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Function-Private Subspace Membership Enc. and Its Applications

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Predicate Encryption [BW07, KSW08]

predicate p: Σ → {0,1}
attribute x in Σ

(pp, msk)

sk_p ↓ ↑ p

pp: public

Enc(pp,x,m) →

Bob can recover m only if p(x)=1

Applications: spam filtering encrypted email
routing encrypted bank transactions

Function Privacy [Boneh-R.-Segev13]

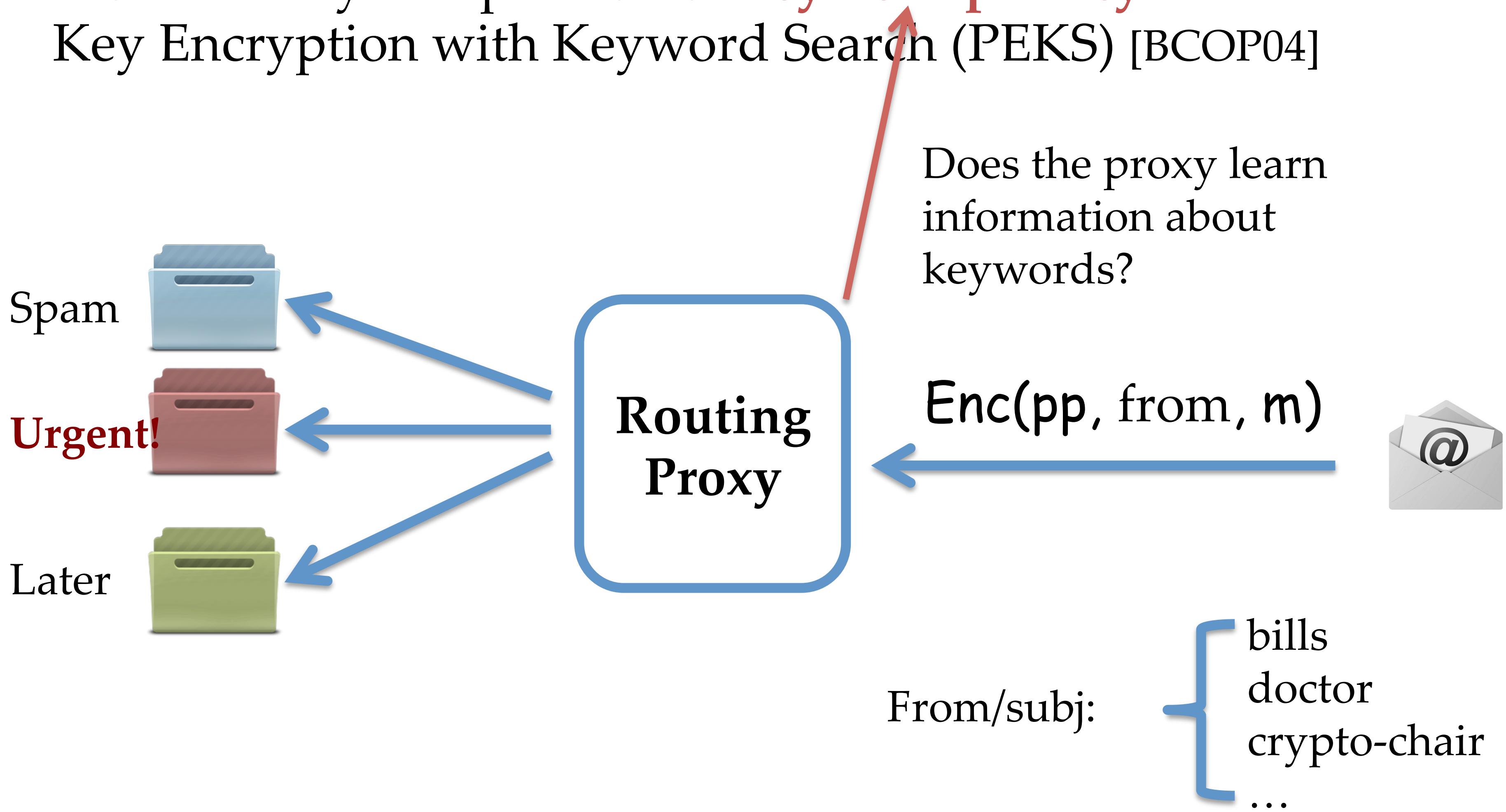
Question: must sk_p reveal p?

