Review of the book

"An Introduction to Kolmogorov Complexity and Its Applications"

by Ming Li and Paul Vitányi

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Jiaxin Pan
HGI, Ruhr-University Bochum, Germany

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1 Summary of the book

This is one of well-known books on *Kolmogorov complexity ("K-complexity" for short)*. Informally, the K-complexity of an object is a measure of computational resources needed to specify the object. This notion has its roots in probability theory, information theory, and philosophical notions of randomness.

The main goal of this book is to give a unified and comprehensive introduction to the central ideas and applications of K-complexity theory. In particular, Chapters 2–4 provide us the mathematical theory of K-complexity and its applications are discussed in Chapters 5–8:

- Chapter 1 gives a brief introduction of the book and recalls useful mathematical notions and basics of probability theory and computability theory. Historical notes and context about the main roots of K-complexity are also included. For those who want to proceed directly to the technical details, this chapter can be safely skipped.

- Chapters 2 and 3 consider the optimal effective description length of objects and treat all aspects of the elementary mathematical theory of K-complexity, which is usually called *algorithmic complexity theory*. Two topics are the main focus here and also main tools in this book: the theory of Martin-Löf tests for randomness of finite objects and infinite sequences, which is inextricably intertwined with the theory of K-complexity, and the statistical properties of finite strings with high K-complexity.

- Chapter 4 considers the optimal (greatest) effective probability of objects. In particular, the theory of effective randomness tests under arbitrary recursive distributions for both finite and infinite sequences is developed.

- Chapter 5 develops a general theory of inductive reasoning and its applications. Chapter 6 introduces the incompressibility method. Chapter 7 studies the resource-bounded K-complexity and its applications in computational complexity theory. Chapter 8 considers advanced applications of K-complexity to relations between physics and computation.

Style of the book. The book is well-structured and self-contained. It contains enough basic knowledge from mathematics, information theory and computer science, which is used quite frequently in the book. The central theory of K-complexity and its applications are covered.

In order to elaborate the theory better, plenty of examples are given and also there are non-indented and smaller-font paragraphs interspersed in the main text, which are used to give explanatory, or raise some important technical issues, or point out alternative views of the content without distracting readers. Exercises are given at the end of main sections, which contain much of the technical content of the literature on K-complexity and related issues. Difficulty of exercises is rated by the rating system of Knuth [1]. I think that is really good for its readers, since they can see if an exercise is trivial, or medium, or even a research problem and then they can decide which exercise to solve and know how good they can understand the main text. At the end of every chapter, a historical section is presented.

2 Recommendation

This is an excellent introductory book written by two experts in the field. I think the book is a nice textbook for lectures at graduate level on Kolmogorov complexity or lectures with special interest in learning theory, or randomness, or information theory. As explained before, the book contains plenty of examples and exercises. Thus I think this book is also interesting for those who want to learn basics of K-complexity on their own. Moreover, although I am not an expert in this field, I suppose that researchers working on K-complexity will still benefit from reading the book.

The reviewer is a Ph.D. student at the Ruhr-University Bochum, Germany.

References

[1] D. E. Knuth. *The Art of Computer Programming, Volume I: Fundamental Algorithms, 2nd Edition.* Addison-Wesley, 1973.