How to Construct an Ideal Cipher from a Small Set of Public Permutations

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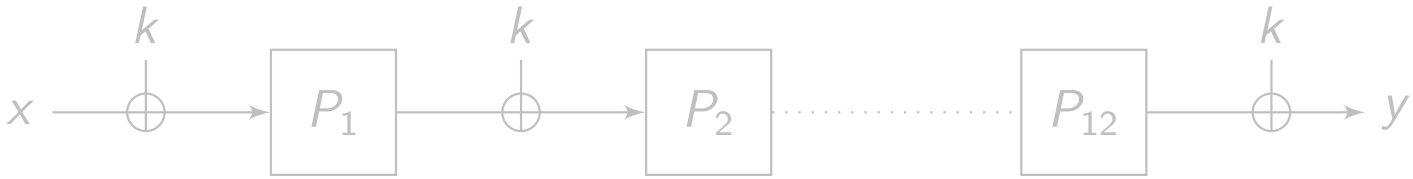
Summary

- We show how to construct an ideal cipher from a small set of n-bit public random permutations $\{P_1, \ldots, P_r\}$
- The construction we consider is the single-key iterated Even-Mansour cipher (*aka* key-alternating cipher) with 12 rounds:



$\Rightarrow$ this yields a family of 2^n permutations indexed by the n-bit key k from only 12 public n-bit permutations

- We show that this construction "behaves" as an ideal cipher with n-bit blocks and n-bit keys using the indifferentiability framework
- We also show that at least 4 rounds are necessary to achieve indifferentiability from an ideal cipher

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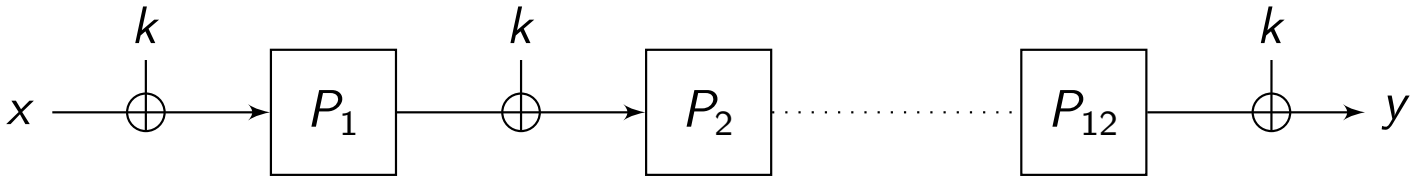

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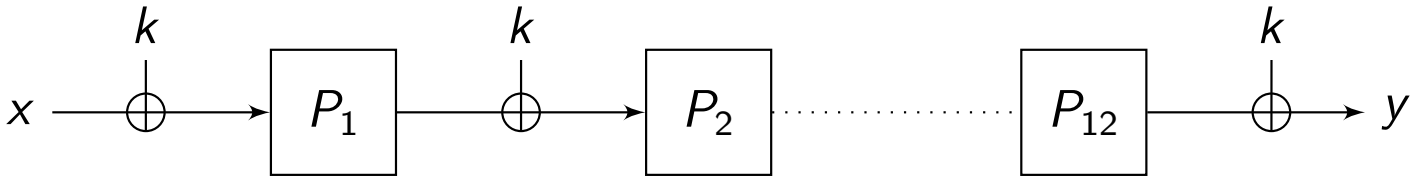

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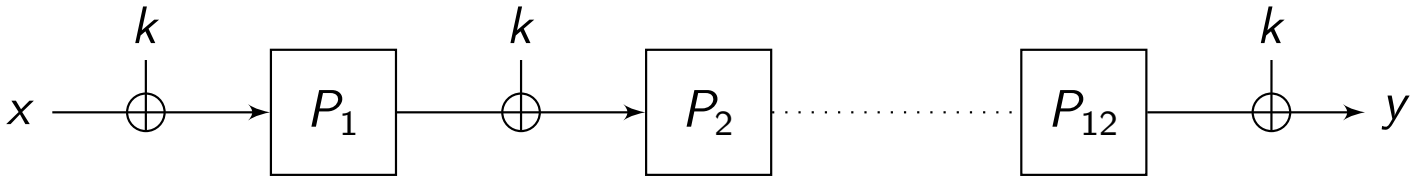

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Outline

1. Background on the Iterated Even-Mansour Cipher

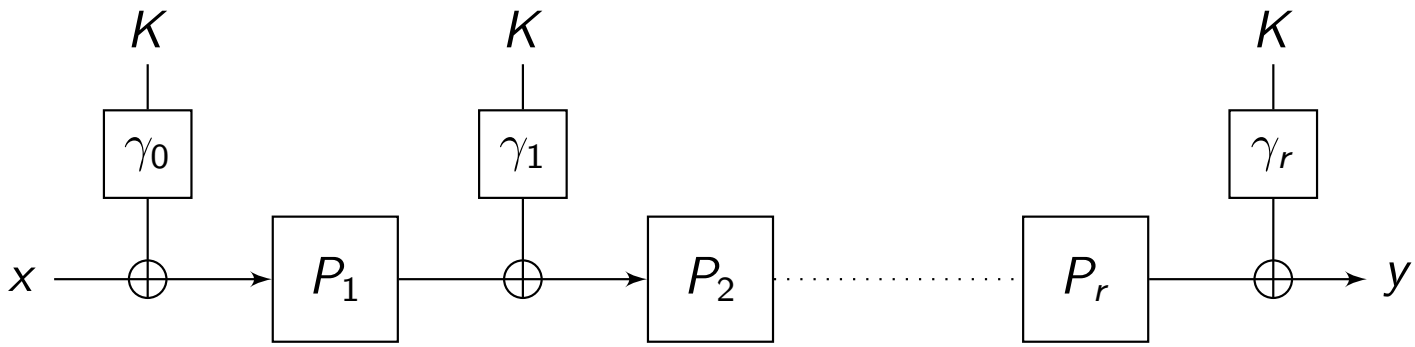
2. Indifferentiability of the IEM cipher
   - Formalizing the problem
   - Which key schedule?
   - At least 4 rounds are necessary

3. Indifferentiability proof for 12 rounds

Outline

Iterated Even-Mansour cipher (*aka* key-alternating cipher)

Iterated Even-Mansour (IEM) with r rounds:



- The P_i's are public permutations on $\{0,1\}^n$
- $K \in \{0,1\}^\ell$ is the (master) key
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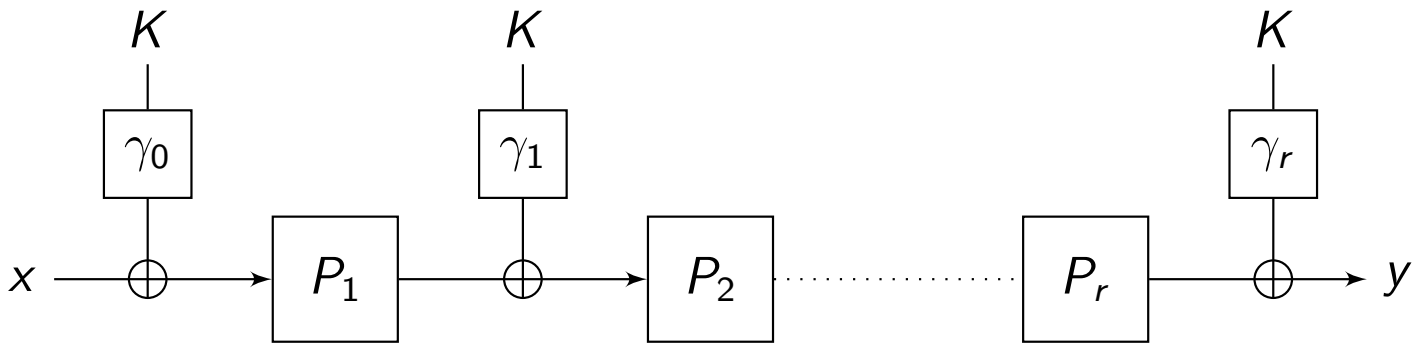

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Most (if not all) SPN ciphers can be described as key-alternating ciphers. E.g. for AES-128, one has $r = 10$, the γ_i's are efficiently invertible permutations, and:

$$P_1 = \ldots = P_9 = \texttt{SubBytes} \circ \texttt{ShiftRows} \circ \texttt{MixColumns}$$
$$P_{10} = \texttt{SubBytes} \circ \texttt{ShiftRows}$$

When the P_i's are fixed permutations, one can prove results like:

- the best differential characteristic over $r' < r$ rounds has probability at most p
- the best linear approximation over $r' < r$ rounds has probability at most p'

This gives upper bounds on the distinguishing probability of very specific adversaries

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Analysis in the Random Permutation Model (RPM)

Recently, a lot of results have been obtained in the Random Permutation Model: the P_i's are viewed as oracles to which the adversary can make black-box queries (both to P_i and P_i^{-1}).

Interpretation: gives a guarantee against any adversary which does not use particular properties of the P_i's

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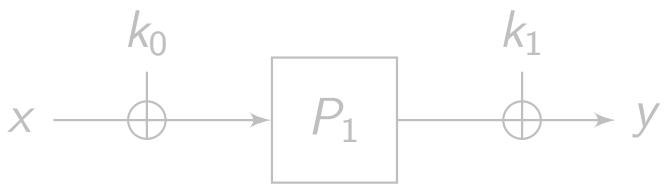

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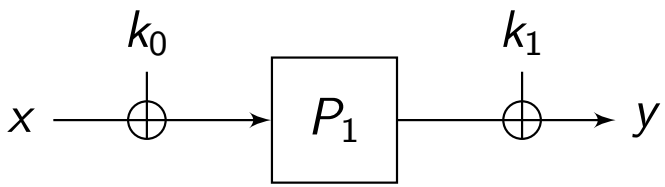

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- for any even r, security up to $\mathcal{O}(N^{\frac{r}{r+2}})$ queries [LPS12]
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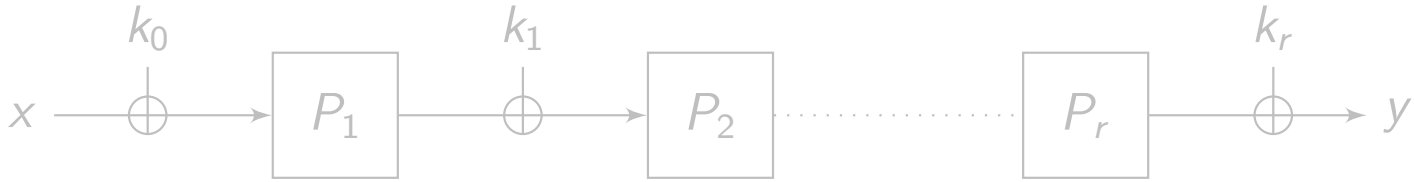

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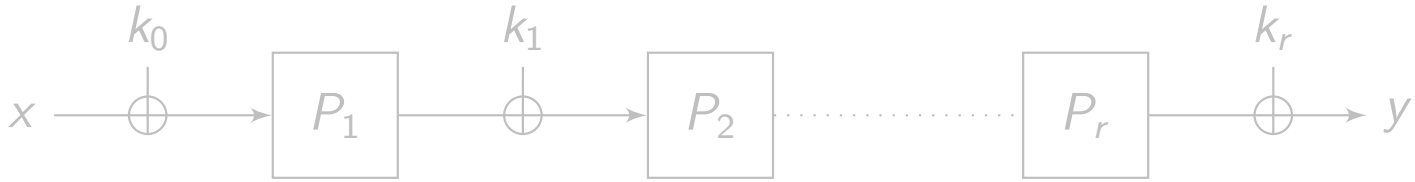

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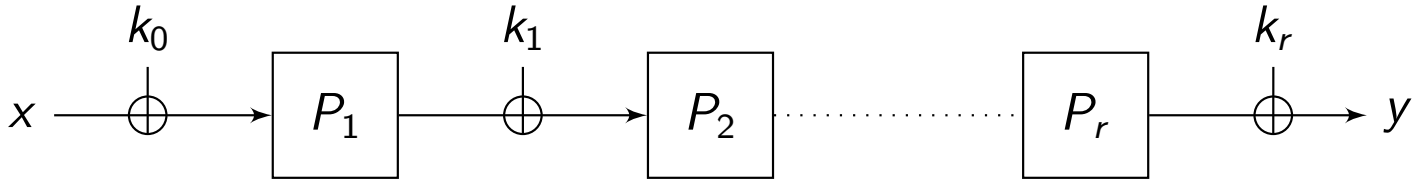

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Previous results state that the IEM cipher is a (strong) pseudorandom permutation (in the random permutation model)
= usual single, secret-key security model

Question

What about related-, known- or chosen-key attacks?
Can we even hope to prove that the IEM "behaves" as (*is indifferentiable from*) an ideal cipher?