Can we build schemes where sk_p reveals no information about p

In previous works, sk_p may leak p.
In several schemes, p is leaked explicitly

Function Privacy [Boneh-R.-Segev13]

Motivated by the question of **keyword privacy** in Public Key Encryption with Keyword Search (PEKS) [BCOP04]



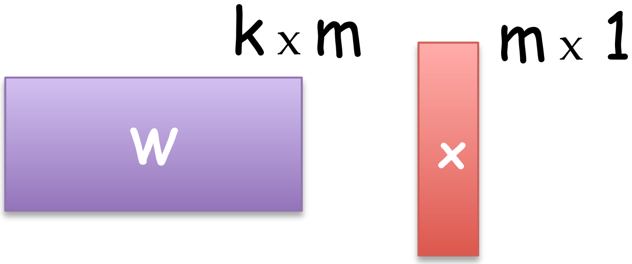
Function Privacy [Boneh-R.-Segev13]

(In a nutshell)

- Define function privacy of Identity-Based Encryption (IBE implies encrypted keyword search [BCOP04])
- Observe that given $\mathsf{sk}_{\mathsf{id}}$, semantic security for id is not possible (due to the public-key nature of encryption)
- Construct IBE schemes where the secret key reveals no information about the identity
  - identity must have some min-entropy
  - constructions in RO and STD model
  - constructions from pairings and lattices

Subspace-Membership Enc.

$$p_W(x) = \begin{cases} 1 \text{ if } (W \cdot x = 0 \text{ in } F_q) \\ 0 \text{ otherwise} \end{cases}$$



- Predicate p corresponds to matrix W over F_q
- Ciphertext attribute x is a vector over F_q

  sk_p can decrypt if $W \cdot x = 0$

- $k=1$ is inner-product encryption [KSW08, Fre10, AFV11]
- Subspace membership with delegation [OT09,OT12]
- **Security requirement:**
  given secret keys for predicates $p_1, \ldots, p_Q$,
  semantic security for ciphertexts with attribute x where $p_i(x)=0$ (for all i)

SME – Applications

- Predicates that are *roots of polynomials*
  - ciphertexts encrypted to an attribute x in F_q
  - secret keys derived for polynomial predicates $p(x) = 1$ iff $(p_0 + p_1 x + p_2 x^2 + \ldots + p_d x^d = 0)$
  - *Basic idea:* encrypt to vector $(1 \quad x \quad x^2 \quad \ldots \quad x^d)$ subspace is orthogonal to $(p_0 \quad p_1 \quad p_2 \quad \ldots \quad p_d)$

Vandermonde vector

- Hidden Vector Encryption [BW07]
  - predicates for comparison and set membership queries
- Subsumes Identity-Based Encryption
  - attribute $x = (1, id)$, subspace is $W = (-id, 1)$
- Predicates with conjunction and disjunctions

This Paper

- Extend the framework and techniques of [BRS13] to subspace membership encryption (SME)
- Define function-private SME:
  schemes where the secret key reveals no information about the subspace
  - identify *minimal necessary* restrictions
- Black-box constructions of function-private SME from non-function-private inner-product encryption schemes
  - *First* black-box constructions of function-private schemes
- Applications with function privacy (discussed later)

Function Privacy for SME

- What information does sk_W leak about W?
- Given sk_W and a guess for W, due to the public-key nature of Enc, guess can be verified (up to constant factors)

(assume W is a vector)

sk_W

1	w_1	w_2	…		w_{m-1}

guess

encrypt m with $x=$

w_1	-1	0	…	0	0

If decryption recovers m then w_1 guessed correctly!

Function Privacy for SME

- What information does sk_W leak about W?
- Given sk_W and a guess for W, due to the public-key nature of Enc, guess can be verified (up to constant factors)

(assume W is a vector)

sk_W

1	w_1	w_2	…		w_{m-1}

guess

Next,

encrypt m with $x=$

w_2	0	-1	…	0	0

If decryption recovers m then w_2 guessed correctly!

