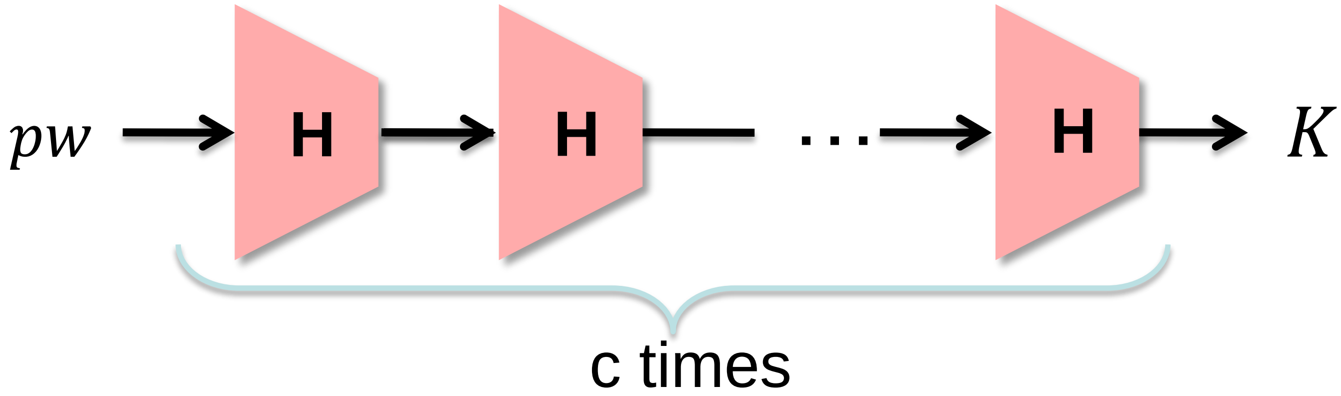

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M_1

Work $\boldsymbol{c} \times \boldsymbol{N}$ to retrieve M_1

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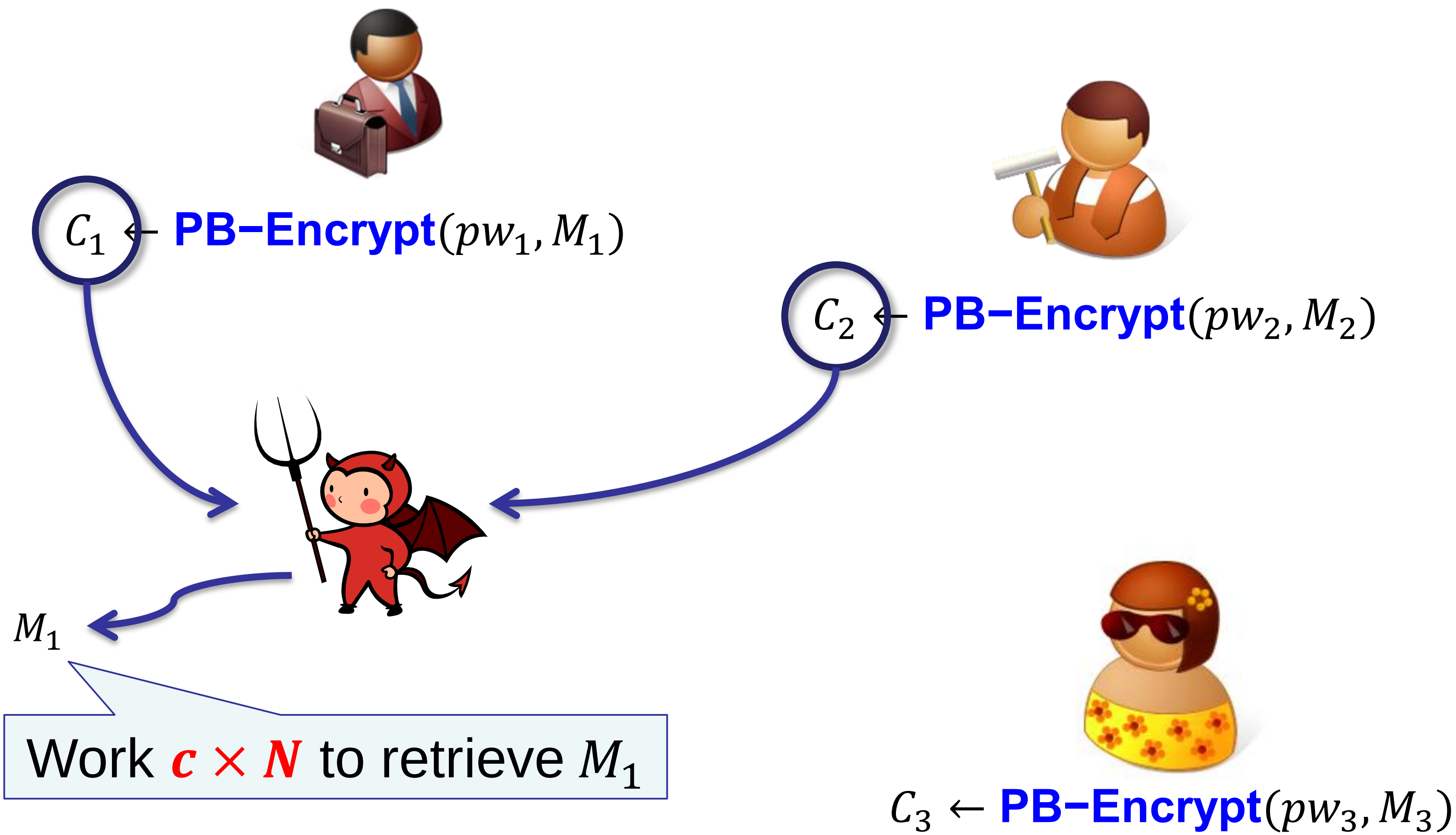

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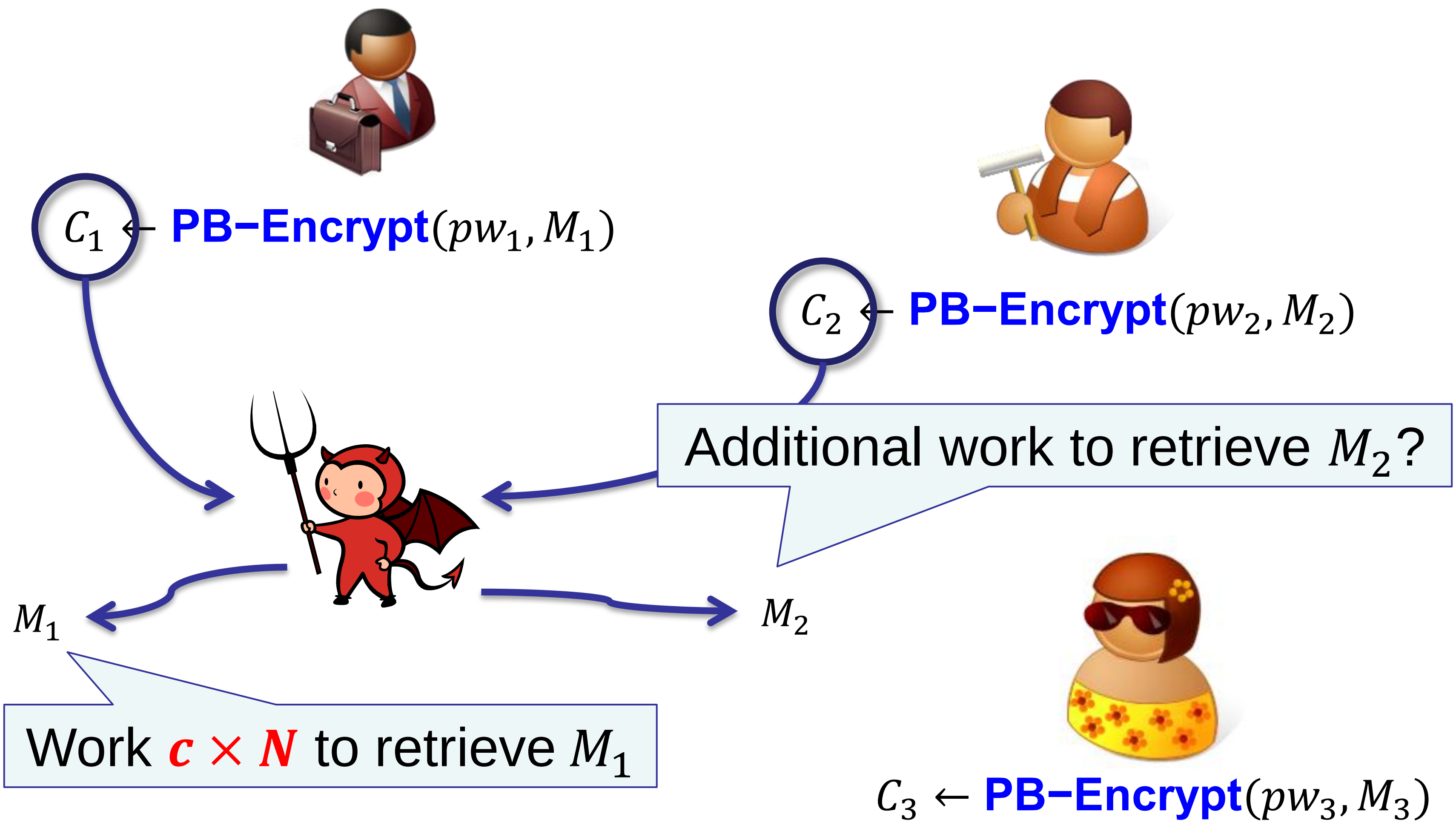

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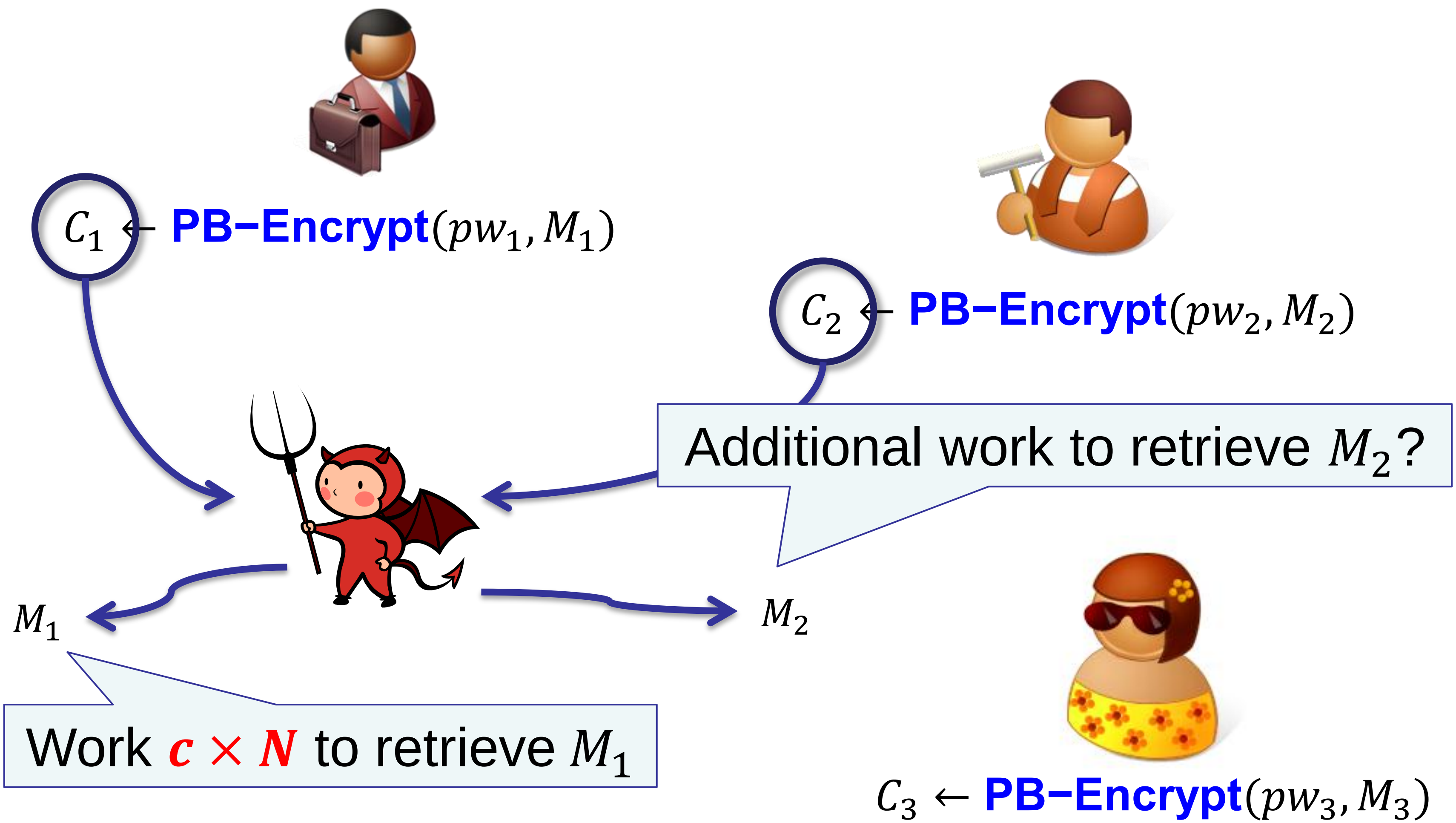

Ideally: Work $m \times c \times N$ to retrieve m plaintexts!

