Limitations on Transformations from Composite-Order to Prime-Order Groups: The Case of Round-Optimal Blind Signatures

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Elliptic curves: what are they and why do we care?

Bilinear groups are cyclic groups G of some finite order that admit a nondegenerate bilinear map $e: G \times G \to G_T$

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Historically, we use elliptic curves for two main reasons:

- Efficiency: discrete log problem is harder, can use smaller parameters

- Functionality: IBE [BF01], predicate encryption [KSW08], etc.

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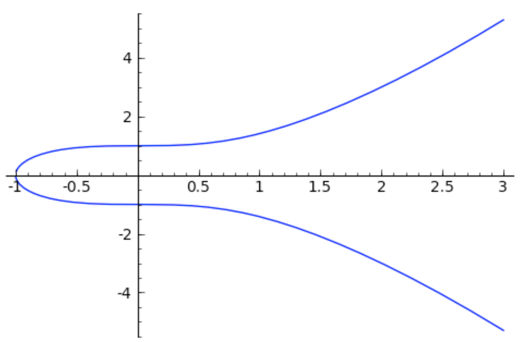

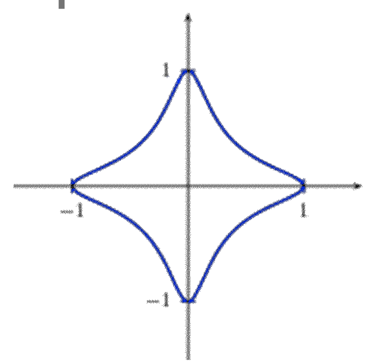

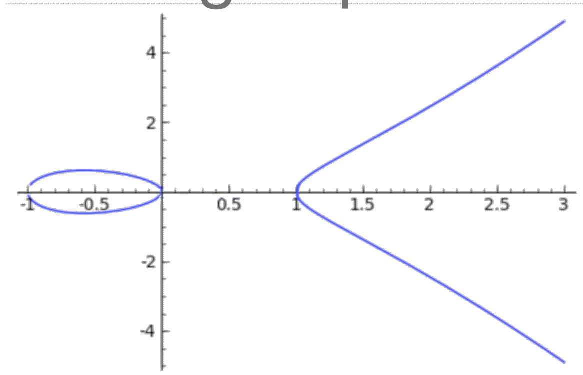

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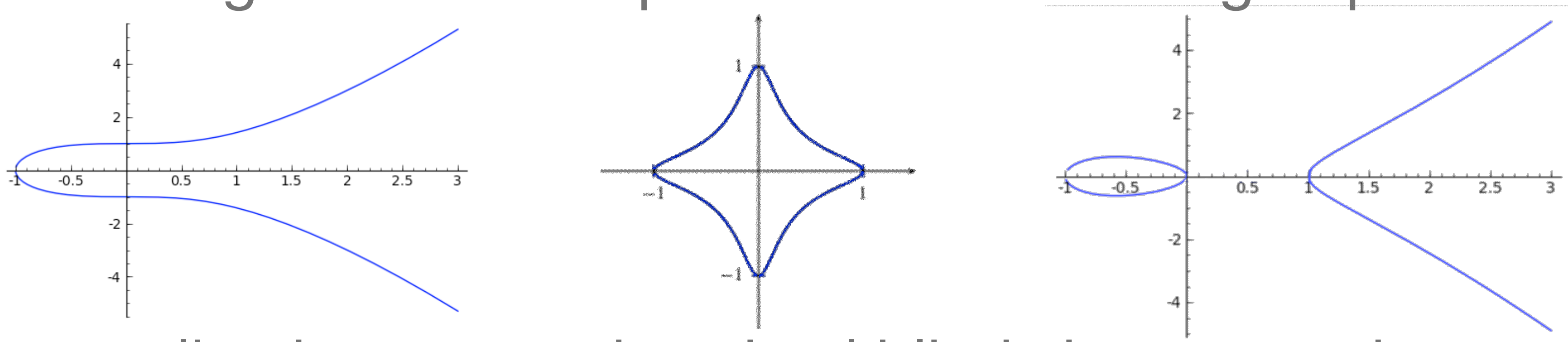

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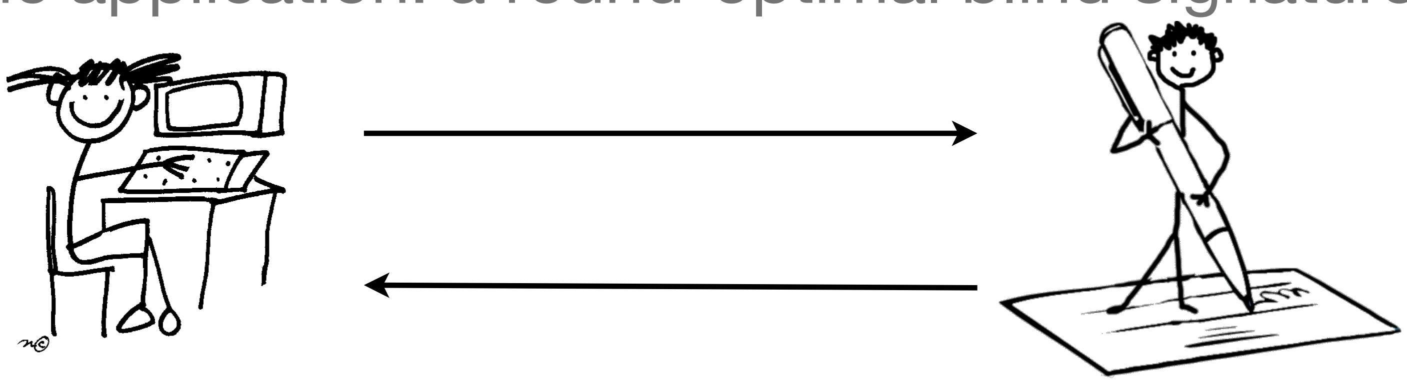