Function Privacy for SME

- What information does sk_W leak about W?
- Given sk_W and a guess for W, due to the public-key nature of Enc, guess can be verified (up to constant factors)

(assume W is a vector)

sk_W

1	w_1	w_2	…		w_{m-1}

guess

Finally,

encrypt m with $x=$

w_{m-1}	0	0	…	0	-1

Can verify guess only given sk_W!

Function Privacy for SME

- Is *unpredictability* of W sufficient (like in IBE)?
- **No!**
  Following test works even if w_1 and w_2 are unpredictable so long as $w_1/w_2 = a$

sk_W

1	w_1	w_2	…		w_{m-1}

← guess

encrypt m with $x=$

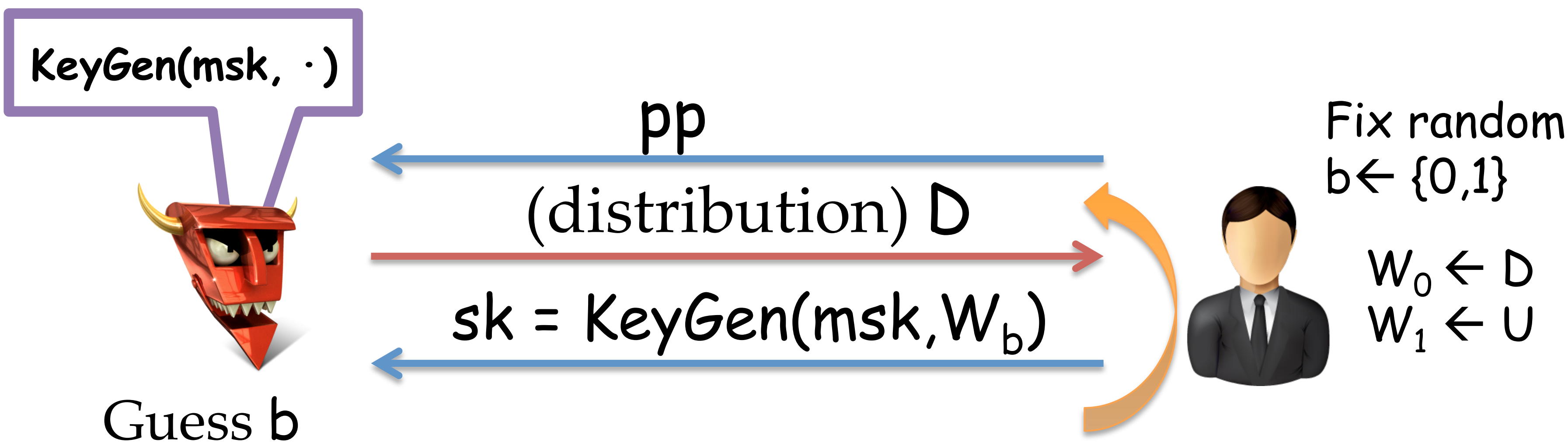
0	-1	a	…	0	0

Can ***still*** verify guess only given sk_W!

Function Privacy for SME

Minimal necessary restriction:

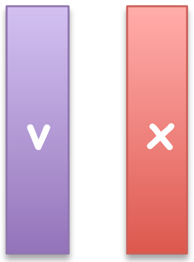
sk_W reveals no information *if* columns of W come from a distribution with *conditional min-entropy*, *i.e.*, j^{th} column still unpredictable given $w_1,\ldots,w_{j-1}$



Adversary cannot guess b with probability better than 1/2

Construction from Inner Prod Enc.

Inner Product Predicate Encryption



$$p_v(x) = \begin{cases} 1 \text{ if } (v^T \cdot x = 0 \text{ in } F_q) \\ 0 \text{ otherwise} \end{cases}$$

- Predicate p corresponds to a vector v over F_q
- Ciphertext attribute x is a vector over F_q

