Illegitimi non carborundum

Ronald L. Rivest

Viterbi Professor of EECS
MIT, Cambridge, MA

CRYPTO 2011
2011-08-15
Illegitimi non carborundum
(Don’t let the bastards grind you down!)

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Overview and Context

The Game of “FLIP IT”

Non-Adaptive Play

Adaptive Play

Lessons and Open Questions
Cryptography is mostly about using *mathematics* and *secrets* to achieve confidentiality, integrity, or other security objectives.
We make *assumptions* as necessary, such as ability of parties to generate unpredictable keys and to keep them secret, or inability of adversary to perform certain computations.
Murphy’s Law: “If anything can go wrong, it will!”
Assumptions may fail, badly. (Maginot Line)
Even worse...

In an adversarial situation, assumption may fail *repeatedly*...

(ref Advanced Persistent Threats)
Most crypto is like Maginot line...

We work hard to make up good keys and distribute them properly, then we sit back and wait for the attack.

There is a line we assume adversary can not cross (theft of keys).
Partial key theft

Much research allows adversary to steal some portion of key(s).

- secret-sharing [S79,...]
- proactive crypto [HJKY95,...]
- signer-base intrusion-resilience [IR04,...]
- leakage-resilient crypto [MR04,...]

But adversary isn’t allowed to steal everything, all at once. (Some exceptions, e.g. intrusion-resilient secure channels [IMR’05])

This just moves the line in the digital sand a bit...
To be a good security professional, there shouldn’t be limits on your paranoia!
(The adversary won’t respect such limits...) Are we being sufficiently paranoid??
Lincoln’s Riddle

Q: “If I call the dog’s tail a leg, how many legs does it have?”

A: Four. It doesn’t matter what you call the tail; it is still a tail.
Lincoln’s Riddle

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A: “Four. It doesn’t matter what you call the tail; it is still a tail.”
Corollary to Lincoln’s Riddle

Calling a bit-string a “secret key” doesn’t actually make it secret...
Corollary to Lincoln’s Riddle

Calling a bit-string a “secret key” doesn’t actually make it secret...

Rather, it just identifies it as an interesting target for the adversary!
Our goal

To develop new models for scenarios involving total key loss. Especially those scenarios where theft is *stealthy* or *covert* (not immediately noticed by good guys).
The Game of “FLIPIT”
(aka “Stealthy Takeover”)

joint work with
Ari Juels, Alina Oprea, Marten van Dijk
of RSA Labs
Flip It is a two-player game

- Defender = Player 0 = Blue
- Attacker = Player 1 = Red
FLIPIT is a two-player game

- Defender = Player 0 = Blue
- Attacker = Player 1 = Red

FLIPIT is rather symmetric, and we say “player i” to refer to an arbitrary player.
There is a contested critical secret or resource

Examples:

▶ A password
▶ A digital signature key
▶ A computer system
▶ A mountain pass
There is a contested critical secret or resource

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A player can “move” (take control) at any time

- Defender move puts resource into Good state
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Time is continuous, not discrete.
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Time is *continuous*, not discrete.
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- Defender move puts resource into Good state
  = Initialize Reset Recover Dis-infect

- Attacker move puts resource into Bad state
  = Compromise Corrupt Steal Infect

Time is *continuous*, not discrete.
Players move at same time with probability 0.
Examples of moves:

- Create new password or signing key.
- Steal password or signing key.
Examples of moves:

- Create new password or signing key.
- Steal password or signing key.
- Re-install system software.
- Use zero-day attack to install rootkit.
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- Send soldiers to mountain pass.
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Continual back-and-forth warfare...

- Note that Attacker can take over at any time.

- The game may go on forever...

Attacker: \(\text{control}(10.0) - \text{moves}(2) \times \text{cost}(3) = \text{score}(4.0)\)

Defender: \(\text{control}(15.0) - \text{moves}(3) \times \text{cost}(1) = \text{score}(12.0)\)
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- There is no “perfect defense”.

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Moves are “stealthy”

- In practice, compromise is often undetected...

- Player’s uncertainty about system state increases with time since his last move.

- A move may take control (“flip”) or have no effect (“flop”).

- Uncertainty means flops are unavoidable.
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- In FLIPIT, players do not immediately know when the other player makes a move!
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Moves may be informative

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- In basic FLIPPIT, each move has feedback that reveals all previous moves.
Moves may be informative

- A player learns the state of the system *only* when he moves.
- In basic FLIPIIT, each move has feedback that reveals all previous moves.
- In variants, move reveals only current state, or time since other player last moved...
Cost of moves and gains for being in control

- Moves aren’t for free!
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- Player $i$ pays $k_i$ points per move:
  - Defender pays $k_0$, Attacker pays $k_1$
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- Being in control yields gain!
Moves aren’t for free!