Multi-instance security amplification

Not true in general:

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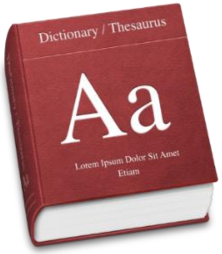

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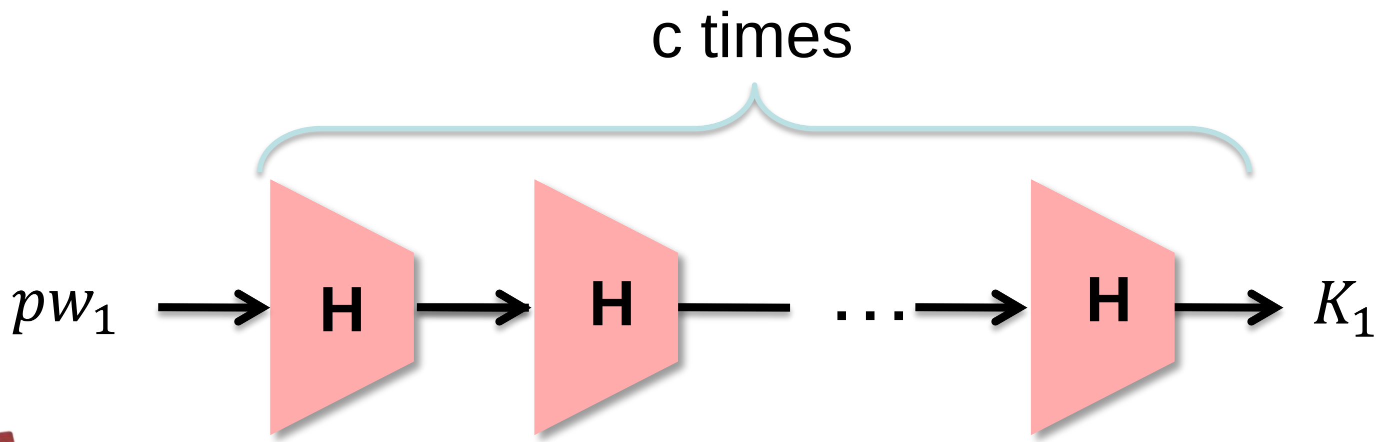

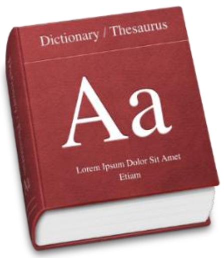

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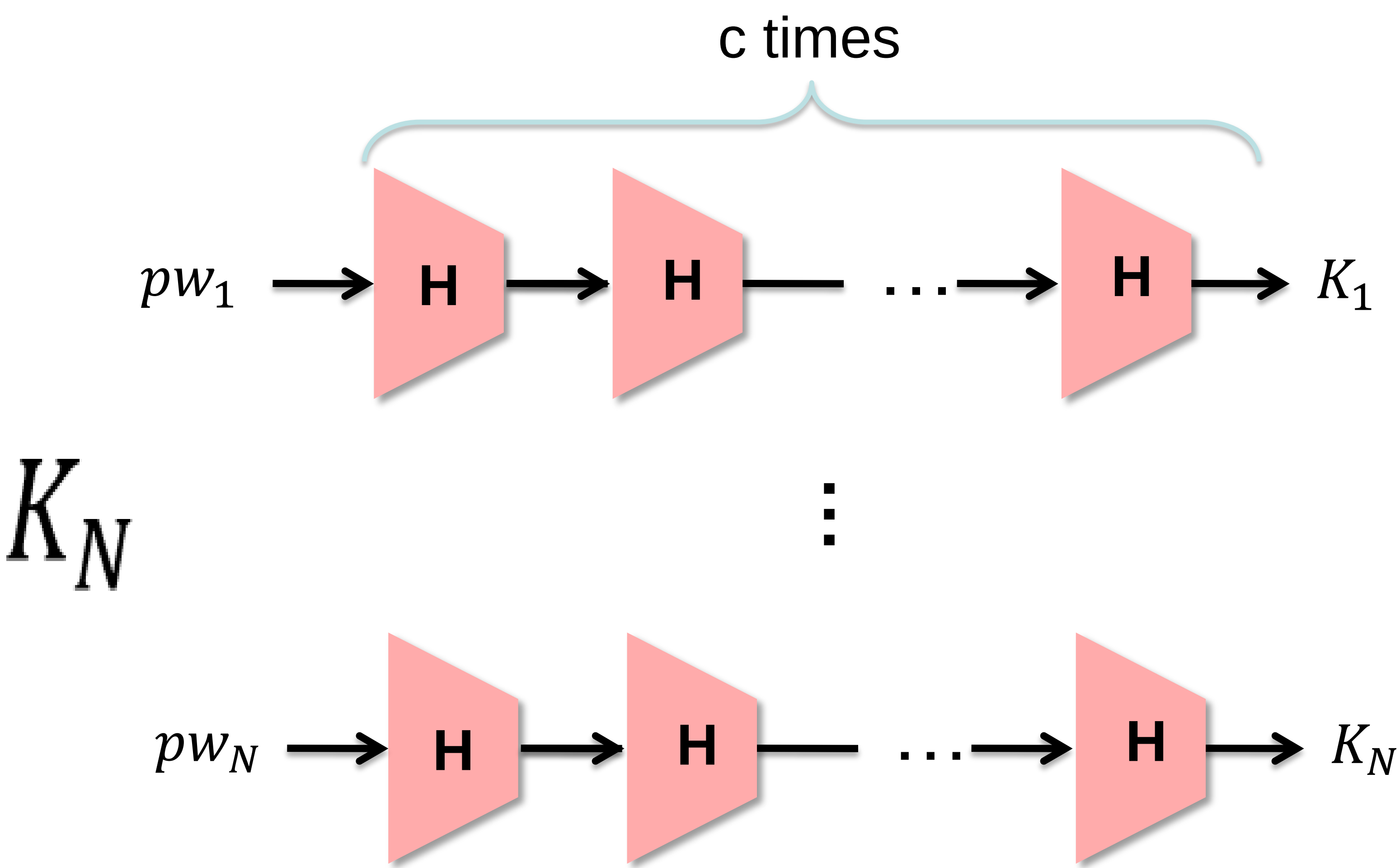

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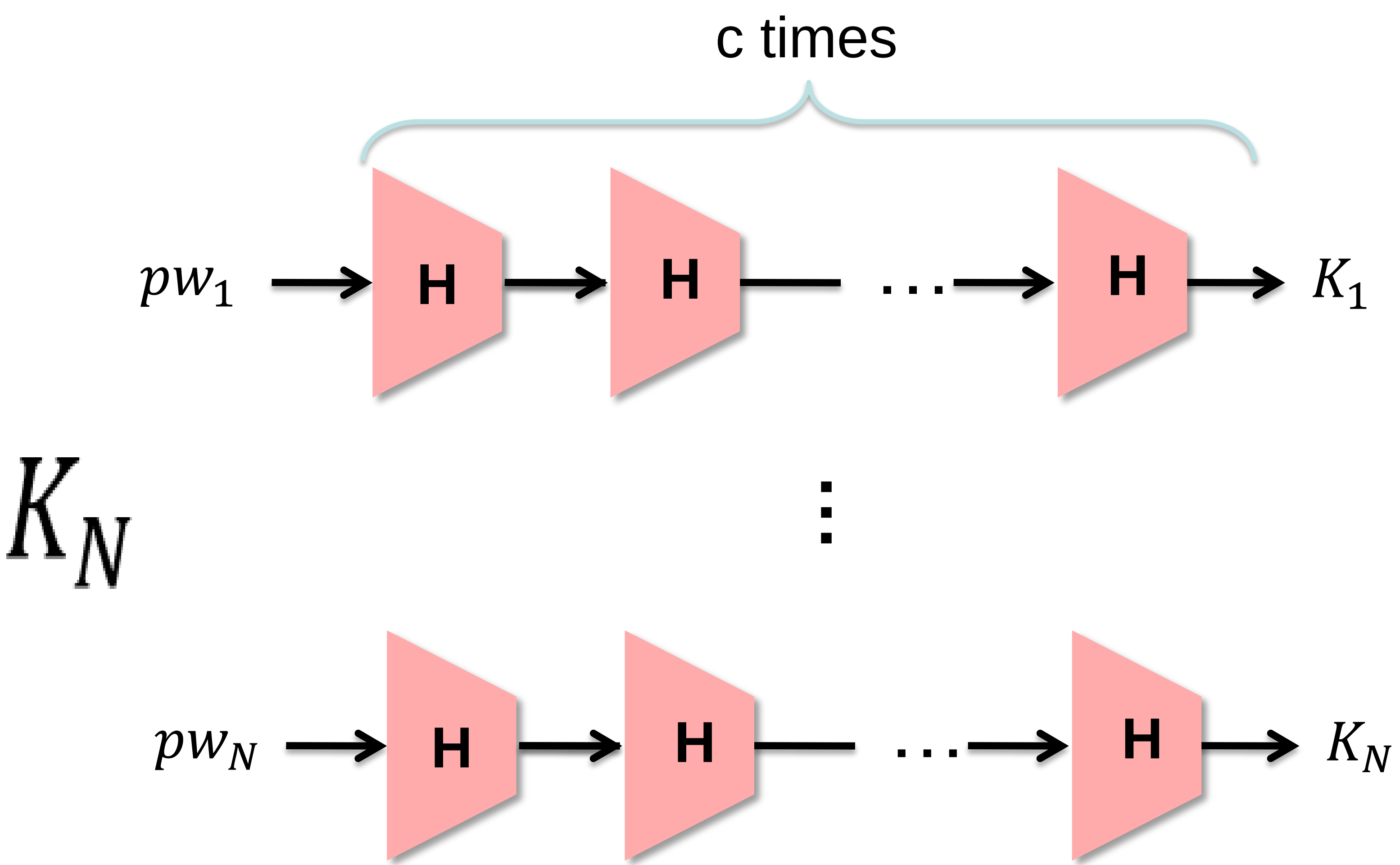
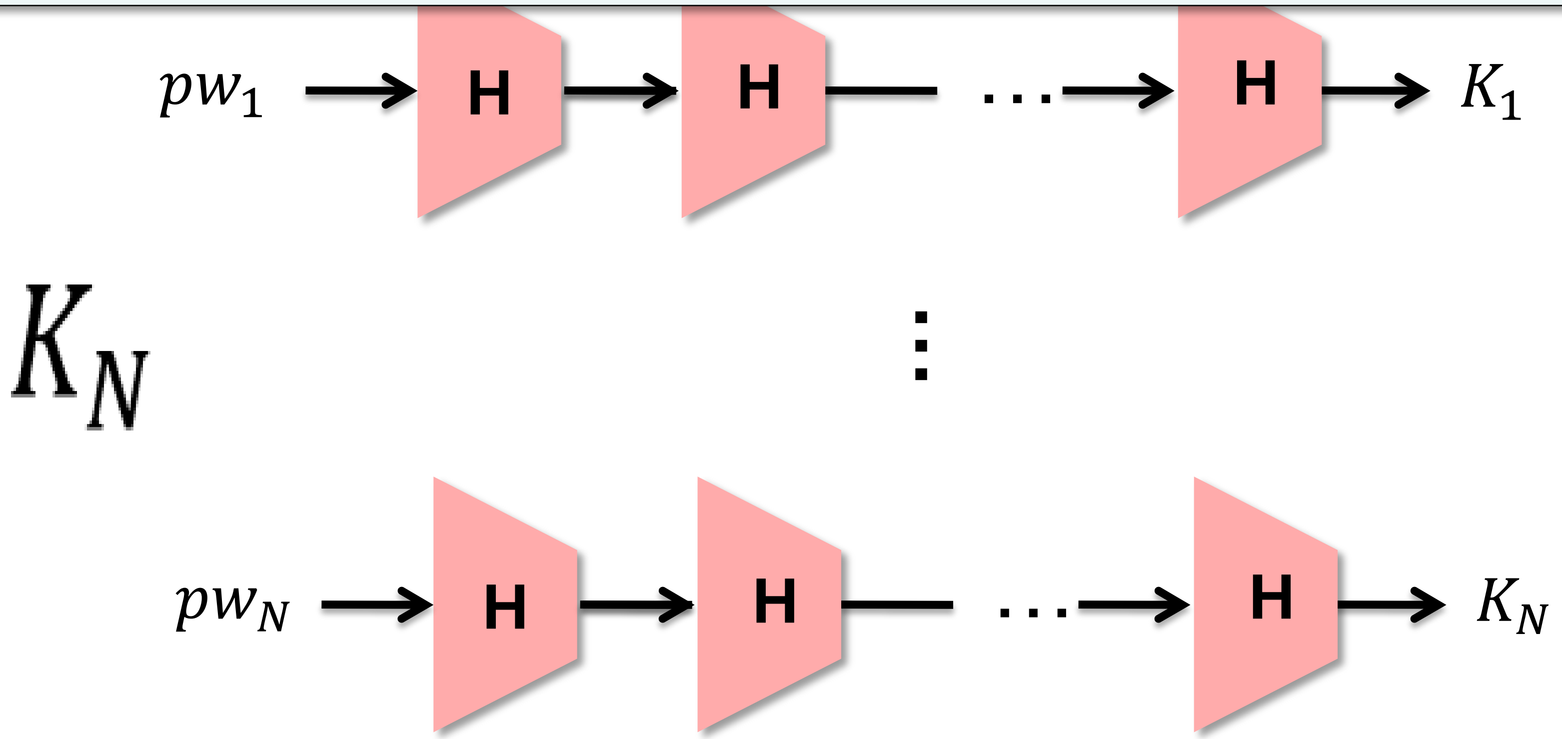