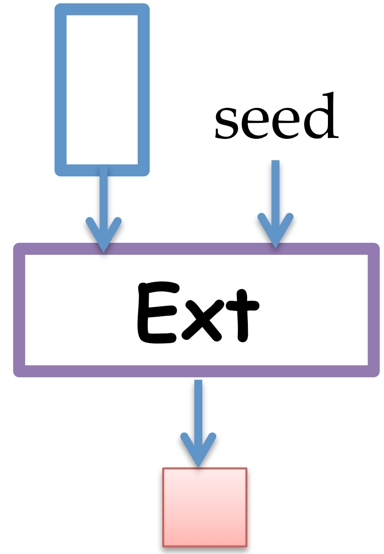
sk_p can decrypt if $v^T \cdot x = 0$

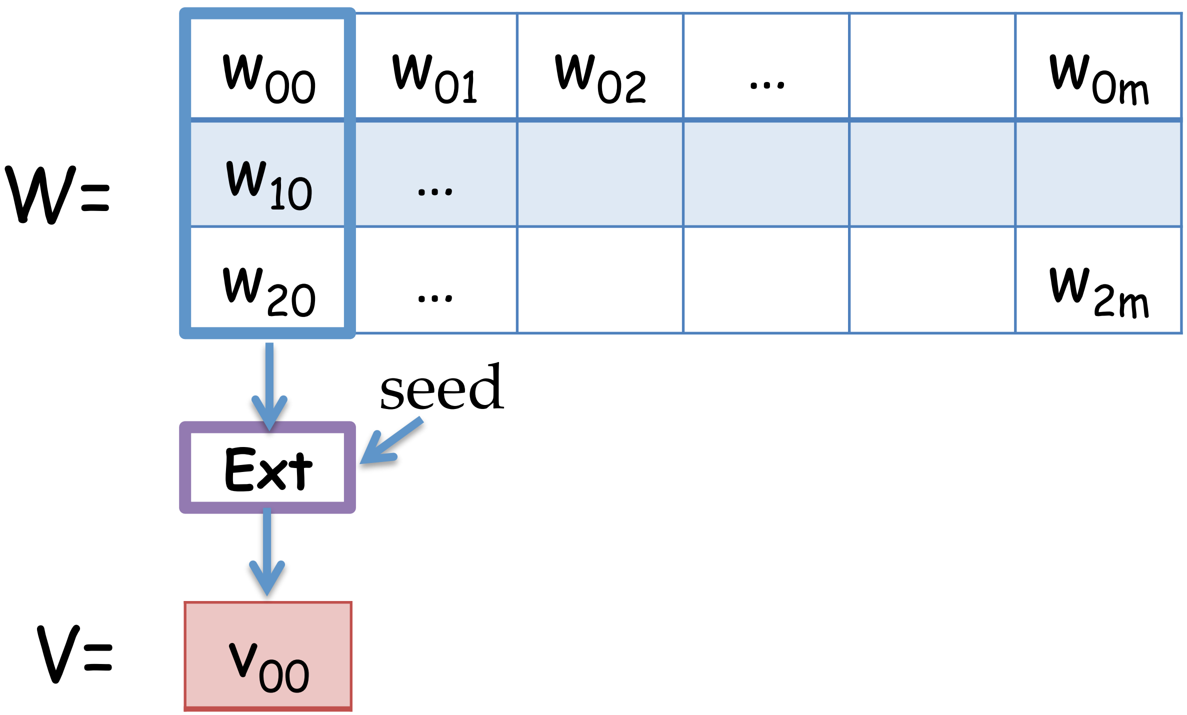
We construct function-private SME from any underlying (non-function-private) inner prod. scheme

- black-box manner
- modify the KeyGen algorithm by *pre-processing* subspace W to derive an inner-prod sk vector v

Construction from Inner Prod. Enc.

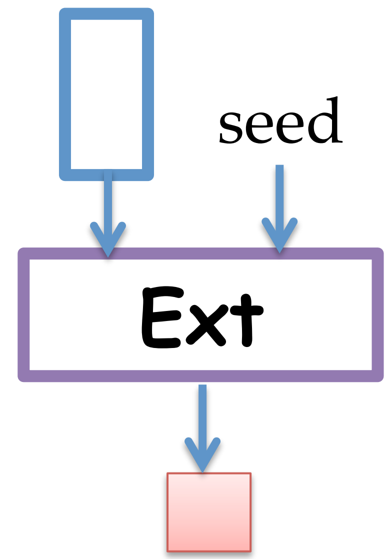
Key idea: apply extractor on *columns* of W
run (underlying) inner prod
KeyGen on extracted vector

