Improved Security Analyses for CBC MACs

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The CBC function

$[n] = \{0,1\}^n$, $\pi : [n] \to [n]$.

The CBC function $\mathrm{CBC}_\pi : [n]^* \to [n]$ is defined as

$\mathrm{CBC}_\pi(M_1 \| M_2 \| \dots \| M_\ell) = C_\ell$ where $C_0 = 0^n$, $C_i = \pi(M_i \oplus C_{i-1})$

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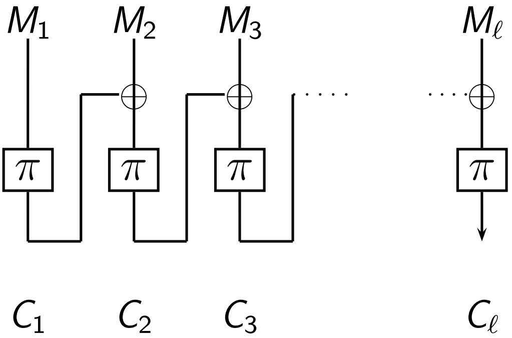

The Encrypted-CBC (ECBC) function

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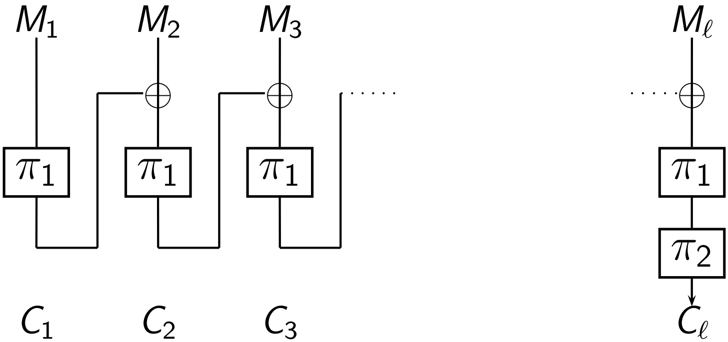

Attack Models

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$$\mathcal{A}[\text{eq}, q, n, \ell] \subseteq \mathcal{A}[\text{pf}, q, n, \ell] \subseteq \mathcal{A}[\text{any}, q, n, \ell]$$

$$\mathbf{Adv}_{\mathrm{CBC}}(A) = \Pr[\pi \xleftarrow{\$} \mathit{Perm}(n); A^{\mathrm{CBC}_\pi} \Rightarrow 1] - \Pr[f \xleftarrow{\$} \mathit{Func}(n); A^{f} \Rightarrow 1]$$

$$\boxed{\mathbf{Adv}^{\text{atk}}_{\mathrm{CBC}}(q, n, \ell) = \max_{A \in \mathcal{A}[\text{atk}, q, n, \ell]} \mathbf{Adv}_{\mathrm{CBC}}(A)}$$

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DGHKR04	$\mathbf{Adv}^{\text{eq}}_{\text{ECBC}}(q, n, \ell) \leq c \cdot q^2 / 2^n$ for $\ell \leq 2^{n/2}$.

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$$\mathbf{Adv}^{\mathrm{eq}}_{\mathrm{CBC}}(2^{n/2}, n, \ell) = \Theta(1)$$

$$\mathbf{Adv}^{\mathrm{eq}}_{\mathrm{ECBC}}(2^{n/2}, n, \ell) = \Theta(1)$$

Our Results

Improve prefix free CBC from $\ell^2 \cdot q^2/2^n$ to:

Theorem

$$\mathbf{Adv}^{\mathrm{pf}}_{\mathrm{CBC}}(q, n, \ell) \leq c \cdot \ell \cdot q^2/2^n \qquad \textit{for} \qquad \ell \leq 2^{n/3}$$

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Improve ECBC from $c \cdot \ell^2 \cdot q^2/2^n$ to:

Theorem

$$\mathbf{Adv}^{\mathrm{any}}_{\mathrm{CBC}}(q, n, \ell) \leq c \cdot \ell^{1/\ln \ln \ell} \cdot q^2/2^n \qquad \textit{for} \qquad \ell \leq 2^{n/4}$$

