Efficient One-Way Secret-Key Agreement and Private Channel Coding via Polarization

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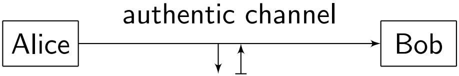
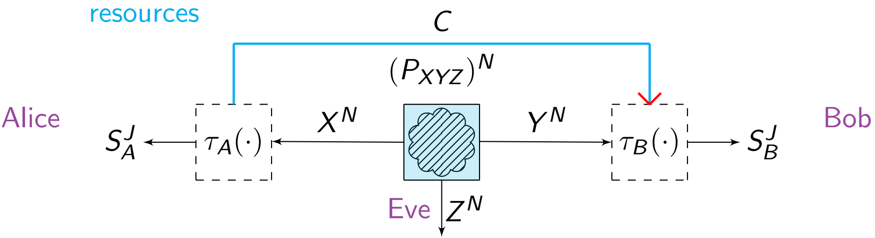
Information Theoretic Cryptography

Goal: information-theoretically secure private communication



- impossible [Shannon'48]
- possible when assuming correlated randomness [Maurer'93]
  - one-way secret key agreement
  - private channel coding over a wiretap channel

One-Way Secret-Key Agreement (SKA)



- reliability $\lim_{J\to\infty} \Pr\left[S_A^J \neq S_B^J\right] = 0$

uniformly distributed

- (strong) secrecy $\lim_{N\to\infty} \| P_{S_A^J, Z^N, C} - \boxed{\overline{P}_{S_A^J}} \times P_{Z^N, C} \|_1 = 0$

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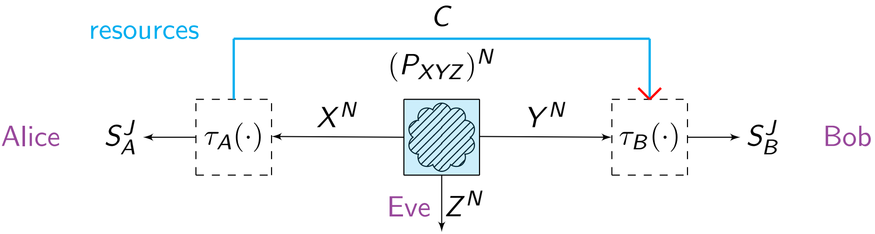

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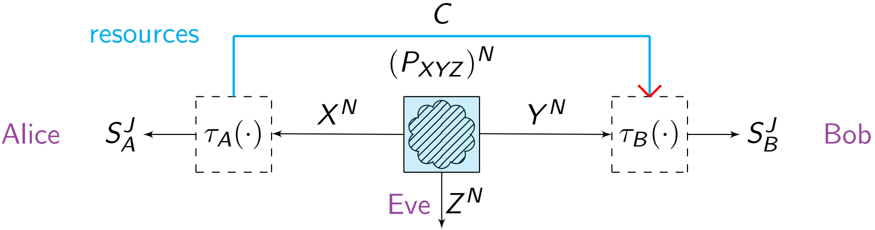
Historically

- (weak) secrecy $\lim_{N\to\infty} \frac{1}{N} I\left(S_A^J; Z^N, C\right) = 0$ insufficient [Maurer&Wolf'00]
- (strong) secrecy $\lim_{N\to\infty} I\left(S_A^J; Z^N, C\right) = 0$

$$\lim_{N\to\infty} \delta\left(P_{S_A^J}, \overline{P}_{S_A^J}\right) = 0$$

One-Way Secret-Key Agreement (SKA)



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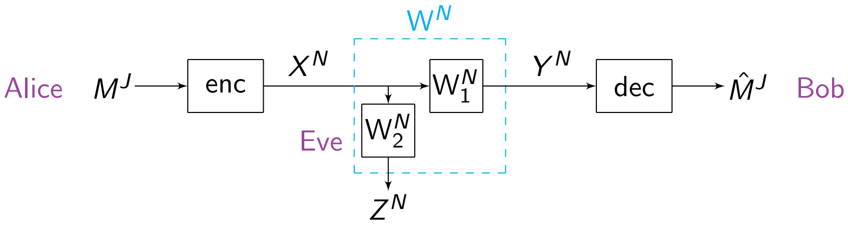
uniformly distributed