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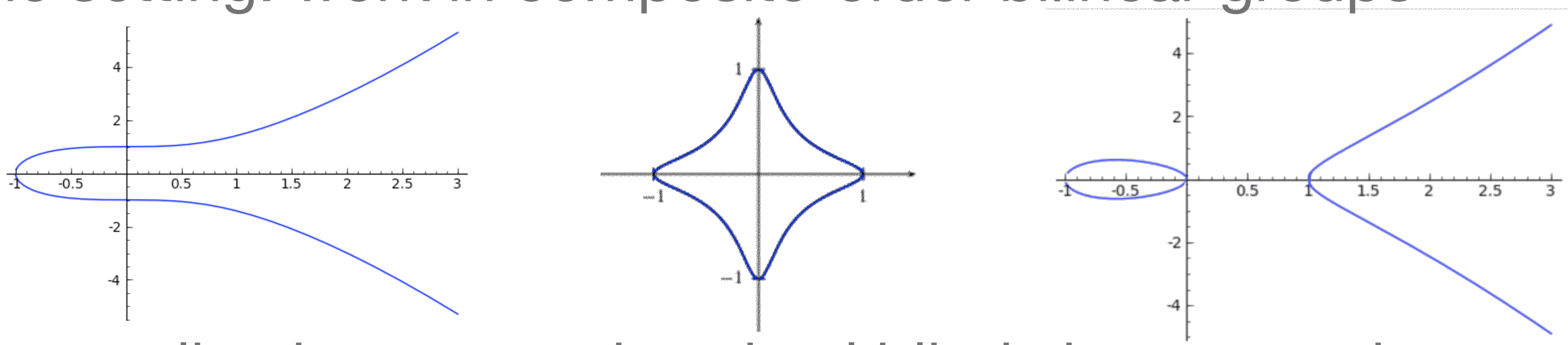

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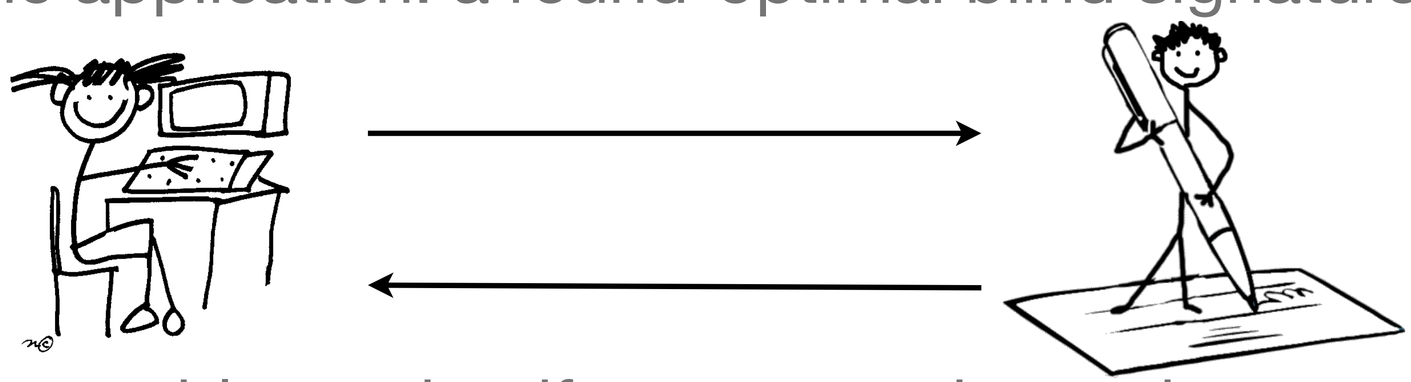

- The problem: what if we want to instantiate our scheme in a prime-order setting instead?

The setting: composite-order bilinear groups

- Cyclic groups G and G_T of order $N = pq$, $G = G_p \times G_q$ but p,q are secret
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“somewhat” homomorphic encryption [BGN05]

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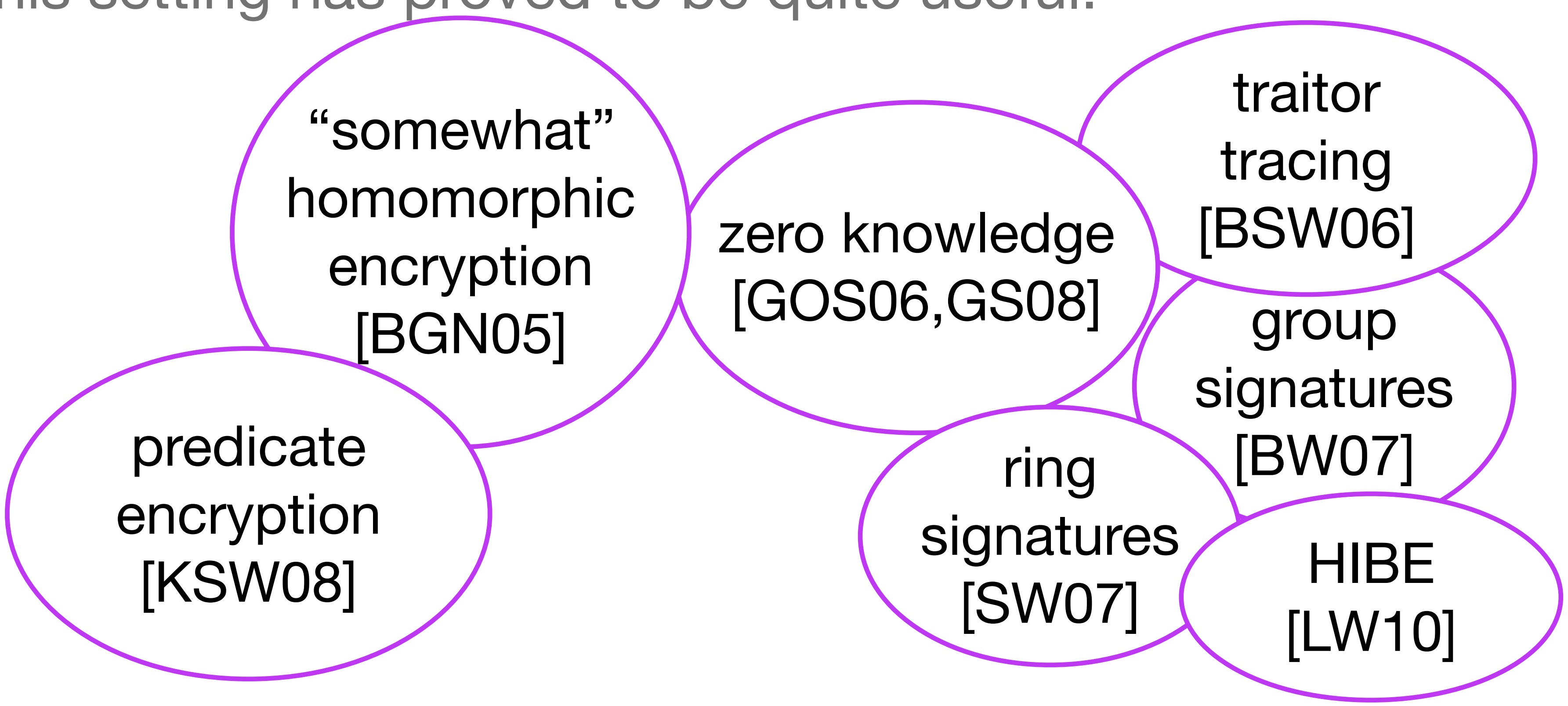