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- the pseudorandomness security notion for a block cipher is sufficient to prove the security of a lot of applications (encryption modes and MACs)
- however, sometimes it is not sufficient (e.g. for block cipher-based hash functions like Davies-Meyer mode)
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- similar to the random oracle model for a hash function
- warning: instantiation problems as well (no concrete block cipher can be proved to be an ideal cipher in any reasonable sense)
- though we cannot prove that a block cipher behaves as an ideal cipher in the standard model, we can prove results in idealized models (e.g. the Random Permutation Model in the case of the IEM cipher)
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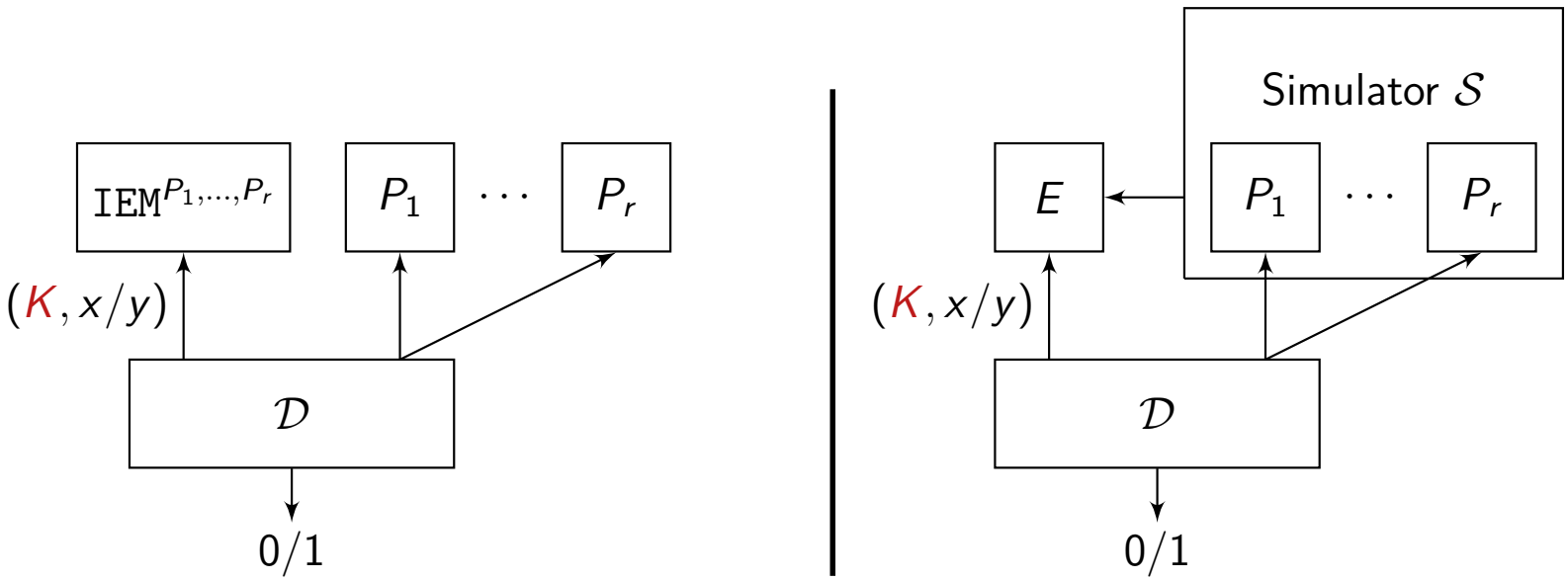
Indifferentiability: definition

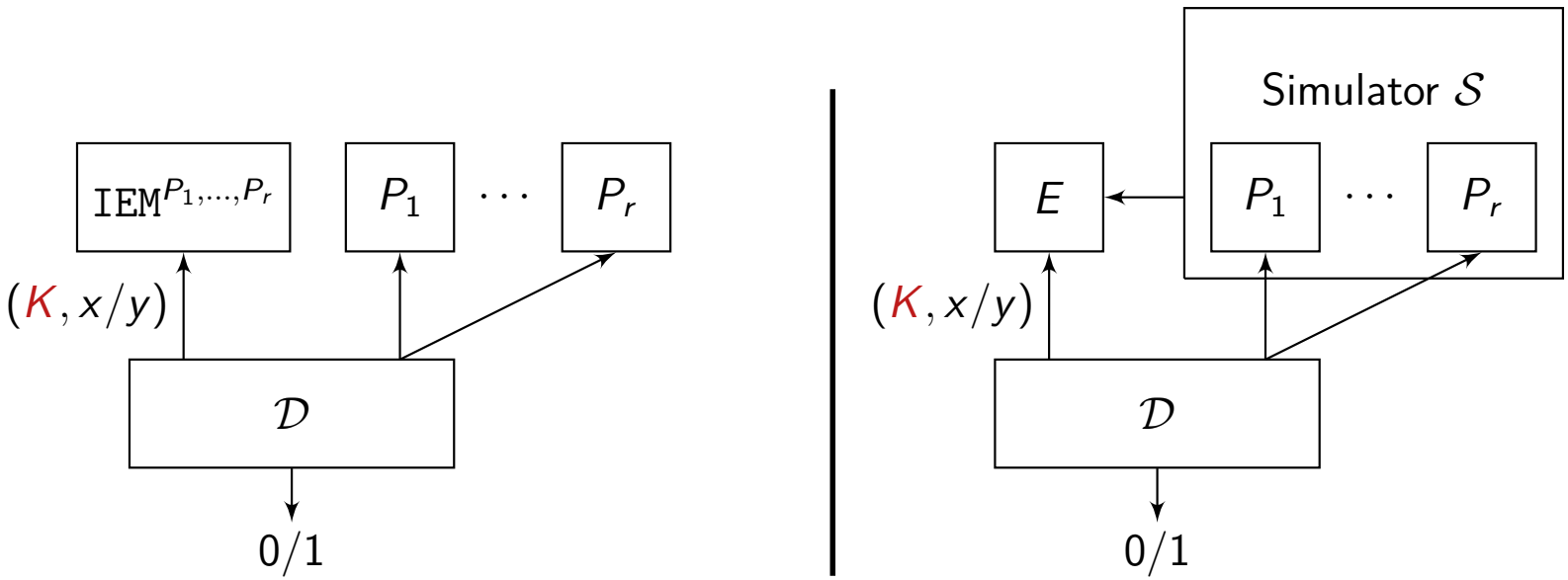
Definition

The IEM cipher $\mathtt{IEM}^{P_1,\ldots,P_r}$ with random permutations $\boldsymbol{P} = (P_1, \ldots, P_r)$ is said indifferentiable from an ideal cipher E if there exists a polynomial time simulator $\mathcal{S}$ with oracle access to E such that the two systems $(\mathtt{IEM}^{\boldsymbol{P}}, \boldsymbol{P})$ and $(E, \mathcal{S}^E)$ are indistinguishable.



Indifferentiability: definition

NB: The distinguisher specifies the plaintext/ciphertext and the key when querying $\mathtt{IEM}^{P_1,\ldots,P_r}$ or E.



The answers of the simulator $\mathcal{S}$ must be:

- coherent with answers the distinguisher can obtain directly from E
- close in distribution to the answers of random permutations

Composition theorem

Usefulness of indifferentiability: composition theorem

Theorem

If a cryptosystem Γ is secure when used with an ideal cipher E, and if $\mathtt{IEM}^{P_1,\ldots,P_r}$ (for sufficiently many rounds) is indifferentiable from E, then Γ is also secure when used with $\mathtt{IEM}^{P_1,\ldots,P_r}$ with random permutations $P_1, \ldots, P_r$ (for single-stage security notions).

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Is the Iterated Even-Mansour cipher, for sufficiently many rounds, and with an adequate key schedule, indifferentiable from an ideal cipher?

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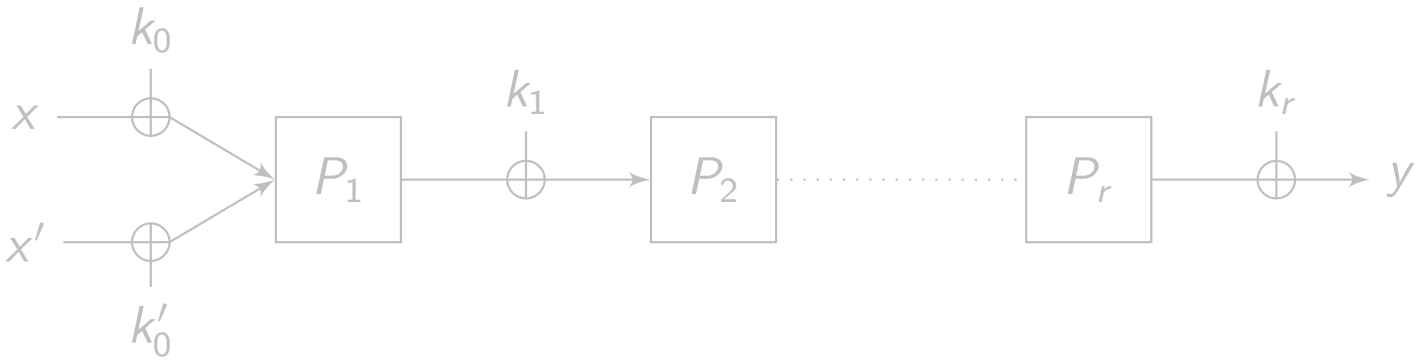

IEM with independent round keys is not indifferentiable from an ideal cipher with key space $\{0,1\}^{(r+1)n}$ because of the following distinguisher:

- choose an arbitrary $x \in \{0,1\}^n$ and $k_0 \in \{0,1\}^n$
- define $x' = x \oplus c$ and $k'_0 = k_0 \oplus c$ with c a non-zero constant
- let $K = (k_0, k_1, \ldots, k_r)$ and $K' = (k'_0, k_1, \ldots, k_r)$
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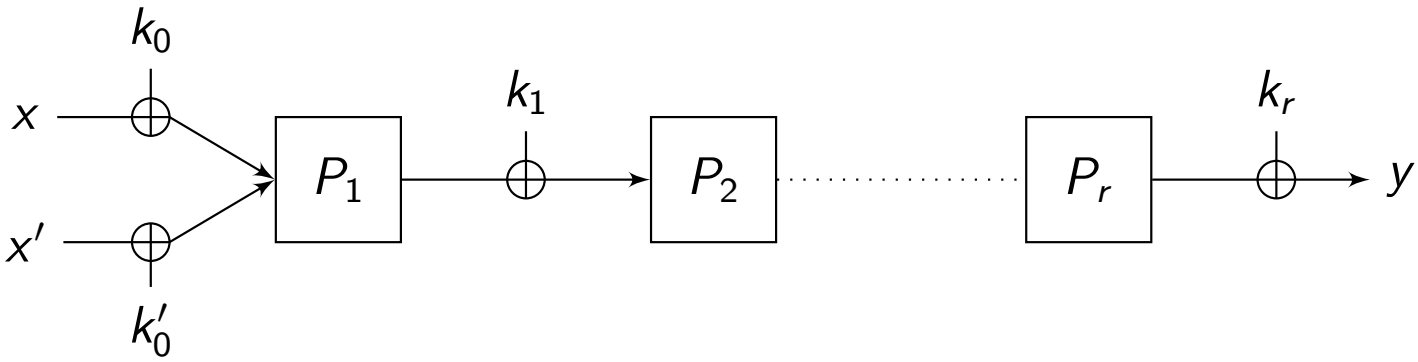

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Proving indifferentiability for the IEM cipher

Independent keys leave too much "freedom" to the adversary.

Two ideas to solve the problem:

1. add a key schedule, and put some cryptographic assumption on it
   $\Rightarrow$ Andreeva et al. CRYPTO 2013 [ABD+13]
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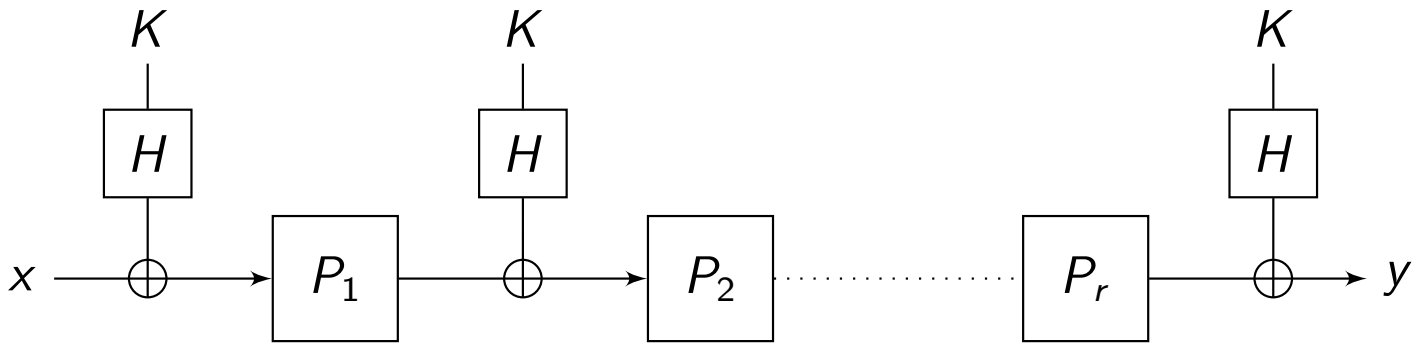

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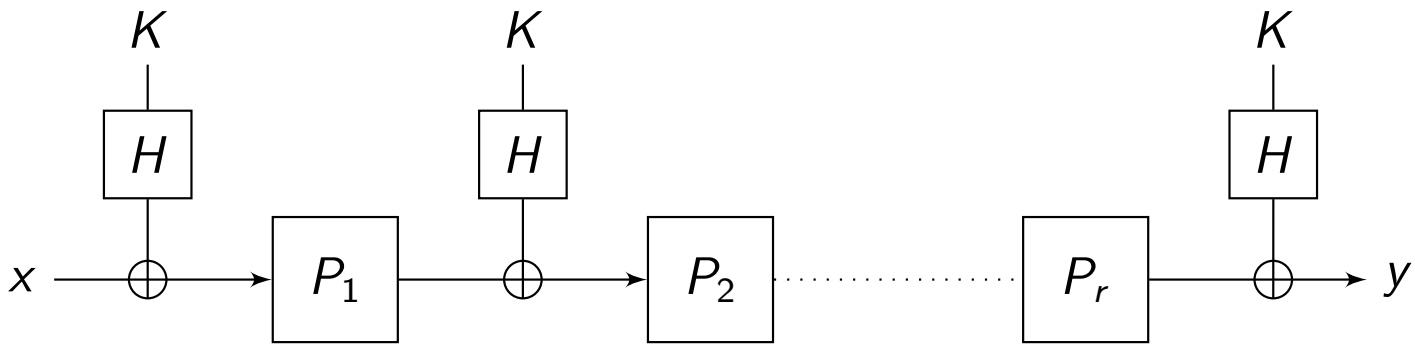

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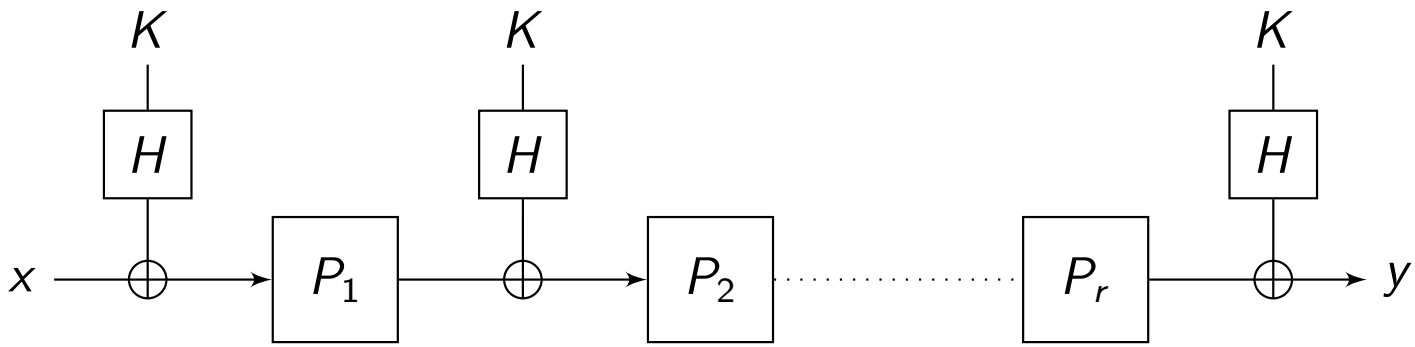

