

Complete Fairness in Multi-Party Computation Without an Honest Majority

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Abstract. Gordon et al. recently showed that certain (non-trivial) functions can be computed with complete fairness in the *two-party* setting. Motivated by their results, we initiate a study of complete fairness in the *multi-party* case and demonstrate the first completely-fair protocols for non-trivial functions in this setting. We also provide evidence that achieving fairness is “harder” in the multi-party setting, at least with regard to round complexity.

1 Introduction

In the setting of secure computation, a group of parties wish to run a protocol for computing some function of their inputs while preserving, to the extent possible, security properties such as privacy, correctness, input independence and others. These requirements are formalized by comparing a real-world execution of the protocol to an *ideal world* where there is a trusted entity who performs the computation on behalf of the parties. Informally, a protocol is “secure” if for any real-world adversary \mathcal{A} there exists a corresponding ideal-world adversary \mathcal{S} (corrupting the same parties as \mathcal{A}) such that the result of executing the protocol in the real world with \mathcal{A} is computationally indistinguishable from the result of computing the function in the ideal world with \mathcal{S} .

One desirable property is *fairness* which, intuitively, means that either *everyone* receives the output, or else *no one* does. Unfortunately, it has been shown by Cleve [1] that complete fairness is impossible *in general* without a majority of honest parties. Until recently, Cleve’s result was interpreted to mean that *no* non-trivial functions could be computed with complete fairness without an honest majority. A recent result of Gordon et al. [2], however, shows that this folklore is wrong; there exist non-trivial functions that *can* be computed with complete fairness in the two-party setting. Their work demands that we re-evaluate our current understanding of fairness.

Gordon et al. [2] deal exclusively with the case of two-party computation, and leave open the question of fairness in the multi-party setting. Their work does *not* immediately extend to the case of more than two parties. (See also the discussion in the section that follows.) An additional difficulty that arises in the

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multi-party setting is the need to ensure *consistency* between the outputs of the honest parties, even after a malicious abort. In more detail: in the two-party setting, it suffices for an honest party’s output (following an abort by the other party) to be consistent only with its own local input. But in the multi-party setting, the honest parties’ outputs must each be consistent with *all* of their inputs. This issue is compounded by the adversary’s ability to adaptively abort the t malicious players in any order and at any time, making fairness in the multi-party setting even harder to achieve.

We initiate a study of complete fairness in the multi-party setting. We focus on the case when a private¹ broadcast channel (or, equivalently, a PKI) is available to the parties; note that Cleve’s impossibility result applies in this case as well. Although one can meaningfully study fairness in the absence of broadcast, we have chosen to assume broadcast so as to separate the question of *fairness* from the question of *agreement* (which has already been well studied in the distributed systems literature). Also, although the question of fairness becomes interesting as soon as an honest majority is not assumed, here we only consider the case of completely-fair protocols tolerating an *arbitrary* number of corrupted parties. We emphasize that, as in [2], we are interested in obtaining *complete* fairness rather than some notion of *partial* fairness.

1.1 Our Results

A natural first question is whether two-party feasibility results [2] can be extended “easily” to the multi-party setting. More formally, say we have a function $f : \{0, 1\} \times \cdots \times \{0, 1\} \rightarrow \{0, 1\}$ taking n boolean inputs. (We restrict to boolean inputs/outputs for simplicity only.) For any non-trivial subset $I \subset [n]$, define the *partition* f_I of f as the two-input function $f_I : \{0, 1\}^{|I|} \times \{0, 1\}^{n-|I|} \rightarrow \{0, 1\}$ given by

$$f_I(y, z) = f(x),$$

where $x \in \{0, 1\}^n$ is such that $x_I = y$ and $x_{\bar{I}} = z$. It is not hard to see that if there exists an I for which f_I *cannot* be computed with complete fairness in the two-party setting, then f cannot be computed with complete fairness in the multi-party setting. Similarly, the round complexity for computing f with complete fairness in the multi-party case must be at least the round complexity of fairly computing each f_I . What about the converse? We show the following negative result regarding such a “partition-based” approach to the problem:

Theorem 1 *(Under suitable cryptographic assumptions) there exists a 3-party function f all of whose partitions can be computed with complete fairness in $\mathcal{O}(1)$ rounds, but for which any multi-party protocol computing f with complete fairness requires $\omega(\log k)$ rounds, where k is the security parameter.*

¹ We assume private broadcast so as to ensure security against passive eavesdroppers (who do not corrupt any parties).

