Notions of Black-Box Reductions, Revisited

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CASED

Cryptoplexity
Cryptography & Complexity Theory
Technische Universität Darmstadt
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Heisenberg-Programm
Deutsche Forschungsgemeinschaft

Introduction

The Cryptographic Zoo

PRP **PKE**

PRF

MAC **KA**

MPC

OWF

OWP

PRG

CRHF

COM

SIG

ZK

- basic issues in cryptography
  - what can be built from what?
  - how (efficient)?

A Typical Theorem in Cryptography

$f \xrightarrow{\text{constr.}} G[f]$

Theorem: Let f be a $\overbrace{P}^{\text{e.g. OWP}}$. Then construction $G[f]$ is a $\overbrace{Q}^{\text{e.g. PRG}}$.

Question 1: what is $G[f]$?

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- construction G uses f as an oracle (G^f)
- construction G uses f in some constricted way
- construction G uses f's code
- ???

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Theorem: Let f be a $\overset{\text{e.g. OWP}}{\overbrace{P}}$. Then construction $G[f]$ is a $\overset{\text{e.g. PRG}}{\overbrace{Q}}$.

(corollary: if P exists, then Q exists.)

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Proving the Theorem

$$\begin{tikzcd} f \arrow[r, "\text{constr.}"] & G[f] \\ \mathcal{S}[\mathcal{A}, f] \arrow[u, dotted] & \mathcal{A} \arrow[l, "\text{red.}"'] \arrow[u, dotted] \end{tikzcd}$$

Theorem: Let f be a P. Then construction $G[f]$ is a Q.

- almost always: proof by reduction (show the contrapositive)
- transform an attack on G into an attack on f
- if algorithm $\mathcal{A}$ breaks G, then algorithm $\mathcal{S}[\mathcal{A}, f]$ breaks f

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- $\mathcal{S}[\mathcal{A}, f]$ is the (constructive) reduction
  - Question 2: what is $\mathcal{S}[\mathcal{A},\ \ ]$?
  - Question 3: what is $\mathcal{S}[\ \ , f]$?

Why We Care About these Questions

- very important for impossibility results / separations
  - i.e., *much weaker* versions of P exists $\not\Rightarrow$ Q exists
  - what exactly is being ruled out?
  - ...and what is left to try?
  - impossibility results are inspiring
- enforces precise definitions of primitives
  - "we separate xyz from OWFs..."
- more black box, more efficient, more practical (usually)
- better understanding of a fundamental technique in our field

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- three* types of reductions:

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semi black box. $\forall \mathcal{A} \exists \mathcal{S}$: if $\mathcal{A}^f$ breaks G^f, then $\mathcal{S}^f$ breaks f.

order switched

f oracle

no $\mathcal{A}$ oracle

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weakly black box. $\forall \mathcal{A} \exists \mathcal{S}$: if $\mathcal{A}$ breaks G^f, then $\mathcal{S}^f$ breaks f.

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  - . . . derived in a systematic way

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- consider, for example,
  - reduction makes non-black-box use of primitive, but black-box use of adversary (think meta reductions)
  - efficient primitives and/or adversaries
  - black-box use, but partial information (run time, #queries, ...)
- [RTV04] too coarse to capture such differences

CAP

Three Questions: A Short Encoding

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Q3: what is $\mathcal{S}[\quad, f]$?

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Q1: what is $G[f]$?

C

Q2: what is $\mathcal{S}[\mathcal{A},\ \ ]$?

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- $C, A, P \in \{\mathrm{N}, \mathrm{B}\}$
- <u>N</u>on black box / <u>B</u>lack box

Obtaining Actual Definitions

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example: BBB

1. what is $G[f]$? B "$\exists G$" $\prec$ "$\forall f$"
   what is $\mathcal{S}[\mathcal{A},\ \ ]$? B
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2. "$\exists G$", "$\exists\mathcal{S}$" $\prec$ "$\forall f$", "$\forall\mathcal{A}$"

3. $\exists G, \mathcal{S}\, \forall f, \mathcal{A}$ $\quad$ $\mathcal{A}^{f, G^f}$ breaks $G^f \Longrightarrow \mathcal{S}^{\mathcal{A}^f, f}$ breaks f

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example: NBB

1. what is $G[f]$? N "$\forall f$" $\prec$ "$\exists G$"
   what is $\mathcal{S}[\mathcal{A},\ \ ]$? B "$\exists \mathcal{S}$" $\prec$ "$\forall \mathcal{A}$"
   what is $\mathcal{S}[\ \ , f]$? B "$\exists \mathcal{S}$" $\prec$ "$\forall f$"

2. "$\exists \mathcal{S}$" $\prec$ "$\forall f$" $\prec$ "$\exists G$" and "$\exists \mathcal{S}$" $\prec$ "$\forall \mathcal{A}$"

3. $\exists \mathcal{S}\, \forall f \exists G \forall \mathcal{A}$ $\qquad$ $\mathcal{A}^{f,G^f}$ breaks $G^f \Longrightarrow \mathcal{S}^{\mathcal{A}^f, f}$ breaks f

