

# Memory Delegation

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# Delegation of Computation

- Emerging scenarios

I'd like to verify the answer!

FTI@Home, Folding@Home, etc...

Here is a proof  $\pi$



$F, x$

$y$

$\pi$

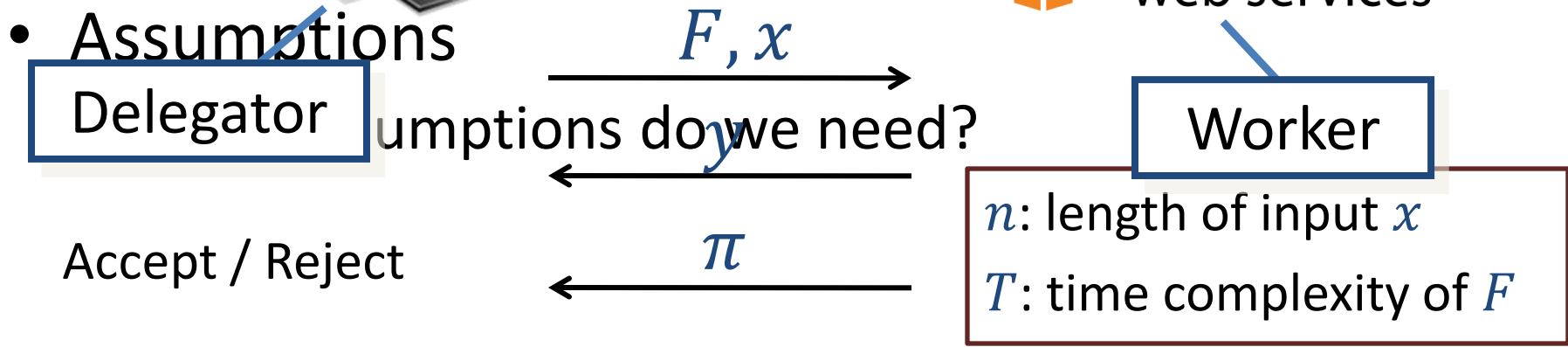
Accept / Reject

Delegator

Worker

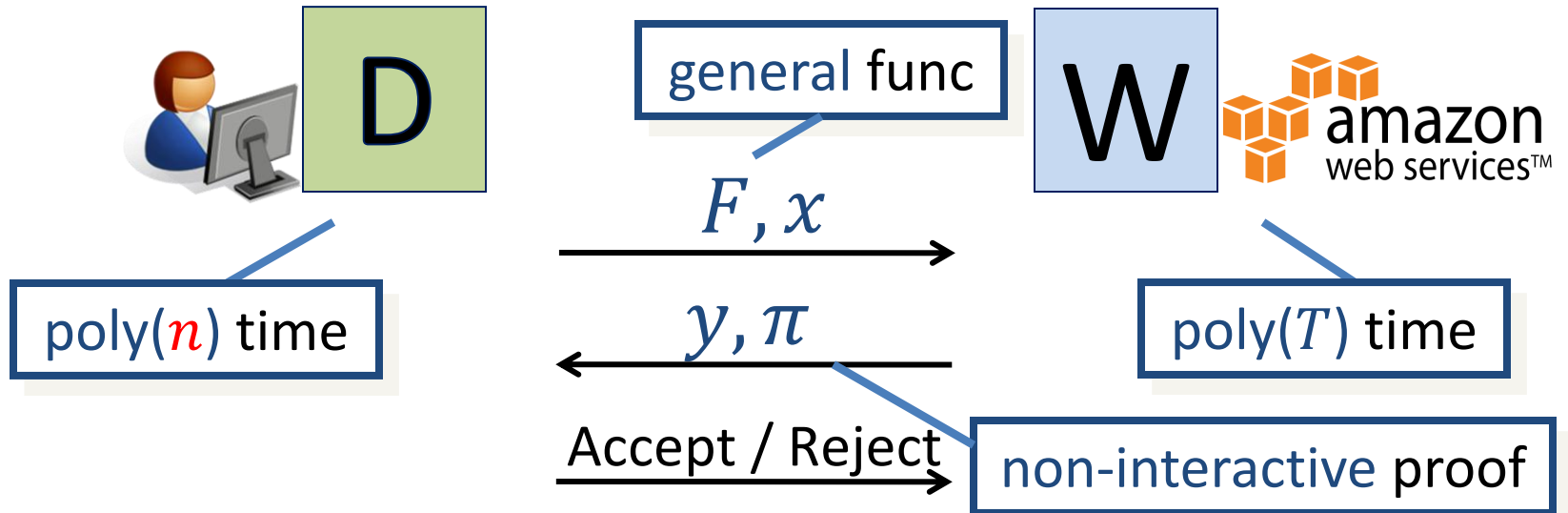
# Important Properties

- Computational Efficiency
  - verification *must be faster* than computation
  - want small overhead for the worker
- Interaction
  - can the proofs be non-interactive ?
- Generalization
  - can we delegate all functions?



