New Generic Attacks on Hash-based MACs

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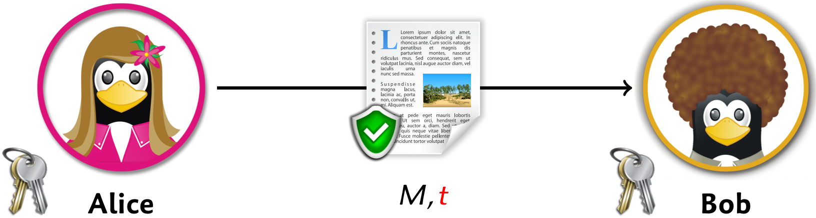
Message Authentication Codes



- Alice sends a message to Bob
- Bob wants to authenticate the message.
- Alice use a key k to compute a tag: $t = \mathrm{MAC}_k(M)$
- Bob verifies the tag with the same key k: $t \overset{?}{=} \mathrm{MAC}_k(M)$
- Symmetric equivalent to digital signatures

MAC Constructions

- Dedicated designs
  - Pelican-MAC, SQUASH, SipHash
- From universal hash functions
  - UMAC, VMAC, Poly1305
- From block ciphers
  - CBC-MAC, OMAC, PMAC
- From hash functions
  - HMAC, Sandwich-MAC, Envelope-MAC

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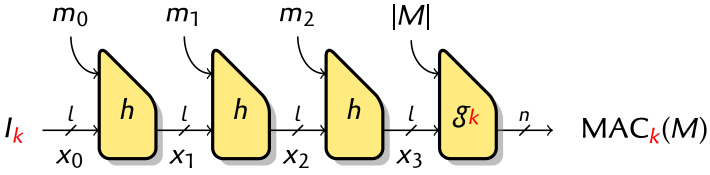
HMAC

- HMAC has been designed by Bellare, Canetti, and Krawczyk in 1996
- Standardized by ANSI, IETF, ISO, NIST.
- Used in many applications:
  - To provide authentication:
    - SSL, IPSEC, ...
  - To provide identification:
    - Challenge-response protocols
    - CRAM-MD5 authentication in SASL, POP3, IMAP, SMTP, ...
  - For key-derivation:
    - HMAC as a PRF in IPsec
    - HMAC-based PRF in TLS

Hash-based MACs



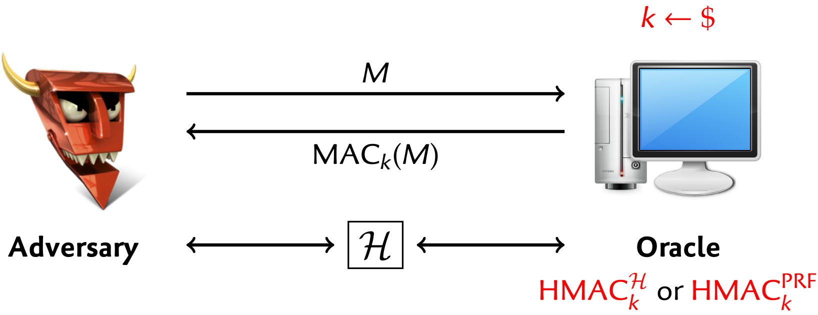
- l-bit chaining value
- n-bit output
- k-bit key

- Key-dependant initial value I_k
- Unkeyed compression function h
- Key-dependant finalization, with message length g_k

Security of HMAC

	Security proof	/ Attack
Existential forgery: Forge a valid pair	$2^{l/2}$	$2^{l/2}$
Universal forgery: Predict the MAC of a challenge	$2^{l/2}$	2^{n}
Distinguishing-R: Distinguish HMAC from a PRF	$2^{l/2}$	$2^{l/2}$
Distinguishing-H: Distinguish HMAC-SHA1 from HMAC-PRF	$2^{l/2}$	2^{l}
State-recovery: Find the internal state after some message	$2^{l/2}$	2^{l}
Key-recovery: Extract the key from a MAC oracle	$2^{l/2}$	2^{k}