Player $i$ pays $k_i$ points per move:
Defender pays $k_0$, Attacker pays $k_1$

Being in control yields gain!

Player earns one point for each second he is in control.
How well are you playing? (Notation)

- Let $N_i(t)$ denote number moves by player $i$ up to time $t$. His average rate of play is

$$\alpha_i(t) = \frac{N_i(t)}{t}.$$
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- Let $N_i(t)$ denote number moves by player $i$ up to time $t$. His average rate of play is

$$\alpha_i(t) = \frac{N_i(t)}{t}.$$

- Let $G_i(t)$ denote the number of seconds player $i$ is in control, up to time $t$. His rate of gain up to time $t$ as

$$\gamma_i(t) = \frac{G_i(t)}{t}.$$
How well are you playing? (Notation)

- Score (net benefit) $B_i(t)$ up to time $t$ is 
  \[
  \text{TimeInControl} - \text{CostOfMoves}:
  \]
  \[
  B_i(t) = G_i(t) - k_i \cdot N_i(t)
  \]

- Benefit rate is 
  \[
  \beta_i(t) = \frac{B_i(t)}{t} = \gamma_i(t) - k_i \cdot \alpha_i(t)
  \]

- Player wishes to maximize $\beta_i = \lim_{t \to \infty} \beta_i(t)$. 
Movie of **FLIP IT** Game – Global View

Attacker: control(10.0) - moves(2)*cost(3) = score(4.0)

Defender: control(15.0) - moves(3)*cost(1) = score(12.0)
Movie of FLIP IT Game – Defender View

Attacker: control(10.0) - moves(2)*cost(3) = score(4.0)

Defender: control(15.0) - moves(3)*cost(1) = score(12.0)
How to play well?
Non-Adaptive Play
Non-adaptive strategies

- A *non-adaptive strategy* plays blindly, independent of other player’s moves.
Non-adaptive strategies

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- In principle, a non-adaptive player can pre-compute his entire (infinite!) list of moves before the game starts.
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  - *Periodic play*
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  - *Periodic* play
  - *Exponential* (memoryless) play
Non-adaptive strategies

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- Some interesting non-adaptive strategies:
  - *Periodic* play
  - *Exponential* (memoryless) play
  - *Renewal* strategies: iid intermove times
Periodic play

Player $i$ may play *periodically* with rate $\alpha_i$ and period $1/\alpha_i$
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E.g. for $\alpha_0 = 1/3$, we might have:
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E.g. for $\alpha_0 = 1/3$, we might have:

![Periodic Play Diagram](image)

It is convenient to assume that periodic play involves miniscule amounts of jitter or drift; play is effectively periodic but will drift out of phase with truly periodic.
Adaptive play against a periodic opponent

An *adaptive* Attacker can easily learn the period and phase of a periodic Defender, so that periodic play is useless against an adaptive opponent, unless it is very fast. Examples:

- a sentry make his regular rounds
- 90-day password reset
Periodic Attacker

Theorem

If Attacker moves periodically at rate $\alpha_1$ (and period $1/\alpha_1$, with unknown phase), then optimum non-adaptive Defender strategy is

- if $\alpha_1 > 1/2k_0$, don’t play(!),
- if $\alpha_1 = 1/2k_0$, play periodically at any rate $\alpha_0$, $0 \leq \alpha_0 \leq 1/2k_0$,
- if $\alpha_1 < 1/2k_0$, play periodically at rate

$$\alpha_0 = \sqrt{\frac{\alpha_1}{2k_0}} > \alpha_1$$
Graph for Periodic Attacker and Periodic Defender

\((k_0 = 1, k_1 = 1.5)\)
Graph for Periodic Attacker and Periodic Defender

\( k_0 = 1, k_1 = 1.5 \)

\[ \alpha_0^2 \leq 1 \]

if \( \alpha_1 > \frac{1}{2} k_0 \) Attacker too fast for Defender

if \( \alpha_1 = \frac{1}{2} k_0 \) Defender can play with 0 benefit

if \( \alpha_1 < \frac{1}{2} k_0 \) Defender maximizes benefit with \( \alpha_0 = \sqrt{\alpha_1^2 k_0} \)

Optimal Attacker play

Nash equilibrium at \( (\alpha_0, \alpha_1) = (\frac{1}{3}, \frac{2}{3}) \)

\( (\gamma_0, \gamma_1) = (\frac{2}{3}, \frac{1}{3}) \)