This indicates that fairness in the multi-party setting is qualitatively harder than fairness in the two-party setting. (A somewhat analogous result in a different context was shown by Chor et al. [3].)

The function f for which we prove the above theorem is interesting in its own right: it is the 3-party *majority* function (i.e., voting). Although the $\omega(\log k)$ -round lower bound may seem discouraging, we are able to show a positive result for this function; to the best of our knowledge, this represents the first non-trivial feasibility result for complete fairness in the multi-party setting.

Theorem 2 *(Under suitable cryptographic assumptions) there exists an $\omega(\log k)$ -round protocol for securely computing 3-party majority with complete fairness.*

Our efforts to extend the above result to n -party majority have been unsuccessful. One may therefore wonder whether there exists *any* (non-trivial) function that can be computed with complete fairness for general n . Indeed, there is:

Theorem 3 *(Under suitable cryptographic assumptions) for any number of parties n , there exists an $\Theta(n)$ -round protocol for securely computing boolean OR with complete fairness.*

OR is non-trivial in our context: OR is complete for multi-party computation (without fairness) [4], and cannot be computed with information-theoretic privacy even in the two-party setting [5].

Relation to prior work. At a superficial level, the proof of the $\omega(\log k)$ -round lower bound of Theorem 1 uses an approach similar to that used to prove an analogous lower bound in [2]. We stress, however, that our theorem does not follow as a corollary of that work (indeed, it cannot since each of the partitions of f can be computed with complete fairness in $\mathcal{O}(1)$ rounds). We introduce new ideas to prove the result in our setting; in particular, we rely in an essential way on the fact that the output of any two honest parties must agree (whereas this issue does not arise in the two-party setting considered in [2]).

Ishai et al. [6] propose a protocol that is resilient to a dishonest majority in a weaker sense than that considered here. Specifically, their protocol achieves the following guarantee (informally): when $t < n$ parties are corrupted then a real execution of the protocol is as secure as an execution in the ideal world with complete fairness *where the adversary can query the ideal functionality $\mathcal{O}(t)$ times* (using different inputs each time). While this definition may guarantee privacy for certain functions (e.g., for the sum function), it does not prevent the malicious parties from *biasing* the output of the honest parties. We refer the reader to their work for further discussion.

1.2 Outline of the Paper

We include the standard definitions of secure multi-party computation in the full version of this paper [7]. We stress that although the definitions are standard,

what is *not* standard is that we are interested in attaining complete fairness even though we do not have an honest majority.

We begin with our negative result, showing that any completely-fair protocol for 3-party majority requires $\omega(\log k)$ rounds. Recall that what is especially interesting about this result is that it demonstrates a gap between the round complexities required for completely-fair computation of a function and its (two-party) partitions. In Section 3, we show an $\omega(\log k)$ -round protocol for completely-fair computation of 3-party majority. In Section 4 we describe our feasibility result for the case of boolean OR.

2 A Lower Bound on the Round Complexity of Majority

2.1 Proof Overview

In this section, we prove Theorem 1 taking as our function f the three-party majority function \mathbf{maj} . That is, $\mathbf{maj}(x_1, x_2, x_3) = 0$ if at least two of the three values $\{x_1, x_2, x_3\}$ are 0, and is 1 otherwise. Note that any partition of \mathbf{maj} is just (isomorphic to) the greater-than-or-equal-to function, where the domain of one input can be viewed as $\{0, 1, 2\}$ and the domain of the other input can be viewed as $\{0, 1\}$ (in each case, representing the number of ‘1’ inputs held). Gordon et al. [2] show that, under suitable cryptographic assumptions, the greater-than-or-equal-to function on constant-size domains can be securely computed with complete fairness in $\mathcal{O}(1)$ rounds.