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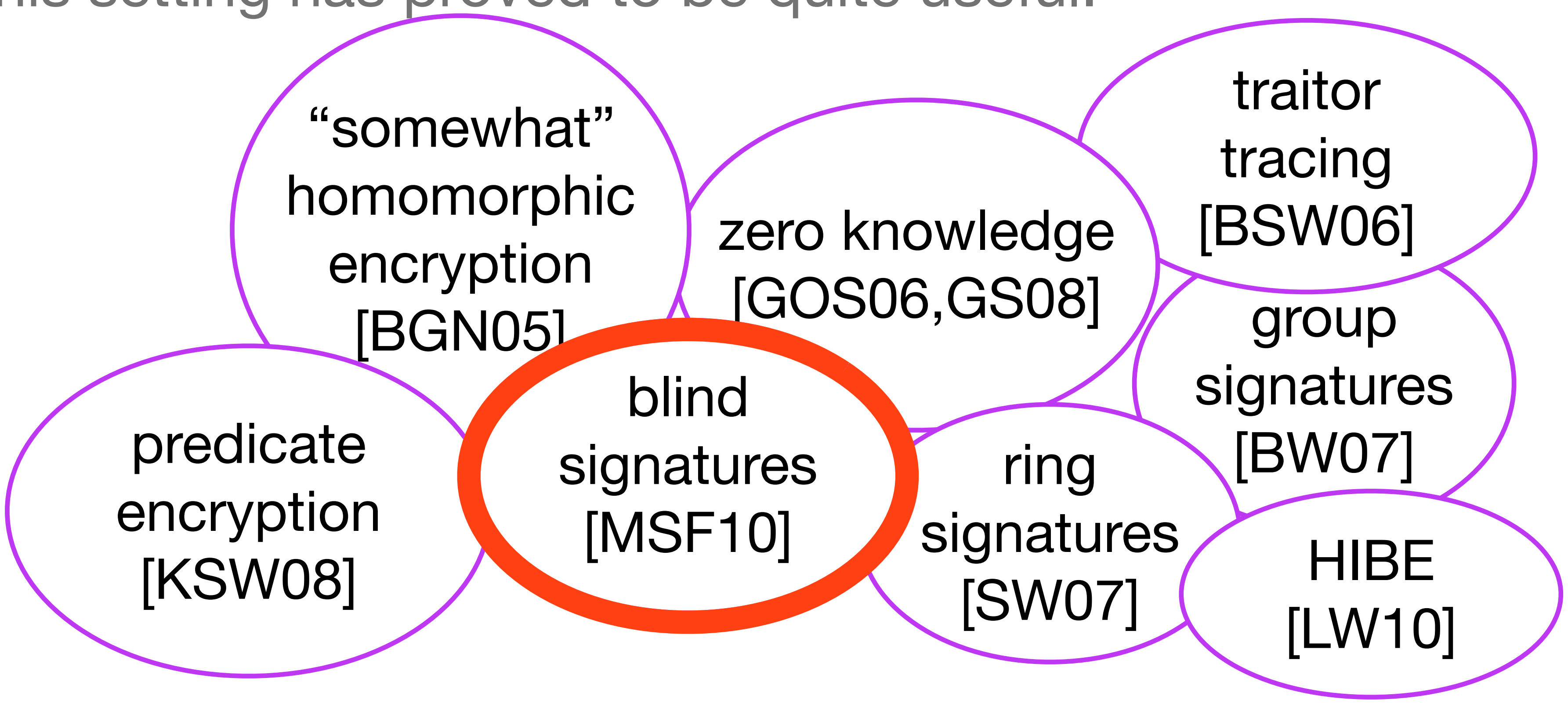

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- Composite-order means bigger: in prime-order groups, can use group of size ~160 bits; in composite-order groups need ~1024 bits (discrete log vs. factoring)

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Previously, people converted schemes in an ad-hoc way [W09,GSW09,LW10]

Freeman [F10] is first to provide a general conversion method

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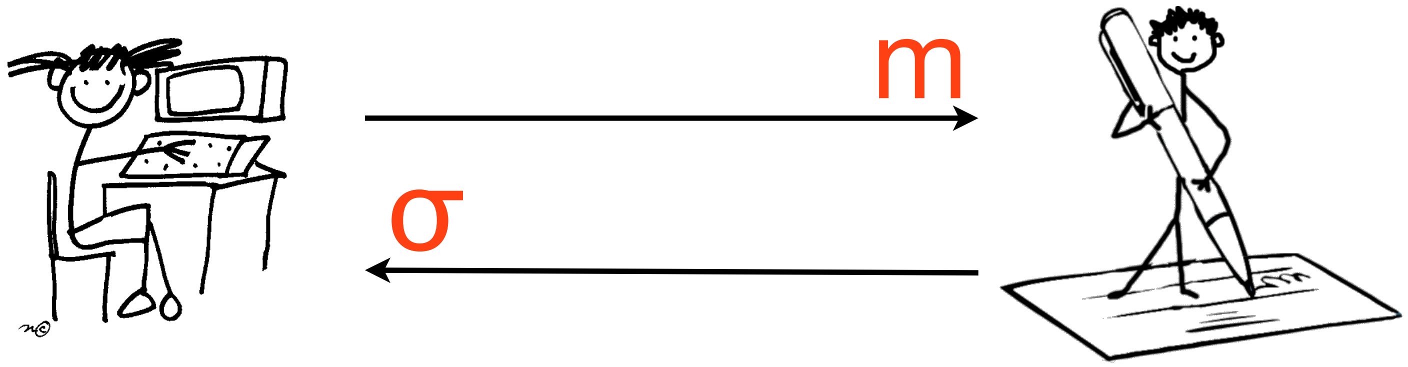

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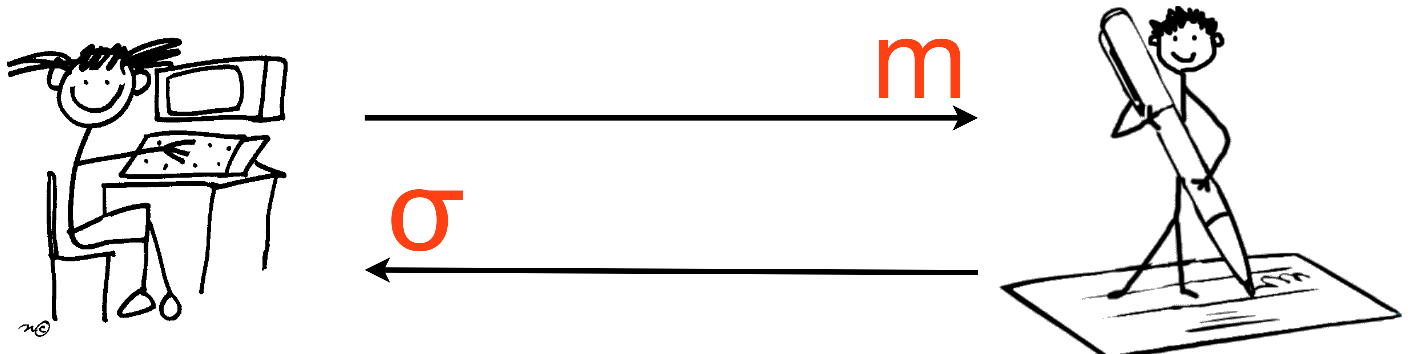