Distinguishing-H attack



$k \leftarrow \$$

M

$\mathrm{MAC}_k(M)$

Adversary

$\boxed{\mathcal{H}}$

Oracle

$\mathrm{HMAC}_k^{\mathcal{H}}$ or $\mathrm{HMAC}_k^{\mathrm{PRF}}$

- Security notion from PRF
- Distinguish HMAC using $\mathcal{H}$ from HMAC with a PRF

Distinguishing-H attack

- Collision-based attack does not work:
  - Any compression function has collisions
  - Secret key prevents pre-computed collisions

- Folklore assumption: distinguishing-H attack should require 2^{l}

*"If we can recognize the hash function inside HMAC,
it must be a bad hash function"*

Outline

Introduction

MACs

HMAC

New generic attacks

Cycle detection

Distinguishing-H attack

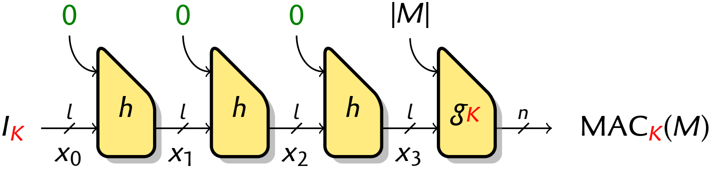
State recovery attack

Key-recovery Attack on HMAC-GOST

HMAC-GOST

Key recovery

Main Idea



- Using a fixed message block, we iterate a fixed function
- Starting point and ending point unknown because of the key

Can we detect properties of the function $h_0 : x \mapsto h(x, 0)$?

- Study the cycle structure of random mappings
- Used to attack HMAC in related-key setting

[Peyrin, Sasaki & Wang, Asiacrypt 12]

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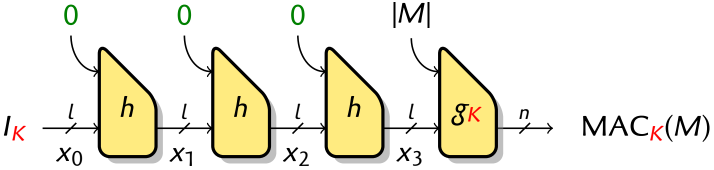

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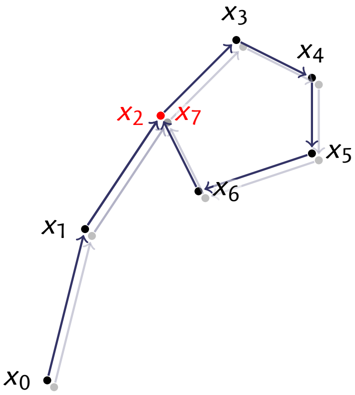
Random Mappings



- Functional graph of a random mapping $x \rightarrow f(x)$
- Iterate f: $x_i = f(x_{i-1})$
- Collision after $\approx 2^{l/2}$ iterations
  - Cycles
- Trees rooted in the cycle
- Several components

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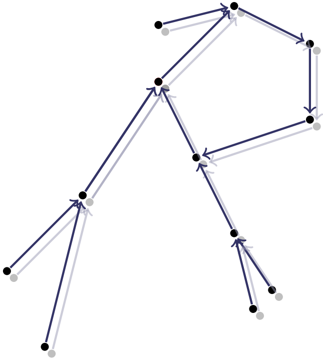

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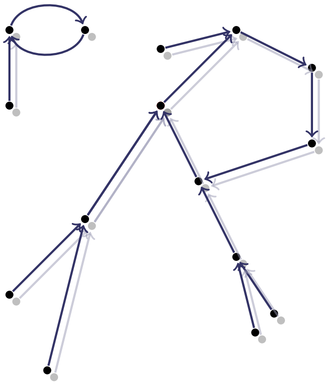

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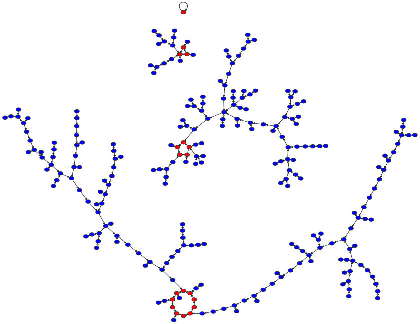
Cycle structure