We prove Theorem 1 by showing that any completely-fair 3-party protocol for \mathbf{maj} requires $\omega(\log k)$ rounds. The basic approach is to argue that if Π is any protocol for securely computing \mathbf{maj} , then eliminating the last round of Π results in a protocol Π' that still computes \mathbf{maj} correctly “with high probability”. Specifically, if the error probability in Π is at most μ (that we will eventually set to some negligible function of k), then the error probability in Π' is at most $c \cdot \mu$ for some constant c . If the original protocol Π has $r = \mathcal{O}(\log k)$ rounds, then applying this argument inductively r times gives a protocol that computes \mathbf{maj} correctly on all inputs with probability significantly better than guessing *without any interaction at all*. This gives the desired contradiction.

To prove that eliminating the last round of Π cannot affect correctness “too much”, we consider a constraint that holds for the ideal-world evaluation of \mathbf{maj} . (Recall, we are working in the ideal world where complete fairness holds.) Consider an adversary who corrupts two parties, and let the input of the honest party P be chosen uniformly at random. The adversary can learn P ’s input by submitting $(0, 1)$ or $(1, 0)$ to the trusted party. The adversary can also try to bias the output of \mathbf{maj} to be the opposite of P ’s choice by submitting $(0, 0)$ or $(1, 1)$; this will succeed in biasing the result half the time. But the adversary cannot both learn P ’s input and *simultaneously* bias the result. (If the adversary submits $(0, 1)$ or $(1, 0)$, the output of \mathbf{maj} is always equal to P ’s input; if the adversary submits $(0, 0)$ or $(1, 1)$ the the output of \mathbf{maj} reveals nothing about P ’s input.) Concretely, for any ideal-world adversary the *sum* of the probability

that the adversary guesses P 's input and the probability that the output of maj is not equal to P 's input is at most 1. In our proof, we show that if correctness holds with significantly lower probability when the last round of Π is eliminated, then there exists a real-world adversary violating this constraint.

2.2 Proof Details

We number the parties P_1, P_2, P_3 , and work modulo 3 in the subscript. The input of P_j is denoted by x_j . The following claim formalizes the ideal-world constraint described informally above.

Claim 4 *For all $j \in \{1, 2, 3\}$ and any adversary \mathcal{A} corrupting P_{j-1} and P_{j+1} in an ideal-world computation of maj , we have*

$$\Pr[\mathcal{A} \text{ correctly guesses } x_j] + \Pr[\text{OUTPUT}_j \neq x_j] \leq 1,$$

where the probabilities are taken over the random coins of \mathcal{A} and random choice of $x_j \in \{0, 1\}$.

Proof. Consider an execution in the ideal world, where P_j 's input x_j is chosen uniformly at random. Let EQUAL be the event that \mathcal{A} submits two equal inputs (i.e., $x_{j-1} = x_{j+1}$) to the trusted party. In this case, \mathcal{A} learns nothing about P_j 's input and so can guess x_j with probability at most $1/2$. It follows that:

$$\Pr[\mathcal{A} \text{ correctly guesses } x_j] \leq \frac{1}{2} \Pr[\text{EQUAL}] + \Pr[\overline{\text{EQUAL}}].$$

Moreover, $\Pr[\text{OUTPUT}_j \neq x_j] = \frac{1}{2} \Pr[\text{EQUAL}]$ since $\text{OUTPUT}_j \neq x_j$ occurs only if \mathcal{A} submits $x_{j-1} = x_{j+1} = \bar{x}_j$ to the trusted party. Therefore:

$$\begin{aligned} & \Pr[\mathcal{A} \text{ correctly guesses } x_j] + \Pr[\text{OUTPUT}_j \neq x_j] \\ & \leq \frac{1}{2} \Pr[\text{EQUAL}] + \Pr[\overline{\text{EQUAL}}] + \frac{1}{2} \Pr[\text{EQUAL}] \\ & = \Pr[\text{EQUAL}] + \Pr[\overline{\text{EQUAL}}] = 1, \end{aligned}$$

proving the claim.