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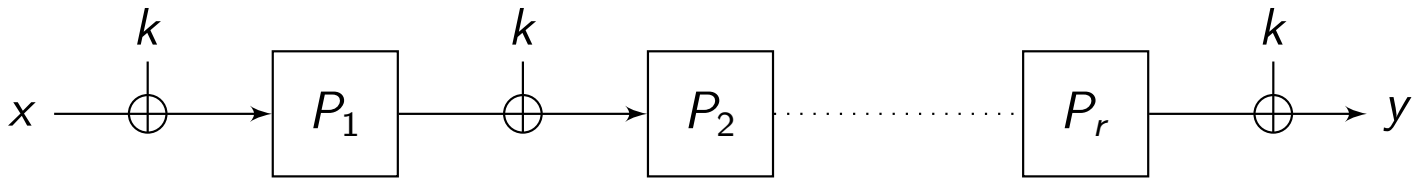

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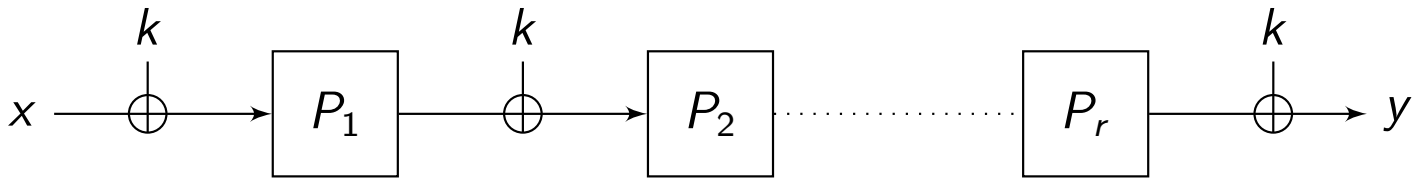

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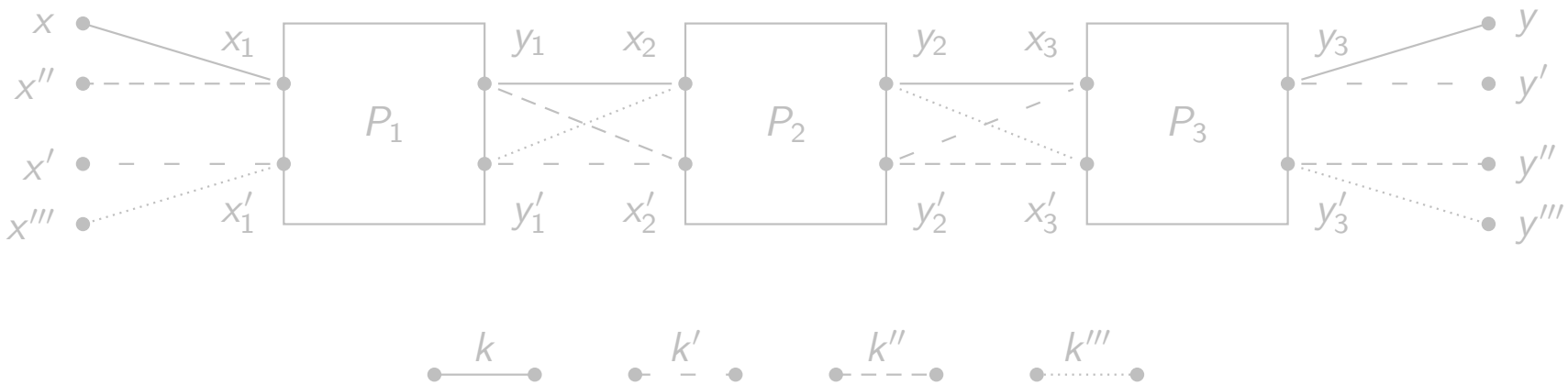

One can (easily) find (x, x', x'', x'''), (y, y', y'', y''') and (k, k', k'', k''') such that $y = \mathtt{IEM}^{(P_1,P_2,P_3)}(k, x)$, etc. and:

$$\begin{cases} k \oplus k' \oplus k'' \oplus k''' = 0 \\ x \oplus x' \oplus x'' \oplus x''' = 0 \\ y \oplus y' \oplus y'' \oplus y''' = 0 \end{cases}.$$

Finding such values can be showed to be hard for an ideal cipher.

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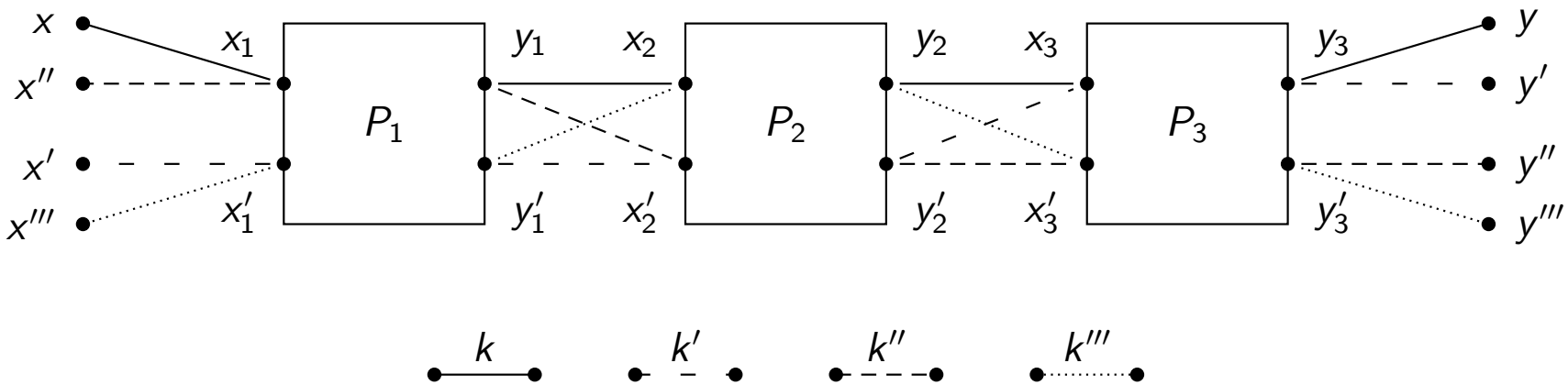

One can (easily) find (x, x', x'', x'''), (y, y', y'', y''') and (k, k', k'', k''') such that $y = \mathtt{IEM}^{(P_1,P_2,P_3)}(k, x)$, etc. and:

$$\begin{cases} k \oplus k' \oplus k'' \oplus k''' = 0 \\ x \oplus x' \oplus x'' \oplus x''' = 0 \\ y \oplus y' \oplus y'' \oplus y''' = 0 \end{cases}.$$

Finding such values can be showed to be hard for an ideal cipher.

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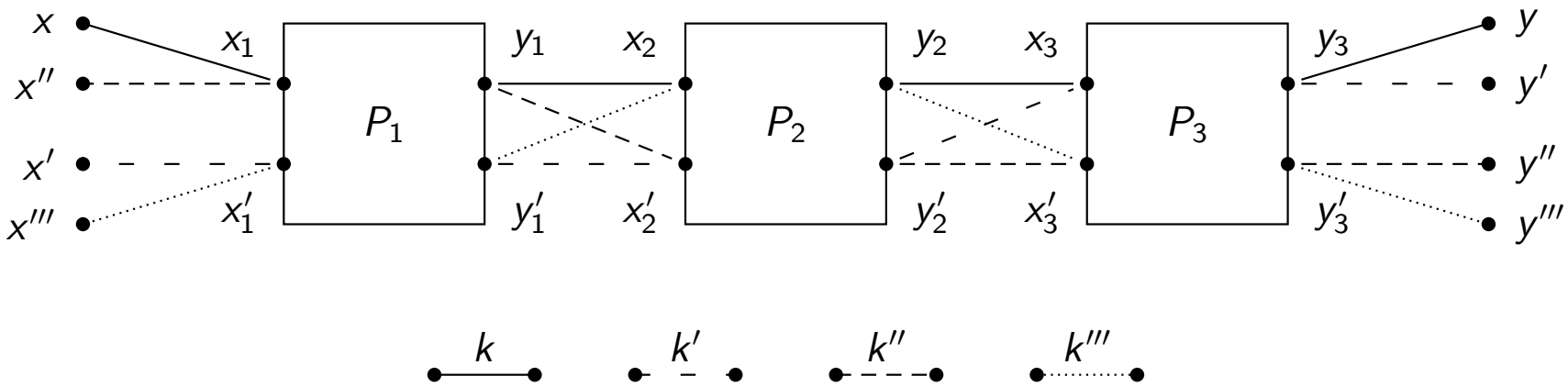

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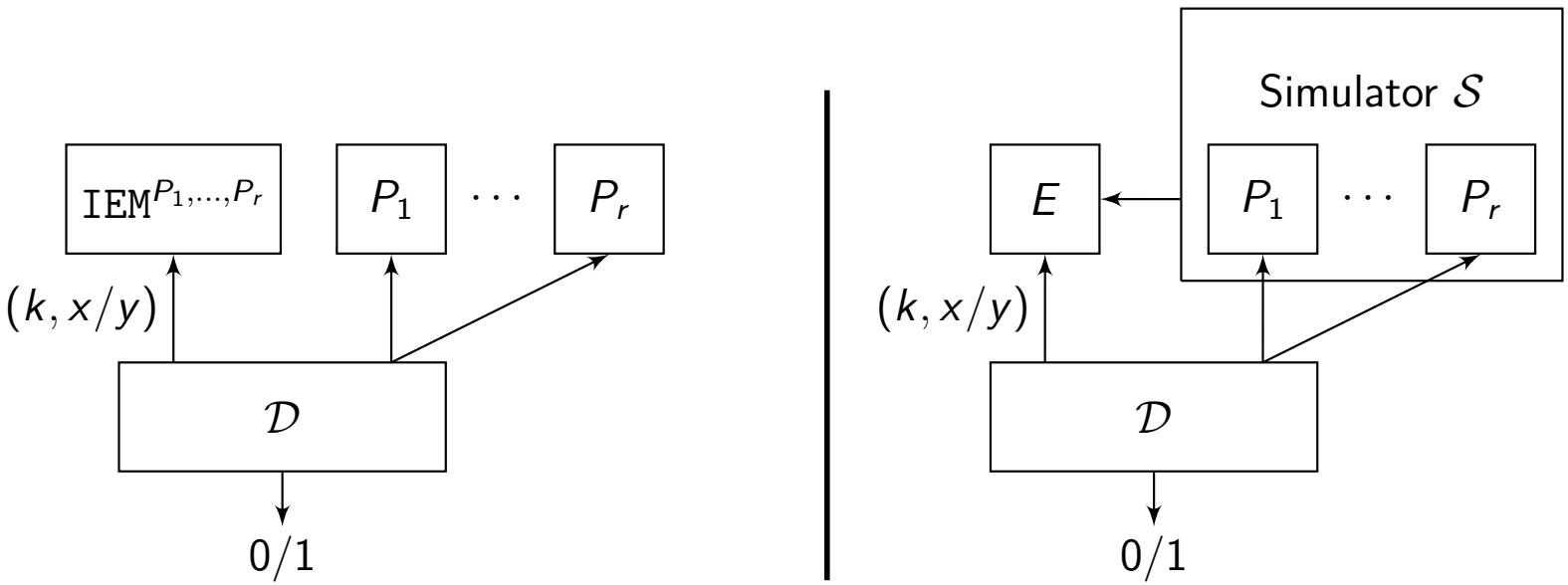
Outline

Reminder: the indifferentiability setting



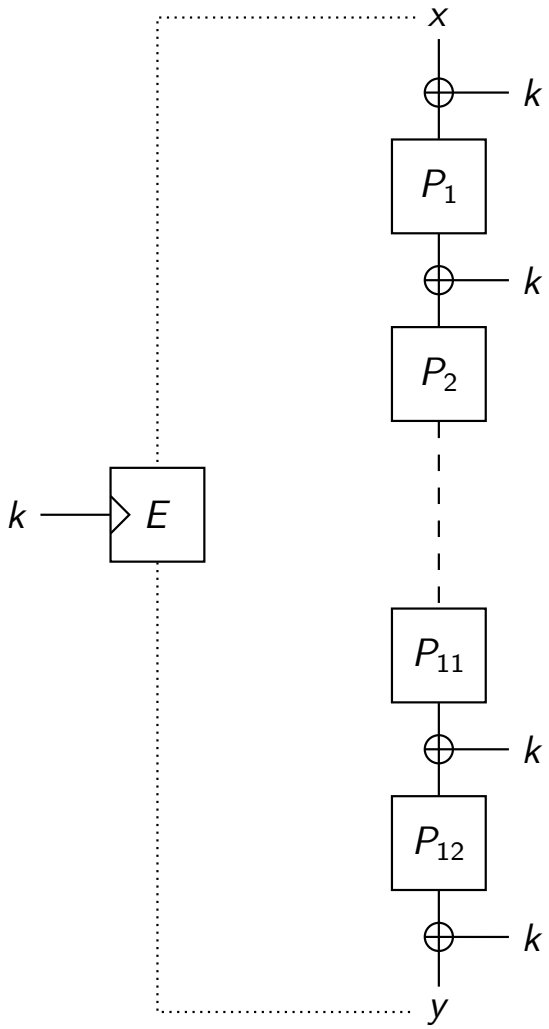
Simulation: general strategy

The simulator must return answers that are coherent with what the distinguisher can obtain from the ideal cipher E, i.e.:

$$\mathtt{IEM}^{P_1,\ldots,P_{12}}(k,x) = E(k,x)$$

For this, the simulator must adapt at least one permutation to "match" what is given by the ideal cipher.

The general strategy is close to the one used for the indifferentiability of the Feistel permutation [CPS08, HKT11].