Thm[Csiszár&Körner'78]**:** One-way secret-key rate

$$S_\rightarrow(X;Y|Z) = \begin{cases} \max\limits_{P_{U,V}} & H(U|Z,V) - H(U|Y,V) \\ \text{s.t.} & V \multimap U \multimap X \multimap (Y,Z), \\ & |\mathcal{V}| \leqslant |\mathcal{X}|,\ |\mathcal{U}| \leqslant |\mathcal{X}|^2. \end{cases}$$

Private Channel Coding (PCC)



- reliability $\lim_{J\to\infty} \Pr\left[M^J \neq \hat{M}^J\right] = 0$
- (strong) secrecy $\lim_{N\to\infty} \|P_{M^J,Z^N,C} - P_{M^J} \times P_{Z^N,C}\|_1 = 0$

Thm[Csiszár&Körner'78]**:** Secrecy capacity

$$C_s = \begin{cases} \max\limits_{P_{V,X}} & H(V|Z) - H(V|Y) \\ \text{s.t.} & V \multimap X \multimap (Y,Z), \\ & |\mathcal{V}| \leqslant |\mathcal{X}|. \end{cases}$$

Efficient, Optimal Protocols

essentially linear complexity

- efficient $\neq$ practically efficient
- optimal $=$ achieve the highest possible rate
- (practically) efficient one-way secret-key agreement
  - only weak secrecy, degradability assumptions [Abbe'12]
  - shared key, degradability assumptions [Chou *et al.*'13]
- (practically) efficient private channel coding
  - only weak secrecy, degradability assumptions [Mahdavifar&Vardy'11]
  - binary symmetric wiretap channels (degradablity?!) [Bellare *et al.*'12]
  - degraded wiretap channels [Sasoglu&Vardy'13]

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getting rid of these assumptions

Polarization Phenomenon - Polar Codes

polar transform

- let $(X^N, Y^N) \sim (P_{X,Y})^N$
  let $U^N = G_N X^N$, where $\boxed{G_N} := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{\otimes \log N}$
- For $\epsilon \in (0,1)$, define a high- and a low-entropy set

$$\mathcal{R}_\epsilon^N(X|Y) := \left\{ i \in [N] : H\left(U_i \middle| U^{i-1}, Y^N\right) \geqslant 1 - \epsilon \right\}$$
$$\mathcal{D}_\epsilon^N(X|Y) := \left\{ i \in [N] : H\left(U_i \middle| U^{i-1}, Y^N\right) \leqslant \epsilon \right\}$$

Polarization Phenomenon - Polar Codes

- let $(X^N, Y^N) \sim (P_{X,Y})^N$ polar transform
  let $U^N = G_N X^N$, where $G_N := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{\otimes \log N}$
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$$\mathcal{D}_\epsilon^N(X|Y) := \left\{ i \in [N] : H\left(U_i \middle| U^{i-1}, Y^N\right) \leqslant \epsilon \right\}$$

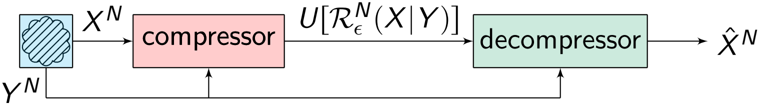
Thm[Arıkan'09]**:** Polarization Phenomenon: For any $\epsilon \in (0,1)$

$$\lim_{N\to\infty} \frac{|\mathcal{R}_\epsilon^N(X|Y)|}{N} = H(X|Y) \text{ and } \lim_{N\to\infty} \frac{|\mathcal{D}_\epsilon^N(X|Y)|}{N} = 1 - H(X|Y)$$

- Heart of polar codes (for source and channel coding)