Let Π be a protocol that securely computes maj using $r = r(k)$ rounds. Consider an execution of Π in which all parties run the protocol honestly except for possibly aborting in some round. We denote by $b_j^{(i)}$ the value that P_{j-1} and P_{j+1} both² output if P_j aborts the protocol after sending its round- i message (and then P_{j-1} and P_{j+1} honestly run the protocol to completion). Similarly, we denote by $b_{j-1}^{(i)}$ (resp., $b_{j+1}^{(i)}$) the value output by P_j and P_{j+1} (resp., P_j and P_{j-1}) when P_{j-1} (resp., P_{j+1}) aborts after sending its round- i message. Note that an

² Security of Π implies that the outputs of P_{j-1} and P_{j+1} in this case must be equal with all but negligible probability. For simplicity we assume this to hold with probability 1 but our proof can be modified easily to remove this assumption.

adversary who corrupts, e.g., both P_{j-1} and P_{j+1} can compute $b_j^{(i)}$ immediately after receiving the round- i message of P_j .

Since Π securely computes maj with complete fairness, the ideal-world constraint from the previous claim implies that for all $j \in \{1, 2, 3\}$, any inverse polynomial $\mu(k)$, and any poly-time adversary \mathcal{A} controlling players P_{j-1} and P_{j+1} , we have:

$$\Pr_{x_j \leftarrow \{0,1\}} [\mathcal{A} \text{ correctly guesses } x_j] + \Pr_{x_j \leftarrow \{0,1\}} [\text{OUTPUT}_j \neq x_j] \leq 1 + \mu(k) \quad (1)$$

for k sufficiently large. Security of Π also guarantees that if the inputs of the honest parties agree, then with all but negligible probability their output must be their common input regardless of when a malicious P_j aborts. That is, for k large enough we have

$$x_{j+1} = x_{j-1} \Rightarrow \Pr [b_j^{(i)} = x_{j+1} = x_{j-1}] \geq 1 - \mu(k) \quad (2)$$

for all $j \in \{1, 2, 3\}$ and all $i \in \{0, \dots, r(k)\}$.

The following claim represents the key step in our lower bound.

Claim 5 *Fix a protocol Π , a function μ , and a value k such that Equations (1) and (2) hold, and let $\mu = \mu(k)$. Say there exists an i , with $1 \leq i \leq r(k)$, such that for all $j \in \{1, 2, 3\}$ and all $c_1, c_2, c_3 \in \{0, 1\}$ it holds that:*

$$\Pr [b_j^{(i)} = \text{maj}(c_1, c_2, c_3) \mid (x_1, x_2, x_3) = (c_1, c_2, c_3)] \geq 1 - \mu. \quad (3)$$

Then for all $j \in \{1, 2, 3\}$ and all $c_1, c_2, c_3 \in \{0, 1\}$ it holds that:

$$\Pr [b_j^{(i-1)} = \text{maj}(c_1, c_2, c_3) \mid (x_1, x_2, x_3) = (c_1, c_2, c_3)] \geq 1 - 5\mu. \quad (4)$$

Proof. When $j = 1$ and $c_2 = c_3$, the desired result follows from Equation (2); this is similarly true for $j = 2$, $c_1 = c_3$ as well as $j = 3$, $c_1 = c_2$.

Consider the real-world adversary \mathcal{A} that corrupts P_1 and P_3 and sets $x_1 = 0$ and $x_3 = 1$. Then:

- \mathcal{A} runs the protocol honestly until it receives the round- i message from P_2 .
- \mathcal{A} then locally computes the value of $b_2^{(i)}$.
 - If $b_2^{(i)} = 0$, then \mathcal{A} aborts P_1 *without sending its round- i message* and runs the protocol (honestly) on behalf of P_3 until the end. By definition, the output of P_2 will be $b_1^{(i-1)}$.
 - If $b_2^{(i)} = 1$, then \mathcal{A} aborts P_3 *without sending its round- i message* and runs the protocol (honestly) on behalf of P_1 until the end. By definition, the output of P_2 will be $b_3^{(i-1)}$.
- After completion of the protocol, \mathcal{A} outputs $b_2^{(i)}$ as its guess for the input of P_2 .