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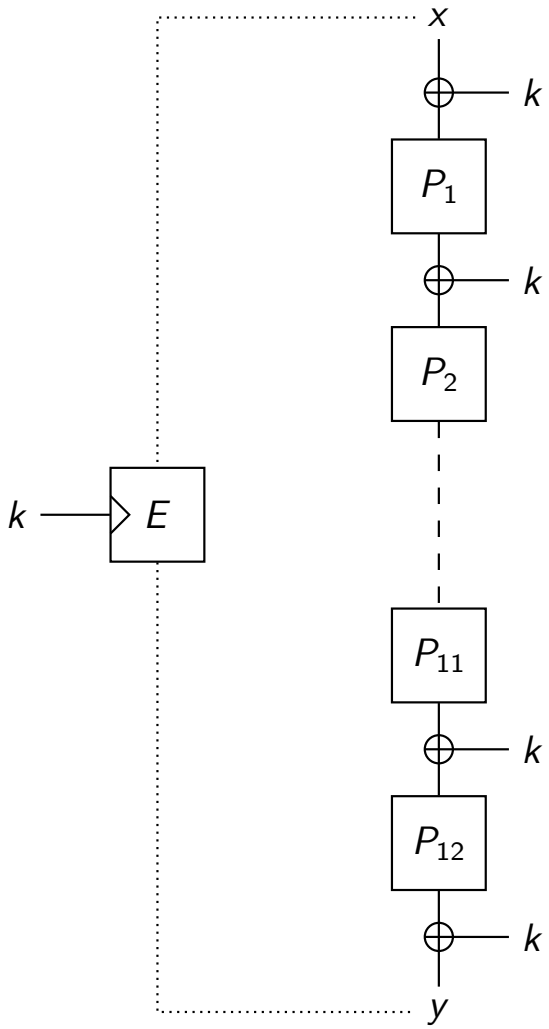

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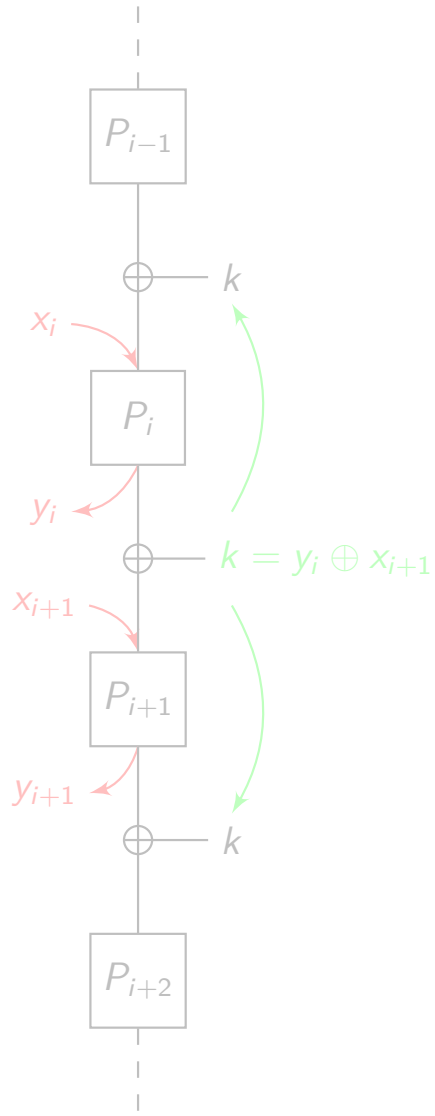
- the simulator maintains an history for each simulated permutation P_i
- the simulator detects and completes "partial chains" = queries to two adjacent perm. $P_i(x_i) = y_i$ and $P_{i+1}(x_{i+1}) = y_{i+1}$
- for any partial chain the key is uniquely defined: $k = y_i \oplus x_{i+1}$
- queries to any two consecutive permutations uniquely define the computations path in the construction (not true for independent keys!)



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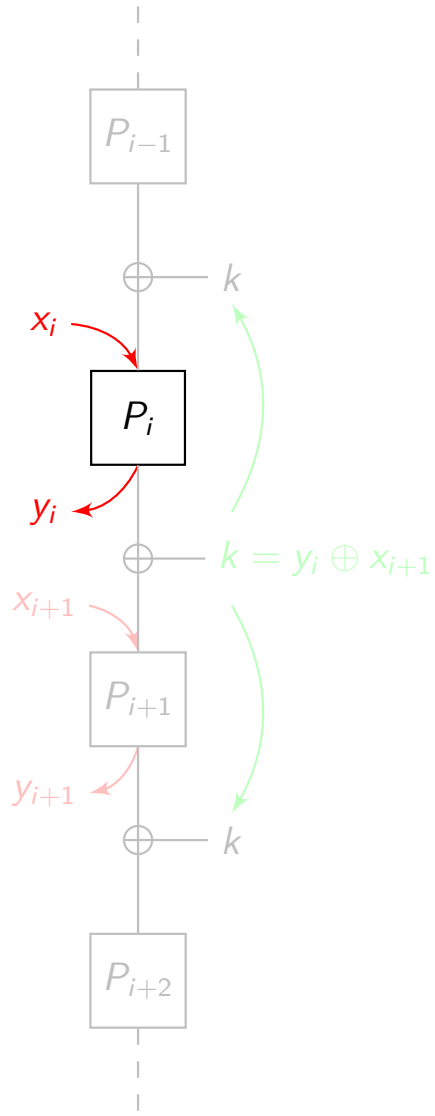

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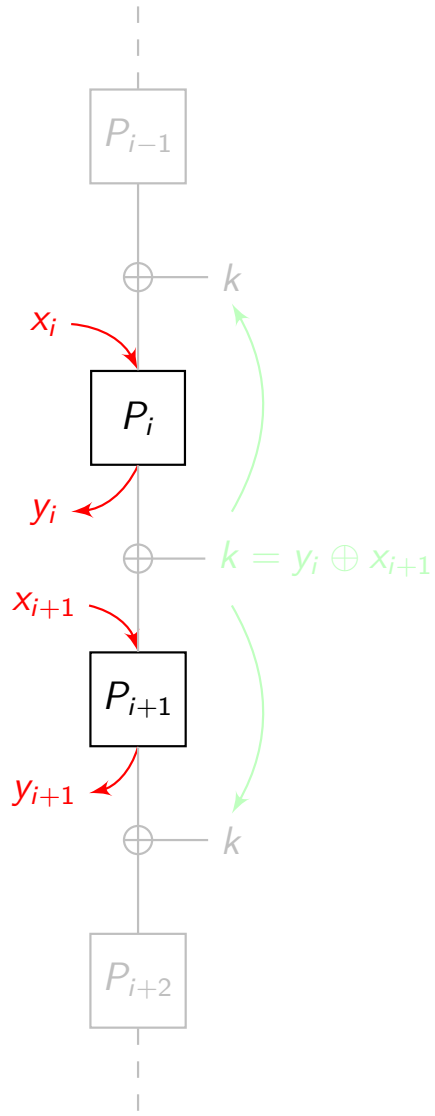

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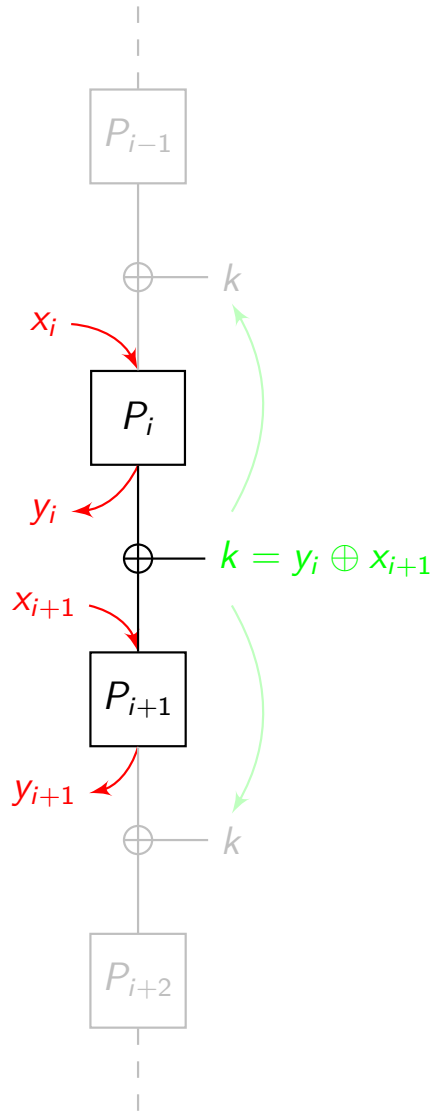

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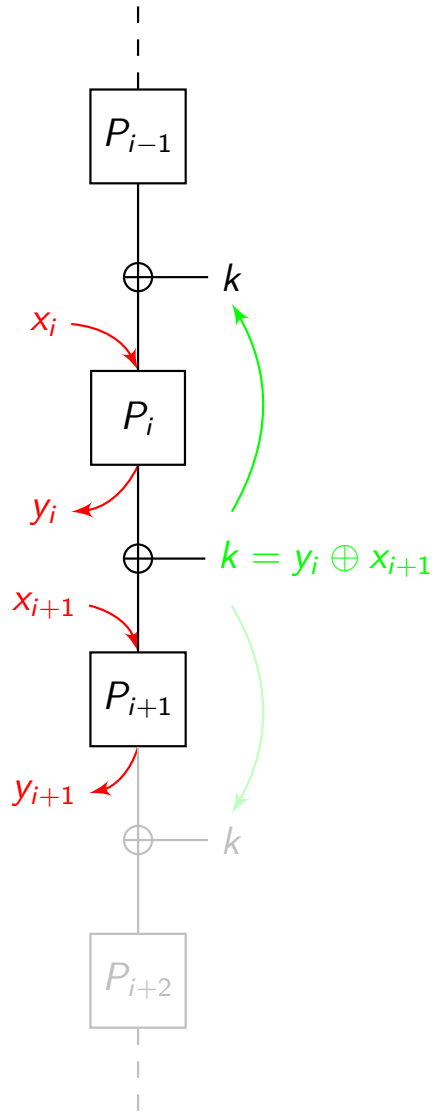

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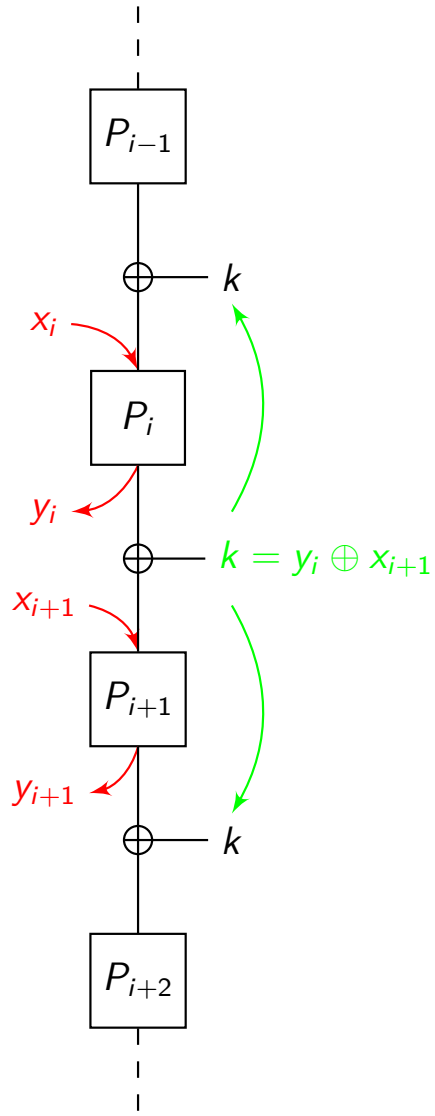

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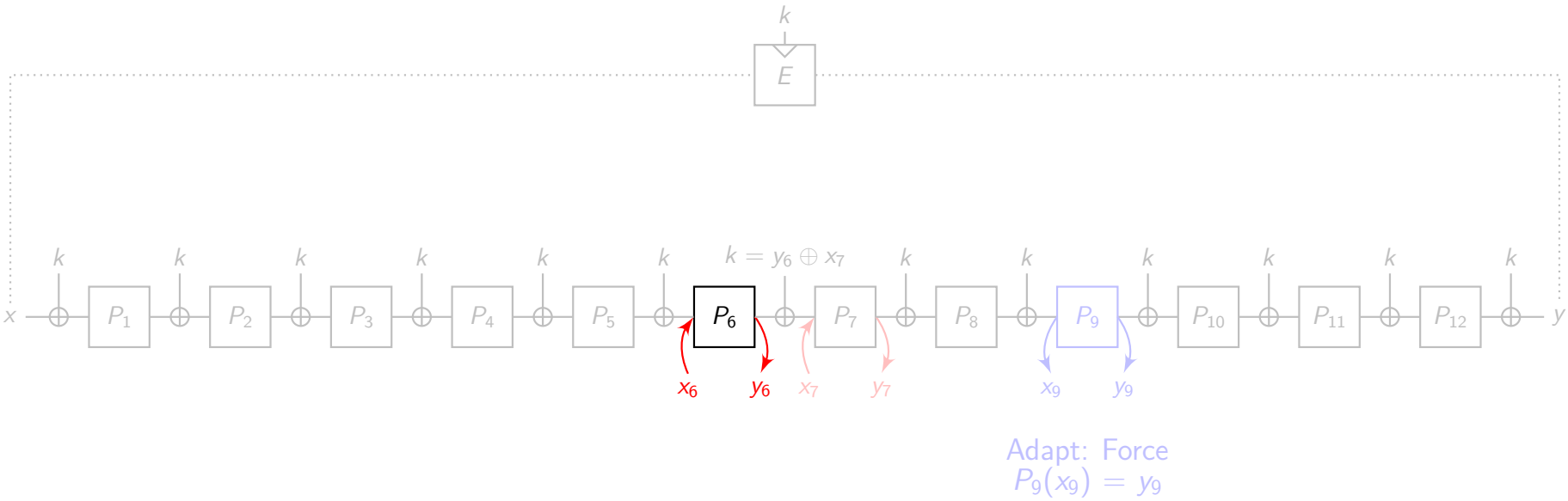

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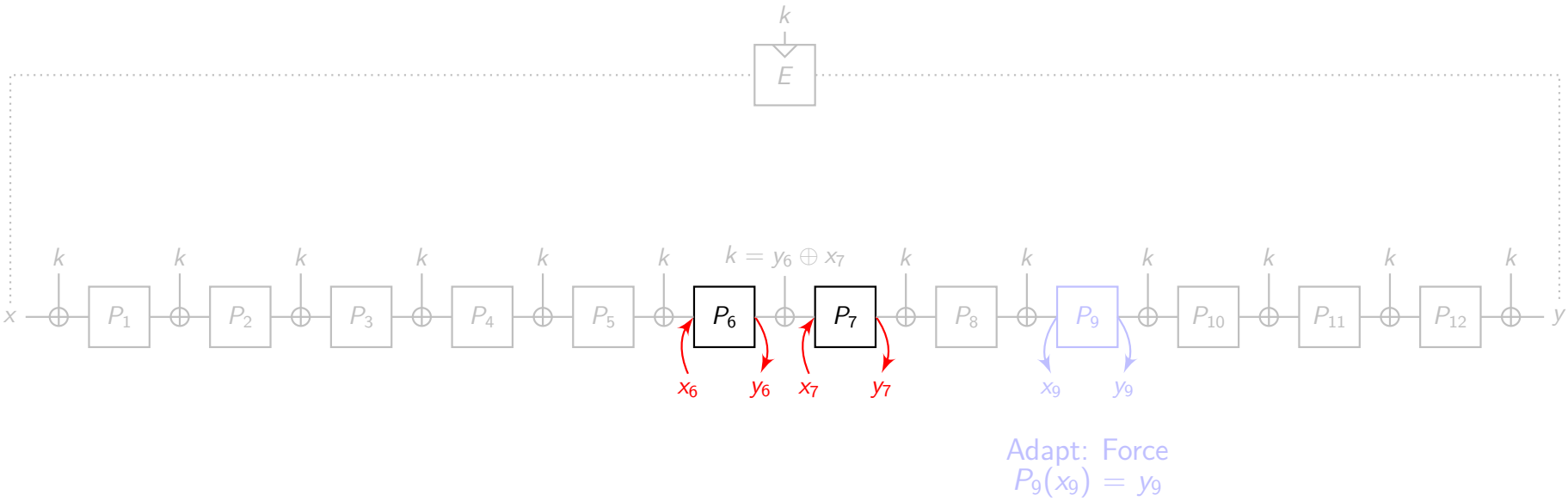