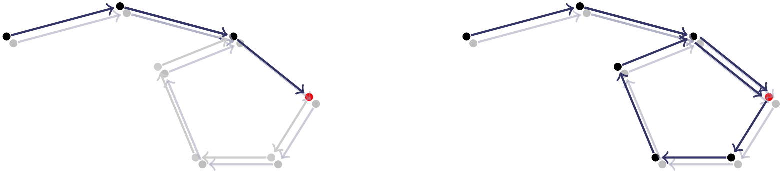


Expected properties of a random mapping over N points:

- # Components: $\frac{1}{2}\log N$
- # Cyclic nodes: $\sqrt{\pi N/2}$
- Tail length: $\sqrt{\pi N/8}$
- Rho length: $\sqrt{\pi N/2}$
- Largest tree: $0.48N$
- Largest component: $0.76N$

Using the cycle length

1. Offline: find the cycle length L of the main component of h_0
2. Online: query $t = \mathrm{MAC}(r \parallel [0]^{2^{\ell/2}})$ and $t' = \mathrm{MAC}(r \parallel [0]^{2^{\ell/2}+L})$

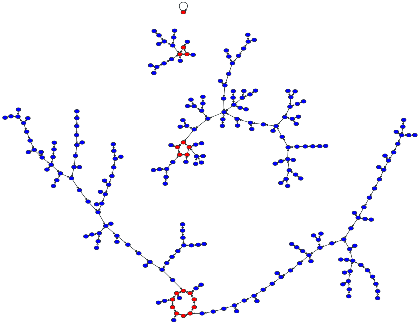


Success if

- The starting point is in the main component $p = 0.76$
- The cycle is reached with less than $2^{\ell/2}$ iterations $p \geq 0.5$

Randomize starting point

Cycle structure



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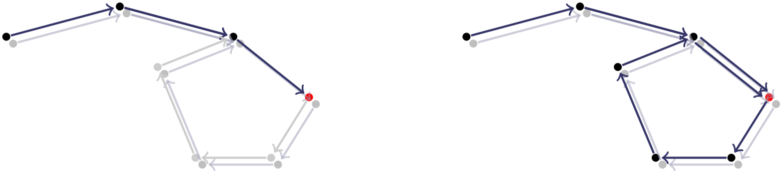

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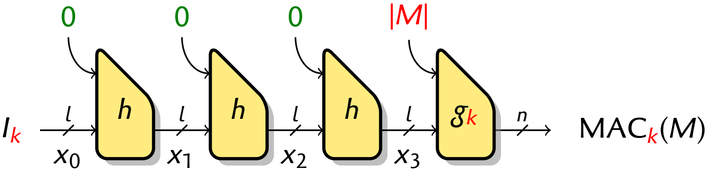
Randomize starting point

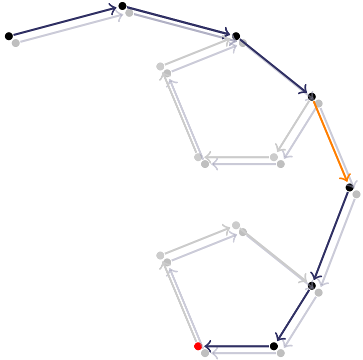
Dealing with the message length

Problem: most MACs use the message length.



Dealing with the message length

Solution: reach the cycle twice



$$M = r \parallel [0]^{2^{\ell/2}} \parallel [1] \parallel [0]^{2^{\ell/2}}$$

Dealing with the message length

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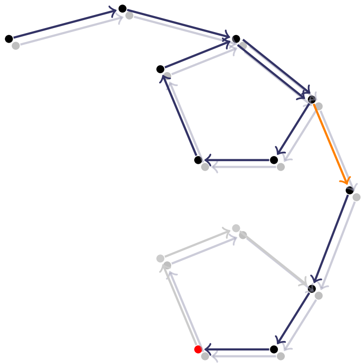
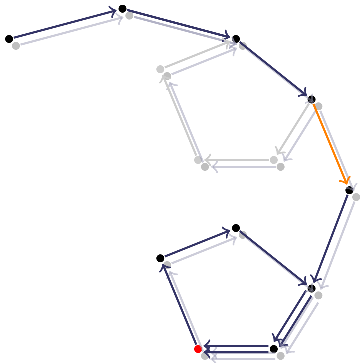