Consider an experiment in which the input x_2 of P_2 is chosen uniformly at random, and then \mathcal{A} runs protocol Π with P_2 . Using Equation (3), we have:

$$\begin{aligned} \Pr[\mathcal{A} \text{ correctly guesses } x_2] &= \Pr[b_2^{(i)} = x_2] \\ &= \Pr[b_2^{(i)} = f(0, x_2, 1)] \geq 1 - \mu. \end{aligned} \quad (5)$$

We also have:

$$\begin{aligned} \Pr[\text{OUTPUT}_2 \neq x_2] &= \frac{1}{2} \cdot \Pr[\text{OUTPUT}_2 = 1 \mid (x_1, x_2, x_3) = (0, 0, 1)] \\ &\quad + \frac{1}{2} \cdot \Pr[\text{OUTPUT}_2 = 0 \mid (x_1, x_2, x_3) = (0, 1, 1)] \\ &= \frac{1}{2} \left(\Pr[b_1^{(i-1)} = 1 \wedge b_2^{(i)} = 0 \mid (x_1, x_2, x_3) = (0, 0, 1)] \right. \\ &\quad + \Pr[b_3^{(i-1)} = 1 \wedge b_2^{(i)} = 1 \mid (x_1, x_2, x_3) = (0, 0, 1)] \\ &\quad + \Pr[b_3^{(i-1)} = 0 \wedge b_2^{(i)} = 1 \mid (x_1, x_2, x_3) = (0, 1, 1)] \\ &\quad \left. + \Pr[b_1^{(i-1)} = 0 \wedge b_2^{(i)} = 0 \mid (x_1, x_2, x_3) = (0, 1, 1)] \right). \end{aligned} \quad (6)$$

From Equation (1), we know that the sum of Equations (5) and (6) is upper-bounded by $1 + \mu$. Looking at the first summand in Equation (6), this implies that

$$\Pr[b_1^{(i-1)} = 1 \wedge b_2^{(i)} = 0 \mid (x_1, x_2, x_3) = (0, 0, 1)] \leq 4\mu. \quad (7)$$

Probabilistic manipulation gives

$$\begin{aligned} &\Pr[b_1^{(i-1)} = 1 \wedge b_2^{(i)} = 0 \mid (x_1, x_2, x_3) = (0, 0, 1)] \\ &= 1 - \Pr[b_1^{(i-1)} = 0 \vee b_2^{(i)} = 1 \mid (x_1, x_2, x_3) = (0, 0, 1)] \\ &\geq 1 - \Pr[b_1^{(i-1)} = 0 \mid (x_1, x_2, x_3) = (0, 0, 1)] - \Pr[b_2^{(i)} = 1 \mid (x_1, x_2, x_3) = (0, 0, 1)] \\ &\geq 1 - \Pr[b_1^{(i-1)} = 0 \mid (x_1, x_2, x_3) = (0, 0, 1)] - \mu, \end{aligned}$$

where the last inequality is due to the assumption of the claim. Combined with Equation (7), this implies:

$$\Pr[b_1^{(i-1)} = 0 \mid (x_1, x_2, x_3) = (0, 0, 1)] \geq 1 - 5\mu.$$

Applying an analogous argument starting with the third summand in Equation (6) gives

$$\Pr[b_3^{(i-1)} = 1 \mid (x_1, x_2, x_3) = (0, 1, 1)] \geq 1 - 5\mu.$$

Repeating the entire argument, but modifying the adversary to consider all possible pairs of corrupted parties and all possible settings of their inputs, completes the proof of the claim.