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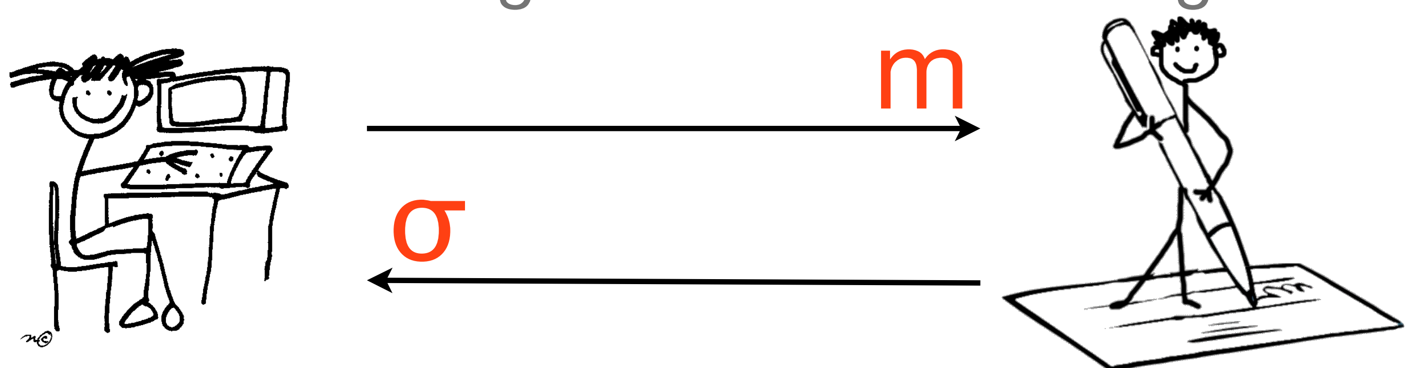

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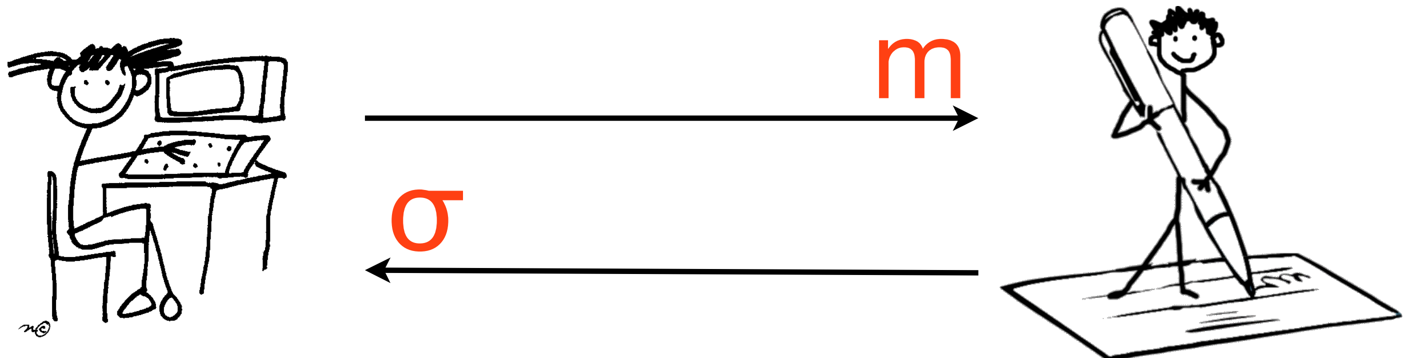

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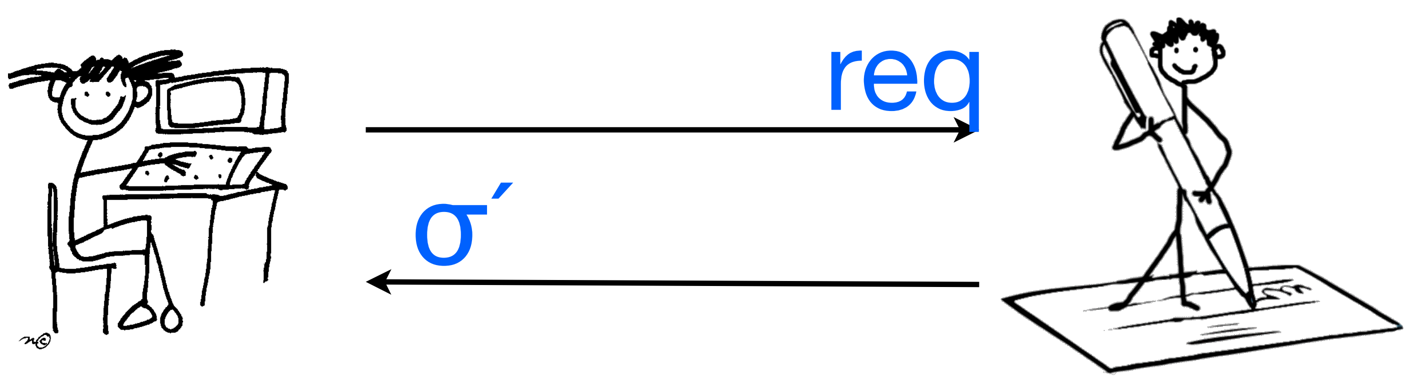

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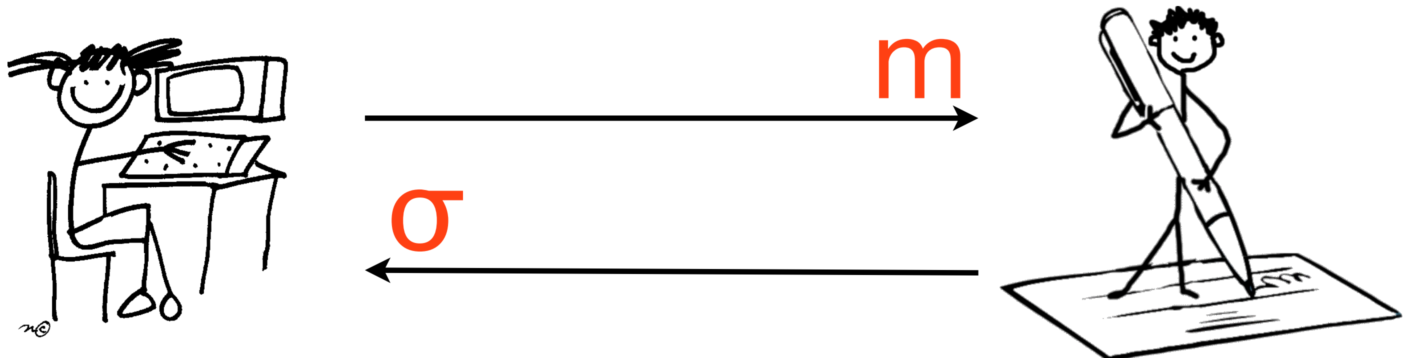