# Holy Grail of Comp. Delegation

$n$ : length of input  $x$ ,  $T$ : time complexity of  $F$



- Completeness:  $D$  accepts correct  $y, \pi$  w.p. 1
- Soundness:  $\forall$  poly( $T$ )-time  $W^*$ ,  
 $\Pr[ D \text{ accepts } \textit{wrong} \text{ answer} ] \leq ngl$

# Previous Results on Comp. Del.

Results	Trade-offs
GKR scheme [GKR '08, KR '09]	<b>Non-interactive</b> proofs 😊 For low-depth functions 😞
Universal Arguments [K '92, M '94, BG '02]	4-message interactive proofs 😞 For <b>general</b> functions 😊
Offline/Online [GGP '10, CKV '10, AIK'10]	With (inefficient) offline preprocessing 😞 <b>Non-interactive</b> & for <b>general</b> functions* 😊

All above results are *efficient*, but require *assumptions*

(\*)  $W^*$  is not allowed to learn the decision bits of  $D$

# The Goal of Delegation

- Holy grail of computation delegation:
  - Can we achieve *efficient* and *non-interactive* computation delegation for *general* functions under reasonable assumptions ?
- We don't know the answer to this question yet. But we want *more*!

~~Delegator runs  
in  $O(n)$  time~~

Delegator should run  
in  $o(n)$  time !

# When data $x$ is large and in the cloud...

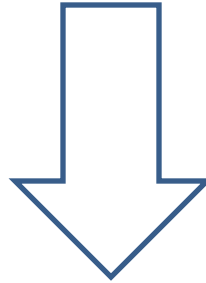
Can  $D$  delegate the data  $x$  as well,  
only keep  $\text{Cert}(x)$  & verify in  $o(N)$  time?



$N$ : length of input  $x$ ,       $T$ : time complexity of  $F$

# Our Main Results

GKR Scheme & Universal Argument  
as **Computation** Delegation Schemes



GKR Scheme & Universal Argument  
as **Memory/Streaming** Delegation Schemes



# Outline

- Computation Delegation
- **Memory Delegation**
- Streaming Delegation
- Conclusion

# Memory Delegation

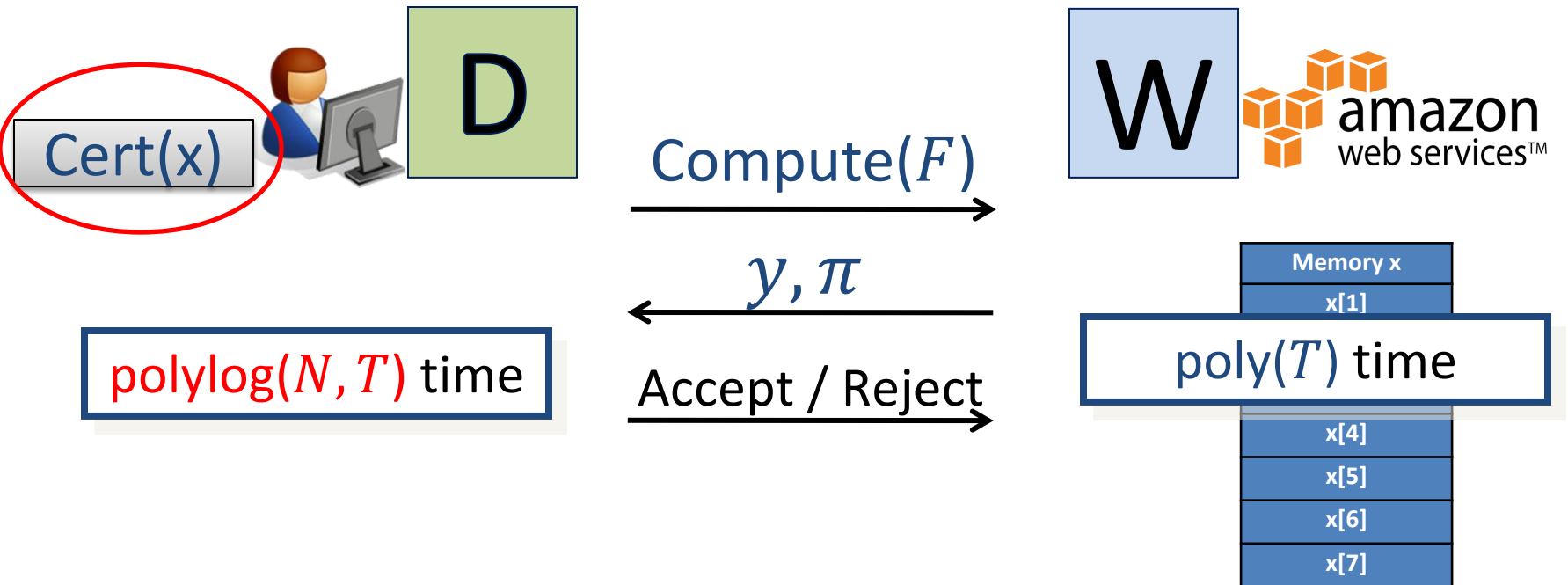
- Initial memory  $x$  holds by delegator  $D$
- $D$  computes a certificate  $\text{Cert}(x)$
- $D$  sends  $x$  to worker  $W$