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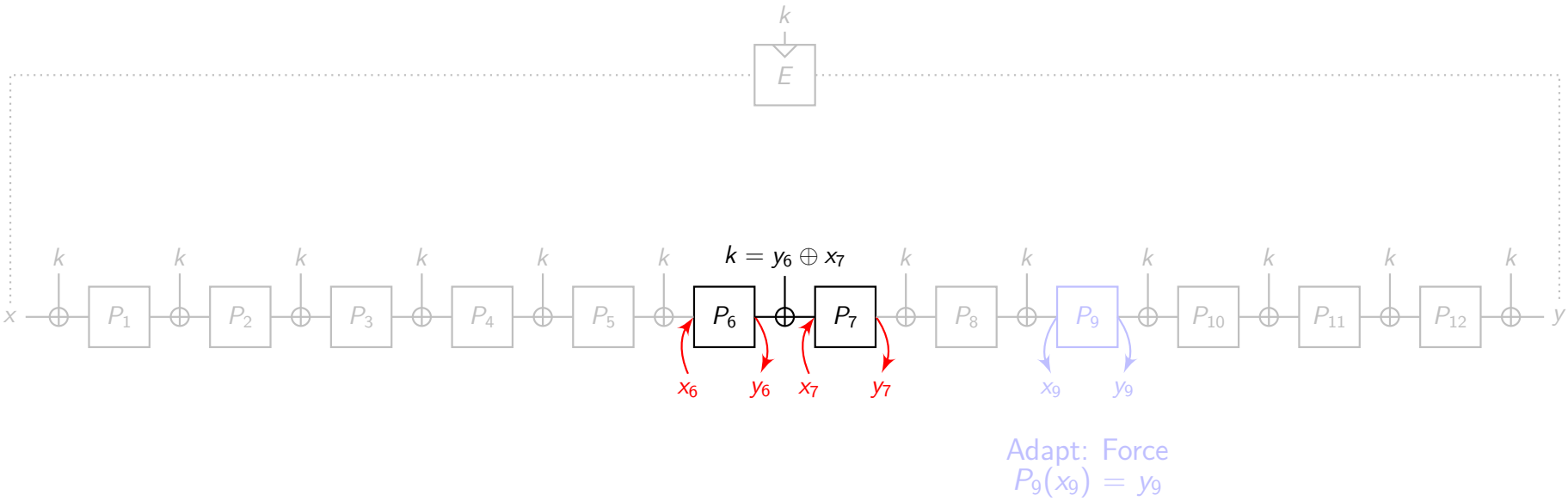

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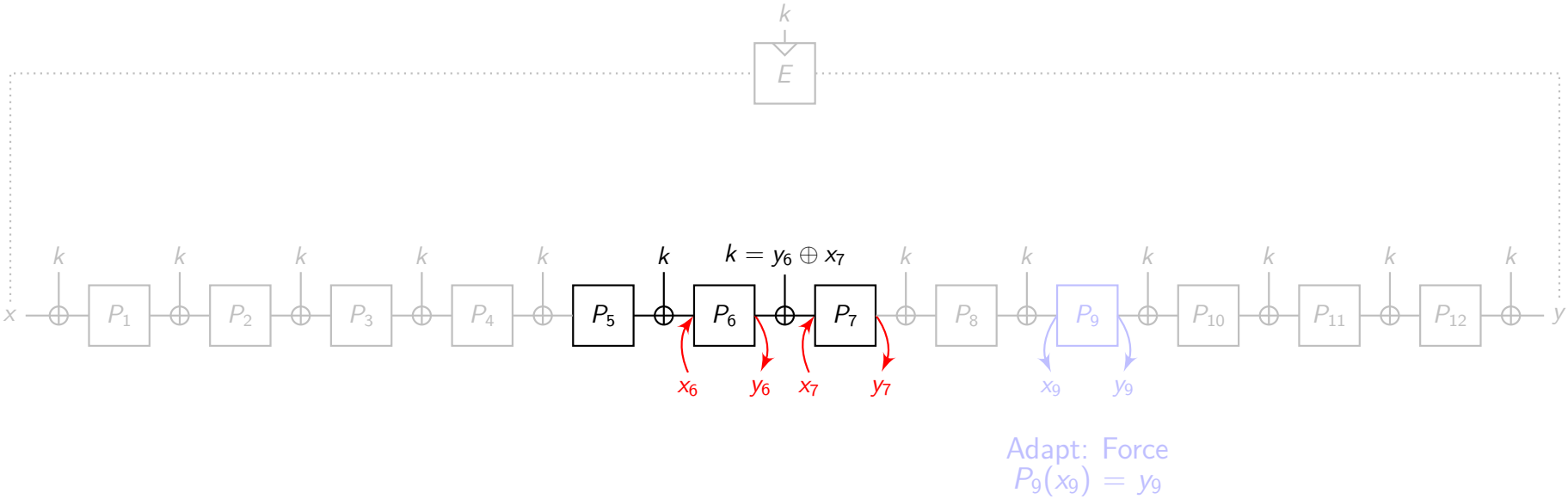

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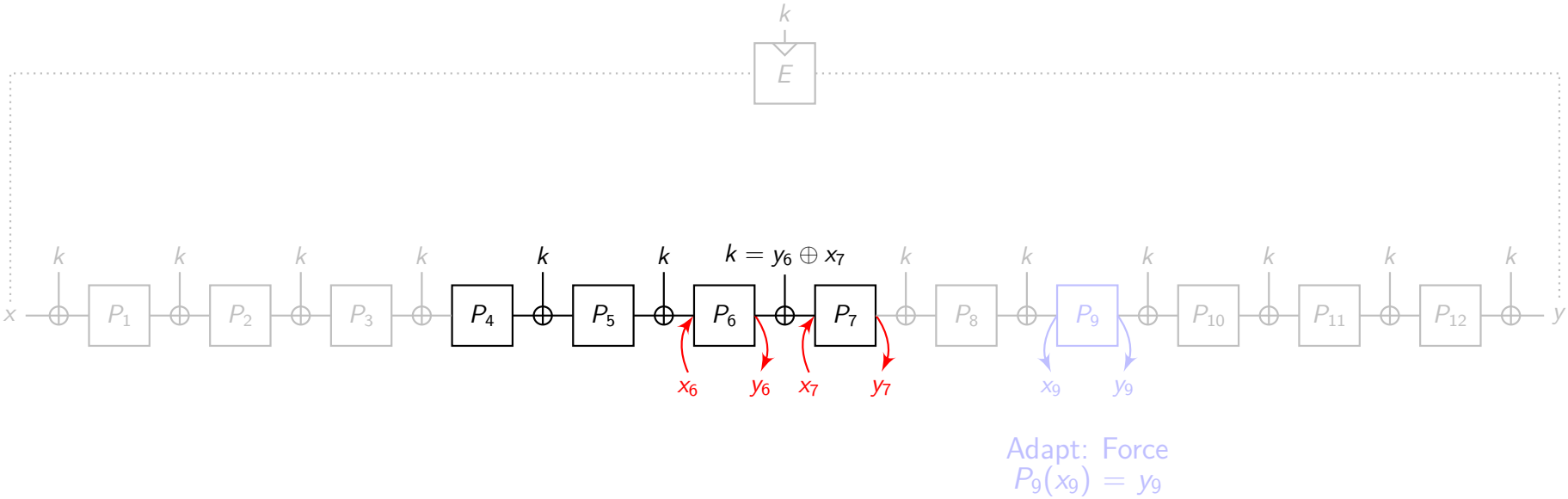

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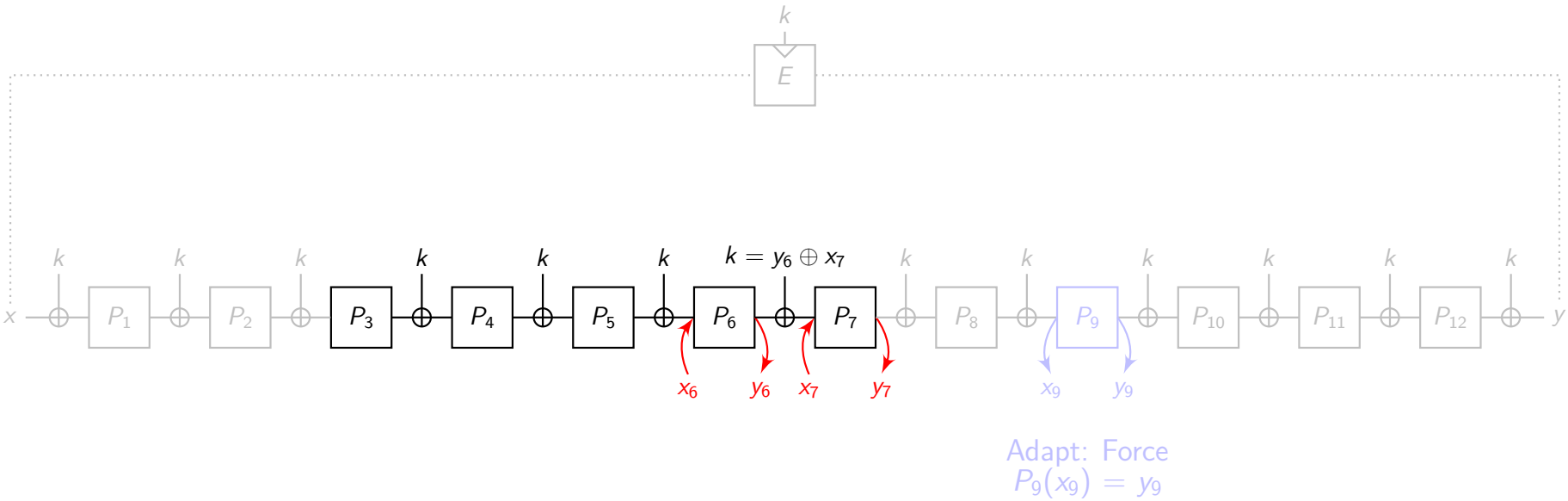

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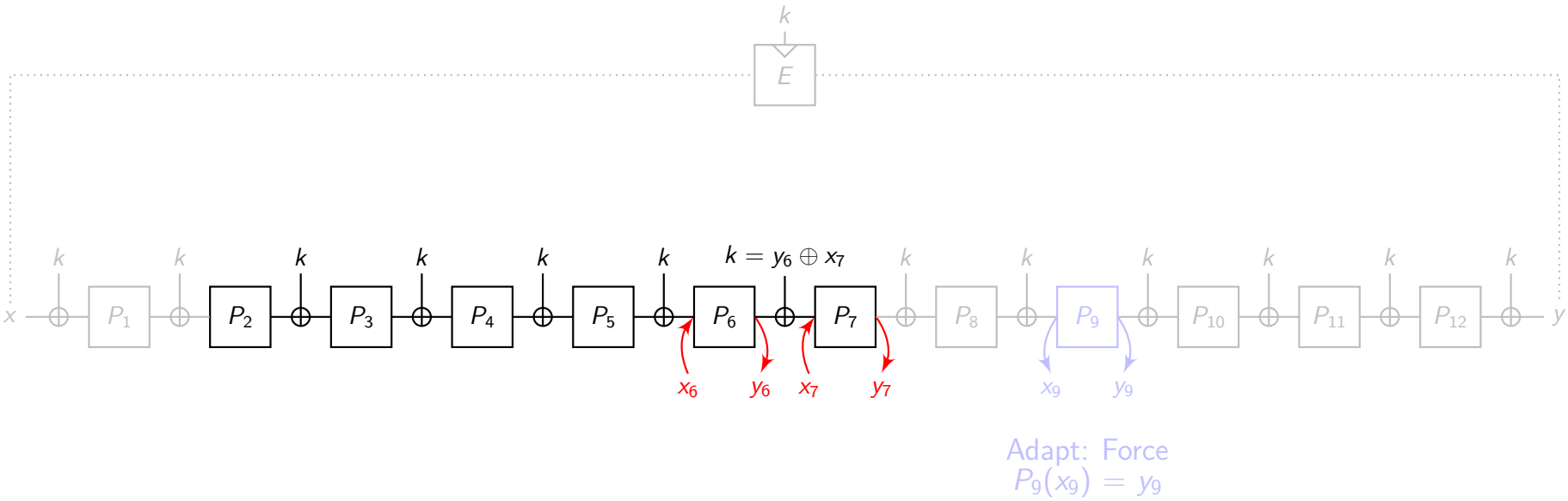

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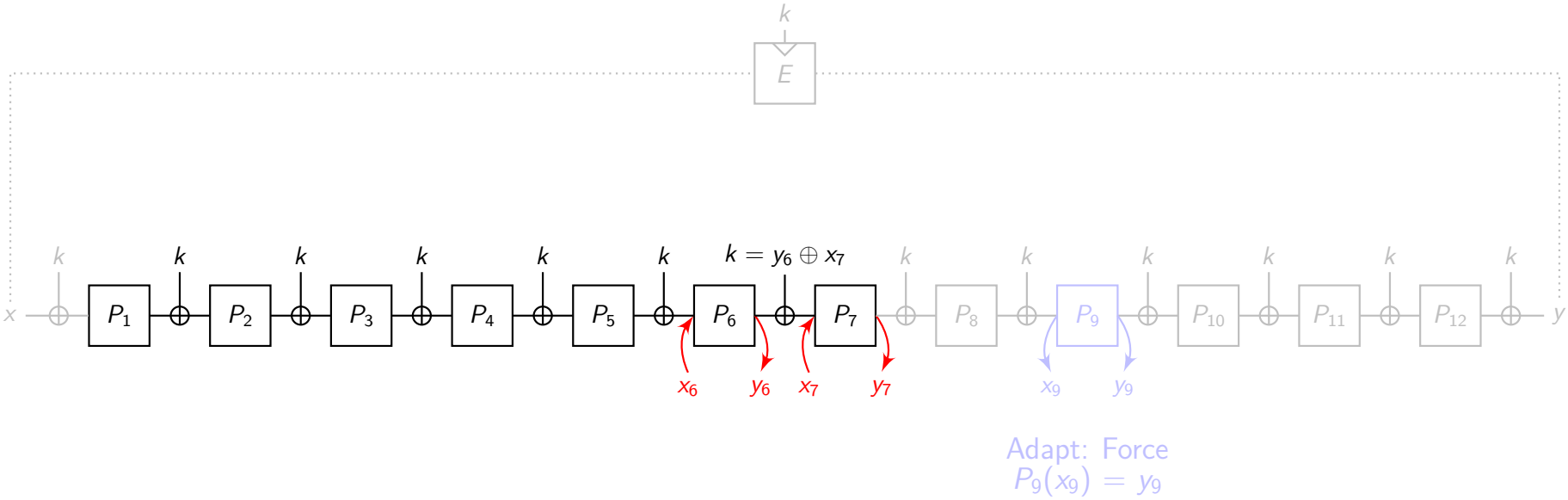