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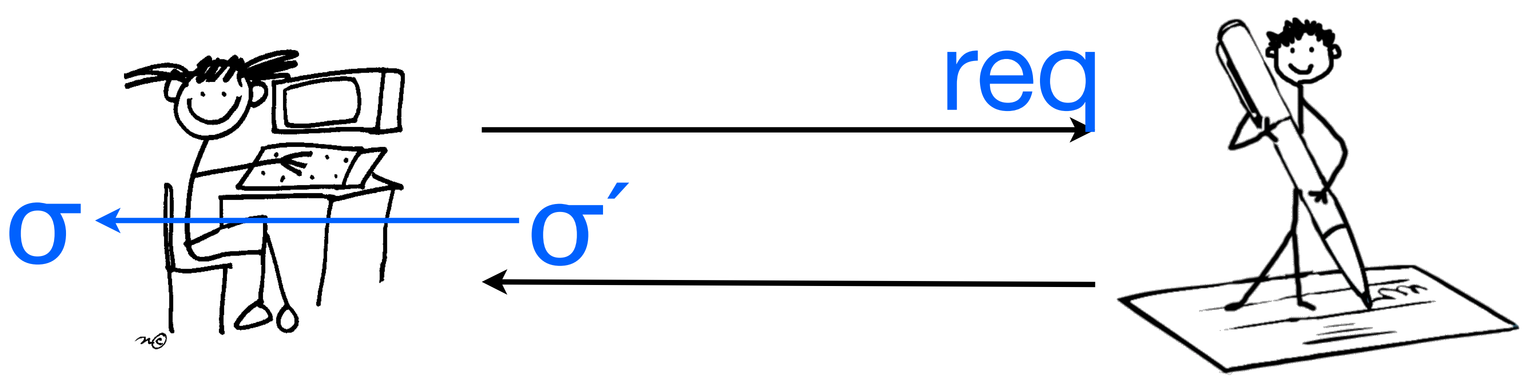

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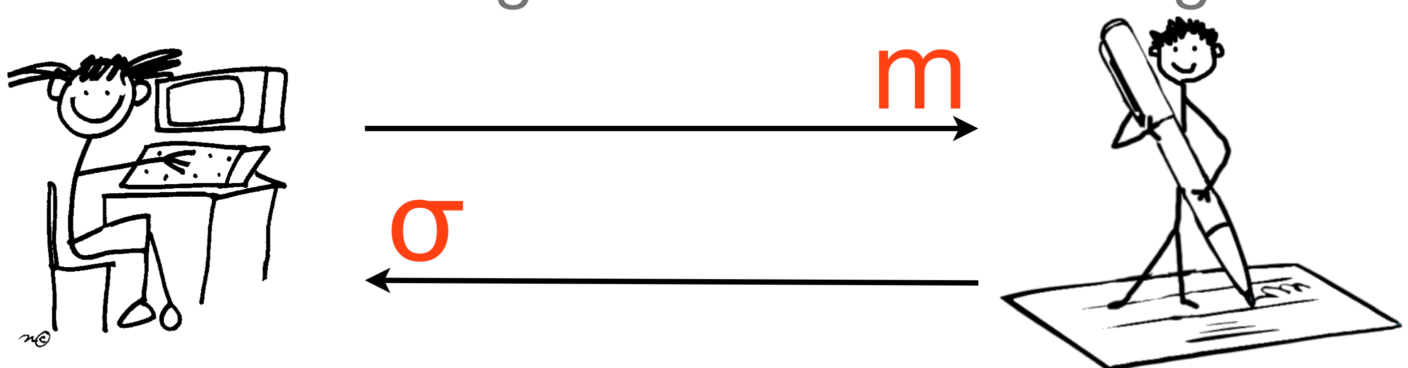

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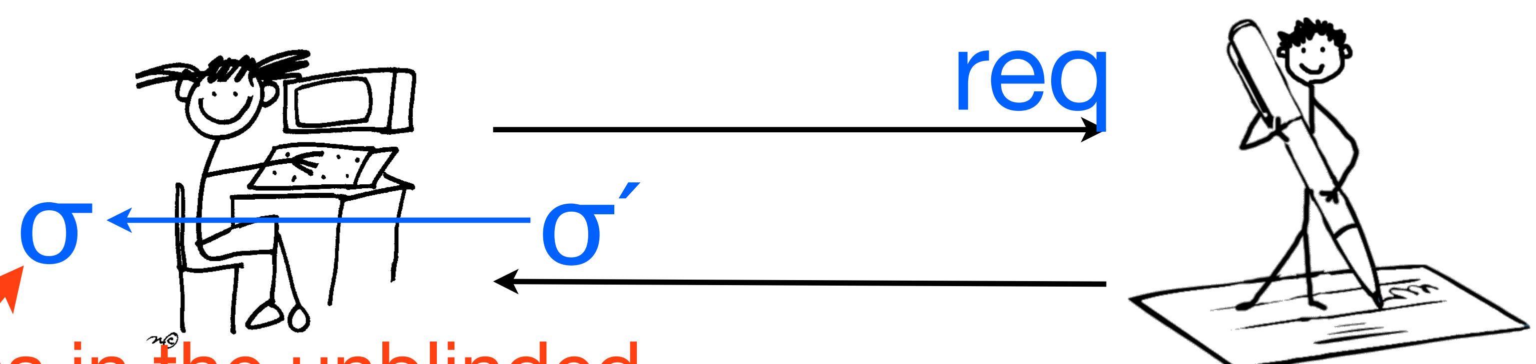