Theorem 6 *Any protocol Π that securely computes maj with complete fairness (assuming one exists³ at all) requires $\omega(\log k)$ rounds.*

Proof. Assume there exists a protocol Π that securely computes maj with complete fairness using $r = \mathcal{O}(\log k)$ rounds. Let $\mu(k) = \frac{1}{4 \cdot 5^{r(k)}}$, and note that μ is noticeable. By the assumed security of Π , the conditions of Claim 5 hold for k large enough; Equation (3), in particular, holds for $i = r(k)$. Fixing this k and applying the claim iteratively $r(k)$ times, we conclude that P_{j-1} and P_{j+1} can correctly compute the value of the function, on all inputs, with probability at least $3/4$ without interacting with P_j at all. This is clearly impossible.

3 Fair Computation of Majority for Three Players

In this section we describe a completely-fair protocol for computing maj for the case of $n = 3$ parties. The high-level structure of our protocol is as follows: the protocol consists of two phases. In the first phase, the parties run a secure-with-abort protocol to generate (authenticated) shares of certain values; in the second phase some of these shares are exchanged, round-by-round, for a total of m iterations. A more detailed description of the protocol follows.

In the first phase of the protocol the parties run a protocol π implementing a functionality ShareGen that computes certain values and then distributes authenticated 3-out-of-3 shares of these values to the parties. (See Fig. 1.) Three sets of values $\{b_1^{(i)}\}_{i=0}^m$, $\{b_2^{(i)}\}_{i=0}^m$, and $\{b_3^{(i)}\}_{i=0}^m$ are computed; looking ahead, $b_j^{(i)}$ denotes the value that parties P_{j-1} and P_{j+1} are supposed to output in case party P_j aborts after iteration i of the second phase; see below. The values $b_j^{(i)}$ are computed probabilistically, in the same manner as in [2]. That is, a round i^* is first chosen according to a geometric distribution with parameter $\alpha = 1/5$.⁴ (We will set m so that $i^* \leq m$ with all but negligible probability.) Then, for $i < i^*$ the value of $b_j^{(i)}$ is computed using the true inputs of P_{j-1} and P_{j+1} but a random input for P_j ; for $i \geq i^*$ the value $b_j^{(i)}$ is set equal to the correct output (i.e., it is computed using the true inputs of all parties). Note that even an adversary who knows all the parties' inputs and learns, sequentially, the values (say) $b_1^{(1)}, b_1^{(2)}, \dots$ cannot determine definitively when round i^* occurs.

We choose the protocol π computing ShareGen to be secure-with-designated-abort [8] for P_1 . Roughly speaking, this means privacy and correctness are ensured no matter what, and output delivery and (complete) fairness are guaranteed unless P_1 is corrupted; a formal definition is given in the full version [7].

The second phase of the protocol proceeds in a sequence of $m = \omega(\log n)$ iterations. (See Fig. 2.) In each iteration i , each party P_j broadcasts its share of $b_j^{(i)}$. (We stress that we allow rushing, and do not assume synchronous broadcast.) Observe that, after this is done, parties P_{j-1} and P_{j+1} *jointly* have enough

³ In the following section we show that such a protocol does, indeed, exist.

⁴ This is the distribution on $\mathbb{N} = \{1, 2, \dots\}$ given by flipping a biased coin (that is heads with probability α) until the first head appears.