$M_1 = r \parallel [0]^{2^{\ell/2}+L} \parallel [1] \parallel [0]^{2^{\ell/2}}$

$M_2 = r \parallel [0]^{2^{\ell/2}} \parallel [1] \parallel [0]^{2^{\ell/2}+L}$

Distinguishing-H attack

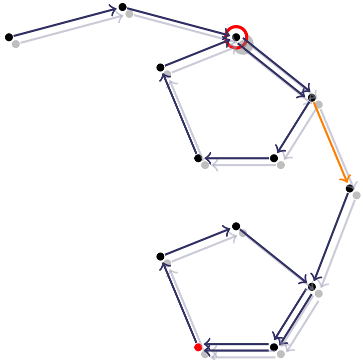
1. Offline: find the cycle length L of the main component of h_0
2. Online: query
$$t = \mathrm{MAC}(r \,\|\, [0]^{2^{\ell/2}} \,\|\, [1] \,\|\, [0]^{2^{\ell/2}+L})$$
$$t' = \mathrm{MAC}(r \,\|\, [0]^{2^{\ell/2}+L} \,\|\, [1] \,\|\, [0]^{2^{\ell/2}})$$
3. If $t = t'$, then h is the compression function in the oracle

Analysis

- Complexity: $2^{\ell/2}$ compression function calls
- Success probability: $p \simeq 0.14$
  - Both starting point are in the main component $p = 0.76^2$
  - Both cycles are reached with less than $2^{\ell/2}$ iterations $p \geq 0.5^2$

State recovery attack

- Consider the first cyclic point
- With high pr., root of the giant tree

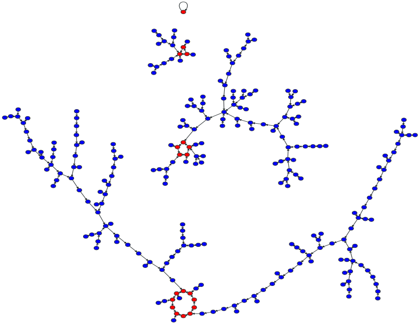


1. **Offline:** find cycle length L, and root of giant tree α
2. **Online:** Binary search for smallest z with collisions: $\mathrm{MAC}(r \,\|\, [0]^{z} \,\|\, [x] \,\|\, [0]^{2^{\ell/2}+L})$, $\mathrm{MAC}(r \,\|\, [0]^{z+L} \,\|\, [x] \,\|\, [0]^{2^{\ell/2}})$
3. State after $r \,\|\, [0]^{z}$ is α (with high pr.)

Analysis

- Complexity $2^{\ell/2} \times \ell \times \log(\ell)$

Cycle structure



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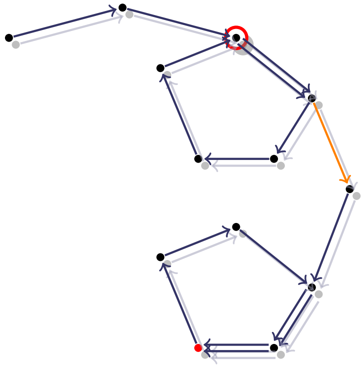

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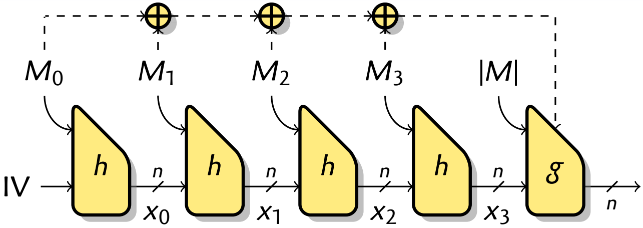
HMAC-GOST