Work $N \times c$ + Work N / ciphertext = $N \times (c + m)$ vs $N \times c \times m$

Multi-instance security amplification

Not true in general:

New design goal: Multi-instance security amplification
"Hardness of breaking multiple instances must increase linearly in the number of instances."



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PKCS#5 – Password-based cryptography standard

Salting as suggested in PKCS#5 prevents attack

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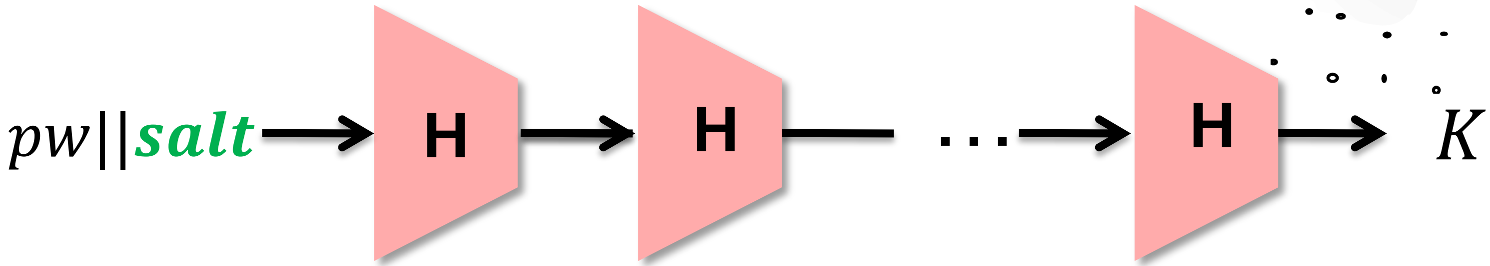

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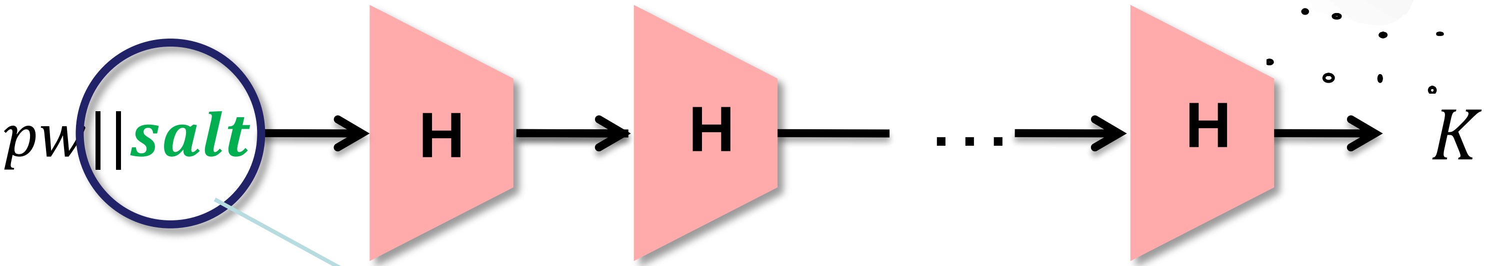

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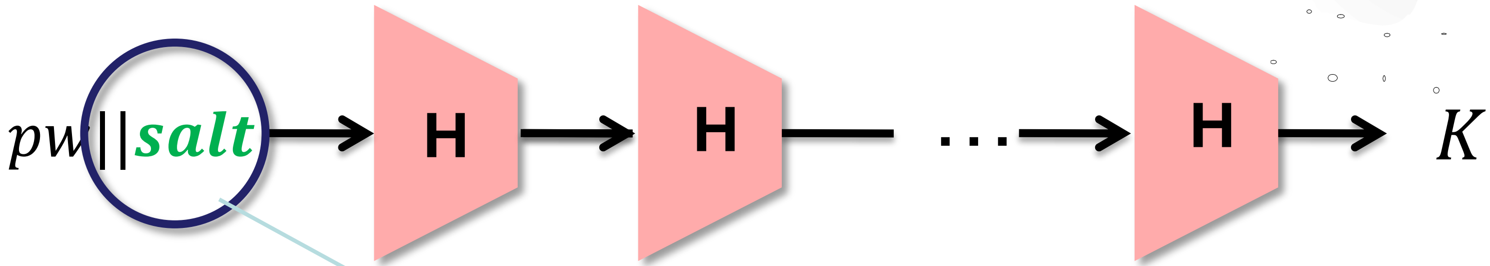

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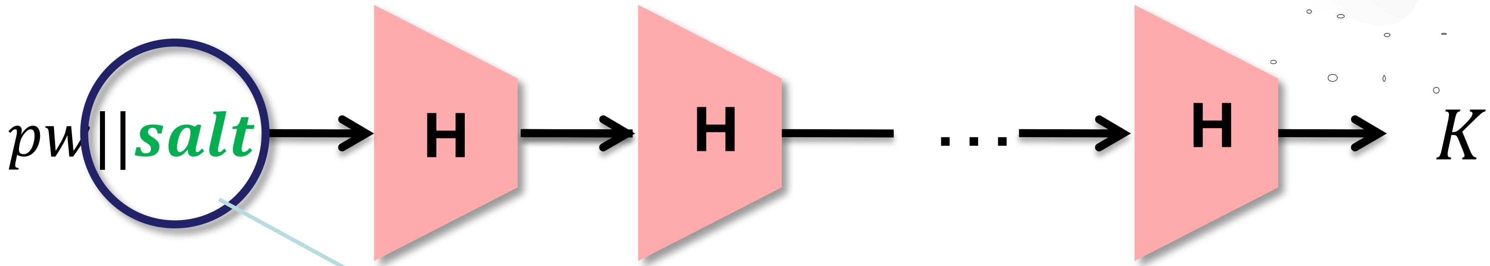

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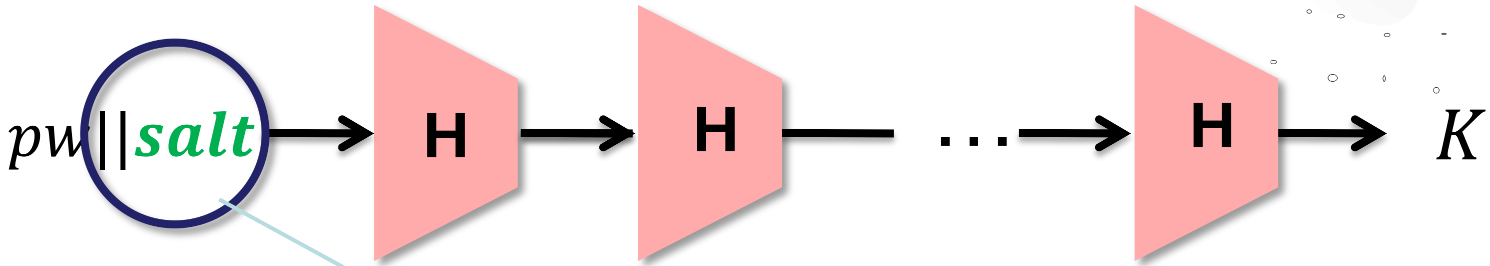

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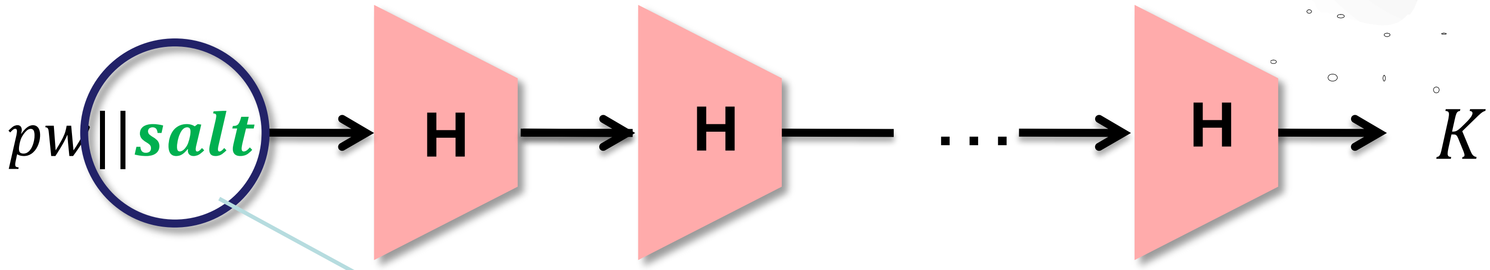

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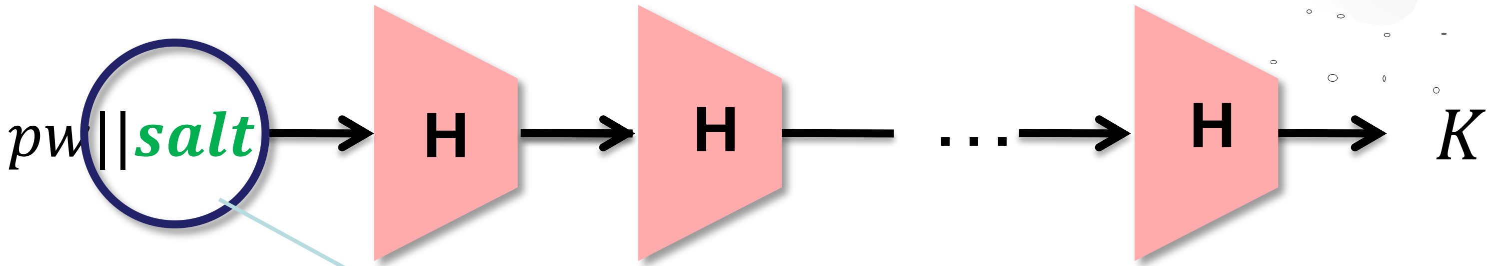

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Allows decryption

Question: Does salting provably ensure multi-instance security amplification?

Iteration and salting in the real world

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6.46 million LinkedIn passwords leaked online

Summary: *More than 6.4 million LinkedIn pa... hack. Though some login details are encrypted,*

By Zack Whittaker for Between the Lin...

Follow @zackwhittaker

A user on a Russian forum has claimed to have ... from LinkedIn.

It looks as though some of the weaker passwo... cracked already. Other users have been seen r... seek help in cracking the encryption.

No salting!

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LinkedIn Password Leak: Salt Their Hide

view issue

by Poul-Henning Kamp | June 7, 2012

Topic: Security

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If it does not take a full second to calculate the password hash, it is too weak.