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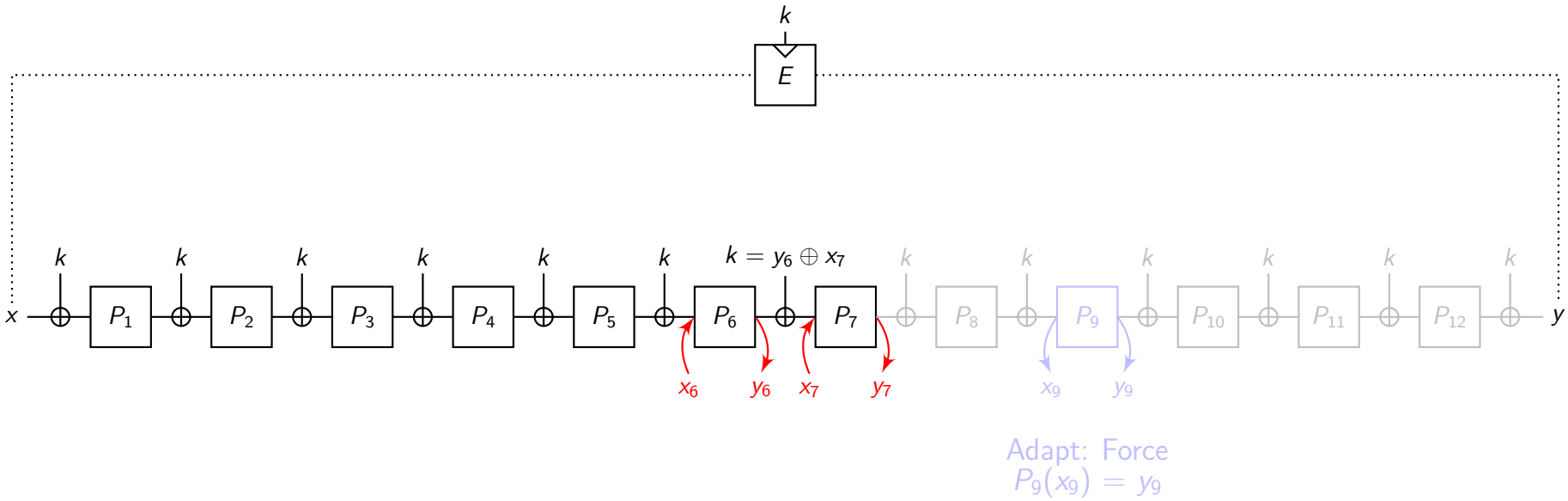

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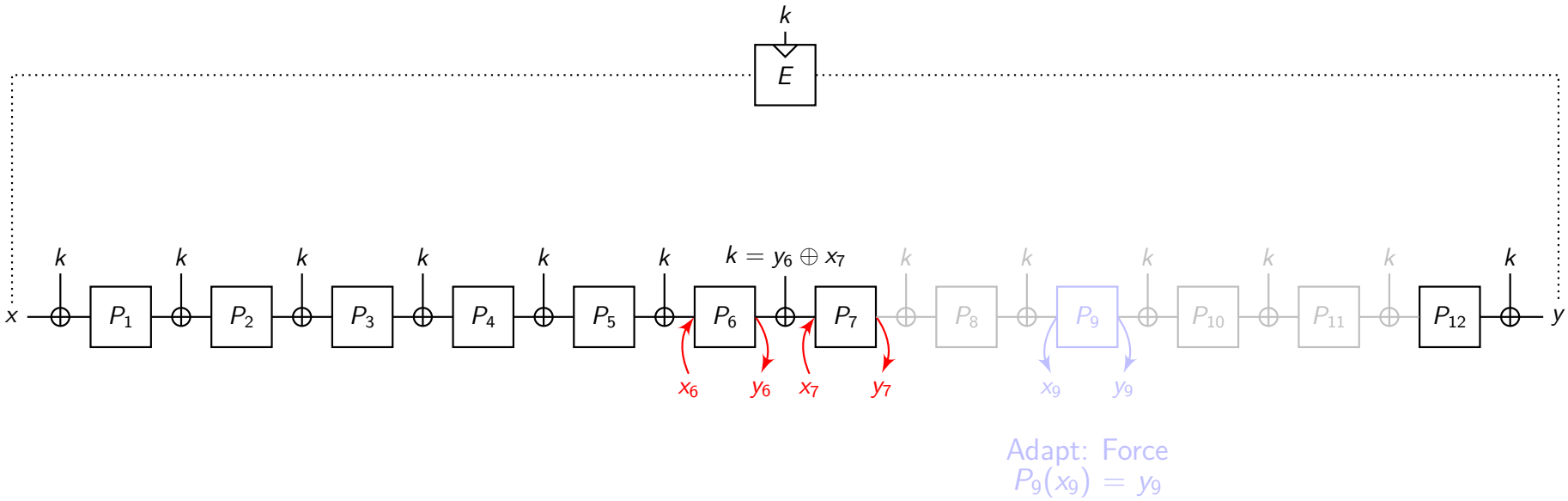

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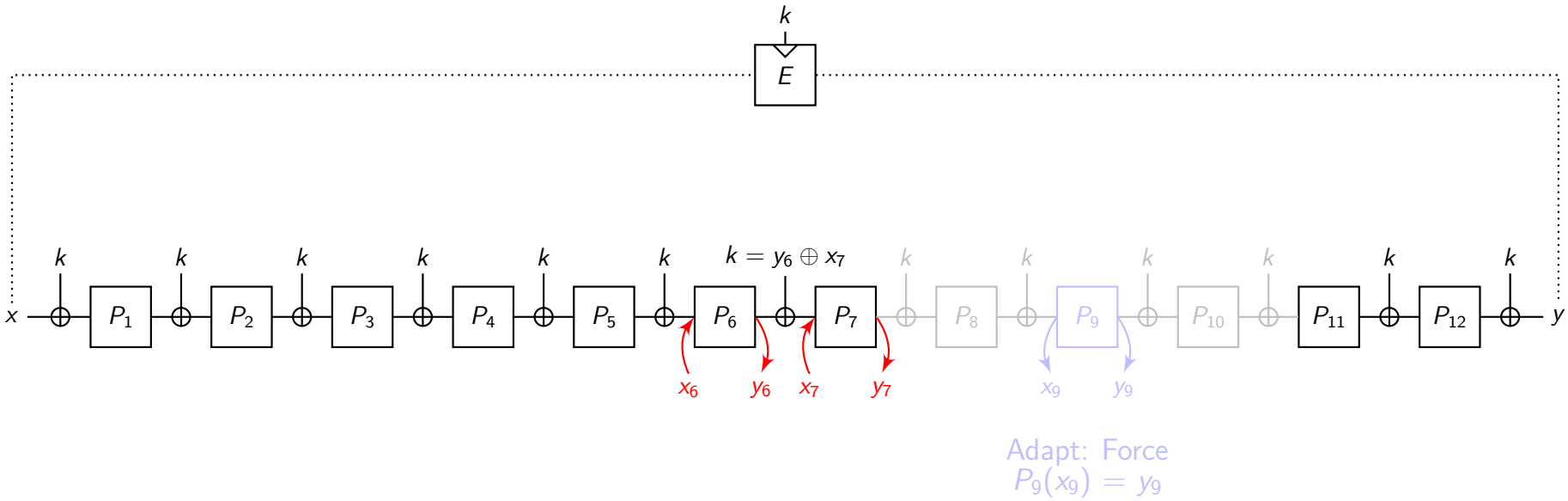

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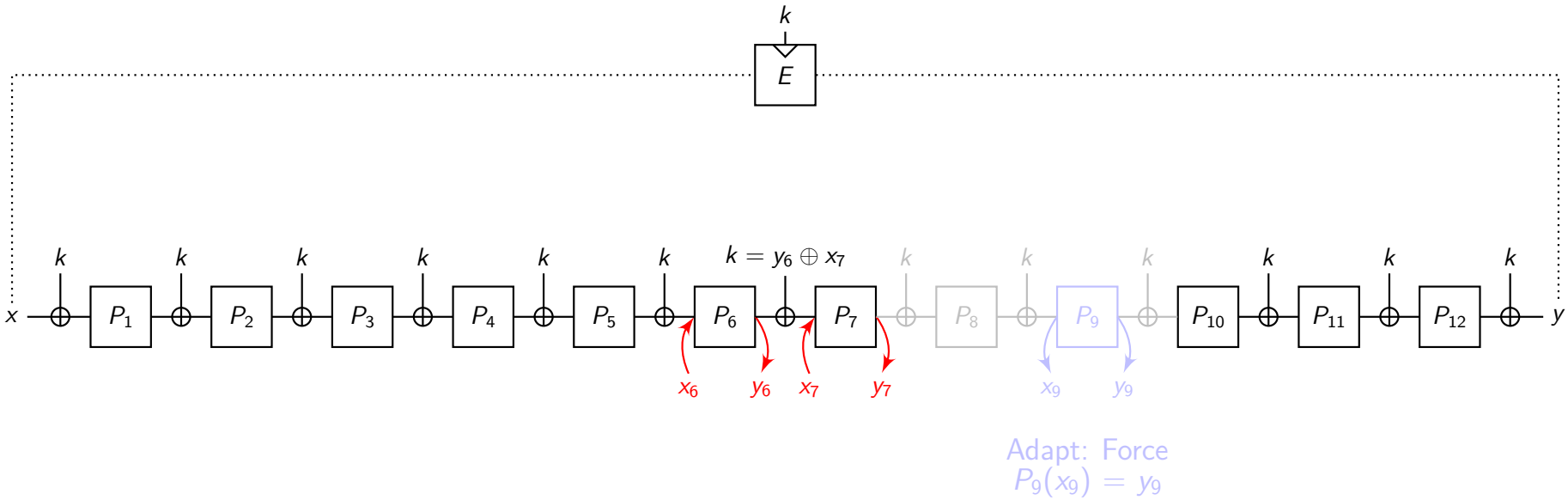

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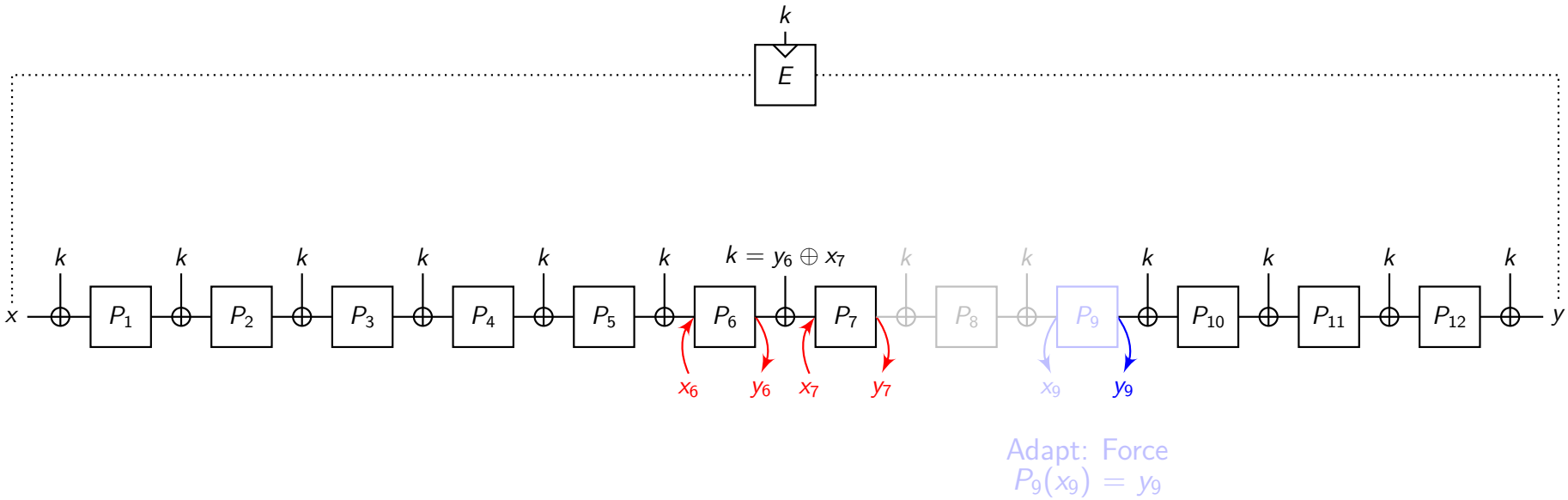

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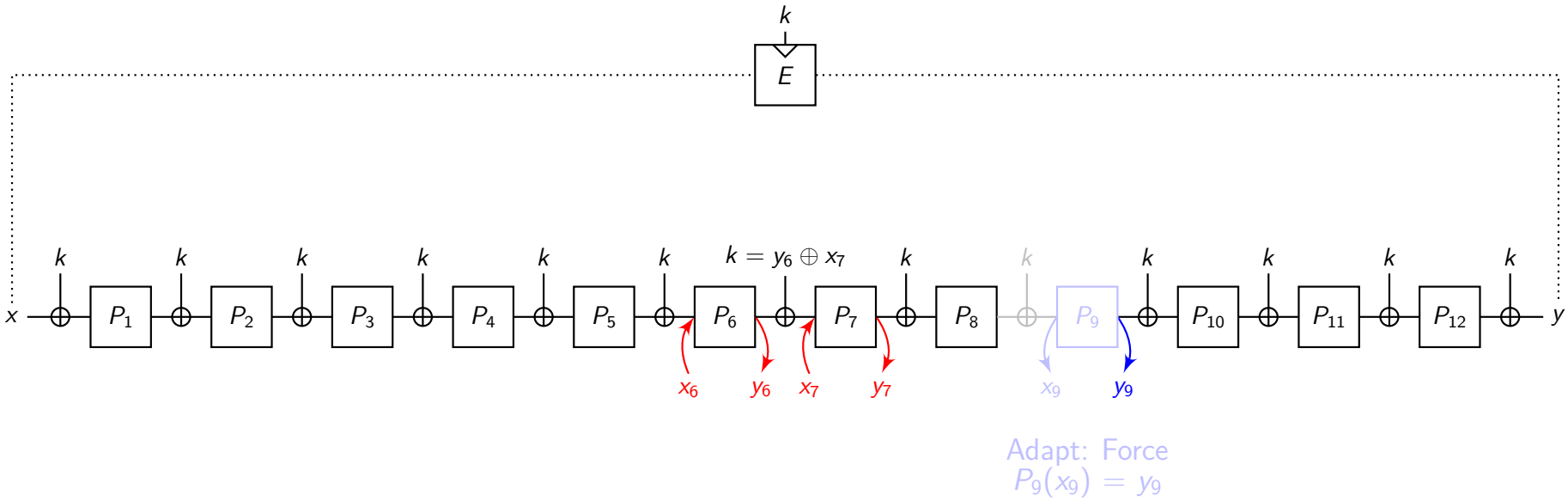