Memory $x$
$x[1]$
$x[2]$
$x[3]$
$x[4]$
$x[5]$
$x[6]$
$x[7]$

# Compute Operation

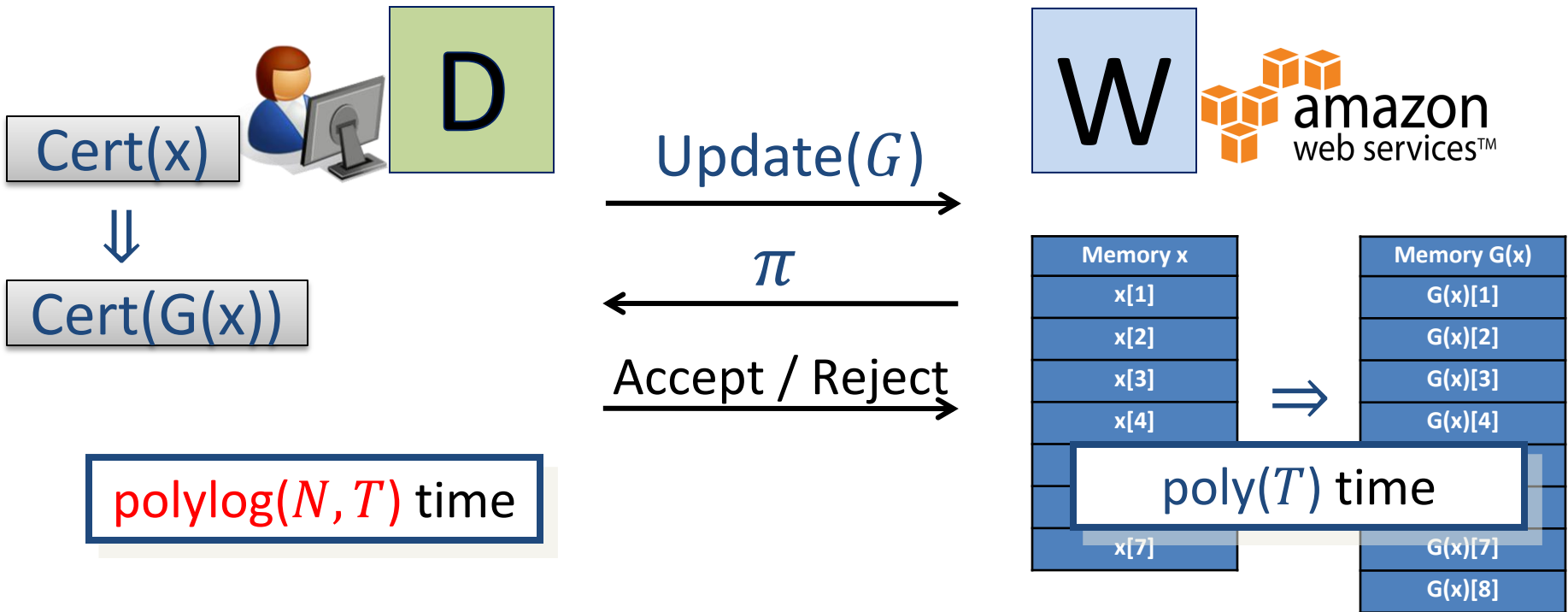
- $D$  can verify  $\pi$  using certificate  $\text{Cert}(x)$
- Efficiency:  $D$  should run in time  $\text{polylog}(N, T)$
- $W$  should run in time  $\text{poly}(T)$