Permutation vs. Functions

$$\mathrm{CBC} = \{\mathrm{CBC}_\pi; \pi \xleftarrow{\$} Perm(n)\}$$

$$\mathrm{CBC}' = \{\mathrm{CBC}_f; f \xleftarrow{\$} Func(n)\}$$

$$\mathbf{Adv}^{\mathrm{pf}}_{\mathrm{CBC}}(q = 2^{n/4}, n, \ell = 2^{n/4}) \approx \ell \cdot q^2/2^n \leq 2^{-n/4}$$

$$\mathbf{Adv}^{\mathrm{pf}}_{\mathrm{CBC}'}(q = 2^{n/4}, n, \ell = 2^{n/4}) = \Theta(1) \qquad \text{[Berke04]}$$

ECBC and the Carter-Wegman Paradigm

$$\mathrm{ECBC}_{\pi_1,\pi_2}(.) = \pi_2(\mathrm{CBC}_{\pi_1}(.))$$

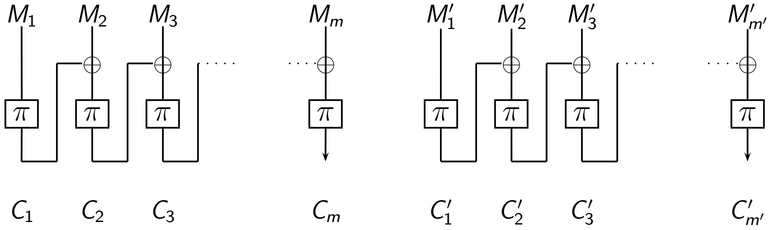
$$\mathbf{CP}_n(M, M') = \Pr[\pi \leftarrow \mathit{Perm}(n); \mathrm{CBC}_\pi(M) = \mathrm{CBC}_\pi(M')]$$

$$\mathbf{CP}^{\mathrm{any}}_{n,\ell} = \max_{M, M', |M| \le \ell n, |M'| \le \ell n} \mathbf{CP}_n(M, M')$$

$$\boxed{\mathbf{Adv}^{\mathrm{any}}_{\mathrm{ECBC}}(q, n, \ell) \le q^2 \cdot \mathbf{CP}^{\mathrm{any}}_{n,\ell}}$$

CBC and the Full Collision Probability

$$M = M_1 \| M_2 \| \ldots M_m \text{ and } M' = M'_1 \| M'_2 \| \ldots M'_{m'}$$



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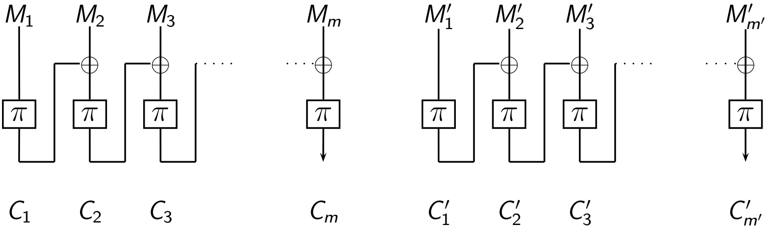

$$\mathbf{FCP}_n(M, M') = \Pr[\pi \leftarrow Perm(n);\, C'_{m'} \in \{C_1, \ldots, C_m, C'_1, \ldots, C'_{m'-1}\}]$$

$$\mathbf{FCP}^{\mathrm{pf}}_{n,\ell} = \max_{M, M', |M| \leq \ell n, |M'| \leq \ell n} \mathbf{FCP}_n(M, M')$$

$$\boxed{\mathbf{Adv}^{\mathsf{any}}_{\mathsf{ECBC}}(q,n,\ell) \le q^2 \cdot \mathbf{CP}^{\mathsf{any}}_{n,\ell}}$$

$$\boxed{\mathbf{Adv}^{\mathsf{pf}}_{\mathsf{CBC}}(q,n,\ell) \le q^2 \cdot \mathbf{FCP}^{\mathsf{pf}}_{n,\ell} + \frac{4mq^2}{2^n}}$$

$$\boxed{\mathbf{Adv}_{\mathrm{ECBC}}^{\mathrm{any}}(q,n,\ell) \le q^2 \cdot \mathbf{CP}_{n,\ell}^{\mathrm{any}}}$$

Lemma

$$\mathbf{CP}_{n,\ell}^{\mathrm{any}} \le \frac{2d(\ell)}{2^n} + \frac{8\ell^4}{2^{2n}}$$

Where $d(\ell) \le \ell^{1/\ln\ln\ell} = o(\ell)$ is the maximum number of divisors of any $m \le \ell$.