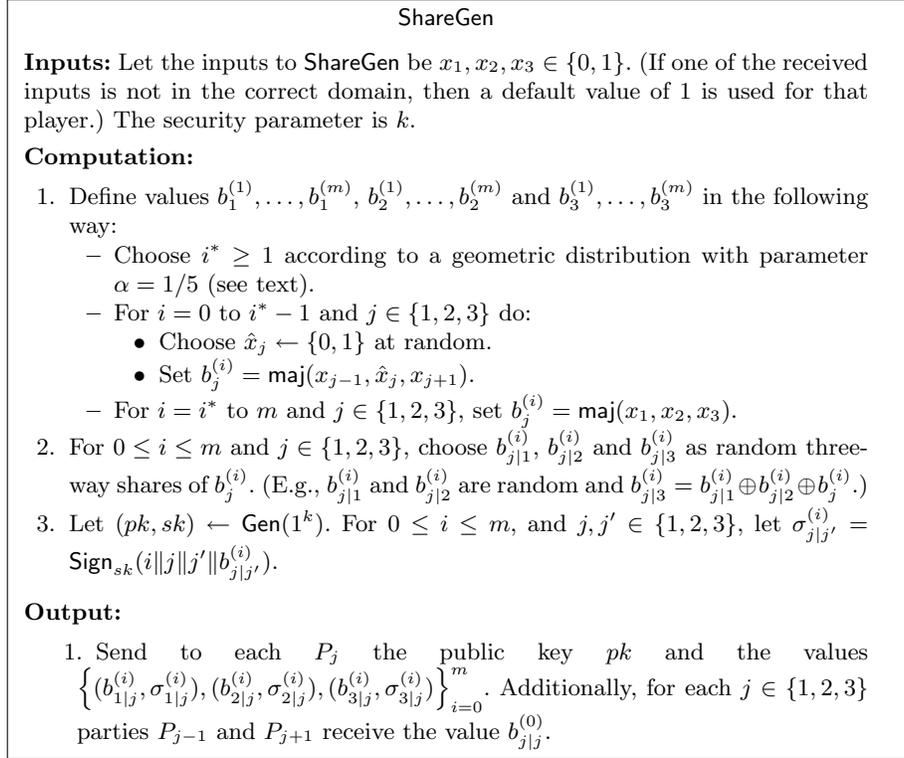


Fig. 1. Functionality **ShareGen**.

information to reconstruct $b_j^{(i)}$, but neither party has any information about $b_j^{(i)}$ on its own. If all parties behave honestly until the end of the protocol, then in the final iteration all parties reconstruct $b_1^{(m)}$ and output this value.

If a single party P_j aborts in some iteration i , then the remaining players P_{j-1} and P_{j+1} jointly reconstruct the value $b_j^{(i-1)}$ and output this value. (These two parties jointly have enough information to do this.) If two parties abort in some iteration i (whether at the same time, or one after the other) then the remaining party simply outputs its own input.

We refer to Fig. 1 and Fig. 2 for the formal specification of the protocol. We now prove that this protocol securely computes **maj** with complete fairness.

Theorem 7 *Assume that $(\text{Gen}, \text{Sign}, \text{Vrfy})$ is a secure signature scheme, that π securely computes **ShareGen** with designated abort, and that π_{OR} securely computes **OR** with complete fairness.⁵ Then the protocol in Figure 2 securely computes **maj** with complete fairness.*

Proof. Let Π denote the protocol of Figure 2. Observe that Π yields the correct output with all but negligible probability when all players are honest.

⁵ It is shown in [2] that such a protocol exists under standard assumptions.

Protocol 1

Inputs: Party P_i has input $x_i \in \{0, 1\}$. The security parameter is k .

The protocol:

1. **Preliminary phase:**

- (a) Parties P_1, P_2 and P_3 run a protocol π for computing **ShareGen**. Each player uses their respective inputs, x_1, x_2 and x_3 , and security parameter k .
- (b) If P_2 and P_3 receive \perp from this execution, then P_2 and P_3 run a two-party protocol π_{OR} to compute the logical-or of their inputs. Otherwise, continue to the next stage.

In what follows, parties always verify signatures; invalid signatures are treated as an abort.