POUL-HENNING KAMP

6.5 million unsalted SHA1 hashed LinkedIn passwords have appeared in the criminal underground. There are two words in that sentence that should cause LinkedIn no end of concern: "unsalted" and "SHA1."

No iteration!

Our results

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Answer: We do not really know!

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Our contributions:

1) **General definitional framework for multi-instance security** of arbitrary cryptographic primitives.
2) **Case study:** Security analysis of PKCS#5 within our framework.

Outline

1. Multi-instance security

2. Security of PKCS#5 – A case study

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2. Security of PKCS#5 – A case study

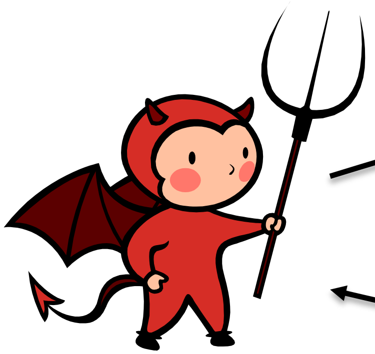
Single-instance security – PB-Encryption

LOR-Security

$b \leftarrow \{0,1\}$
$pw \leftarrow PWD$

Single-instance security – PB-Encryption

LOR-Security



m_0, m_1

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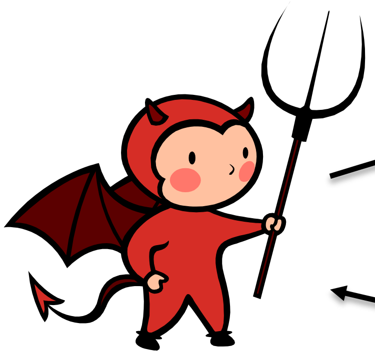

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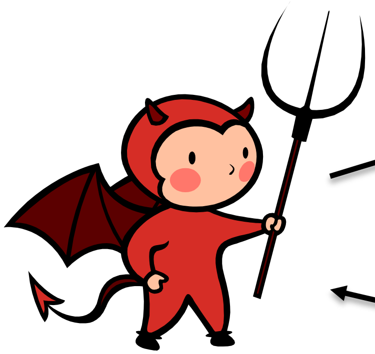

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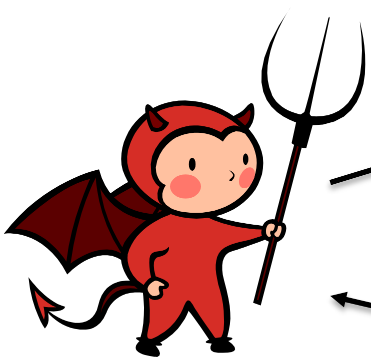

$\boldsymbol{m}$

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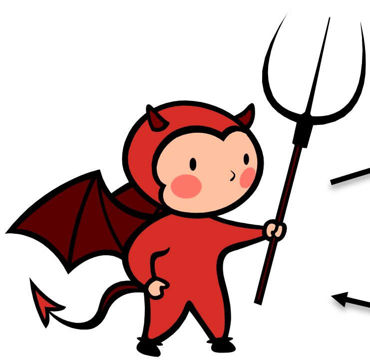

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PWR-Security

m

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pw'

$$\mathbf{Adv}^{\mathrm{pwr}}(A) = \Pr[pw' = pw]$$

The multi-instance (mi) security vista

Our goal: Define security metric for scheme **S** wrt property **P** to measure success of an adversary that:

- instances of the scheme **concurrently**.
- **Corrupts** up to $t < m$ instances of the scheme (e.g., learns passwords).
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The multi-instance (mi) security vista

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PWR security

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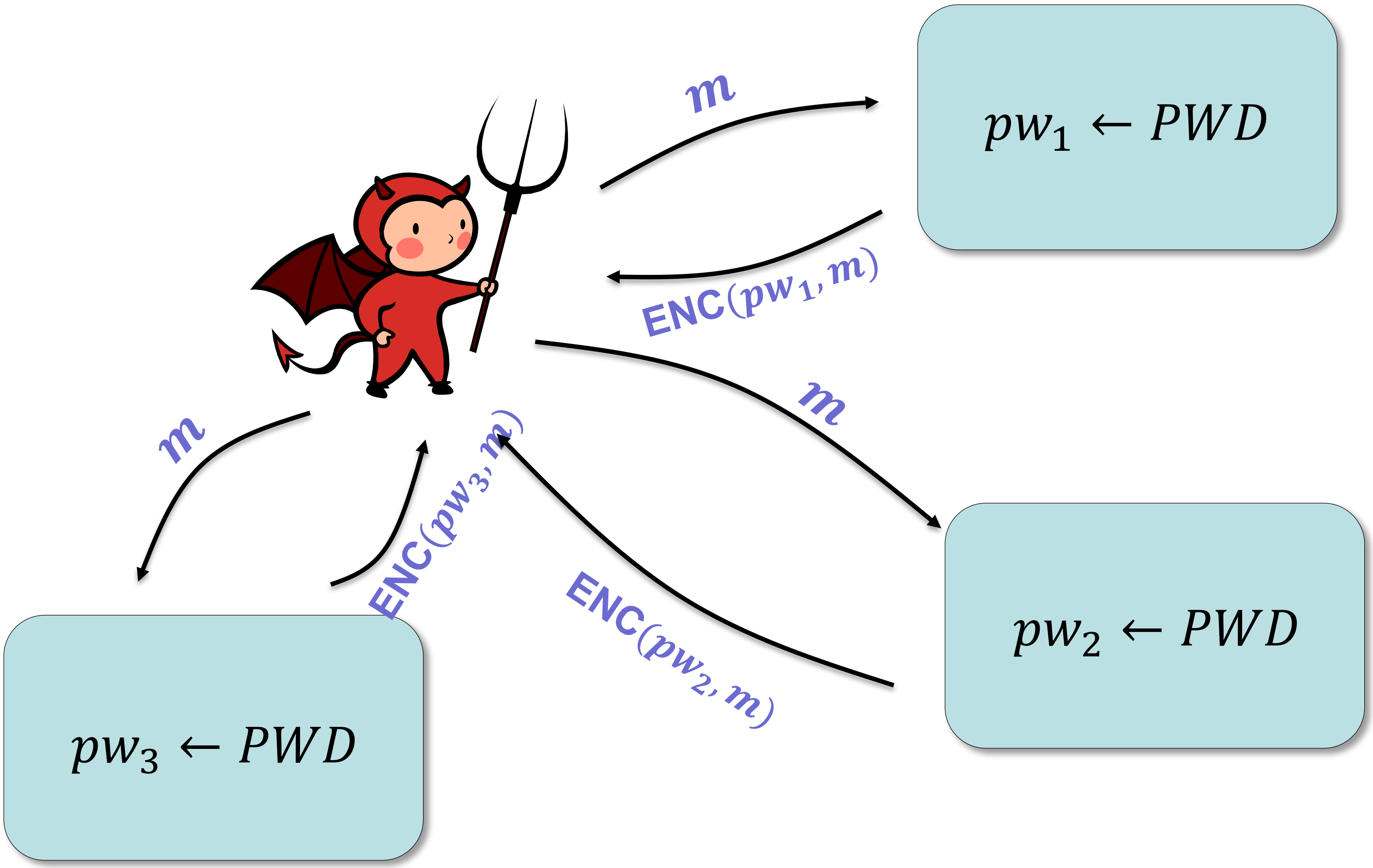
PWR security



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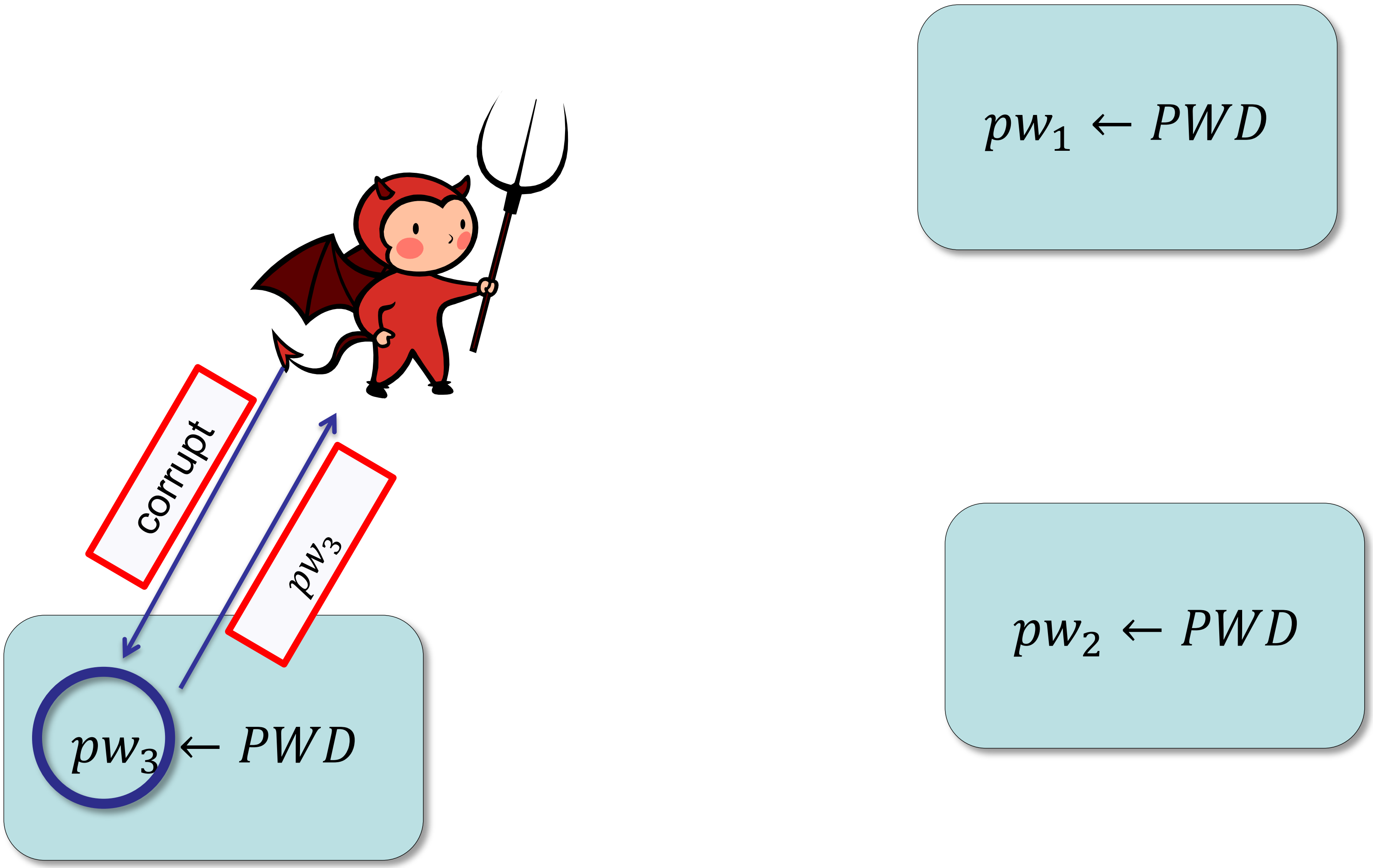
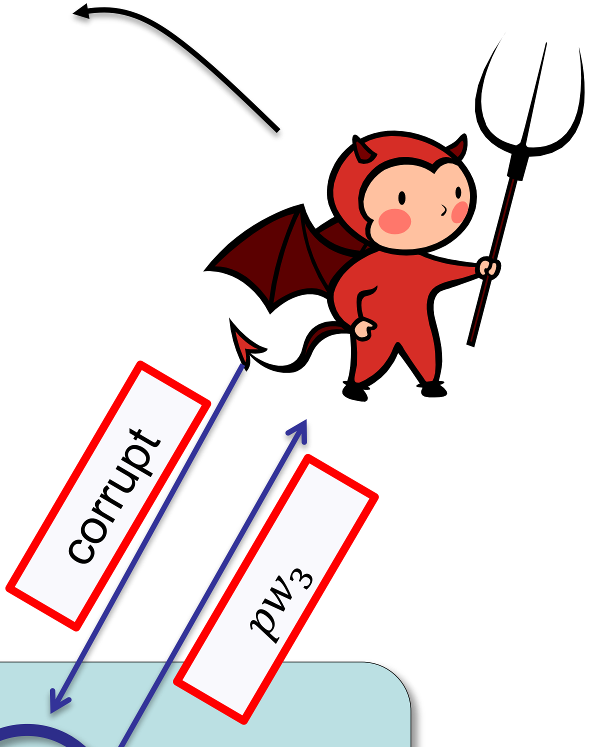


PWR security

(pw'_1, pw'_2, pw'_3)