\( (\beta_0, \beta_1) = (\frac{1}{3}, 0) \)
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If \(\alpha_1 > \frac{1}{2k_0}\), Attacker too fast for Defender.
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Optimal Attacker play
Nash equilibrium at $(\alpha_0, \alpha_1) = (\frac{1}{3}, \frac{2}{9})$ $(\gamma_0, \gamma_1) = (\frac{2}{3}, \frac{1}{3})$ $(\beta_0, \beta_1) = (\frac{1}{3}, 0)$
Graph for Periodic Attacker and Periodic Defender

($k_0 = 1$, $k_1 = 1.5$)

if $\alpha_1 > \frac{1}{2k_0}$
Defender too fast for Attacker

if $\alpha_1 = \frac{1}{2k_0}$
Defender can play with 0 benefit

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Defender maximizes benefit with
\[ \alpha_0 = \sqrt{\frac{\alpha_1}{2k_0}} \]

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Graph for Periodic Attacker and Periodic Defender

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Optimal Attacker play

\( \alpha_0 = \sqrt{\alpha_1} \)

Maximizes benefit when \( \alpha_1 < \frac{1}{2} k_0 \)

Attacker too fast for Defender when \( \alpha_1 > \frac{1}{2} k_0 \)

Defender can play with 0 benefit when \( \alpha_1 = \frac{1}{2} k_0 \)
Graph for Periodic Attacker and Periodic Defender

\( (k_0 = 1, k_1 = 1.5) \)

\[
\alpha_1 = \frac{3}{2} \quad \text{if} \quad \alpha_1 > \frac{1}{2} \\
\alpha_1 = k_0 = 1 \quad \text{if} \quad \alpha_1 = \frac{1}{2} \\
\alpha_1 < \frac{1}{2} \quad \text{if} \quad \text{Defender maximizes benefit with} \quad \alpha_0 = \sqrt{\alpha_1 / k_0}
\]

Optimal Attacker play

\[
\left( \gamma_0, \gamma_1 \right) = \left( \frac{2}{3}, \frac{1}{3} \right) \\
\left( \beta_0, \beta_1 \right) = \left( \frac{1}{3}, 0 \right)
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Graph for Periodic Attacker and Periodic Defender

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Optimal Attacker play
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\((\beta_0, \beta_1) = \left(\frac{1}{3}, 0\right)\)
Exponential Attacker

If Attacker plays exponentially with rate $\alpha_1$, then his moves form a memoryless Poisson process; he plays independently in each interval of time of size $dt$ with probability $\alpha_1 \, dt$.

Probability that intermove delay is at most $x$ is

$$1 - e^{-\alpha_1 x}$$

For $\alpha_1 = 0.5$, we might have:
Graph for Exponential Attacker and Defender

\( k_0 = 1, \ k_1 = 1.5 \)

Attacker too fast if \( \alpha_1 > 1 \)

Optimal Defender play for \( \alpha_1 < 1 \)

\[ \alpha_0 = \sqrt{\alpha_1 k_0 - \alpha_1} \]

Optimal Attacker play

Nash equilibrium at \( (\alpha_0, \alpha_1) = (\frac{6}{25}, \frac{4}{25}) \)

\( (\gamma_0, \gamma_1) = (\frac{3}{5}, \frac{2}{5}) \)

\( (\beta_0, \beta_1) = (\frac{9}{25}, \frac{6}{25}) \)
Graph for Exponential Attacker and Defender

\( k_0 = 1, k_1 = 1.5 \)

\( \alpha_0, \alpha_1 \)

\( \gamma_0, \gamma_1 \)

\( \beta_0, \beta_1 \)

Nash equilibrium at \( (\alpha_0, \alpha_1) = (0.625, 0.425) \)

\( (\gamma_0, \gamma_1) = (0.35, 0.25) \)

\( (\beta_0, \beta_1) = (0.925, 0.625) \)
Graph for Exponential Attacker and Defender

\( \alpha_0 = 1, \alpha_1 = 1.5 \)

Attacker too fast if \( \alpha_1 > 1 \)
Graph for Exponential Attacker and Defender

\(k_0 = 1, k_1 = 1.5\)
Graph for Exponential Attacker and Defender

\(k_0 = 1, k_1 = 1.5\)

Optimal Defender play for \(\alpha_1 < 1\)

\[\alpha_0 = \sqrt{\frac{\alpha_1}{k_0}} - \alpha_1\]
(k_0 = 1, k_1 = 1.5)

Optimal Defender play for $\alpha_1 < 1$

$\alpha_0 = \sqrt{k_0 - \alpha_1}$

Optimal Attacker play

$Nash\ equilibrium\ at\ (\alpha_0, \alpha_1) = (\frac{6}{25}, \frac{4}{25})$

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\((\beta_0, \beta_1) = \left( \frac{9}{25}, \frac{6}{25} \right)\)
Renewal Strategies

A renewal strategy is one with iid intermove delays for player $i$’s moves:

$$\Pr(\text{delay } \leq x) = F_i(x)$$

for some distribution $F_i$.