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The Game-Playing Technique [BR05]

On the s^{th} **query** $F(M_s)$ Game D1

$m_s \leftarrow |M_s|_n, \quad C_s^0 \leftarrow 0^n$
for $i \leftarrow 1$ **to** $m_s - 1$ **do**
  $X_s^i \leftarrow C_s^{i-1} \oplus M_s^i$
  if $X_s^i \in \mathrm{Dom}(\pi)$ **then** $C_s^i \leftarrow \pi(X_s^i)$
    else $\pi(X_s^i) \leftarrow C_s^i \xleftarrow{\$} \overline{\mathrm{Ran}}(\pi)$
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D1 implements CBC.

The Game-Playing Technique [BR05]

On the s^{th} **query** $F(M_s)$ — Game D0

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$$\mathbf{Adv}_{\mathrm{CBC}}(A) = \Pr[A^{D0} \Rightarrow 1] - \Pr[A^{D1} \Rightarrow 1]$$

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$$\mathbf{Adv}_{\mathrm{CBC}}(A) = \Pr[A^{D0} \Rightarrow 1] - \Pr[A^{D1} \Rightarrow 1] \le \Pr[A^{D0} \text{ sets } bad]$$

The Game-Playing Technique Cnt.

Game D7

$\pi \xleftarrow{\$} \mathrm{Perm}(n)$

$C_1^0 \leftarrow C_2^0 \leftarrow 0^n$

for $i \leftarrow 1$ **to** $\mathtt{m}_1$ **do**

  $X_1^i \leftarrow C_1^{i-1} \oplus \mathtt{M}_1^i,\ C_1^i \leftarrow \pi(X_1^i)$

for $i \leftarrow 1$ **to** $\mathtt{m}_2$ **do**

  $X_2^i \leftarrow C_2^{i-1} \oplus \mathtt{M}_2^i,\ C_2^i \leftarrow \pi(X_2^i)$

$\mathit{bad} \leftarrow X_2^{\mathtt{m}_2} \in \{X_1^1, \ldots, X_1^{\mathtt{m}_1},$
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$$\Pr[A^{D7} \text{ sets } \color{red}{bad}] = \mathbf{FCP}_n(\mathtt{M}_1^1 \| \ldots \| \mathtt{M}_{\mathtt{m}_1}^1, \mathtt{M}_1^2 \| \ldots \| \mathtt{M}_{\mathtt{m}_2}^2)$$

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  $X_2^1, \ldots, X_2^{\mathtt{m}_2 - 1}\}$

$$\Pr[A^{D1} \text{ sets } \textit{bad}] \leq q^2 \cdot \Pr[A^{D7} \text{ sets } \textit{bad}] + \frac{4\ell q^2}{2^n}$$

$$\mathbf{Adv}^{\mathrm{pf}}_{\mathrm{CBC}}(q, n, \ell) \leq q^2 \cdot \mathbf{FCP}^{\mathrm{pf}}_{n,\ell} + \frac{4\ell q^2}{2^n}$$

$$\boxed{\mathbf{Adv}^{\text{any}}_{\text{ECBC}}(q, n, \ell) \leq q^2 \cdot \mathbf{CP}^{\text{any}}_{n,\ell}}$$

Lemma

$$\mathbf{CP}^{\text{any}}_{n,\ell} \leq \frac{2d(\ell)}{2^n} + \frac{8\ell^4}{2^{2n}}$$

Where $d(\ell) \leq \ell^{1/\ln\ln\ell} = o(\ell)$ is the maximum number of divisors of any $m \leq \ell$.