$pw_3 \leftarrow PWD$

$pw_1 \leftarrow PWD$

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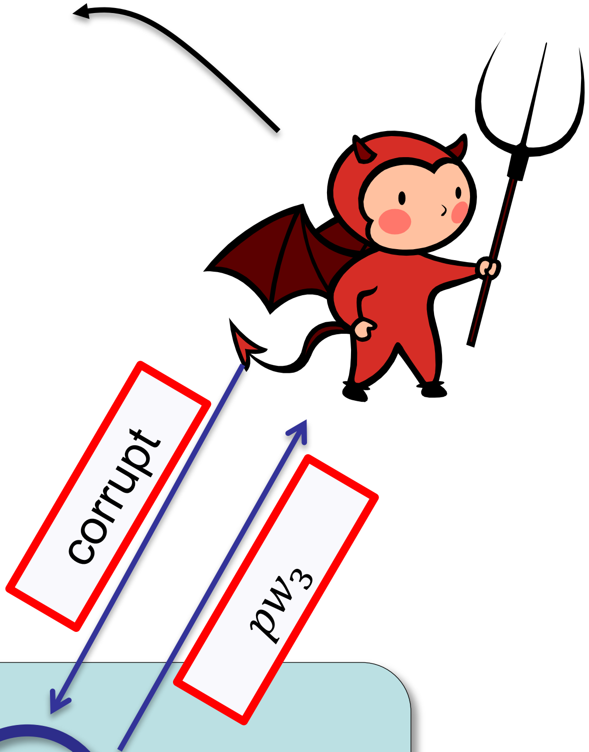

$pw_1 \leftarrow PWD$

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$pw_3 \leftarrow PWD$

$$\mathbf{Adv}^{\mathbf{m-pwr}}(A) = \Pr[pw'_1 = pw_1, \ldots, pw'_m = pw_m]$$

LOR security

$b_1 \leftarrow \{0,1\}$
$pw_1 \leftarrow PWD$

$b_2 \leftarrow \{0,1\}$
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$b_3 \leftarrow \{0,1\}$
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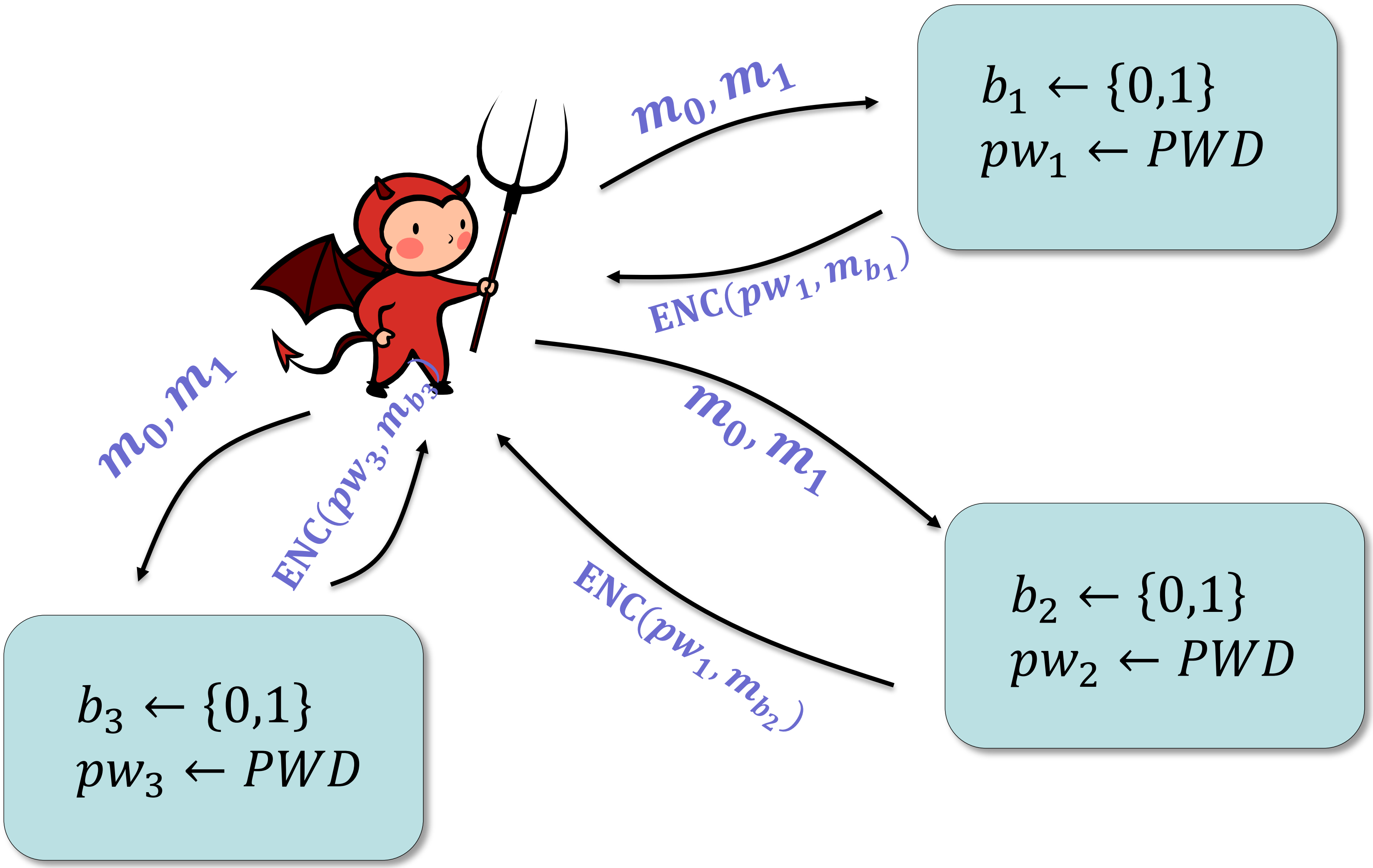
LOR security



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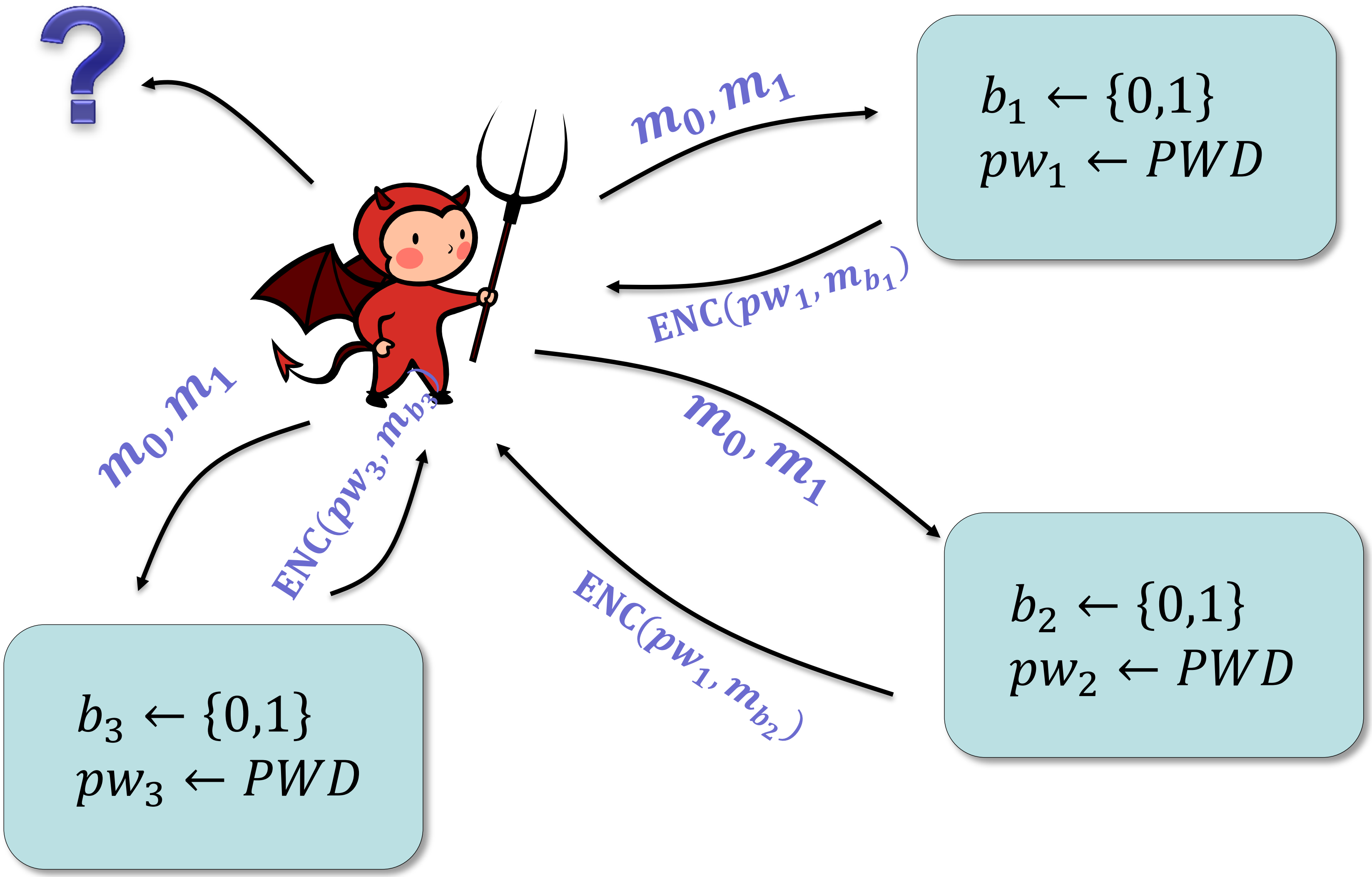

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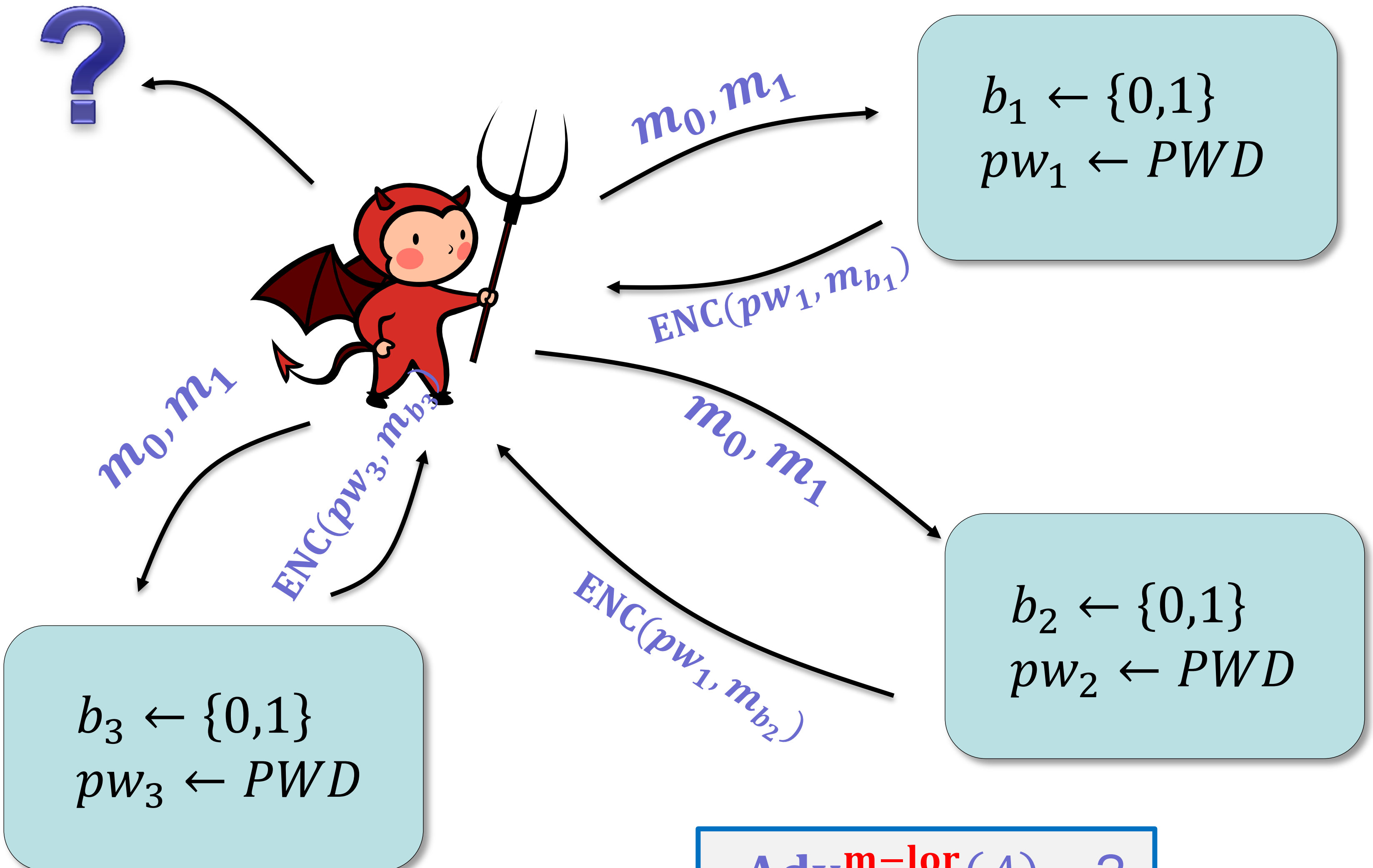

$$\boxed{\mathbf{Adv}^{\mathbf{m-lor}}(A) = ?}$$

Defining mi security for encryption

Attempt #1: **AND-advantage**

Defining mi security for encryption

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LORA-security: **Output:** $(b'_1, \ldots, b'_m)$

