Leakage Resilient ElGamal Encryption

Eike Kiltz and *Krzysztof Pietrzak*

RUHR UNIVERSITÄT BOCHUM RUB

CWI Centrum Wiskunde & Informatica

Asiacrypt 2010, December 9th, Singapore

Outline

1. Hybrid Encryption, the KEM/DEM framework
2. ElGamal KEM
3. Leakage Resilient Crypto
   - Why?
   - How?
   - Other models?
4. Leakage Resilient ElGamal

CCA1 secure KEM (Key Encapsulation Mechanism)

$KEM = \{\mathsf{KG}, \mathsf{Enc}, \mathsf{Dec}\} \approx PKE$ *for* **random** *messages.*
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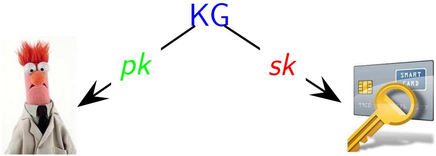

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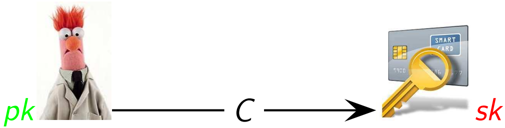

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$$(K_0, C) \leftarrow \textsf{Enc}(pk)\ ,\ K_1 \xleftarrow{\$} \mathcal{K}\ ,\ b \leftarrow \{0, 1\}$$

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$$(K_0, C) \leftarrow \mathsf{Enc}(pk) \ , \ K_1 \xleftarrow{\$} \mathcal{K} \ , \ b \leftarrow \{0, 1\}$$

CCA1 security: $\forall$  : $\Pr[$  guesses $b] - 1/2 = \text{negl}$

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public parameter: Cyclic group $\mathbb{G}$ of prime order p, $g = \langle \mathbb{G} \rangle$

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Side-Channel Attacks

pk 

sk

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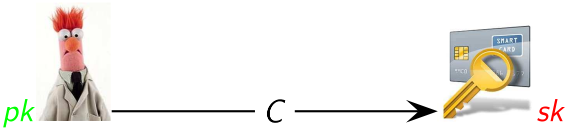

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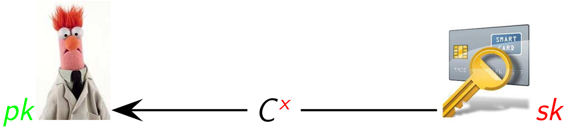

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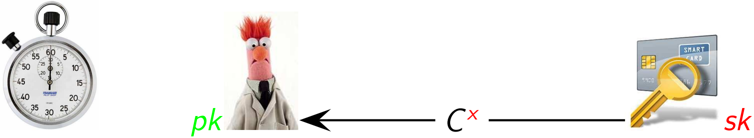

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$pk \longleftarrow C^{x}$ ———  sk

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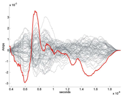
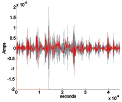
> *Side-Channel Attack: Cryptanalytic attack exploring information leaked from a physical implementation of a cryptosystem.*

More side-channel attacks

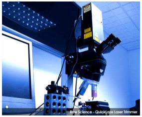
- power analysis

- radiation, sound, heat,. . .



- probing attacks

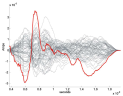


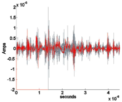
- cold-boot attacks

- cache attacks

More side-channel attacks

- power analysis
  [Eisenbarth et al. CRYPTO'08]
  break wireless car keys





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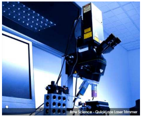

- cold-boot attacks
  [Halderman et al. USENIX'08]
  break disc-encryption schemes


- cache attacks
  [Ristenpart et al. CCS'09]
  break cloud computing

amazon webservices™

Side-Channel Countermeasures

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Implement countermeasures to prevent known attacks.