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A Graph-Based Representation of CBC [DGHKR04]

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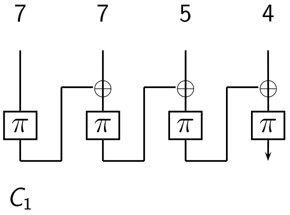
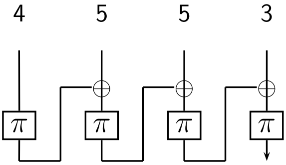
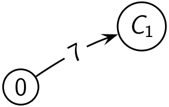
A Graph-Based Representation of CBC [DGHKR04]







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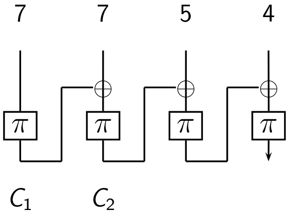

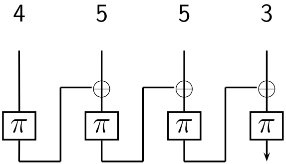

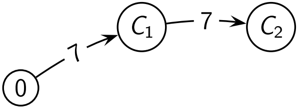

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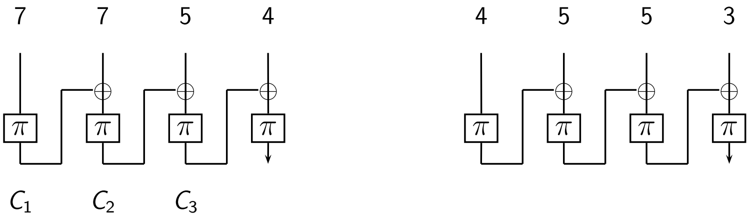

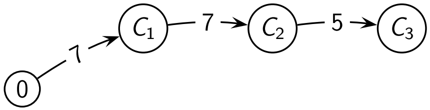

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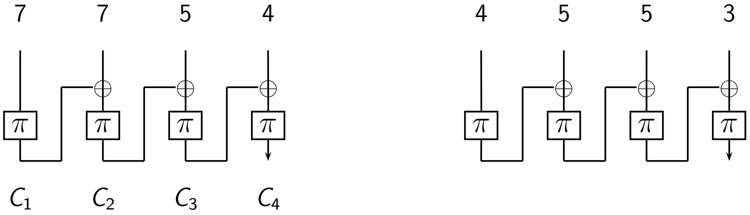

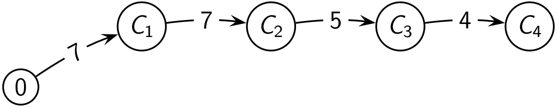

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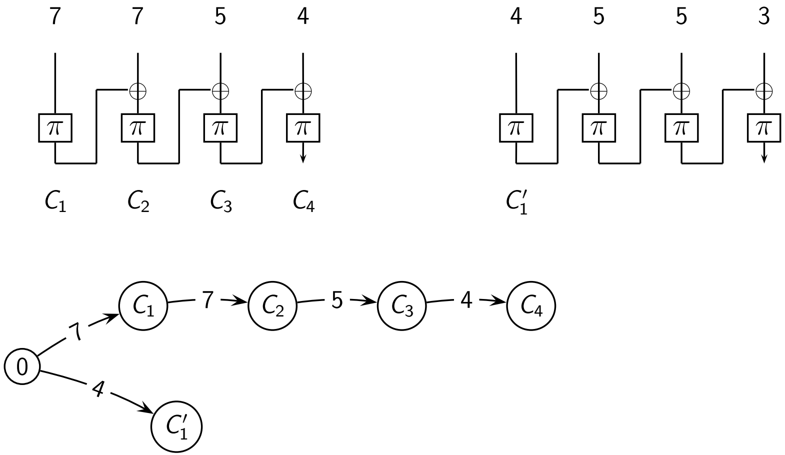

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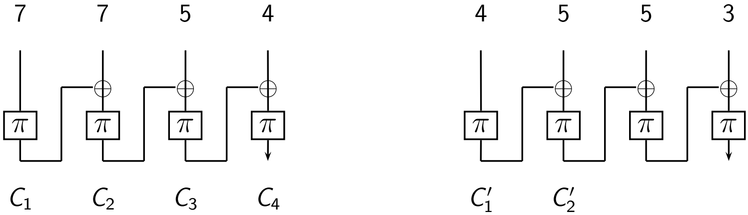
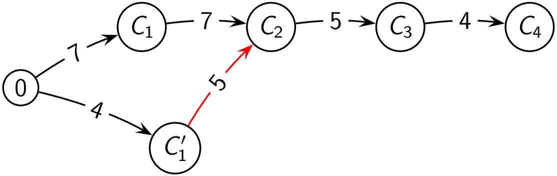