Advantage:

$$\mathbf{Adv}^{\mathbf{m-lora}}(A) = \mathbf{Pr}[(b_1, \ldots, b_m) = (b'_1, \ldots, b'_m)]$$

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Then ∃ adversary guessing second bit at random, with

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Defining mi security for encryption

Attempt #2: **XOR-advantage**

Defining mi security for encryption

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LORX-security: **Output:** b'

Advantage:

$$\mathbf{Adv}^{\mathbf{m-lor}x}(A) = 2 \times \{\mathbf{Pr}[b' = b_1 \oplus \cdots \oplus b_m] - 1/2\}$$

Defining mi security for encryption

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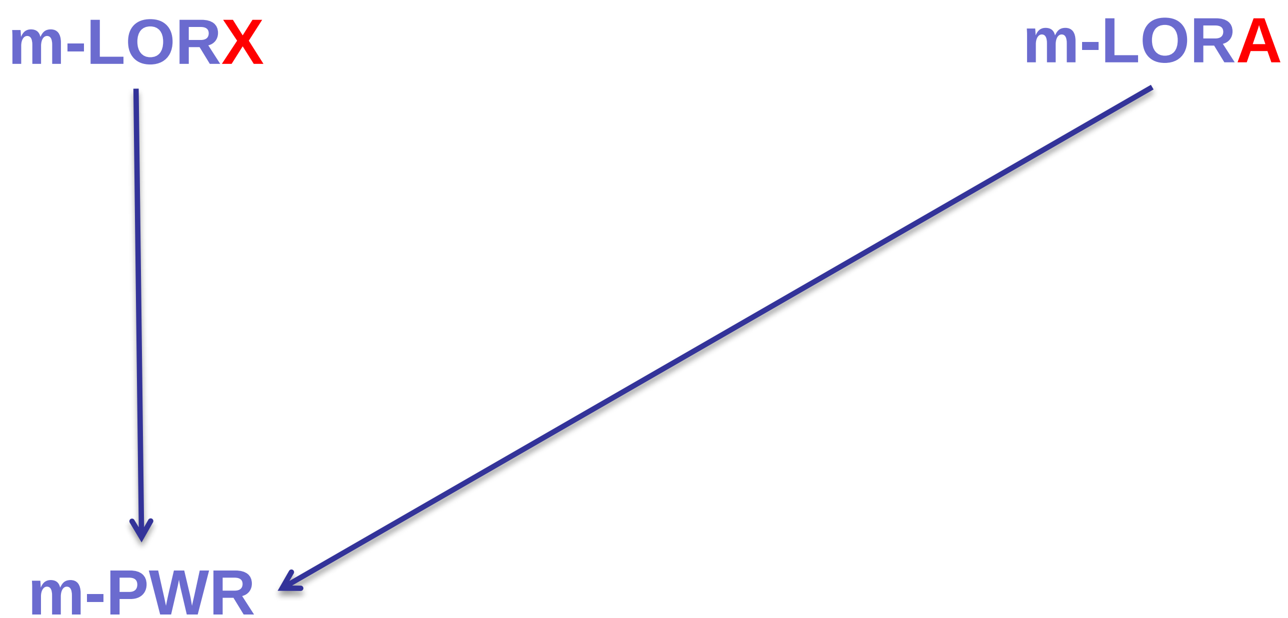
Reason: If ∃ adversary with

$$\mathbf{Pr}[\boldsymbol{b'} = \boldsymbol{b_1}] > \frac{1+\varepsilon}{2}$$

Then: Adversary guessing second bit has **no advantage**

$$\mathbf{Pr}[\boldsymbol{b'} = \boldsymbol{b_1} \oplus \boldsymbol{b_2}] = \frac{1}{2}$$

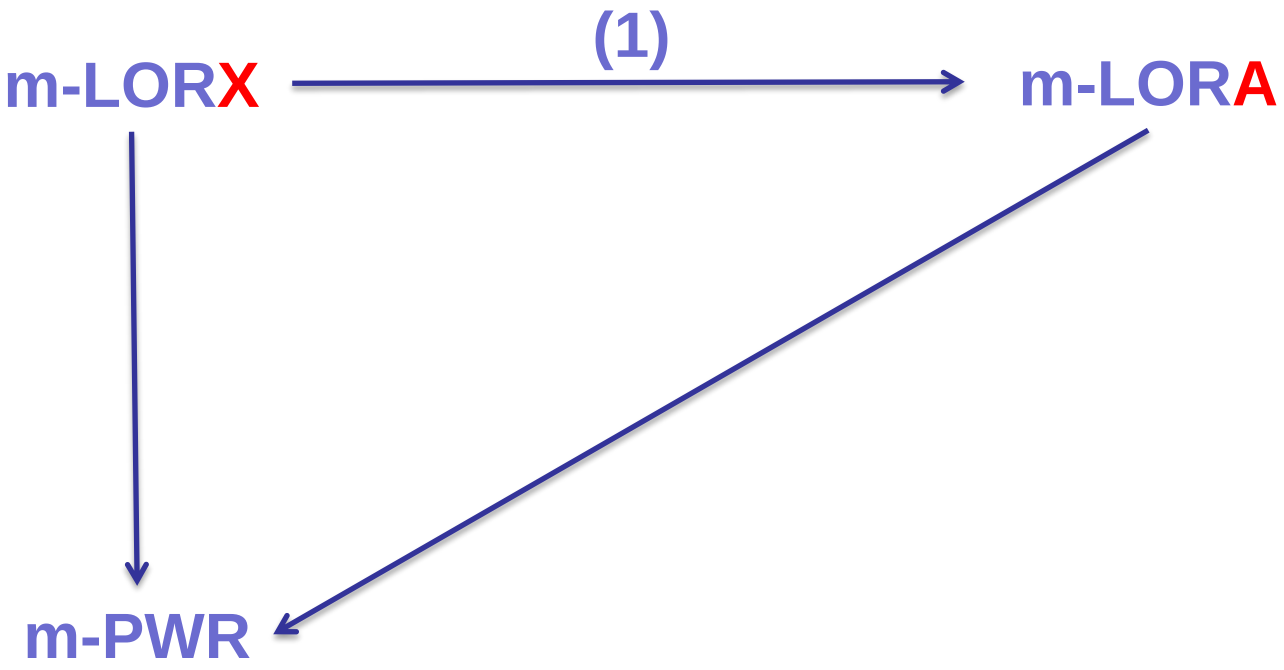
Mi security notions – Relations



Mi security notions – Relations

$$
\begin{tikzcd}
\text{m-LORX} \arrow[r, "(1)"] \arrow[d] & \text{m-LORA} \arrow[dl] \\
\text{m-PWR} &
\end{tikzcd}
$$

Mi security notions – Relations



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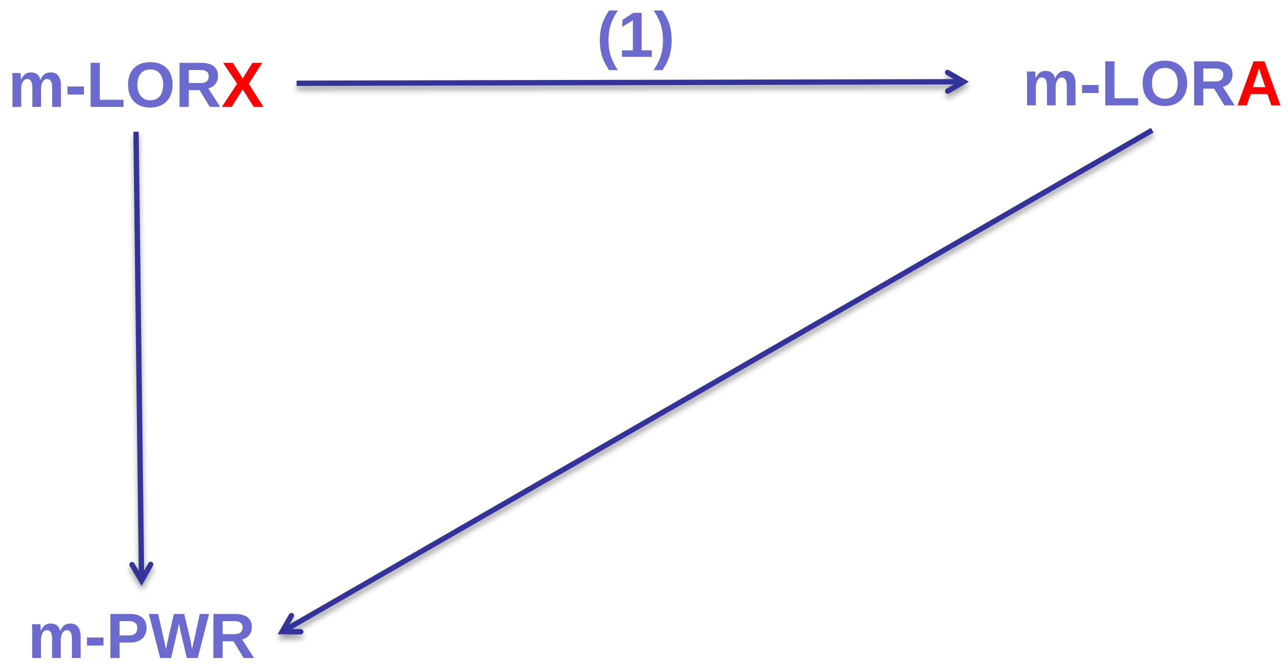

1) Holds in most cases – proof relies on probabilistic lemma from **[U09].**

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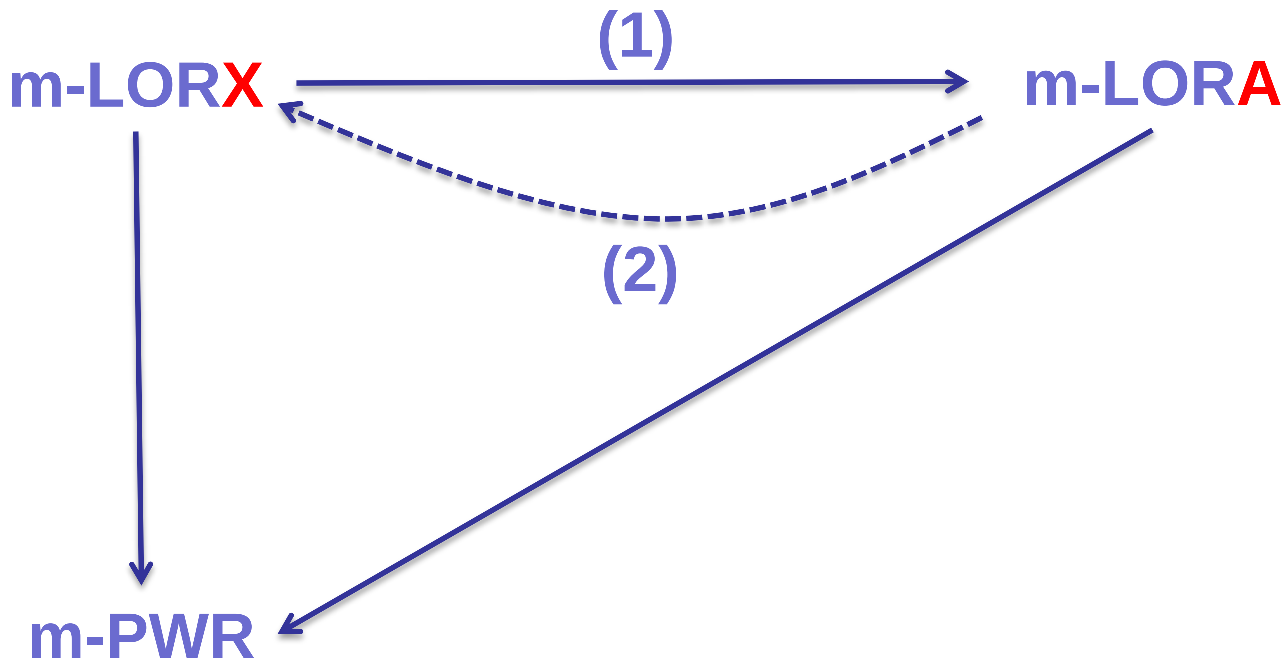