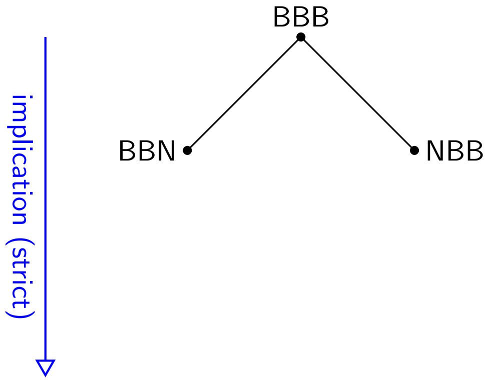
Obtaining Actual Definitions (cont'd)

$$\begin{tikzcd} f \arrow[r, "\text{constr.}"] & G[f] \\ \mathcal{S}[\mathcal{A}, f] \arrow[u, dotted] & \mathcal{A} \arrow[l, "\text{red.}"'] \arrow[u, dotted] \end{tikzcd}$$

Name	Summary of definition				
BBB	$\exists G$	$\exists \mathcal{S}$	$\forall f$	$\forall \mathcal{A}$	$((G^f, \mathcal{A}^f) \Rightarrow (f, \mathcal{S}^{\mathcal{A},f}))$
BNB	$\exists G$	$\forall \mathcal{A}$	$\exists \mathcal{S}$	$\forall f$	$((G^f, \mathcal{A}^f) \Rightarrow (f, \mathcal{S}^{\mathcal{A},f}))$
BBN	$\exists G$	$\forall f$	$\exists \mathcal{S}$	$\forall \mathcal{A}$	$((G^f, \mathcal{A}^f) \Rightarrow (f, \mathcal{S}^{\mathcal{A},f}))$
BNN	$\exists G$	$\forall f$	$\forall \mathcal{A}$	$\exists \mathcal{S}$	$((G^f, \mathcal{A}^f) \Rightarrow (f, \mathcal{S}^{\mathcal{A},f}))$
NBB	$\exists \mathcal{S}$	$\forall f$	$\exists G$	$\forall \mathcal{A}$	$((G^f, \mathcal{A}^f) \Rightarrow (f, \mathcal{S}^{\mathcal{A},f}))$
NBN	$\forall f$	$\exists G$	$\exists \mathcal{S}$	$\forall \mathcal{A}$	$((G^f, \mathcal{A}^f) \Rightarrow (f, \mathcal{S}^{\mathcal{A},f}))$
NNN	$\forall f$	$\exists G$	$\forall \mathcal{A}$	$\exists \mathcal{S}$	$((G^f, \mathcal{A}^f) \Rightarrow (f, \mathcal{S}^{\mathcal{A},f}))$

see page 305 of the proceedings (Part I)

Basic Relations



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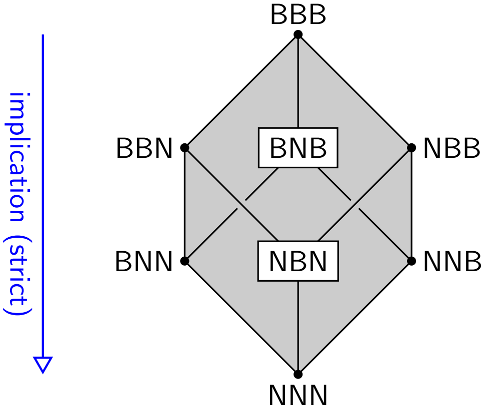

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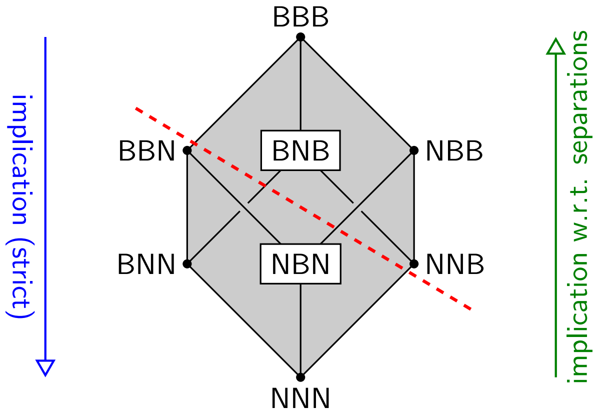
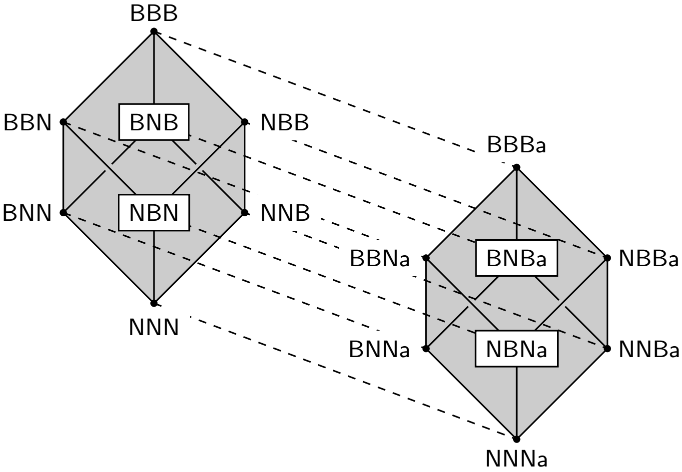


There is More...