Optimal Lossless Source Coding Using Polar Codes

Task: compress X^N w.r.t. side information Y^N



Optimal Lossless Source Coding Using Polar Codes

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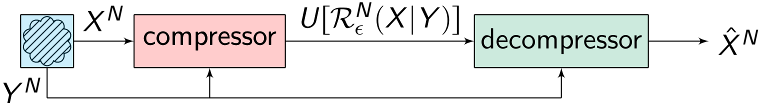

- compression
  - $U^N = G_N X^N$
  - take only $U[\mathcal{R}_\epsilon^N(X|Y)]$
- decompression
  - Likelihood estimation using side information Y^N

Optimal Lossless Source Coding Using Polar Codes

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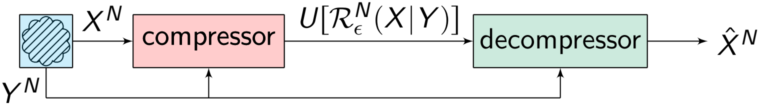


- compression
  - $U^N = G_N X^N$
  - take only $U[\mathcal{R}_\epsilon^N(X|Y)]$

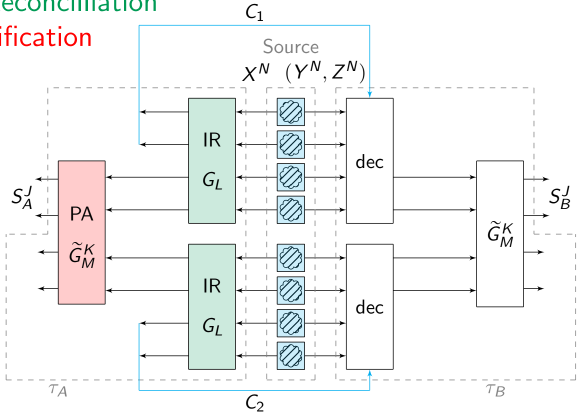
$O(N \log N)$

- decompression
  - Likelihood estimation using side information Y^N
- reliable[Arıkan'10] $\Pr\left[X^N \neq \hat{X}^N\right] = O(2^{-N^\beta})$ for $\beta < \frac{1}{2}$
- optimal [Slepian&Wolf'73], $H(X|Y) = \lim_{N\to\infty} \frac{1}{N}|\mathcal{R}_\epsilon^N(X|Y)|$

One-Way Secret-Key Agreement Protocol (M=2, L=4)

Information Reconcilliation

Privacy Amplification



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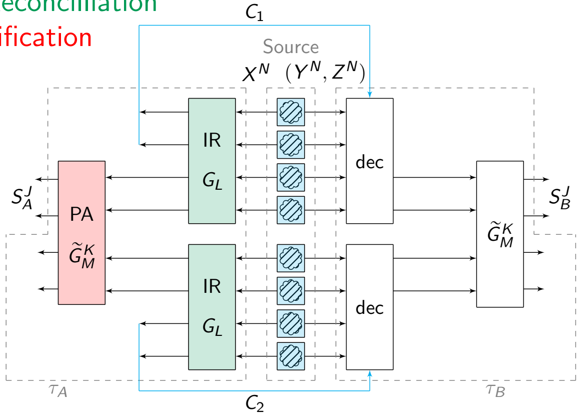


- no degradability assumptions
- no shared key needed

One-Way Secret-Key Agreement Characteristics

For any $\beta < \frac{1}{2}$

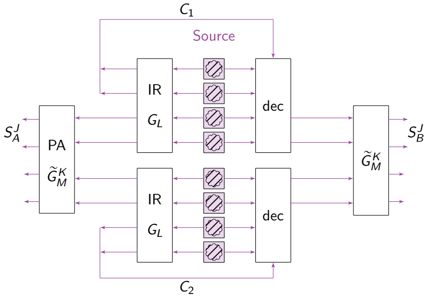
- Reliability: $\Pr\left[S_A^J \neq S_B^J\right] = O\left(M 2^{-L^{\beta}}\right)$
- Secrecy: $\left\| P_{S_A^J, Z^N, C} - \overline{P}_{S_A^J} \times P_{Z^N, C} \right\|_1 = O\left(\sqrt{N} 2^{-\frac{N^{\beta}}{2}}\right)$
- Rate: $R := \frac{J}{N} \geqslant \max\left\{0, H(X|Z) - H(X|Y) - \frac{o(N)}{N}\right\}$
- Complexity: $O(N \log N)$