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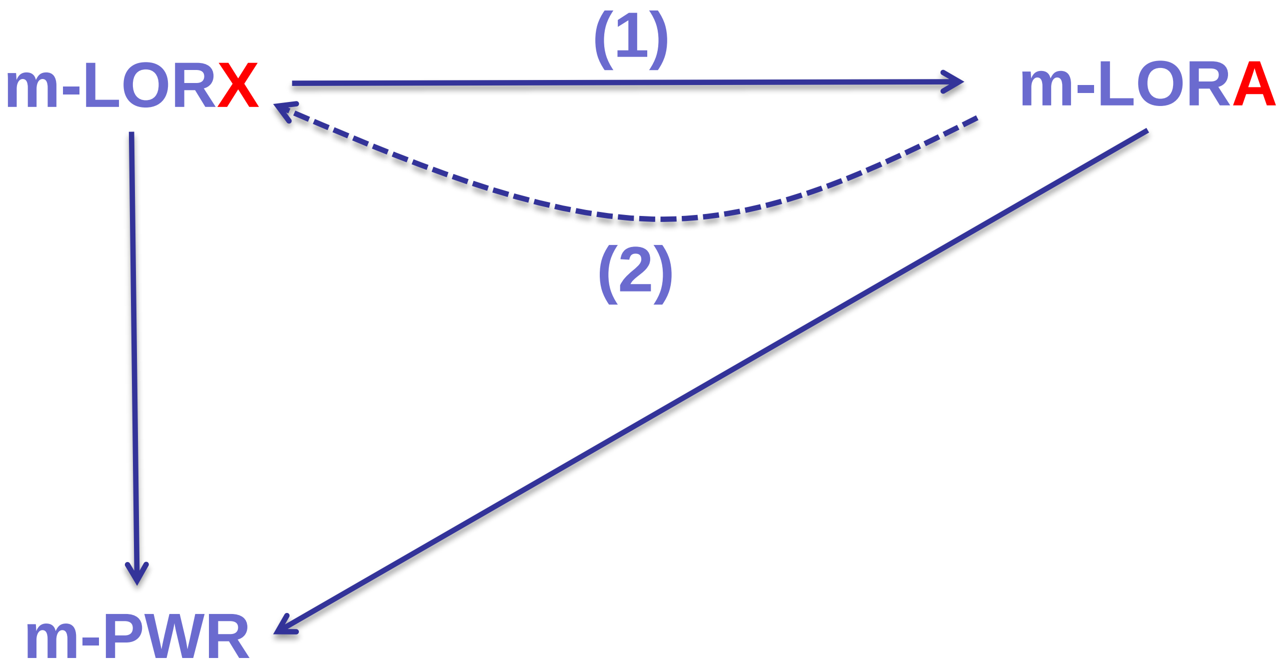
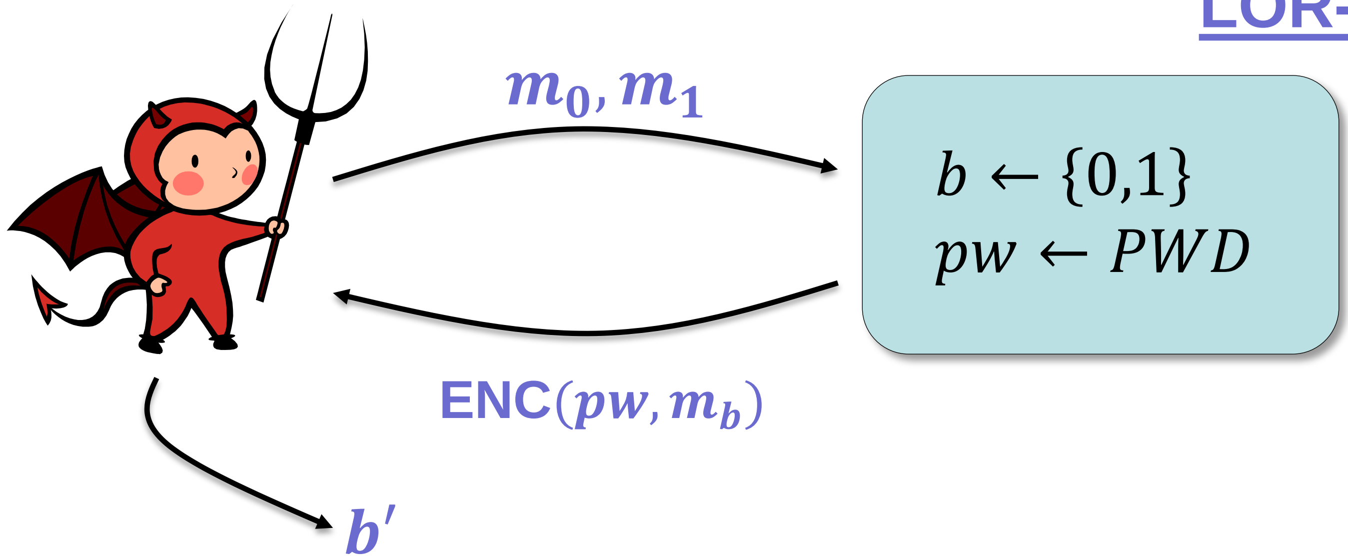


1) Holds in most cases – proof relies on probabilistic lemma from **[U09].**

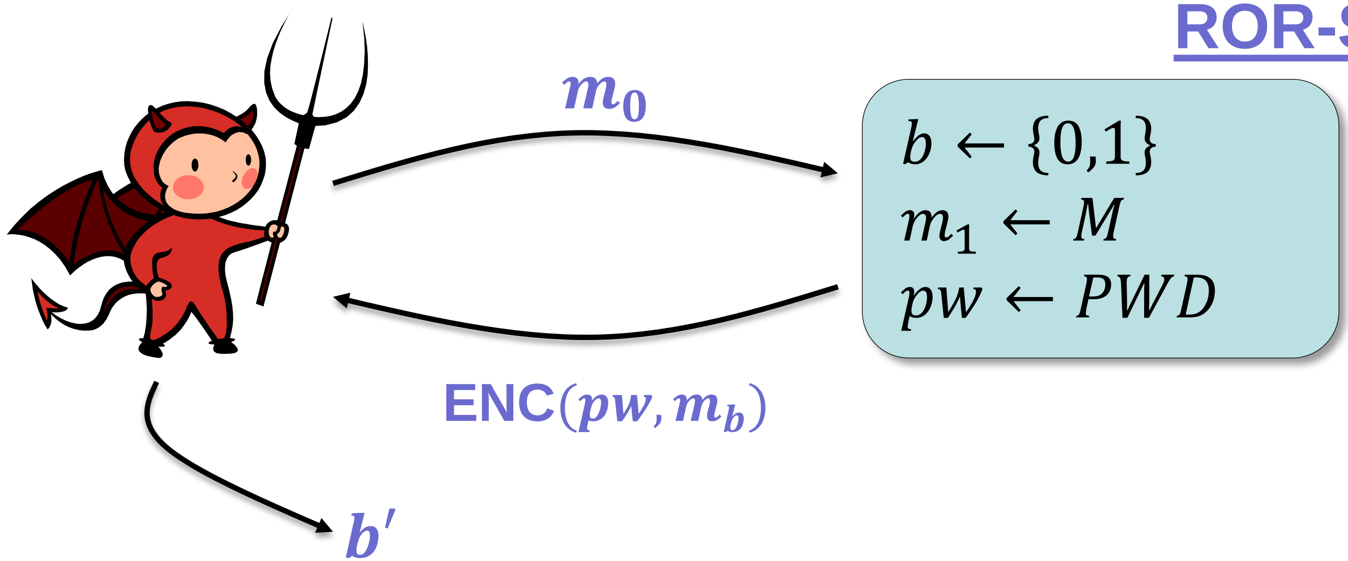
2) Very loose asymptotic implication – based on Goldreich-Levin Theorem **[GL89]**

Relations – LOR vs ROR

LOR-Security



ROR-Security



Relations – LOR vs ROR

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Classical textbook theorem.

$$\mathbf{Adv}^{\mathrm{ror}}(t) \leq \mathbf{Adv}^{\mathrm{lor}}(t) \leq 2 \times \mathbf{Adv}^{\mathrm{ror}}(t)$$

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L **R** $\leq$ **L** **\$** $+$ **\$** **R**

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Mi setting with m instances:

$$\mathbf{Adv}^{\mathrm{m-rorx}}(t) \le \mathbf{Adv}^{\mathrm{m-lorx}}(t) \le 2^{m} \times \mathbf{Adv}^{\mathrm{m-rorx}}(t)$$

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$$\begin{array}{cccccccccccccc} \boxed{L} & \boxed{R} & & \boxed{L} & \boxed{\$} & & \boxed{\$} & \boxed{R} & & \boxed{\$} & \boxed{R} & & \boxed{L} & \boxed{\$} \\ & & \le & & & + & & & + & & & + & & \\ \boxed{L} & \boxed{R} & & \boxed{L} & \boxed{\$} & & \boxed{L} & \boxed{\$} & & \boxed{\$} & \boxed{R} & & \boxed{\$} & \boxed{R} \end{array}$$

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Tight!

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Outline

1. Multi-instance security

2. Security of PKCS#5 – A case study

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PKCS#5 – Defining KDF Security

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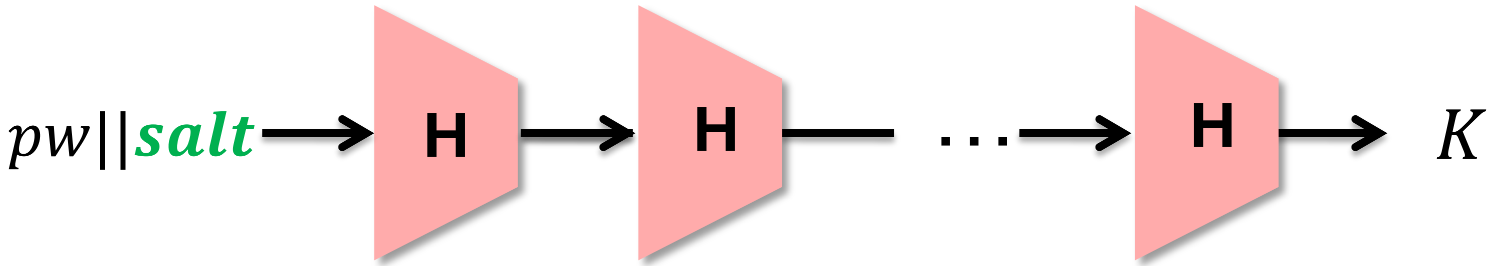