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make physical device look more like a black-box

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extend black-box model to incorporte leakage

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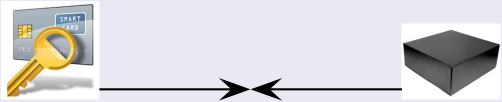

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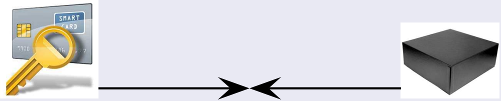

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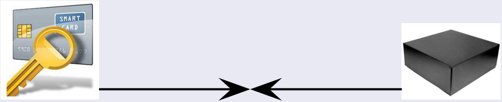

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  (only computation leaks "axiom" [MR04].)

Leakage Resilient Cryptography Cont.

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  LR stream-cipher [DP'08,P09,YSPY'10]
  LR (tree-based) signatures [FKPR'10]
  Evolving PKE sk difficult: must decrypt for fixed pk.

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- Scheme is very efficient ($\approx$ 2x basic ElGamal)
- Security proofs are very limited (generic group.)

Some Related Work

- General Compilers [Goldwasser-Rothblum,Juma-Vahlis Crypto'10]
  General but not practical (One Encryption get gate / Fully homomorphic encryption)
- Non-Continuous leakage (BRM/memory-attacks, auxiliary input), next talk.
- Continuous memory attacks [DHLW,BKKV FOCS'10], [LLW eprint 2010/562].

ElGamal KEM with shared key

g^{x} 

$x \in \mathbb{Z}_p$

ElGamal KEM with shared key

g^{x}  —— C $\longrightarrow$ 

$x \in \mathbb{Z}_p$

ElGamal KEM with shared key

g^{x}  $\leftarrow C^{x}$ —— 

$x \in \mathbb{Z}_p$

ElGamal KEM with shared key

g^x  —— $C, f(.)$ → 

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- Not Leakage-Resilient (learn x bit by bit.)

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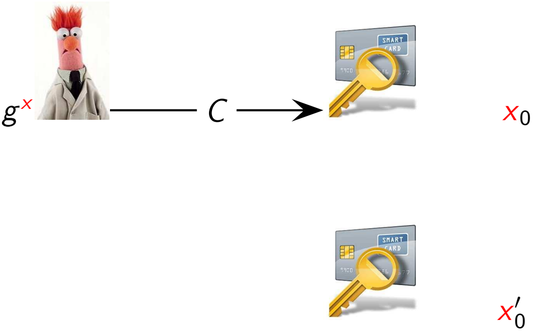

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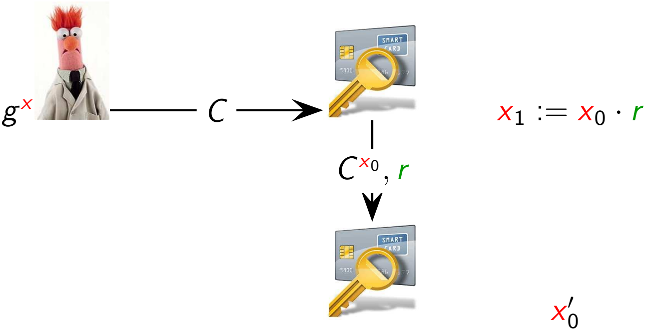

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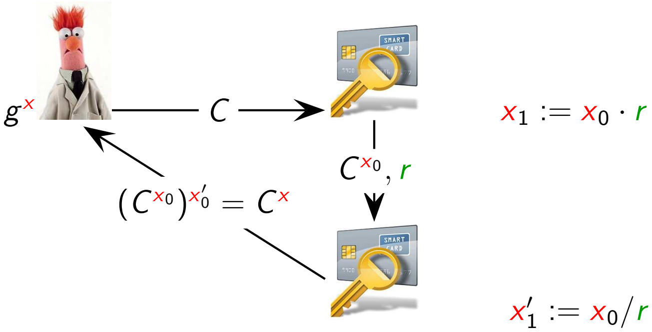