Accident: $C'_2 = C_2$



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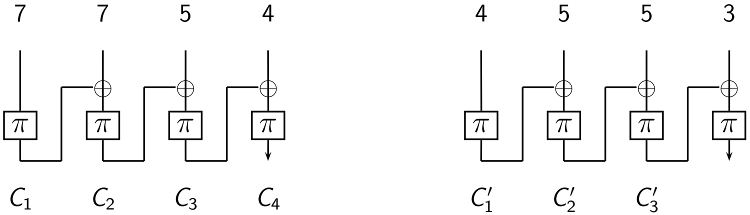
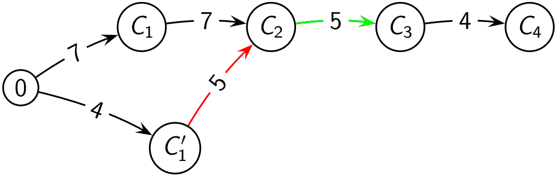


Accident: $C'_2 = C_2$

Induced Collision: $C'_3 = C_3$



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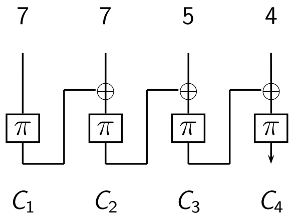
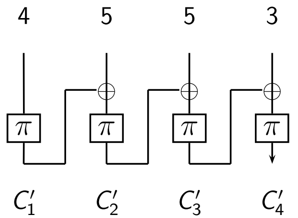
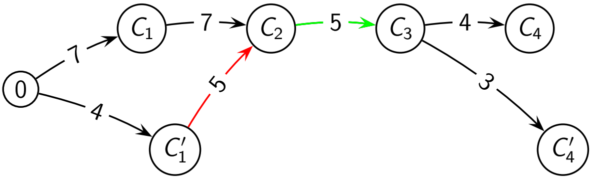




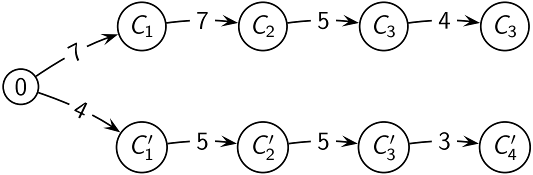
Accident: $C'_2 = C_2$ Induced Collision: $C'_3 = C_3$



Structure Graph G_π, $\mathrm{Acc}(G_\pi) = 1$

Structure Graphs

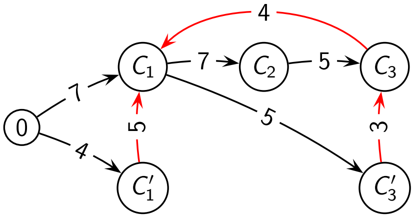
$M = 7\|7\|5\|4 \qquad M' = 4\|5\|5\|3$



$\mathrm{Acc}(H) = 0$

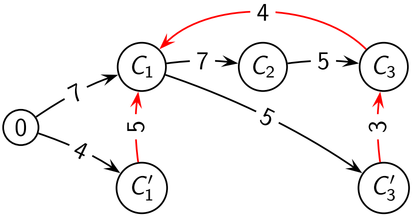
Structure Graphs

$M = 7 \| 7 \| 5 \| 4 \qquad M' = 4 \| 5 \| 5 \| 3 \qquad \mathrm{Acc}(H) = 3$



Structure Graphs

$M = 7 \| 7 \| 5 \| 4$ $\qquad M' = 4 \| 5 \| 5 \| 3$ $\qquad \mathsf{Acc}(H) = 3$



Lemma

$$\Pr[\pi \xleftarrow{\$} \mathrm{Perm}(n);\, G_\pi = H] \leq (2^n - 2\ell)^{-\mathsf{Acc}(H)}$$

$$M = 7\|7\|5\|4 \qquad M' = 4\|5\|5\|3$$

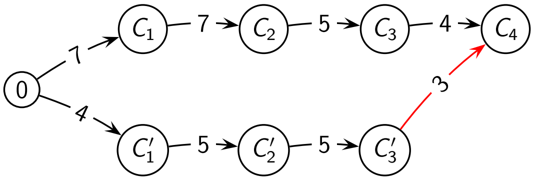
$$\mathbf{CP}_n(M, M') \;=\; \Pr[\pi \xleftarrow{\$} \mathrm{Perm}(n);\, \mathsf{CBC}_\pi(M) = \mathsf{CBC}_\pi(M')]$$