- adversaries $\mathcal{A}$ can be PPT or inefficient
  - [RTV04]: mixed
  - here: inefficient up to now
- all previous notions can be considered for efficient adversaries
- shorthand: CAPa, restricted quantification $\forall\,\text{PPT}\mathcal{A}$

Another Dimension



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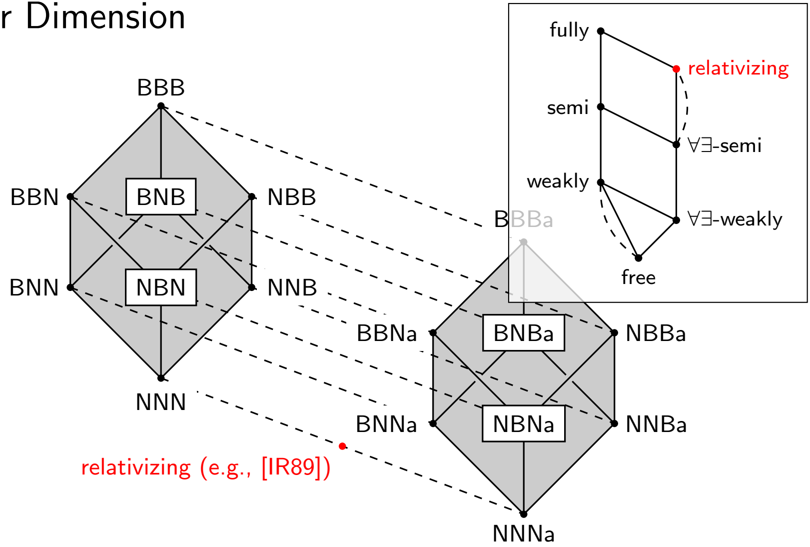

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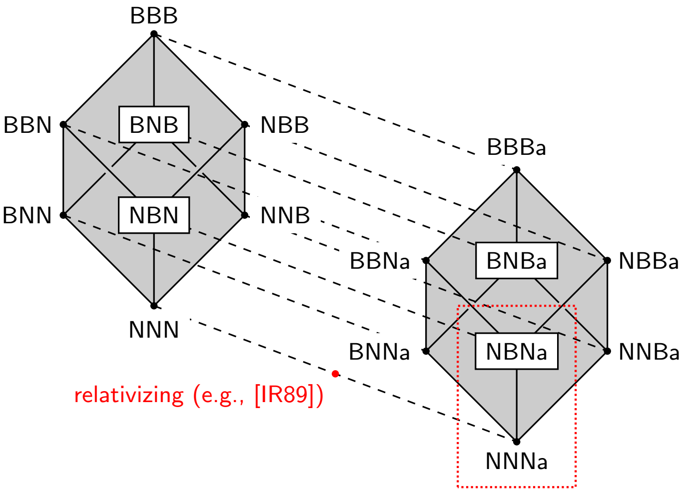
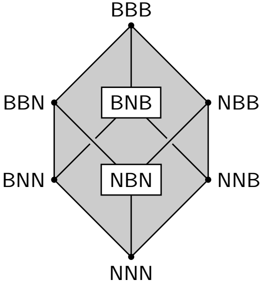


note: not all *CAP*a implications are strict

Neither B nor N

Parameterized Reductions

- consider the Goldreich–Levin hardcore bit [GL89]
- reduction requires success probability of adversary (but nothing else)
- black box? non black box?



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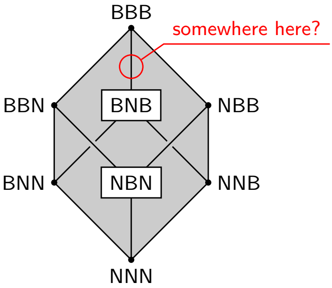

- parameterized reduction
- here: par($\mathcal{A}$) := success probability
- BBB w/ param: $\mathcal{A}^{f,G^f}$ breaks $G^f \Longrightarrow \mathcal{S}^{\mathcal{A}^f,f}(\text{par}(\mathcal{A}))$ breaks f

$\rightarrow$ *parameters made explicit*

Summary

- things I forgot to tell you
  - CAPp: efficient primitives
  - CAPap: efficient adversaries and efficient primitives
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Summary

- things I forgot to tell you
  - *CAP*p: efficient primitives
  - *CAP*ap: efficient adversaries and efficient primitives
  - careful when defining primitives
- things to remember
  - given any reduction/separation, ask three (five) questions
  - "impossibility" rarely means *impossible*
  - look for hidden parameters

The End

Thank you!

?

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Another Dimension for Efficie