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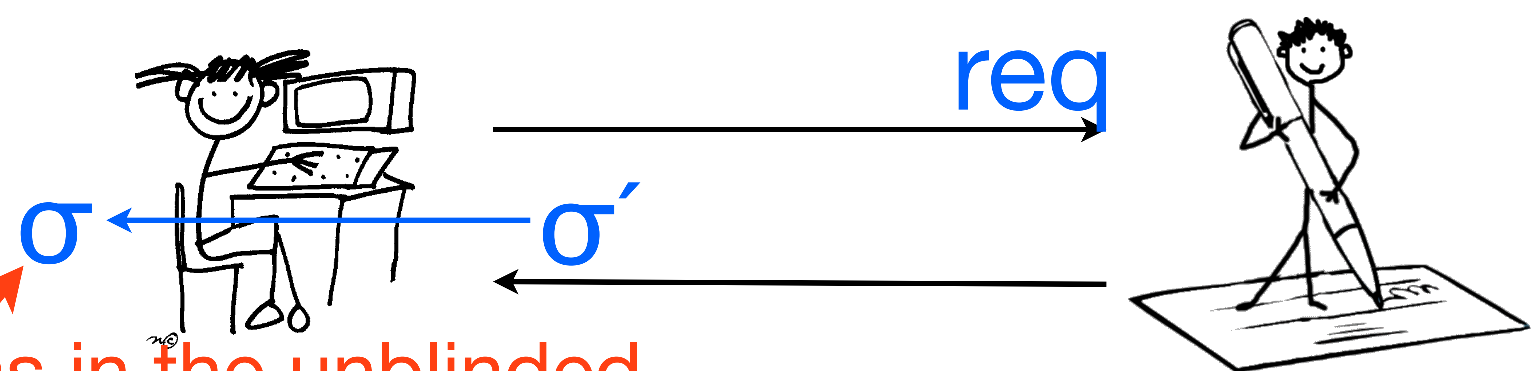

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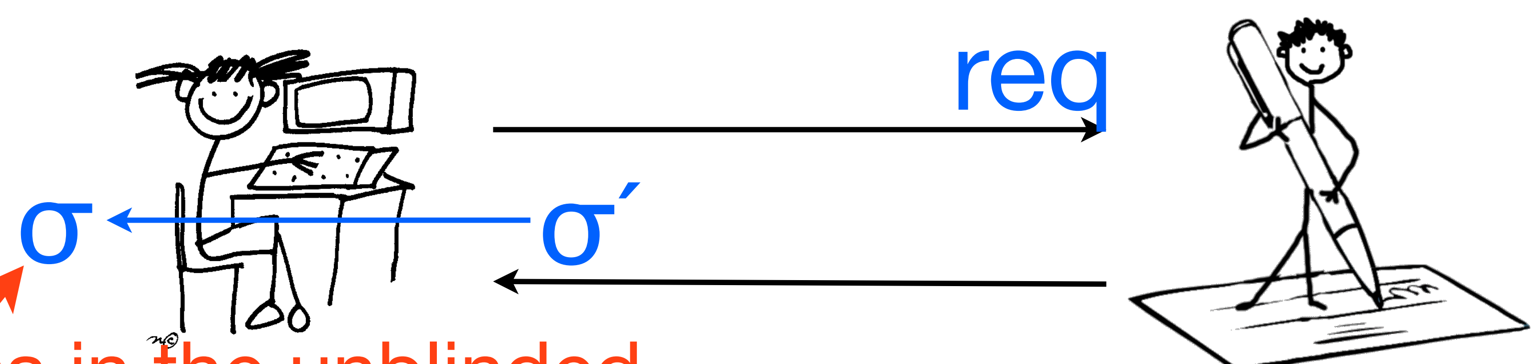

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Still a very active research area [O06,F09,AO10,AHO10,AFGHO10,R10]

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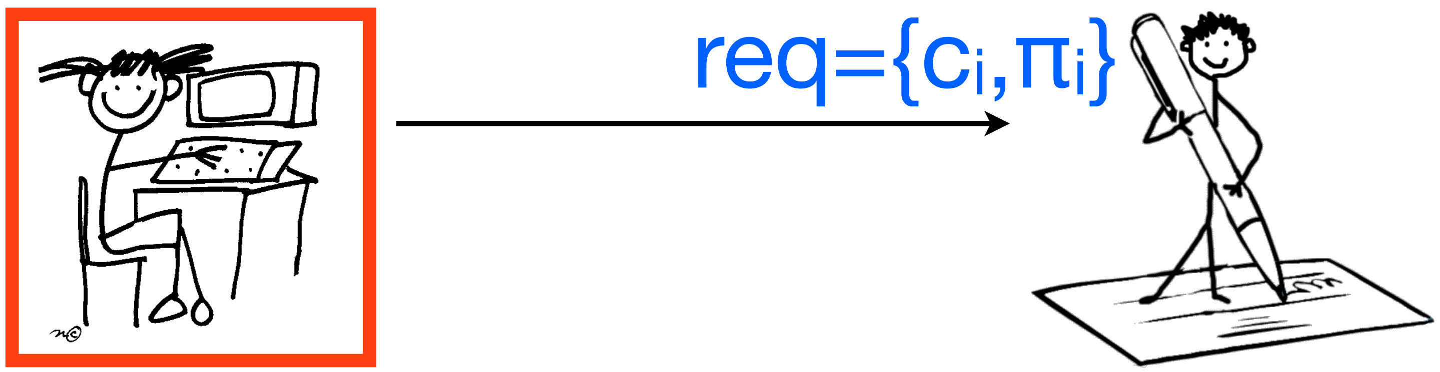

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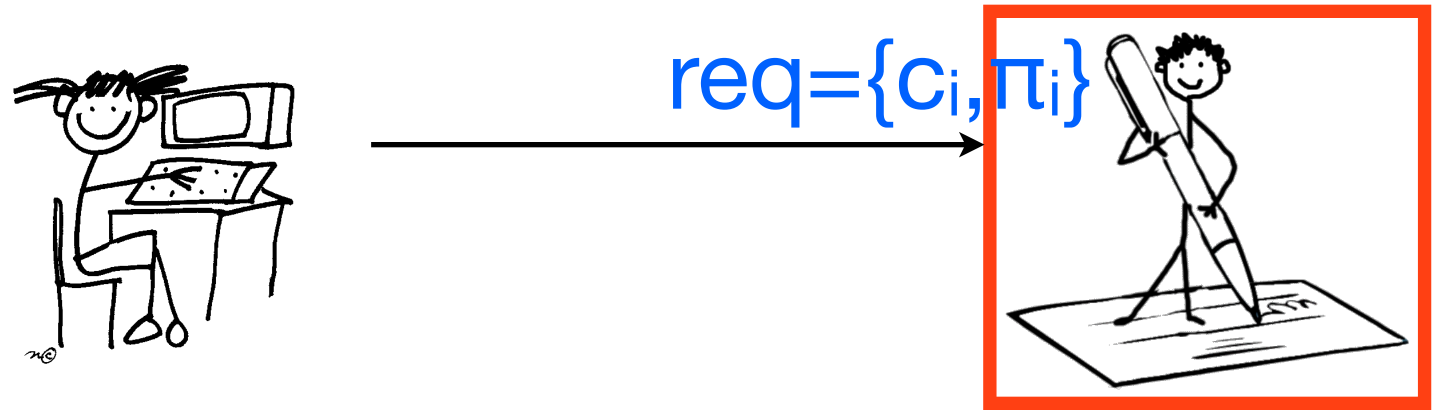