Key recovery

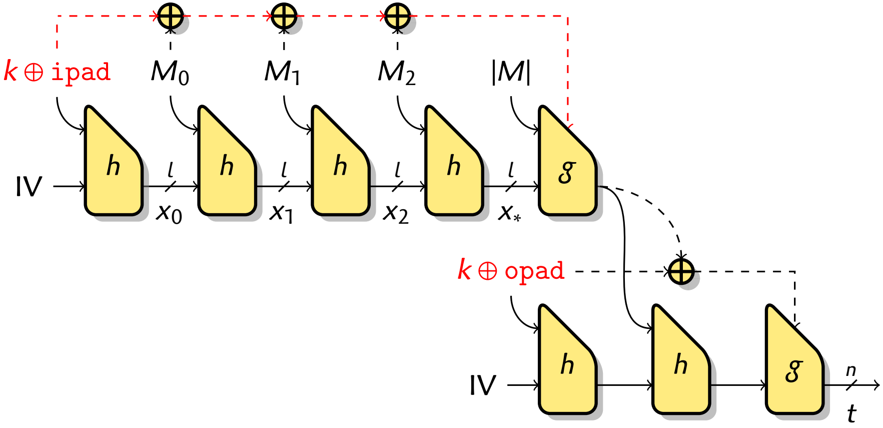
GOST



- Russian standard from 1994
- GOST and HMAC-GOST standardized by IETF
- $n = l = m = 256$

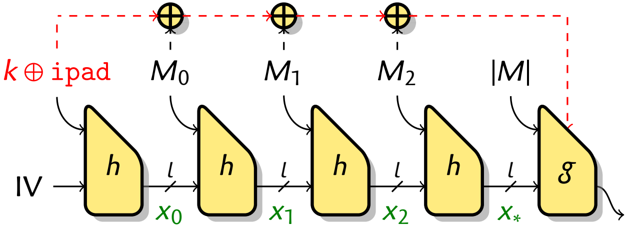
- Checksum (dashed lines)
  - Larger state should increase the security

HMAC-GOST



- In HMAC, key-dependant value used after the message
  - Related-key attacks on the last block

Key recovery attack on HMAC-GOST



1. Recover the state
2. Build a multicollision: $2^{3\ell/4}$ messages with the same x_*
3. Query messages, detect collisions $g(\bar{x}, k \oplus M) = g(\bar{x}, k \oplus M')$

   Store $(M \oplus M', M)$ for $2^{\ell/2}$ collisions
4. Find collisions $g(\bar{x}, y) = g(\bar{x}, y')$ offline

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5. Detect match $M \oplus M' = y \oplus y'$. With high probability $M \oplus k = y$

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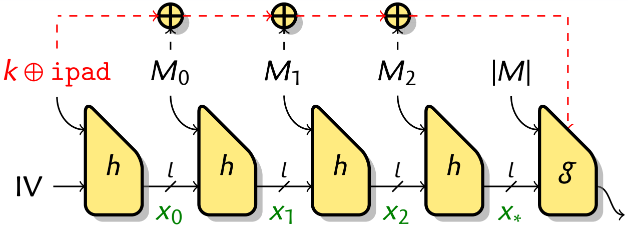

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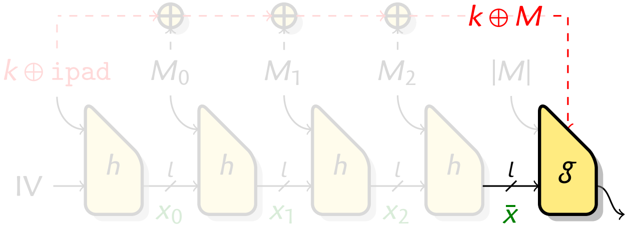

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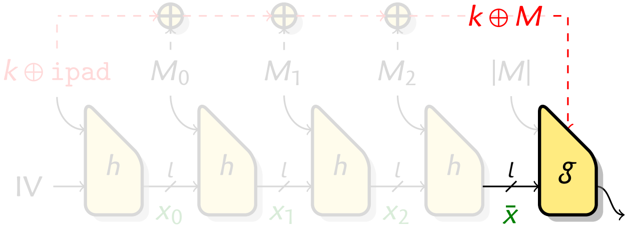

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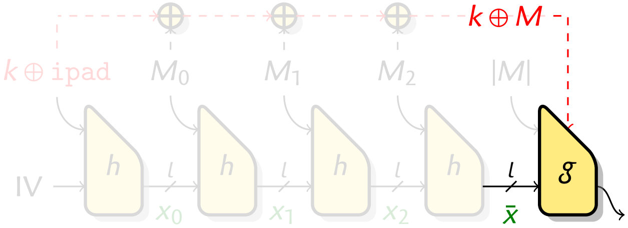

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Conclusion

New generic attacks against hash-based MACs (single-key):

1. Distinguishing-H attack in $2^{l/2}$
   State-recovery attack in $2^{l/2} \times l$
   - Not harder than distinguishing-R.
   - Security proof is tight for these notions.
   - Complexity 2^{l-s} with short messages (length 2^s, $s < l/4$)
2. Key-recovery attack on HMAC-GOST in 2^{192} ($2^{3l/4}$)
   - Generic attack against hash functions with a checksum.
   - The checksum weakens the design!