Renewal strategies form a very large class of (non-adaptive) strategies; periodic, exponential, etc. are special cases...

Origin of term: player’s moves form a renewal process.
Optimal (renewal) play against a renewal strategy.

One of our major results is the following:

**Theorem**

*The optimal renewal strategy against any renewal strategy is either periodic or not playing.*
Proof notes

Average time between buses
≠
Average waiting time for a bus
Average time between buses \neq Average waiting time for a bus

Proof considers *size-biased* interval sizes...
Average time between buses \( \neq \) Average waiting time for a bus

Proof considers \textit{size-biased} interval sizes...

Note that a periodic strategy minimizes variance of interval sizes, and thus minimizes size-biased interval size.
Adaptive Play
Adaptive Strategies

- Periodic strategy not very effective against *adaptive* Attacker, who can learn to move just after each Defender move.
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- **FLIPIT** with adaptive strategies can be complicated – generalizes iterated Prisoner’s Dilemma—e.g. for periodic play:
Adaptive Strategies

- Periodic strategy not very effective against *adaptive* Attacker, who can learn to move just after each Defender move.

- flipIT with adaptive strategies can be complicated – generalizes iterated Prisoner’s Dilemma—e.g. for periodic play:

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<th>fast($\alpha_0 = 0.2$)</th>
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<td>slow($\alpha_1 = 0.1$)</td>
<td>0.40,0.40</td>
</tr>
<tr>
<td>fast($\alpha_1 = 0.2$)</td>
<td>-0.10,0.55</td>
</tr>
<tr>
<td></td>
<td>0.55,-0.10</td>
</tr>
<tr>
<td></td>
<td>0.30,0.30</td>
</tr>
</tbody>
</table>
Exponential works well even against adaptive strategies

Theorem

The optimal strategy (of any sort, even adaptive) against an exponential strategy is either periodic or not playing.

Defender can always play exponential strategy against a potentially adaptive Attacker; Attacker can’t then do better than playing periodically (or not playing).
Defender’s \( (\alpha_0 = 0.25) \) net benefit \( \beta_0 \) against optimal (periodic) Attacker \( (\alpha_1 \text{ variable}) \)

![Graph showing the relationship between \( \beta_0 \) and \( \alpha_1 \)]
Defender’s \((\alpha_0 = 0.25)\) net benefit \(\beta_0\) against optimal (adaptive) Attacker \((\alpha_1\text{ variable})\)
Defender’s ($\alpha_0 = 0.25$) net benefit $\beta_0$ against optimal (adaptive) Attacker ($\alpha_1$ variable)
Lessons and Open Questions
Lessons

- Be prepared to deal with continual repeated failure (loss of control).
Lessons

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- Play fast! Aim to make opponent drop out! (Agility!)
Lessons

- Be prepared to deal with continual repeated failure (loss of control).
- Play fast! Aim to make opponent drop out! (Agility!)
- Arrange game so that your moves cost much less than your opponent’s! (Cheap to refresh passwords or keys, easy to reset system to pristine state (as with a virtual machine))
Conjecture: The optimal non-adaptive strategy against a renewal strategy is periodic.

(We only proved that optimal renewal strategy is periodic.)
Open question 2

What is “optimal” renewal strategy against an adaptive rate-limited Attacker? (e.g. $N_1(t)/t \leq \alpha_1$ for all $t$)?

Perhaps using gamma-distributed intervals or delayed exponentials?
Open question 2

What is “optimal” renewal strategy against an adaptive rate-limited Attacker? (e.g. \(N_1(t)/t \leq \alpha_1\) for all \(t\))?

That is, how to balance trade-off between periodic play, which has low-variance intervals but is predictable, and exponential, which has high-variance intervals but is very unpredictable?

Perhaps using gamma-distributed intervals or delayed exponentials?
Open question 3

Are there information-theoretic bounds on how well a rate-limited Attacker can do against a fixed renewal strategy by Defender?
Open question 4

What learning theory algorithms yield adaptive strategies provably optimal against renewal strategies?
Open questions 5, 6, 7, ...

5 Multi-player FLIP IT

6 Other feedback models (e.g. add low-cost “check”)

7 How to structure PKI when any party (including CA’s) may get “hacked” at any time?

... ...
Online version of FLIPIT

More information on FLIPIT, including an online interactive version of the game, will be available in the next few weeks at:

www.rsa.com/flipit

Enjoy!
The End

Attacker: control(10.0) - moves(2)*cost(3) = score(4.0)

Defender: control(15.0) - moves(3)*cost(1) = score(12.0)