M = # inner blocks

L = # inputs per inner block

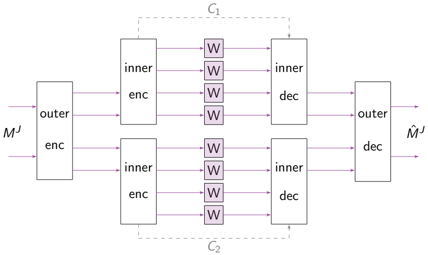
N = ML (blocklength)

Private Channel Coding (L = 4, M = 2)



Source

Secret-key agreement

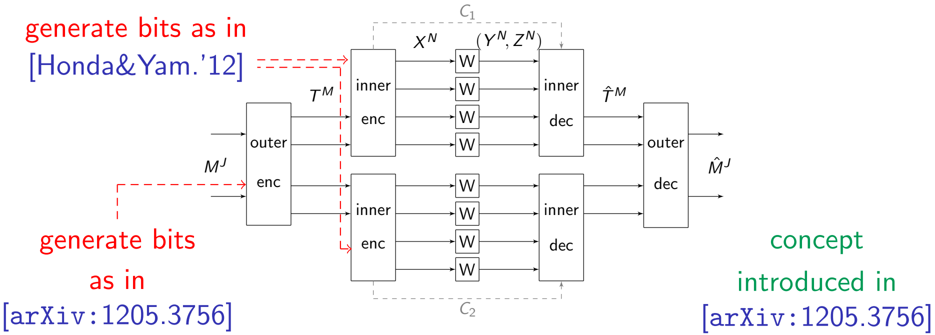


s

Private channel coding

Wiretap channel

Private Channel Coding ($L = 4$, $M = 2$)



- Run secret-key agreement scheme in reverse
- Mimic redundant bits
- Approx. of the secret-key agreement scenario (*shaping*) → same decoder can be used

Private Channel Coding: Characteristics

For any $\beta < \frac{1}{2}$

- Reliability: $\Pr\left[M^J \neq \hat{M}^J\right] = O\left(M2^{-L^\beta}\right)$
- Secrecy: $\|P_{M^J,Z^N,C} - \bar{P}_{M^J} \times P_{Z^N,C}\|_1 = O\left(\sqrt{N}2^{-\frac{N^\beta}{2}}\right)$
- Rate: $R \geqslant \max\left\{0, H(X|Z) - H(X|Y) - \frac{o(N)}{N}\right\}$
- Complexity: $O(N \log N)$

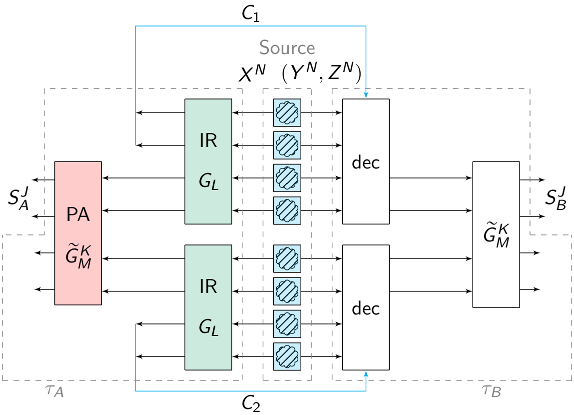
M = # inner blocks
L = # inputs per inner block
N = ML (blocklength)

Summary

One-way secret-key agreement and private channel coding

- at the optimal rate
- strong secrecy
- $O(N \log N)$ computational complexity
- no degradability assumptions
- no preshared key

Code Construction



Find index set at IR and PA layer

- IR: can be done in linear time [Tal&Vardy'11]
- PA: not fully solved yet