Open questions:

- What is the generic security of HMAC above the birthday bound?
- Other applications of state-recovery attack?

Thanks

Questions?

With the support of ERC project CRASH

erc European Research Council
Established by the European Commission
Supporting top researchers
from anywhere in the world

Additional slides

Security of HMAC

Extra slides

Construction of hash-based MACs

Challenge-response Authentication

Security Notions

Generic Attacks

Attacks with short messages

Security of HMAC

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Security of HMAC : new results on GOST

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* checksum, and $l = n$

Comparison of attacks on HMAC

Function	Attack	Complexity	M. len	Notes
HMAC-MD5	dist-H, st. rec.	2^{97}	2	
HMAC-SHA-0	dist-H	2^{100}	2	
HMAC-HAVAL (3-pass)	dist-H	2^{228}	2	
HMAC-SHA-1 62 mid. steps	dist-H	2^{157}	2	
Generic	dist-H, st. rec.	$\tilde{O}(2^{l/2})$	$2^{l/2}$	
	dist-H, st. rec.	$O(2^{l-s})$	2^{s}	$s \leq l/4$
Generic: checksum	key recovery	$O(2^{3l/4})$	$2^{l/4}$	
HMAC-MD5*	dist-H, st. rec.	$2^{66}, 2^{78}$	2^{64}	
		$O(2^{96})$	2^{32}	
HMAC-HAVAL§ (any)	dist-H, st. rec.	$O(2^{202})$	2^{54}	
HMAC-SHA-1§	dist-H, st. rec.	$O(2^{120})$	2^{40}	
HMAC-GOST*	key-recovery	2^{200}	2^{64}	

* MD5, GOST: arbitrary-length; § SHA-1, HAVAL: limited message length.

Hash-based MACs

- Secret-prefix MAC: $\mathrm{MAC}_k(M) = H(k \,\|\, M)$
  - Insecure with MD/SHA: length-extension attack
  - Compute $\mathrm{MAC}_k(M \,\|\, P)$ from $\mathrm{MAC}_k(M)$ without the key
- Secret-suffix MAC: $\mathrm{MAC}_k(M) = H(M \,\|\, k)$
  - Can be broken using offline collisions
- Use the key at the beginning and at the end
  - Sandwich-MAC: $H(k_1 \,\|\, M \,\|\, k_2)$
  - NMAC: $H(k_2 \,\|\, H(k_1 \,\|\, M))$
  - HMAC: $H((k \oplus \mathtt{opad}) \,\|\, H((k \oplus \mathtt{ipad}) \,\|\, M))$
  - Security proofs

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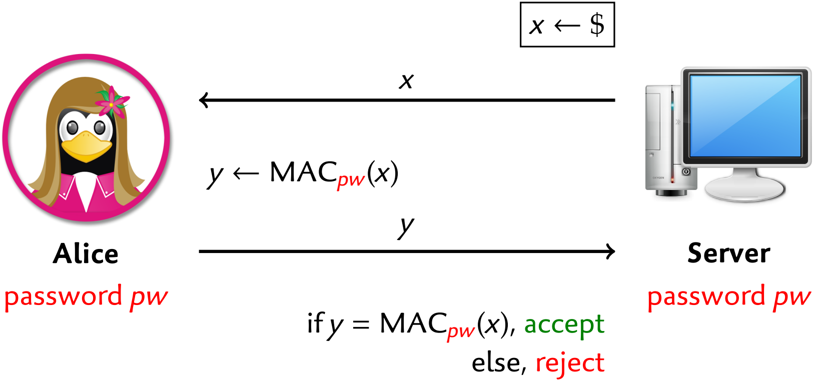
Hash-based MACs