$$M = 7 \| 7 \| 5 \| 4 \qquad M' = 4 \| 5 \| 5 \| 3$$

$$\begin{aligned} \mathbf{CP}_n(M, M') &= \Pr[\pi \xleftarrow{\$} \mathrm{Perm}(n); \mathsf{CBC}_\pi(M) = \mathsf{CBC}_\pi(M')] \\ &= \Pr[\pi \xleftarrow{\$} \mathrm{Perm}(n); G_\pi \text{ satisfies } C_4 = C'_4] \end{aligned}$$

$$M = 7\|7\|5\|4 \qquad M' = 4\|5\|5\|3$$

$$\begin{aligned}\mathbf{CP}_n(M, M') &= \Pr[\pi \xleftarrow{\$} \mathrm{Perm}(n); \mathsf{CBC}_\pi(M) = \mathsf{CBC}_\pi(M')] \\ &= \Pr[\pi \xleftarrow{\$} \mathrm{Perm}(n); G_\pi \text{ satisfies } C_4 = C'_4]\end{aligned}$$



$$M = 7 \| 7 \| 5 \| 4 \qquad M' = 4 \| 5 \| 5 \| 3 \qquad \ell = 4$$

$$\mathbf{CP}_n(M, M') = \Pr[G_\pi \text{ satisfies } C_4 = C'_4]$$

$$\leq \Pr[\mathsf{Acc}(G_\pi) = 1 \text{ and } G_\pi \text{ satisfies } C_4 = C'_4] + \Pr[\mathsf{Acc}(G_\pi) \geq 2]$$

$$\leq \frac{\#G[\text{ with 1 accident where } C_4 = C'_4]}{2^n - 2 \cdot \ell} + \frac{8 \cdot \ell^2}{2^{2n}}$$

Lemma

$$\Pr[\pi \xleftarrow{\$} \mathrm{Perm}(n);\, G_\pi = H] \leq (2^n - 2\ell)^{-\mathsf{Acc}(H)}$$

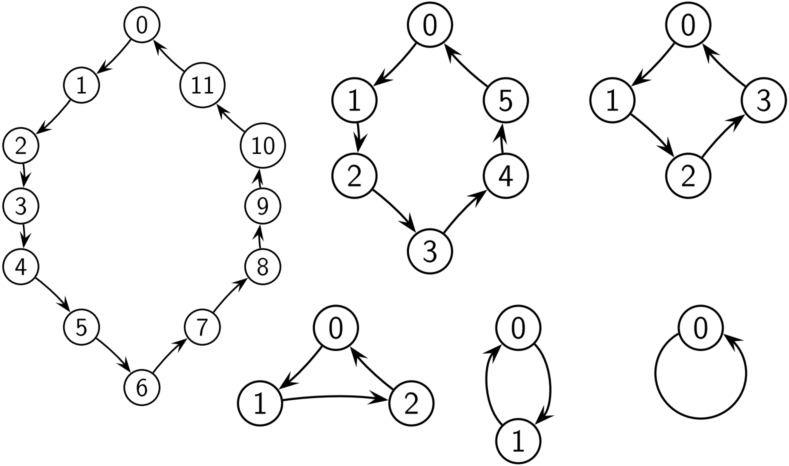
M, M' with $m = |M|, m' = |M'|$, $\ell = \max(m, m')$.

$$\mathbf{CP}_n(M, M') \leq \frac{\boxed{\#G[\text{ with 1 acc. where } C_m = C'_{m'}]}}{2^n - 2 \cdot \ell} + \frac{8 \cdot \ell^2}{2^{2n}}$$

Lemma

$$\#[G \textit{ with 1 acc. where } C_m = C'_{m'}] \leq d(\ell)$$

Where $d(\ell) \leq \ell^{1/\ln\ln\ell} = o(\ell)$ is the maximum number of divisors of any $m \leq \ell$, e.g. $d(15) = 6$ as $12 \leq 15$ has 6 divisors $1, 2, 3, 4, 6, 12$.



Questions?