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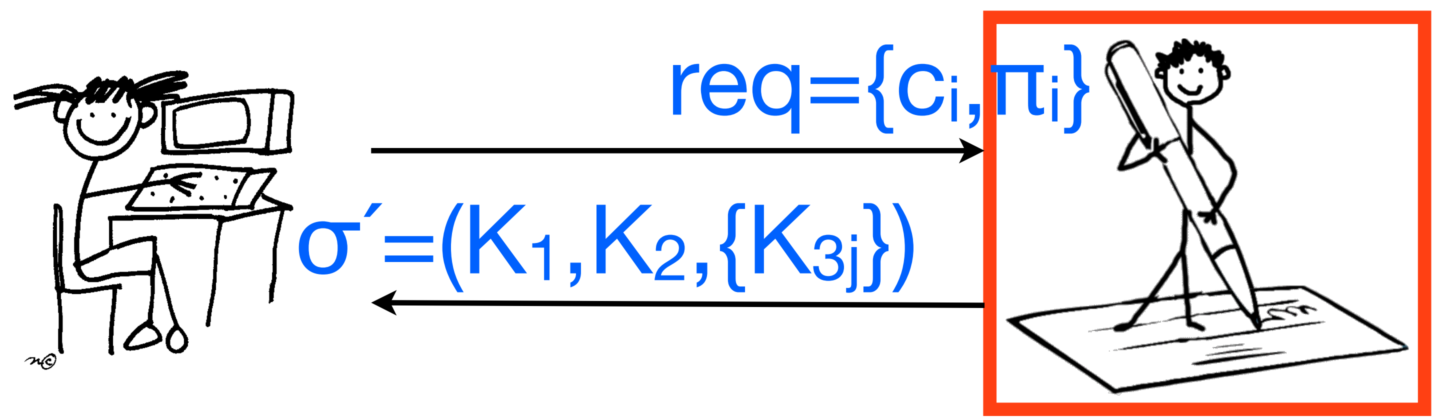

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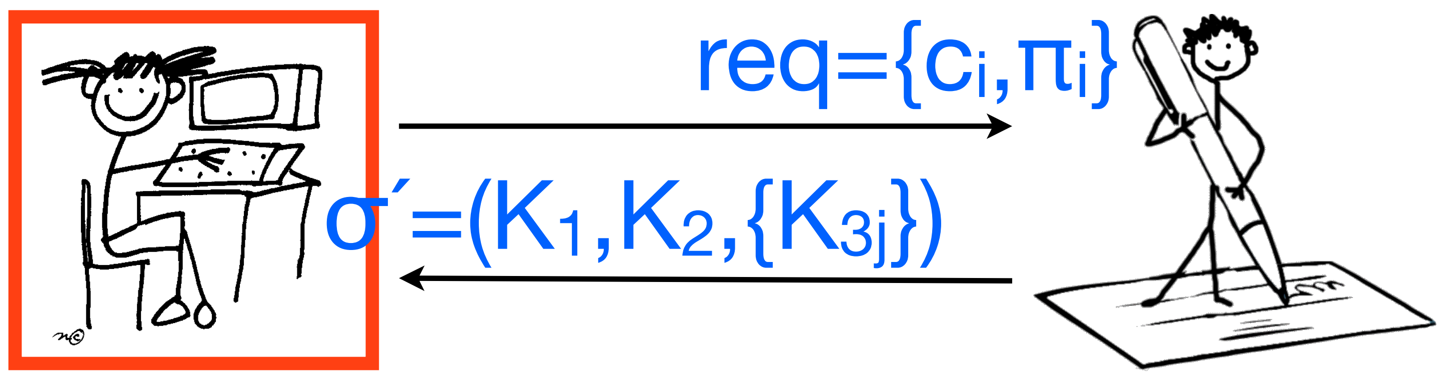

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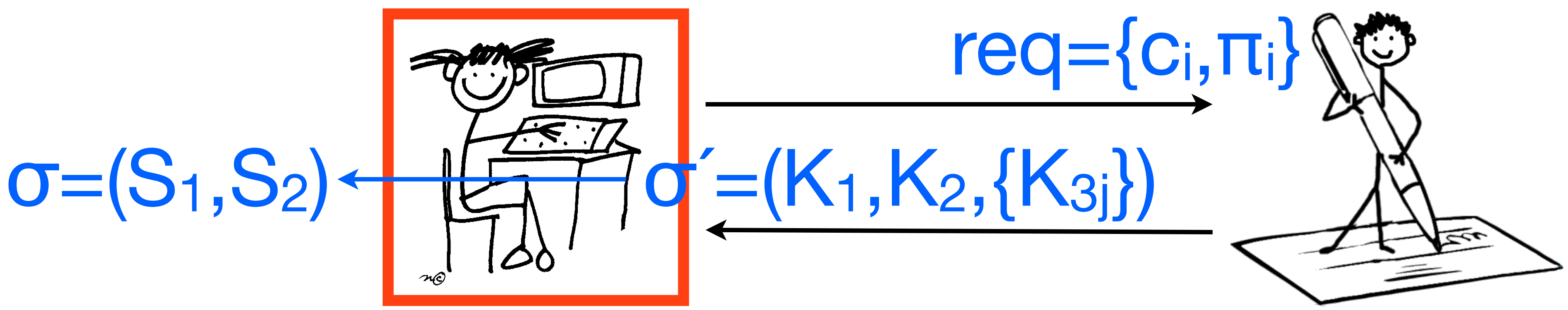

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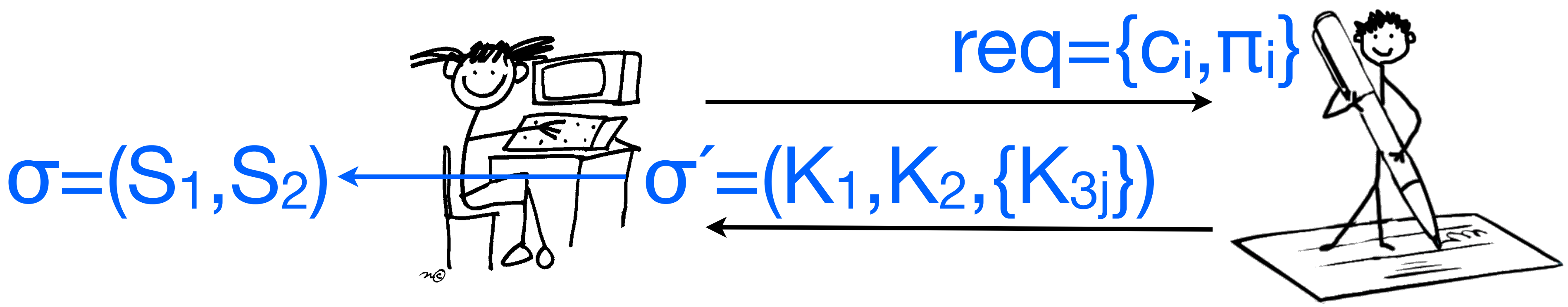