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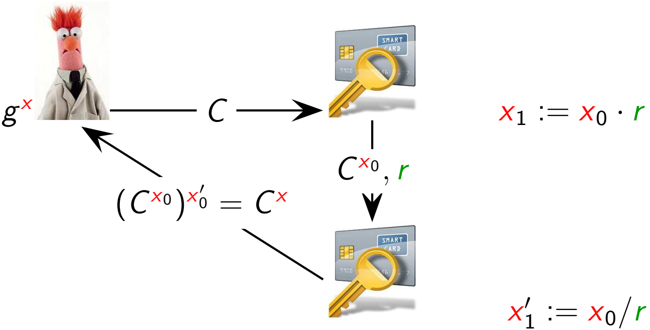

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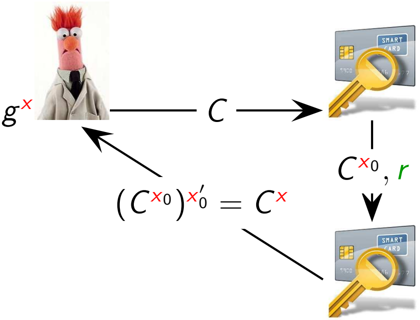
- Multiplicatively Secret-Share $x = x_0 \cdot x'_0$.
- Re-Sharing: $x_{i+1} \leftarrow x_i \cdot r$, $x'_{i+1} \leftarrow x'_i / r$.

ElGamal KEM with shared key



$x_1 := x_0 \cdot r$

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- Re-Sharing: $x_{i+1} \leftarrow x_i \cdot r$, $x'_{i+1} \leftarrow x'_i / r$.
- i'th query:  adaptively chooses $f_i(.)$, $f'_i(.)$.
  Gets leakage $f_i(x_i, r)$, $f'_i(x'_i, r, C^{x_i})$.

Conjecture: ElGamal KEM (as on previous slide) is leakage-resilient if

- *the group order p is not smooth (i.e. $p-1$ has large prime factor.)*
- *Range of leakage functions is bounded to, say $\lambda = 0.25 \cdot \log(p)$ bits.*

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- Scheme if "lifted" to bilinear groups is secure in generic group model (next slides.)

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Bilinear Groups

1. $\mathbb{G}$ is a (multiplicative) cyclic group of prime order p.
2. g is a generator of $\mathbb{G}$.
3. e is a bilinear map $e : \mathbb{G} \times \mathbb{G} \longrightarrow \mathbb{G}_T$
   1. $\forall a, b \in \mathbb{Z}, e(g^a, g^b) = e(g, g)^{ab}$
   2. $e(g, g) \stackrel{\text{def}}{=} g_T \neq 1$.

“lifted” ElGamal KEM

public parameter: $\mathbb{G}, \mathbb{G}_T$ of prime order p, $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$,

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Enc(pk): output $(C := g^r, K := g_T^{rx})$ where $r \xleftarrow{\$} \mathbb{Z}_p$

Dec(sk, C): output $e(C, g^{x})$

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Like for standard ElGamal, can define shared-key version $g^x = g^{x-r} \circ g^r$.

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Theorem

In the bilinear generic group model the lifted, shared-key ElGamal KEM is Leakage-Resilient (CCA1).
The leakage per invocation can be $< .49|\log(p)|$ *bits.*


Questions?

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ICITS 2011, Amsterdam, The Netherlands, May 21 - 24, 2011
5th International Conference on Information Theoretic Security
Submission deadline: Dec 10, 2010

Invited Speakers:

Benny Applebaum, Alexander Barg, Imre Csiszar, Ivan Damgaard, Yuval Ishai, Renato Renner, Leonid Reyzin, Ronald de Wolf