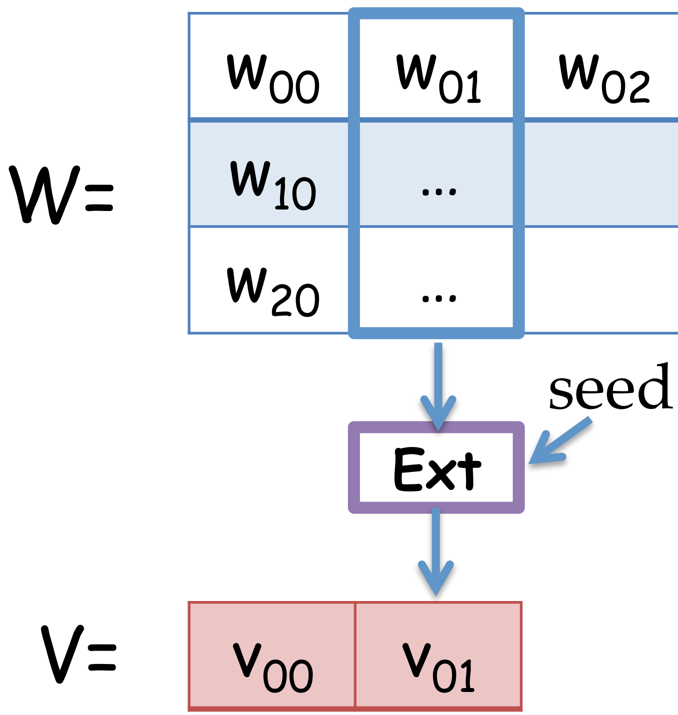




Construction from IPE

Key idea: apply extractor on *columns* of W
run (underlying) inner prod
KeyGen on extracted vector





Construction from IPE

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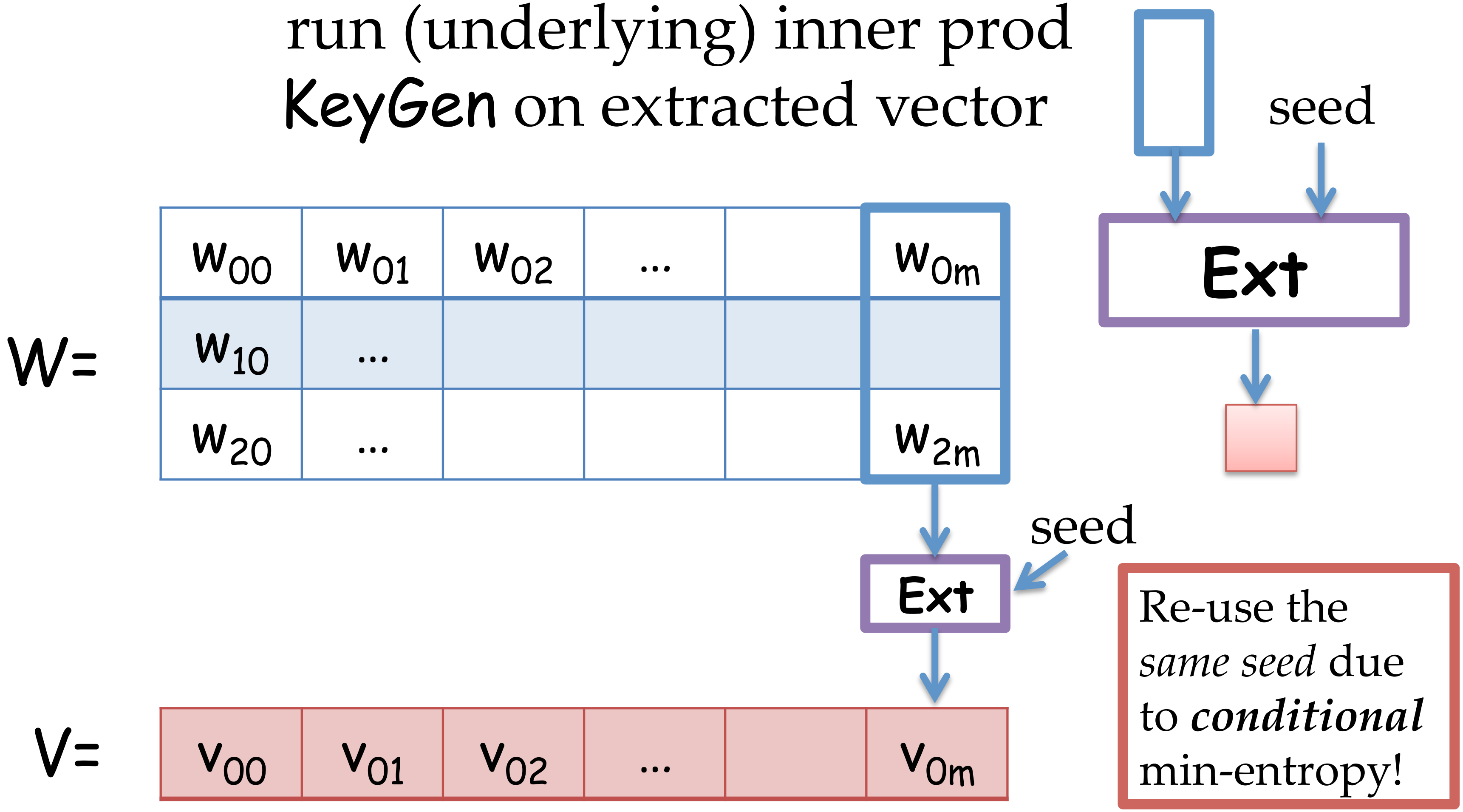