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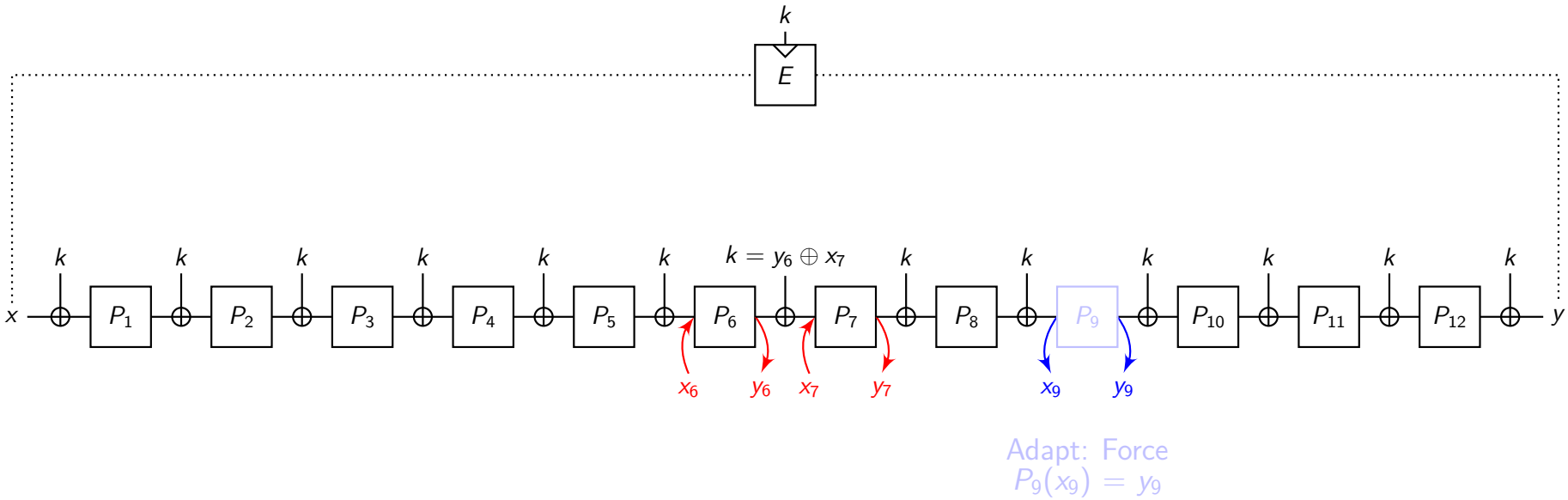

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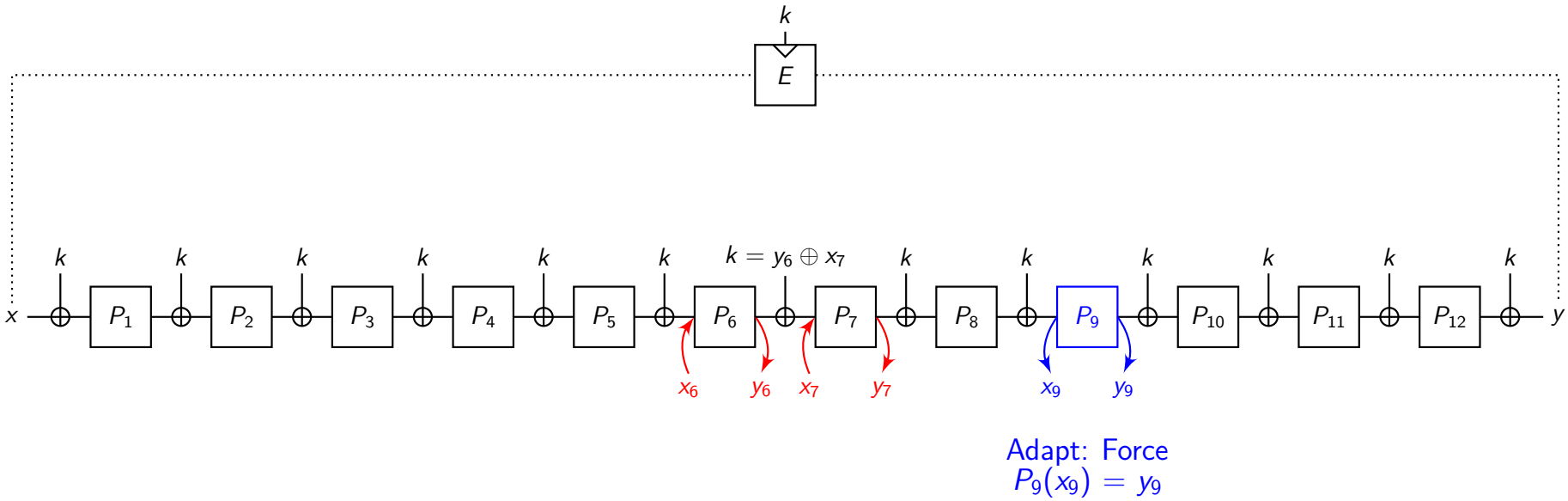

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What could go wrong during simulation

Two problems to deal with:

1. complexity of the simulator:
   - completing a partial chain creates new chains, which must be completed, creating new partial chains, etc.
   - $\Rightarrow$ potential blow-up of the number of chains completed by the simulator
   - but the simulator must be polynomial-time!
2. impossibility to adapt:
   - when the simulator wants to adapt a chain by forcing $P_i(x_i) = y_i$, it might happen that P_i was already defined for x_i or y_i
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Bounding the simulator's complexity

- the simulator only detects and completes partial chains at very specific places:
  - central chains: queries to (P_6, P_7)
  - external chains: queries to $(P_1, P_2, P_{11}, P_{12})$ that matches E
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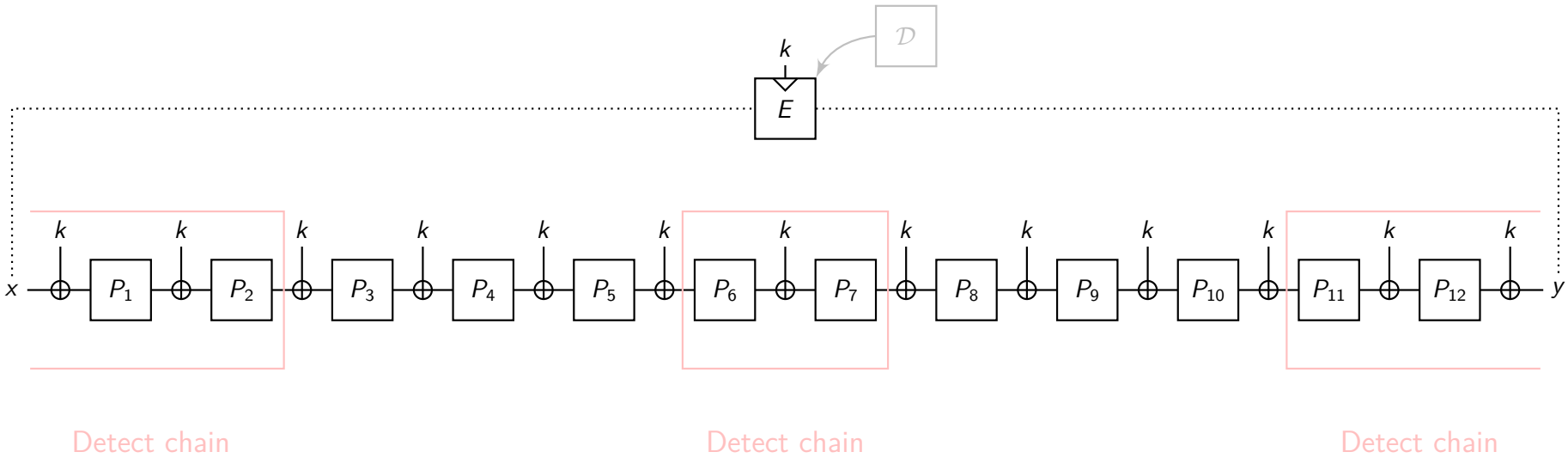

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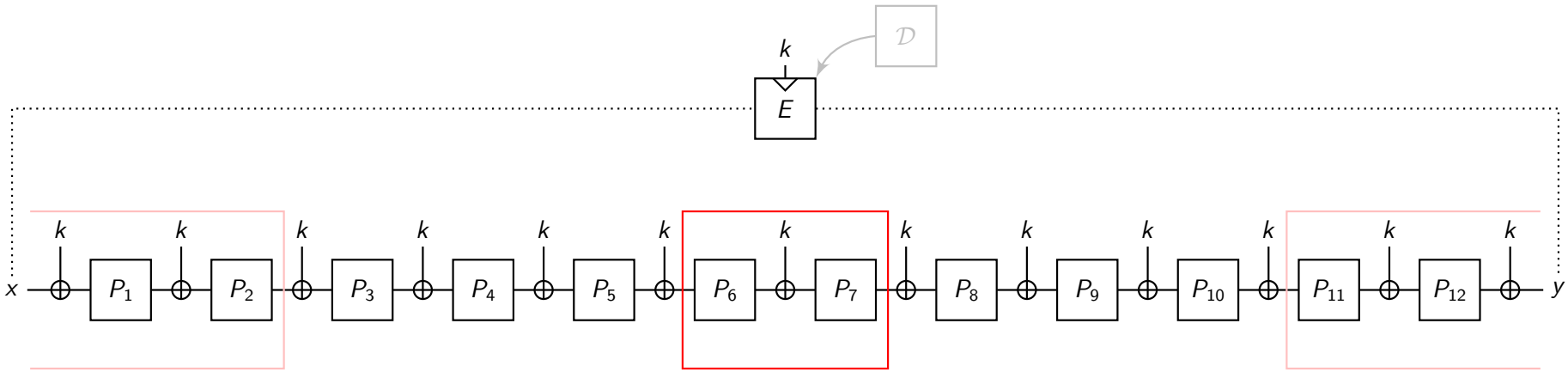

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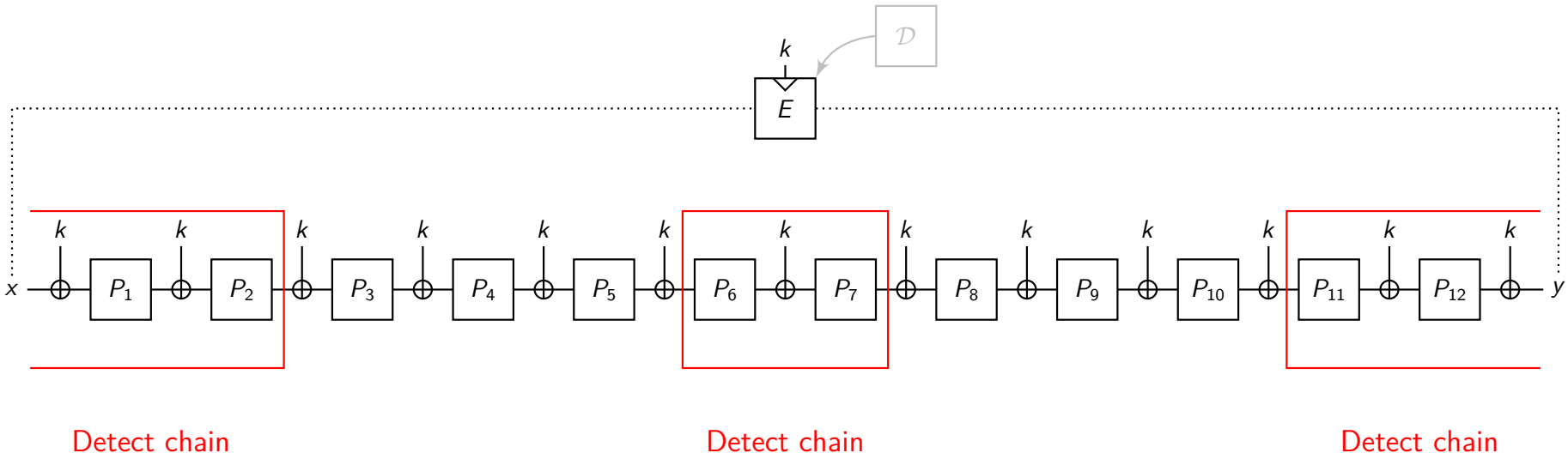

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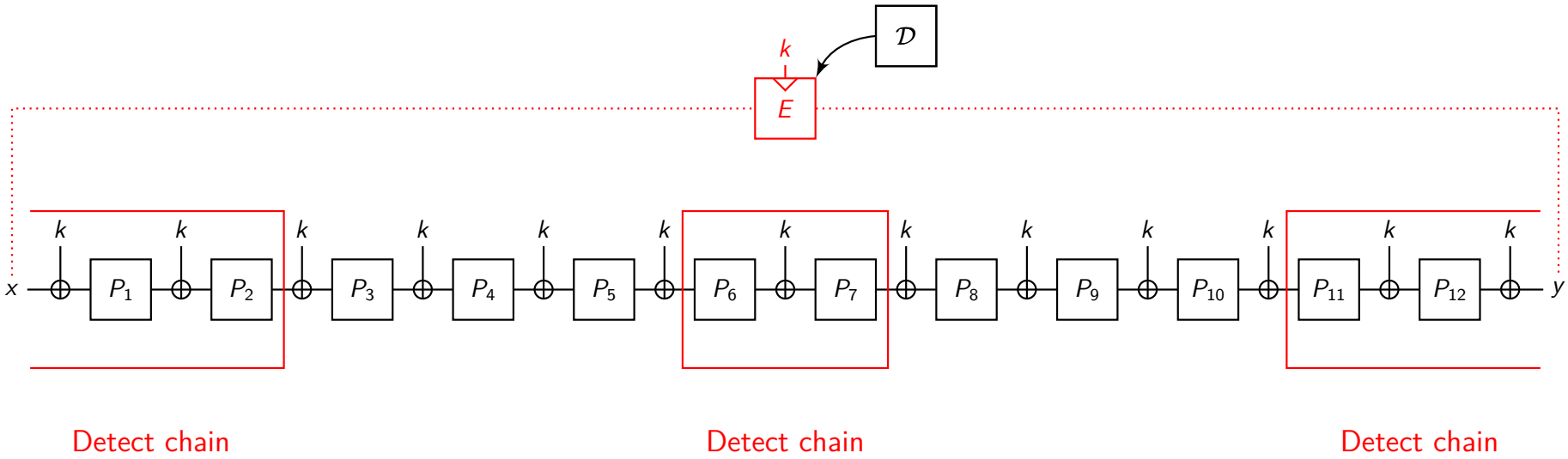