# Update Operation

- Allow **D** sends a general update function  $G$  to **W**
- Allow **W** help **D** update certificate
- *Efficiency*: **D** should run in time  $\text{polylog}(N, T)$
- **W** should run in time  $\text{poly}(T)$

Can you be sure, and here is the update info  $\pi$



# Desired Properties

- **Efficiency**
  - $D$  runs in time  $\text{polylog}(N, T)$
  - $W$  runs in time  $\text{poly}(T)$
- **Completeness**:  $D$  always accepts when  $W$  honest
- **Reusable Soundness**: soundness game for  $D$  and  $W^*$ 
  - $W^*$  can chooses inputs of  $D$  during interaction
  - $W^*$  **learns** the decision of  $D$
  - $W^*$  wins if  $D$  ever accepts mistakenly
  - $\forall$   $\text{poly}(T)$ -time  $W^*$  can win with **negligible** probability

$N$ : length of memory  $x$ ,  $T$ : time complexity of  $F, G$

# Issue of Reusability

- D uses  $\text{cert}(x)$  to compute his decision
  - ⇒ one bit *leakage* info about  $\text{cert}(x)$  *per input*
- Our memory scheme has *public*  $\text{cert}(x)$ 
  - Simple!
- Our streaming scheme has *secret*  $\text{cert}(x)$ 
  - Challenging! Take ideas from continual-leakage model.
  - New *geometric lemma* “dual” to [BKKV ‘10]
  - New *entropy lemma* for lower bounding conditional computational entropy

# Our Memory Delegation Schemes

Under cryptographic assumptions\*, we obtain *efficient* memory delegation schemes with

Our Schemes	Property
Based on GKR scheme	<b>Non-interactive</b> proofs 😊 For low-depth functions 😞
Based on Universal Arguments	4-message interactive proofs 😞 For <b>general</b> functions 😊

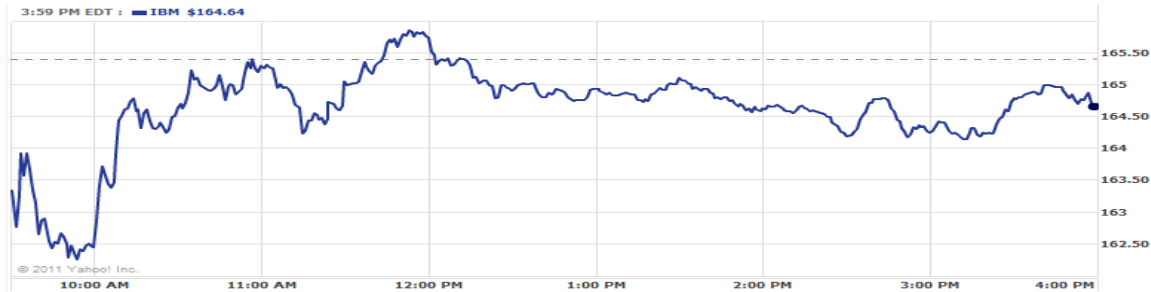
(\*) Based on the same assumptions as the corresponding schemes

# Outline

- Computation Delegation
- Memory Delegation
- **Streaming Delegation**
- Conclusion



# Example: Streaming of Stock Ticks



**-0.1**

Should I buy  
the stock now?



...

# Comparison to Memory Delegation

- Data stream arrives constantly at a high rate
  - ⇒ Ideally, **D** should update certificate by *himself*
- Luckily we can!
  - every update simply appends a data item  $x_t$
- Different from memory delegation
  - Recall update for memory delegation is general
  - **D** gets help from **W**

# Our Streaming Delegation Schemes

Assume the existence of

fully homomorphic encryption schemes [G '09]

Our Schemes	Property
Based on GKR scheme	Non-interactive proofs 😊 For low-depth functions 😞
Based on Universal Arguments	4-message interactive proofs 😞 For <b>general</b> functions 😊

# Outline

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# Conclusion

- We construct **efficient** memory/streaming delegation schemes
  - **non-interactive** for low depth functions
  - 4-message for **general** functions
- Can we achieve the holy grail of computation/memory/ streaming delegation?
  - *efficient* and *non-interactive* schemes for *general* functions

Thanks you!