Construction from IPE

- V extracts entropy from W
- Therefore, sk_V reveals *no information* about W so long as columns of W have conditional min-entropy

Function Privacy!

- Correctness and attribute-hiding security follows from the structure of the extractor:
  $\mathrm{Ext}((w_1, \ldots, w_k), (s_1, \ldots, s_k)) = w_1 s_1 + \ldots + w_k s_k \pmod q$

$$V \cdot x = 0 \quad \textit{iff} \quad s^T \cdot W \cdot x = 0 \quad \textit{iff (w.h.p.)} \quad W \cdot x = 0$$

- **(In the paper)** Additional work to consider the case when q is "small" (poly in security param.)

Applications

- Function privacy when encrypting to *roots of polynomials*
  - ***minimal requirement:***
    coefficients of polynomials ($p_0 \quad p_1 \quad p_2 \quad \ldots \quad p_d$) must come from a distribution with *joint* min-entropy
  - no *conditional* min-entropy (public-key attacks can only use "Vandermonde vectors")
  - ***key idea:*** construct appropriate subspace during key generation with conditional min-entropy property

"Randomizing polynomials"

$$W = \begin{bmatrix} \text{coefficients of } p(x)\cdot r_1(x)\cdot s_1(x) \\ \text{coefficients of } p(x)\cdot r_2(x)\cdot s_2(x) \\ \text{coefficients of } p(x)\cdot r_3(x)\cdot s_3(x) \end{bmatrix}^{3 \times 5}$$

$$p(x) = p_0 + p_1 x + p_2 x^2$$

$$r_i(x) = r_{0,i} + r_{1,i}\, x$$

$$s_i(x) = s_{0,i} + s_{1,i}\, x$$

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  - ***minimal requirement:***
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$$r_i(x) = r_{0,i} + r_{1,i}\, x$$

$$s_i(x) = s_{0,i} + s_{1,i}\, x$$

Applications

- Function-Private IBE with minimal unpredictability

  Basic idea:
  attribute $x = (1, id)$, subspace is $W = (-id, 1)$
  Can "boost" entropy by considering $W = (-r \cdot id, r)$
  for uniformly sampled r from F_q

  Minimal unpredictability required from ID, as compared to [BRS13]

  Tradeoffs: Better function privacy, but stronger assumptions [KSW08] for IBE security

- Conjunctions and Disjunctions

Conclusions

- Extend the work of function privacy [BRS13] to the larger class of subspace-membership predicates
- Construct schemes from any underlying non-function-private inner-product scheme
- Function-private applications of SME
  - Roots of Polynomials
  - Function-Private IBE with minimal unpredictability
  - Conjunctions and Disjunctions

Open Problems

- Function privacy from computational assumptions
  – Recent work by Agrawal et al. [AABKPS13]
- Function privacy for Hidden-Vector Encryption
- Function privacy for larger classes of predicates
- *Enhanced* function privacy
  – preserve function privacy against an adversary that is given ciphertexts on which the challenge predicate evaluates to true

Thank You!
Any Questions?

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