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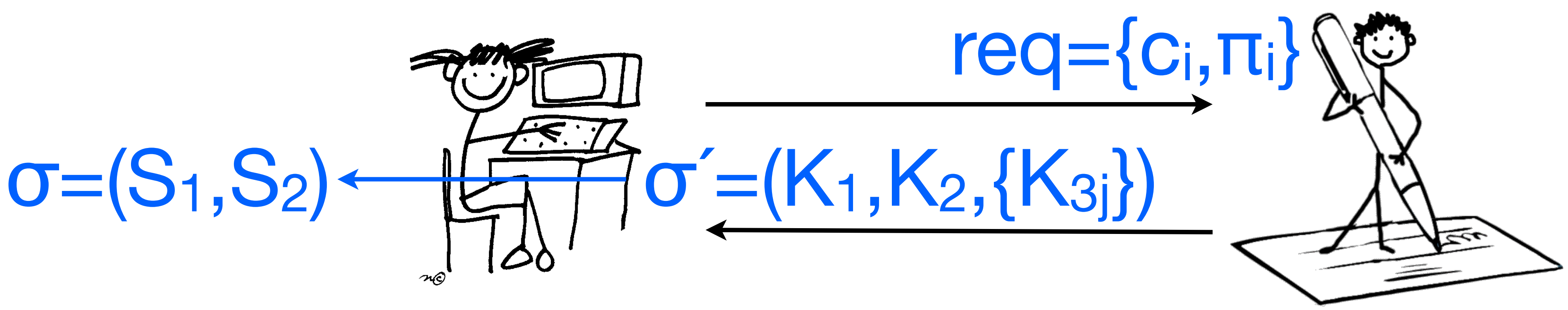

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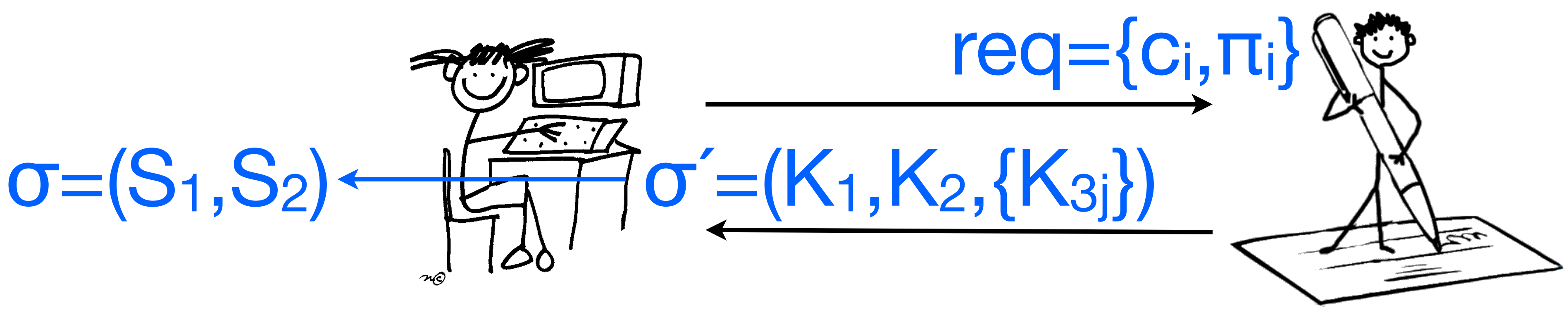

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Can prove the following security theorem:

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- Blindness requires only the abstract assumption,...
- ... but one-more unforgeability requires more.

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- map $\pi: B \rightarrow B_2$ s.t. $\pi(b=b_1 * b_2) = b_2$
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$\lambda = 0 \bmod p$

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Then $\pi(g) = \pi(g_p * g_q) = (g^q * g^p)^\lambda = g_q$

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$E(g_p,g_q) = E(g^q,g^p) = E(g,g)^{pq} = E(g,g)^N = 1$

Freeman [F10] provides generic transformation to prime-order groups for schemes in composite-order groups that require either of these two properties

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Break it up into two lemmas:

- Cancelling shrinks the target space: If we use the DLIN assumption for the indistinguishability of B_1 and B and E is cancelling, then $|E(B,B)| = p$.

- Can't project with small target: If $|E(B,B)| = p$ then E cannot be projecting.

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This turns out to be very tricky!

We want to prove the following theorem:

- If we use the DLIN assumption for the indistinguishability of B_1 and B and E is cancelling, then E cannot be projecting.

Break it up into two lemmas:

- Cancelling shrinks the target space: If we use the DLIN assumption for the indistinguishability of B_1 and B and E is cancelling, then $|E(B,B)| = p$.

✓ Can't project with small target: If $|E(B,B)| = p$ then E cannot be projecting.

The problem: what if we want both properties?

We can prove the following theorem:

- If we use the DLIN assumption* for the indistinguishability of B_1 and B and E is cancelling, then E cannot be projecting with overwhelming probability.

Break it up into two lemmas:

- Let $E: B \times B \to B_T$ be a nondegenerate pairing that is independent of the decomposition $B = B_1 \times B_2$. Then if $B = G^3$, B_1 is a uniformly random rank-2 submodule of B, and E is cancelling, then $|E(B,B)| = p$ with overwhelming probability.

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If B_1 is *not* random, can't be sure indistinguishability still holds

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Conclusions

Showed that if we want projecting and cancelling, generic transformations from composite- to prime-order groups fail

- Can't use DLIN (more generally, k-Linear [HK07,S07])
- This suggests possible functionality gap

Conclusions

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- Can't use DLIN (more generally, k-Linear [HK07,S07])
- This suggests possible functionality gap

Constructed a round-optimal blind signature scheme

- First efficient scheme using 'mild' assumptions (non-interactive, static), even including ones in the random oracle model
- Signature scheme demonstrates potential need for both properties

Open problems

Positive:

Negative:

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups

Negative:

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups

Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups

- Prove our scheme secure in prime-order groups

Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
- Prove our scheme secure in prime-order groups

Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups
- Show our scheme is insecure in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
- Prove our scheme secure in prime-order groups
- Show another general conversion from composite- to prime-order groups

Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups
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Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
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Negative:

- Prove there can be no projecting and cancelling pairing in prime-order groups
- Show our scheme is insecure in prime-order groups
- Prove that some other properties cannot be achieved in prime-order groups

Open problems

Positive:

- Construct a projecting and cancelling pairing in prime-order groups
- Prove our sch[illegible]
- Show another [illegible]e-order groups

Any questions?

Negative:

- Prove there ca[illegible] prime-order groups
- Show our scheme is insecure in prime-order groups
- Prove that some other properties cannot be achieved in prime-order groups