Ensuring that the simulator can always adapt

- chains are always adapted at P_4 or P_9
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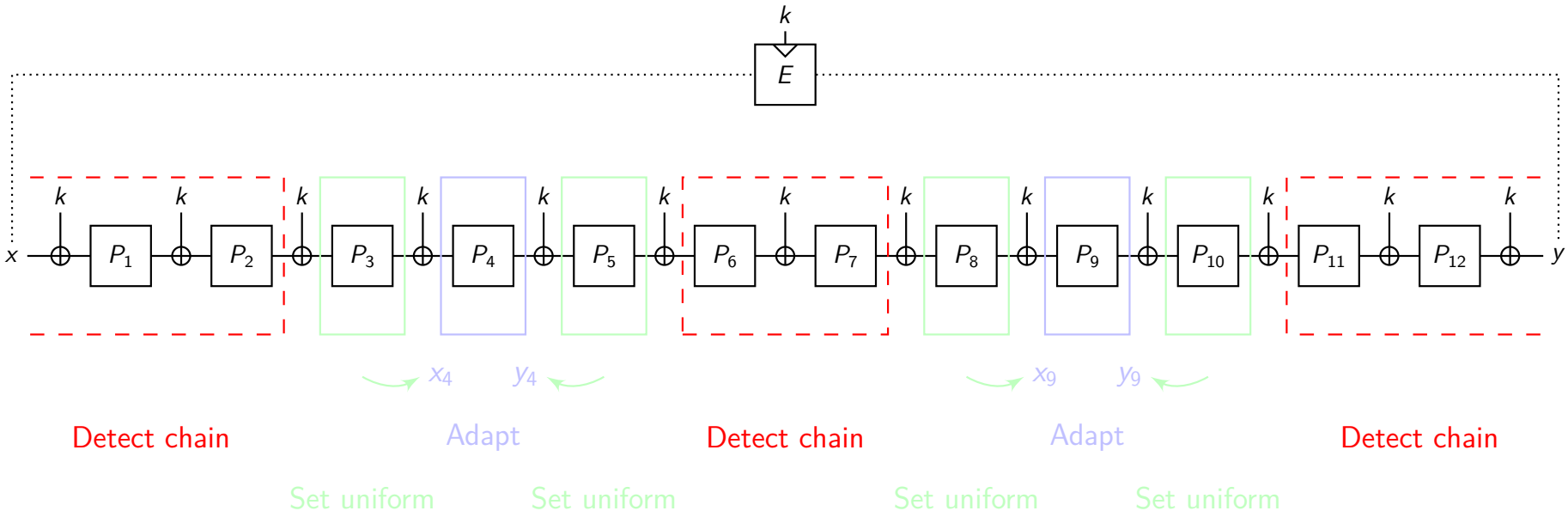

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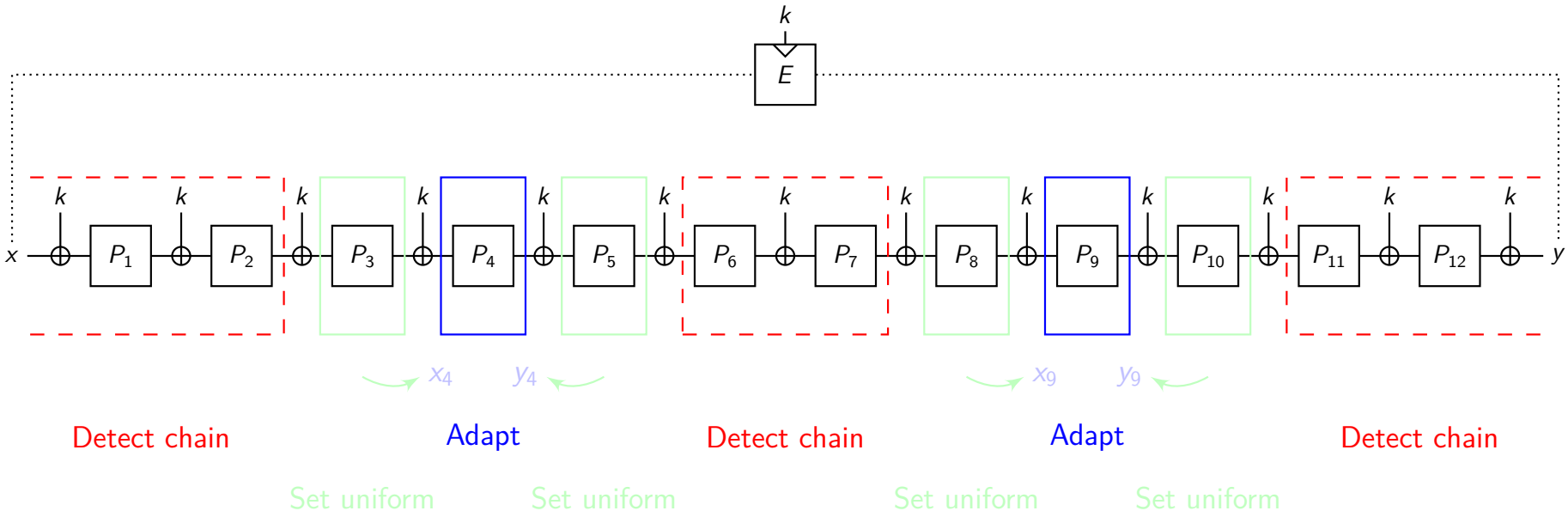

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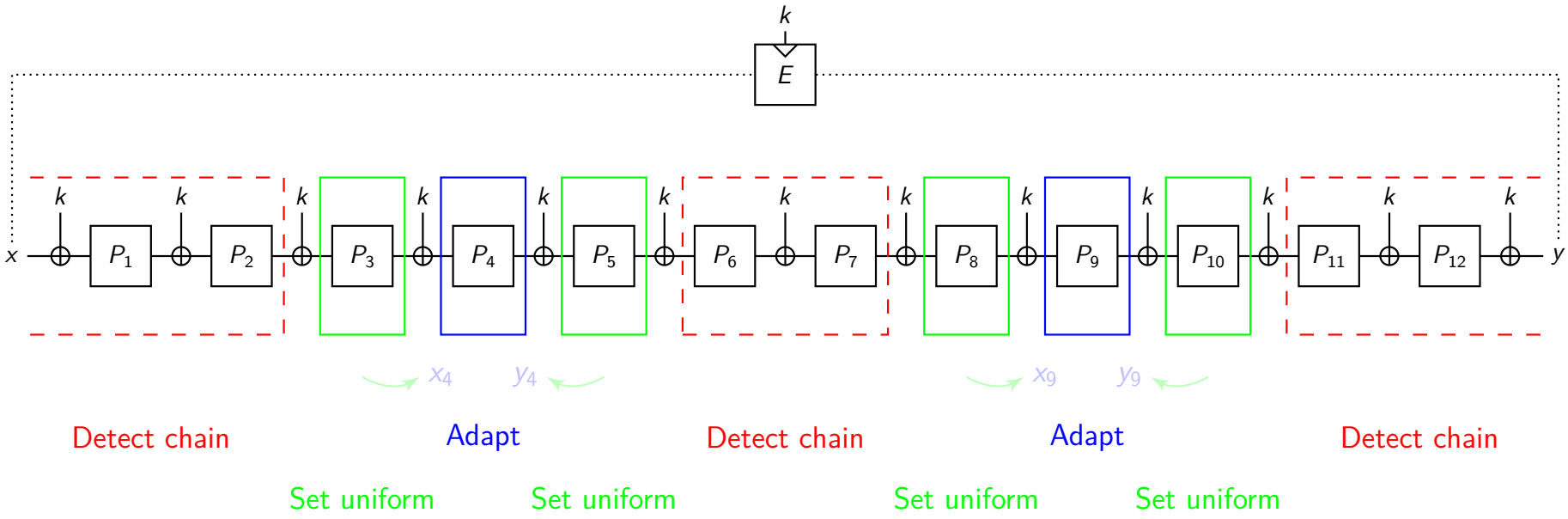
- adaptation rounds are surrounded by buffer rounds whose answers are drawn at random just before adapting
- the values (x_4, y_4) or (x_9, y_9) used to adapt P_4 or P_9 are random
  $\Rightarrow$ in the history of the simulator only with negl. probability



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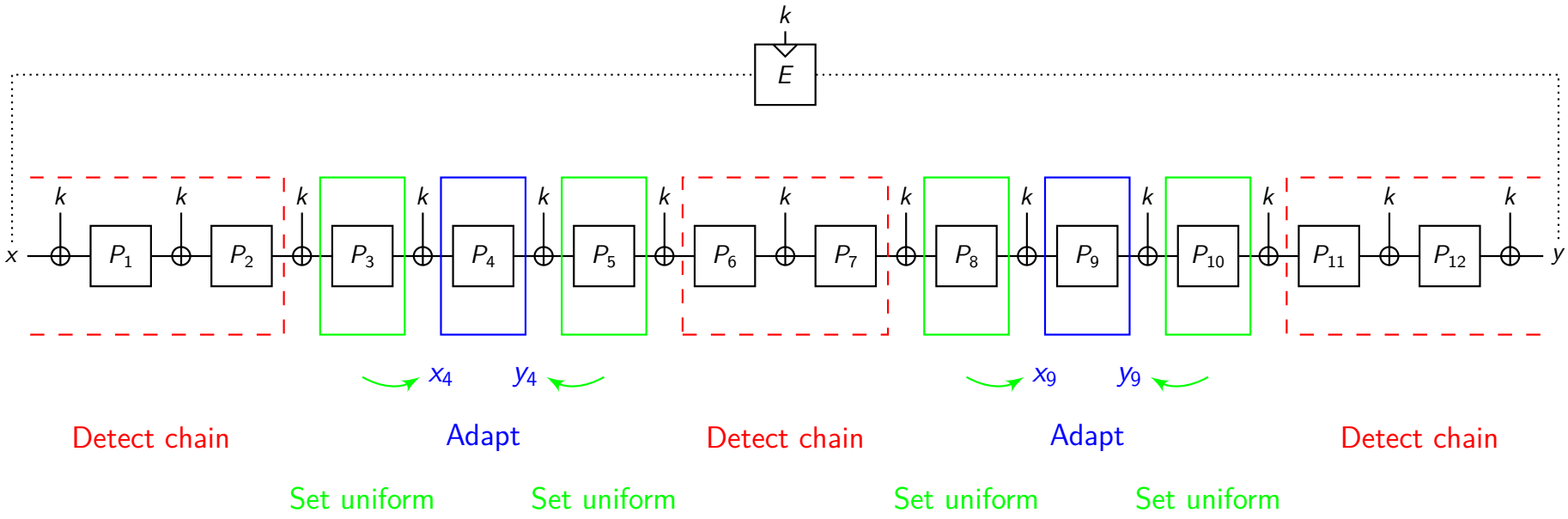

Conclusion

Main result

The single-key IEM cipher with 12 rounds is indifferentiable from an ideal cipher with n-bit keys.

Interpretation of the result:

- shows that the general strategy of building block ciphers from SPNs is sound and may even yield something close to an ideal cipher
- says little about concrete block ciphers: e.g. the permutations P_1, ..., P_{10} of AES-128 are too simple and not independent
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Open problems

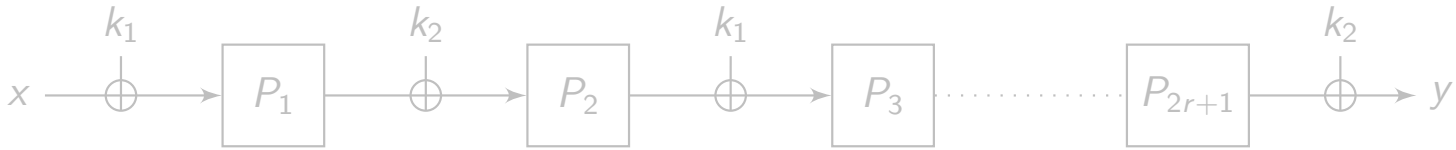
1. exact number of rounds for indifferentiability?
   - The indifferentiability proof requires 12 rounds...
     but the best attack is only on 3 rounds.

Conjecture

The single-key IEM with $3 < r < 12$ rounds is indifferentiable from an ideal cipher with n-bit keys

   - $r = 4$ may well be sufficient
     (we explain which obstacles appear already for $r = 8$ in the full paper)

2. construction with $2n$-bit keys? (or more generally tn-bit keys with $t > 1$)



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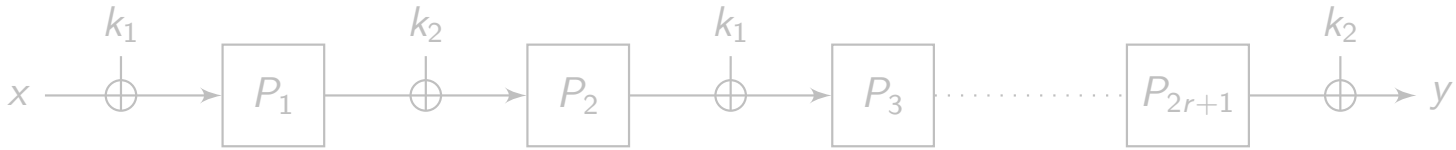

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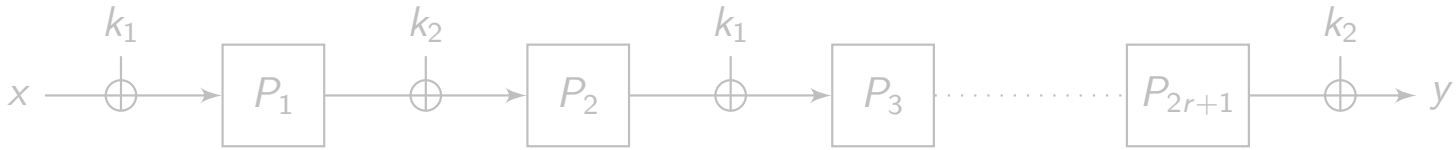

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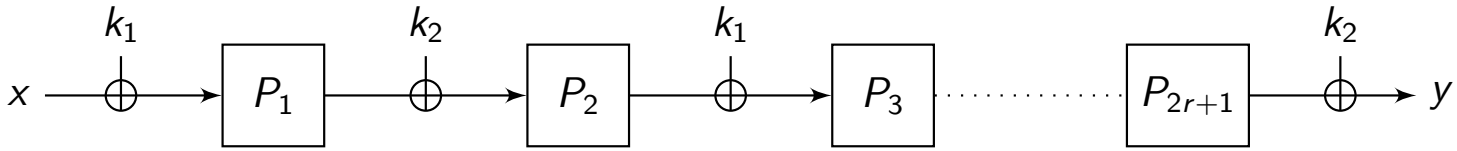

The end...

Thanks for your attention!
Comments or questions?

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