PKCS#5 – Defining KDF Security

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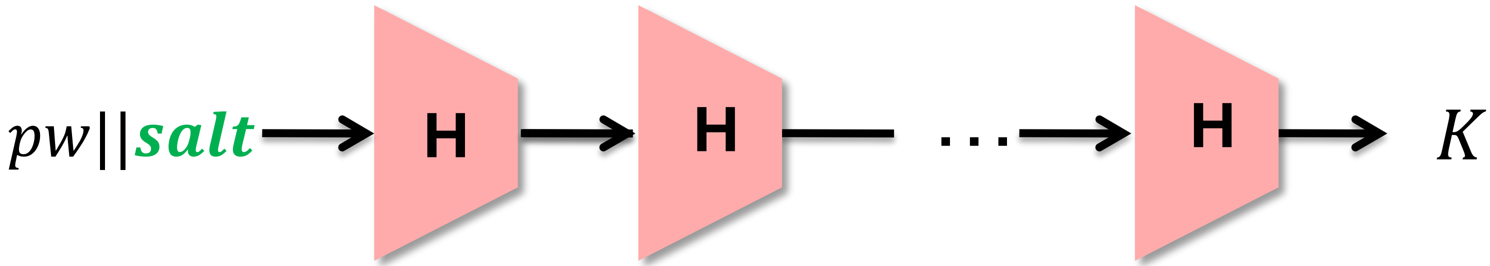
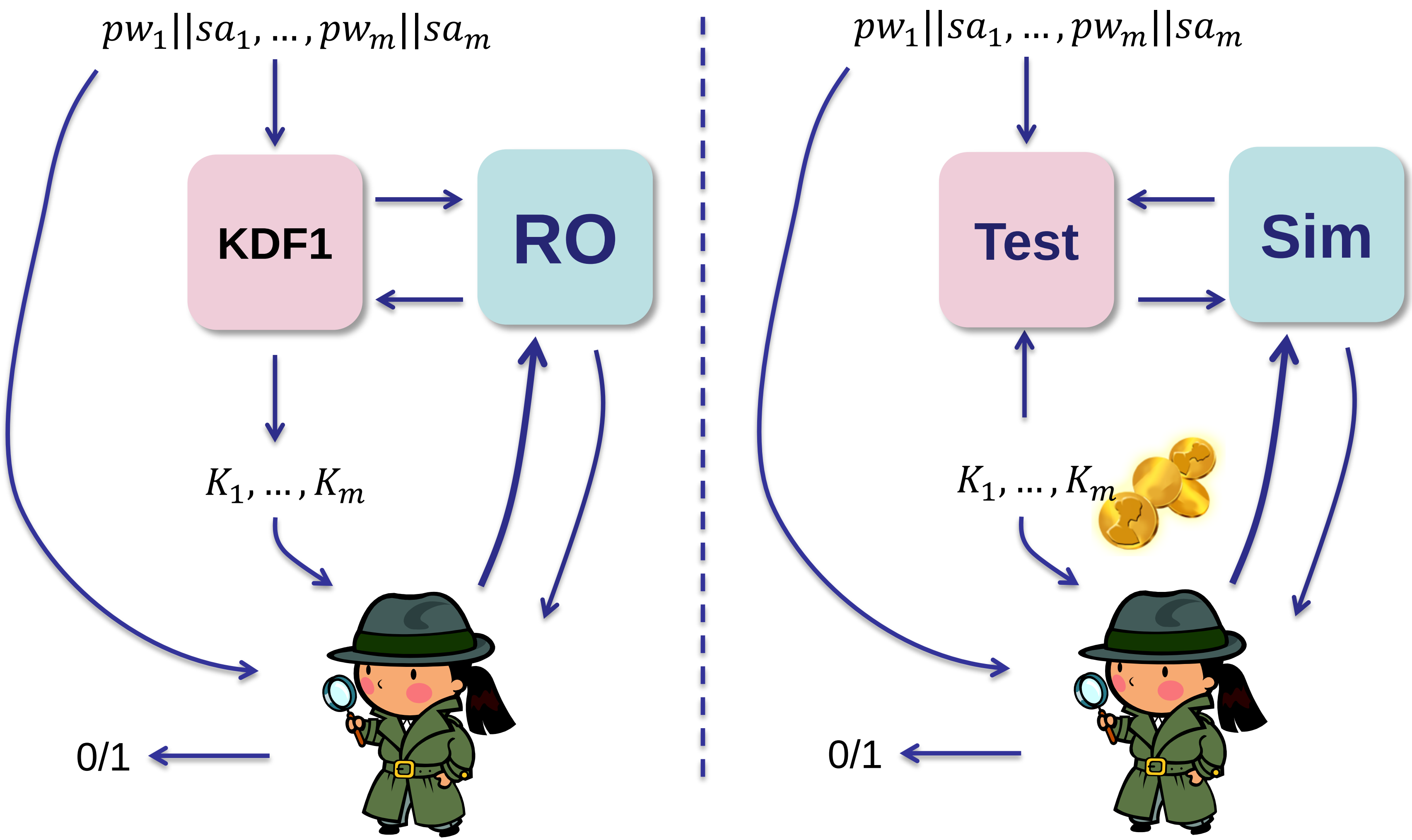


Main step:
Security analysis of **KDF1** for case **H** = **RO**.

KDF Security in the ROM

KDF satisfies indifferentiability-like poperty [MRH04]

∃Sim ∀ password distributions: Left ≈ Right



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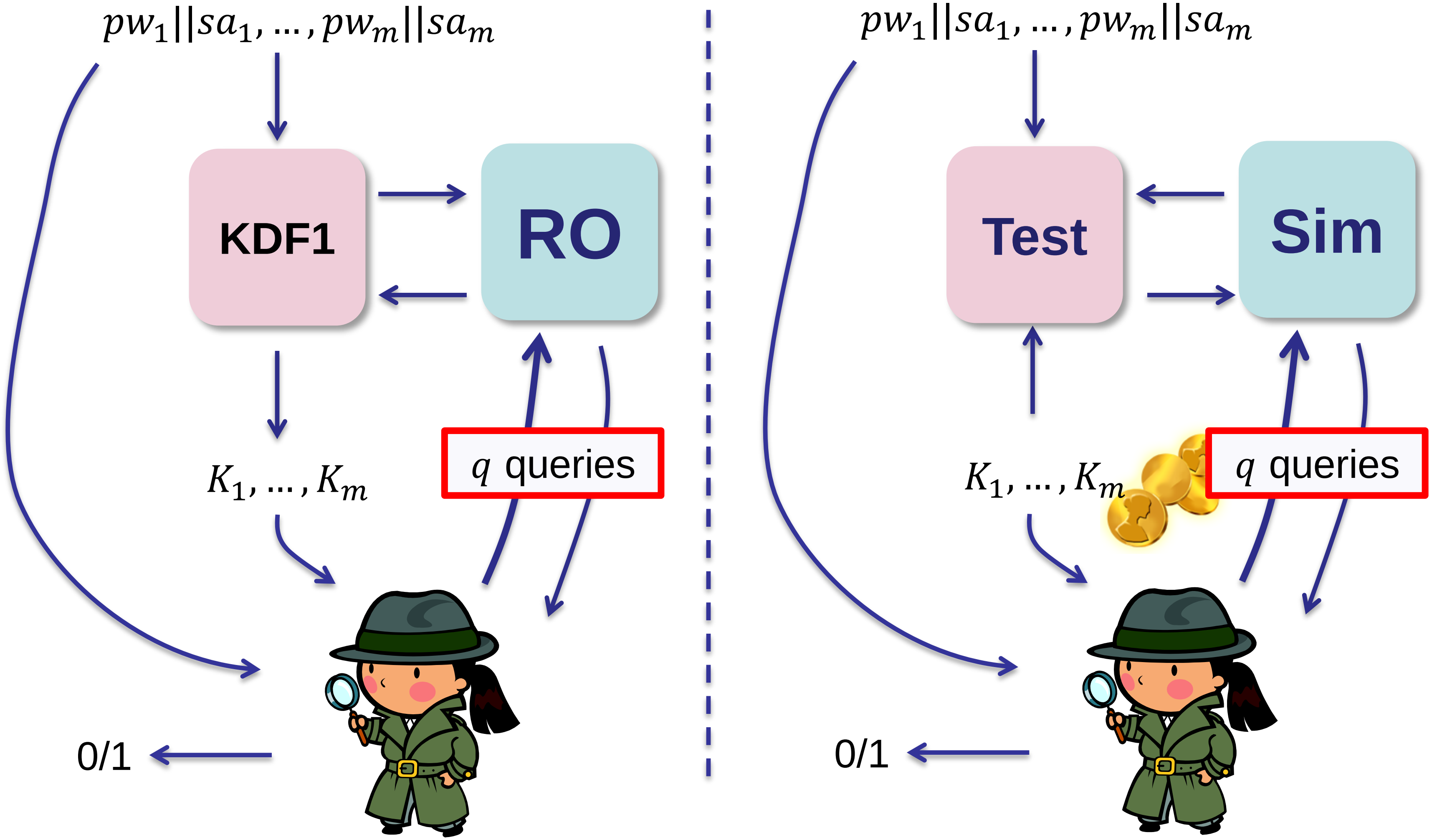

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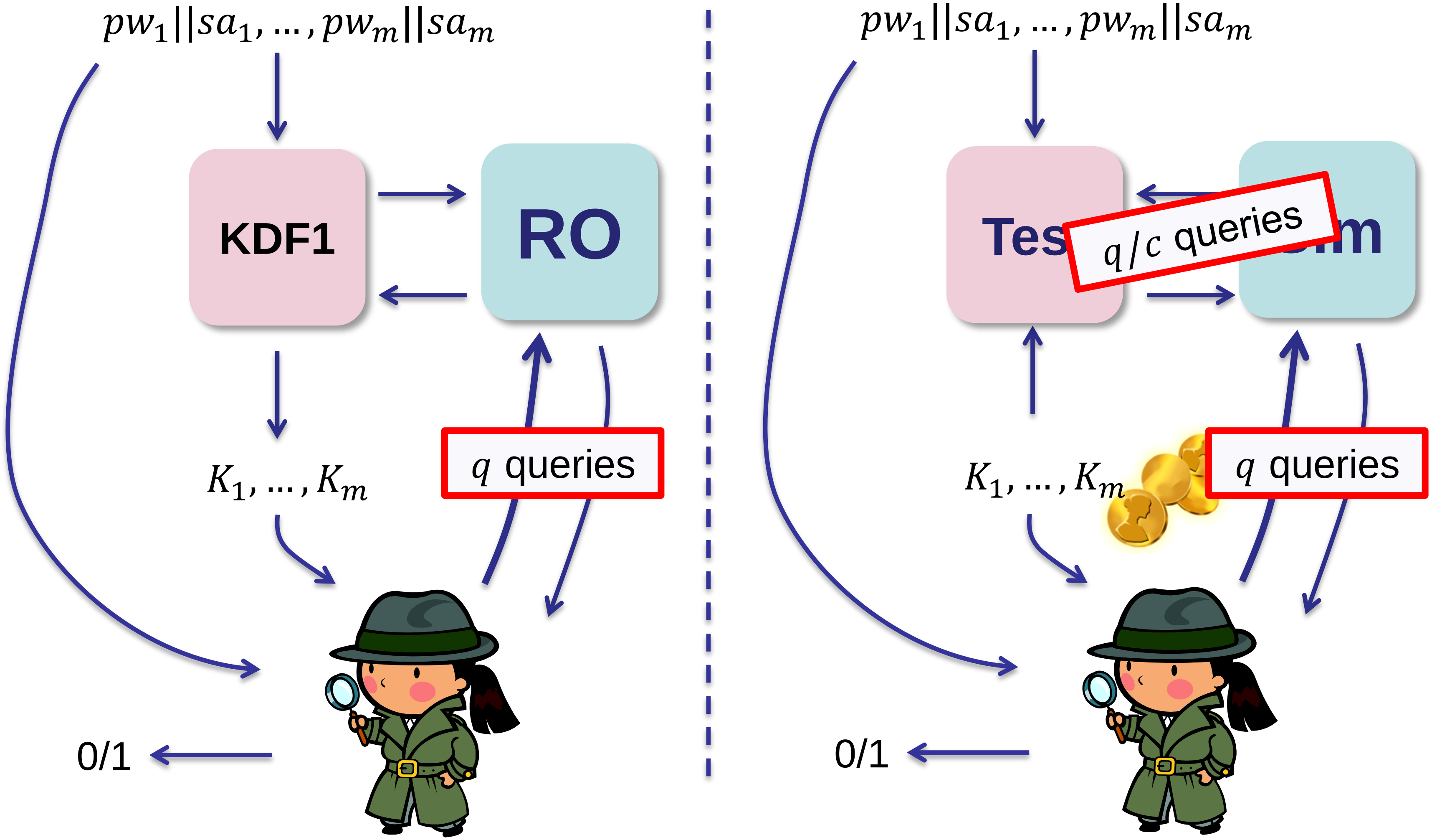

Final result: Security of PB-Encrypt

Question: Does salting deliver multi-instance security amplification for PKCS#5?

PB-Encrypt(pw, M)
$salt \leftarrow \{0,1\}^s$
$K \leftarrow$ **H**$^c(pw||salt)$
$C \leftarrow$ **ENC**(K, M)
Return $C||salt$

Theorem: ∀A making q RO queries, ∃ B such that

$$\mathbf{Adv}^{\mathbf{m-rorx}}_{\text{PB-Encrypt}}(A) < \frac{q}{mcN} + m \cdot \mathbf{Adv}^{\mathbf{ror}}_{\mathbf{ENC}}(B) + \frac{q^2}{2^n} + \frac{q^2}{2^s}$$

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Work $m \times c \times N$ to break encryption (RO queries)

Concluding Remarks

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Thank you!