- Secret-prefix MAC: $\mathrm{MAC}_k(M) = H(k \parallel M)$
  - Insecure with MD/SHA: length-extension attack
  - Compute $\mathrm{MAC}_k(M \parallel P)$ from $\mathrm{MAC}_k(M)$ without the key
- Secret-suffix MAC: $\mathrm{MAC}_k(M) = H(M \parallel k)$
  - Can be broken using offline collisions
- Use the key at the beginning and at the end
  - Sandwich-MAC: $H(k_1 \parallel M \parallel k_2)$
  - NMAC: $H(k_2 \parallel H(k_1 \parallel M))$
  - HMAC: $H((k \oplus \mathtt{opad}) \parallel H((k \oplus \mathtt{ipad}) \parallel M))$
  - Security proofs

Example use: challenge-response authentication



$\boxed{x \leftarrow \$}$

x

$y \leftarrow \mathrm{MAC}_{pw}(x)$

y

Alice

password *pw*

Server

password *pw*

if $y = \mathrm{MAC}_{pw}(x)$, accept

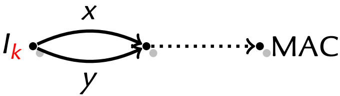
else, reject

- CRAM-MD5 authentication in SASL, POP3, IMAP, SMTP, ...

Security notions

- Key-recovery: given access to a MAC oracle, extract the key
- Forgery: given access to a MAC oracle, forge a valid pair
  - For a message chosen by the adversary: existential forgery
  - For a challenge given to the adversary: universal forgery
- Distinguishing games for hash-based MACs:
  - Distinguish $\mathrm{MAC}_k^{\mathcal{H}}$ from a PRF: distinguishing-R
    e.g. distinguish HMAC from a PRF
  - Distinguish $\mathrm{MAC}_k^{\mathcal{H}}$ from $\mathrm{MAC}_k^{\mathrm{PRF}}$: distinguishing-H
    e.g. distinguish HMAC-SHA1 from HMAC-PRF

Generic Attack on Hash-based MACs

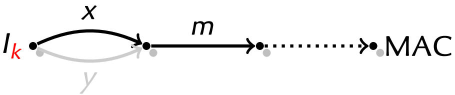


1. Find internal collisions
   - Query $2^{l/2}$ 1-block messages
   - 1 internal collision expected, detected in the output

2. Query $t = \mathrm{MAC}(x \,\|\, m)$

3. $\big(y \,\|\, m, t\big)$ is a forgery

Generic Attack on Hash-based MACs



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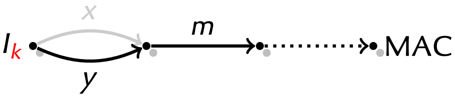

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   - Query $2^{l/2}$ 1-block messages
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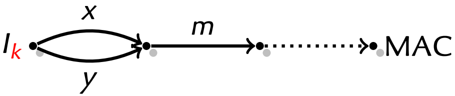
Generic Attack on Hash-based MACs



1. Find internal collisions
   - Query $2^{l/2}$ 1-block messages
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2. Query $t = \mathrm{MAC}(x \parallel m)$ and $t' = \mathrm{MAC}(y \parallel m)$

3. If $t = t'$ the oracle is a hash-based MAC: distinguishing-R

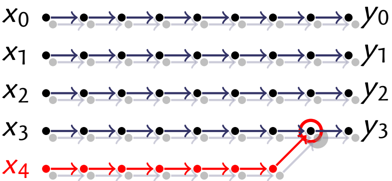
Variant with small messages

- Messages of length $2^{\ell/2}$ are not very practical...
  - SHA-1 and HAVAL limit the message length to 2^{64} bits
- Cycle detection impossible with messages shorter than $L \approx 2^{\ell/2}$

Compare with collision finding algorithms

- Pollard's rho algorithm use cycle detection
- Parallel collision search for van Oorschot and Wiener uses shorter chains

Collision finding with small chains



1. Compute chains $x \rightsquigarrow y$
   Stop when y distinguished
2. If $y \in \{y_i\}$, collision found

Using collisions for state recovery

- Collision points are not random
- Longer chains give more biased distribution
- Precompute collisions offline, and test online