2. **For $i = 1, \dots, m - 1$ do:**

Broadcast shares:

- (a) Each P_j broadcasts $(b_{j|j}^{(i)}, \sigma_{j|j}^{(i)})$.
- (b) If (only) P_j aborts:
 - i. P_{j-1} and P_{j+1} broadcast $(b_{j|j-1}^{(i-1)}, \sigma_{j|j-1}^{(i-1)})$ and $(b_{j|j+1}^{(i-1)}, \sigma_{j|j+1}^{(i-1)})$, respectively.
 - ii. If one of P_{j-1}, P_{j+1} aborts in the previous step, the remaining player outputs its own input value. Otherwise, P_{j-1} and P_{j+1} both output $b_j^{(i-1)} = b_{j|1}^{(i-1)} \oplus b_{j|2}^{(i-1)} \oplus b_{j|3}^{(i-1)}$. (Recall that if $i = 1$, parties P_{j-1} and P_{j+1} received $b_{j|j}^{(0)}$ as output from π .)
- (c) If two parties abort, the remaining player outputs its own input value.

3. **In round $i = m$ do:**

- (a) Each P_j broadcasts $b_{1|j}^{(m)}, \sigma_{1|j}^{(m)}$.
- (b) If no one aborts, then all players output $b_1^{(m)} = b_{1|1}^{(m)} \oplus b_{1|2}^{(m)} \oplus b_{1|3}^{(m)}$. If (only) P_j aborts, then P_{j-1} and P_{j+1} proceed as in step 2b. If two players abort, the remaining player outputs its own input as in step 2c.

Fig. 2. A protocol for computing majority.

This is because, with all but negligible probability, $i^* \leq m$, and then $b_j^{(m)} = \text{maj}(x_1, x_2, x_3)$. We thus focus on security of Π .

When no parties are corrupt, security is straightforward since we assume the existence of a private broadcast channel. We therefore consider separately the cases when a single party is corrupted and when two parties are corrupted. Since the entire protocol is symmetric except for the fact that P_1 may choose to abort π , without loss of generality we may analyze the case when the adversary corrupts P_1 and the case when the adversary corrupts $\{P_1, P_2\}$. In each case, we prove security of Π in a hybrid world where there is an ideal functionality computing **ShareGen** (with abort) as well as an ideal functionality computing **OR** (with complete fairness). Applying the composition theorem of [9] then gives the desired result. A proof for the case where only P_1 is corrupted turns out to be fairly straightforward, and is given in Appendix A.1.

Claim 8 For every non-uniform, poly-time adversary \mathcal{A} corrupting P_1 and P_2 and running Π in a hybrid model with access to ideal functionalities computing ShareGen (with abort) and OR (with complete fairness), there exists a non-uniform, poly-time adversary \mathcal{S} corrupting P_1 and P_2 and running in the ideal world with access to an ideal functionality computing maj (with complete fairness), such that

$$\begin{aligned} & \left\{ \text{IDEAL}_{\text{maj}, \mathcal{S}}(x_1, x_2, x_3, k) \right\}_{x_i \in \{0,1\}, k \in \mathbb{N}} \\ & \stackrel{s}{\equiv} \left\{ \text{HYBRID}_{\Pi, \mathcal{A}}^{\text{ShareGen}, \text{OR}}(x_1, x_2, x_3, k) \right\}_{x_i \in \{0,1\}, k \in \mathbb{N}}. \end{aligned}$$

Proof. This case is significantly more complex than the case when only a single party is corrupted, since here \mathcal{A} learns $b_3^{(i)}$ in each iteration i of the second phase. As in [2], we must deal with the fact that \mathcal{A} might abort exactly in iteration i^* , after learning the correct output but before P_3 has enough information to compute the correct output.

We describe a simulator \mathcal{S} who corrupts P_1 and P_2 and runs \mathcal{A} as a black-box. For ease of exposition in what follows, we refer to the actions of P_1 and P_2 when more formally we mean the action of \mathcal{A} on behalf of those parties.

1. \mathcal{S} invokes \mathcal{A} on the inputs x_1 and x_2 , the auxiliary input z , and the security parameter k .
2. \mathcal{S} receives x'_1 and x'_2 from P_1 and P_2 , respectively, as input to ShareGen. If $x'_1 \notin \{0, 1\}$ (resp., $x'_2 \notin \{0, 1\}$), then \mathcal{S} sets $x'_1 = 1$ (resp., $x'_2 = 1$).
3. \mathcal{S} computes $(sk, pk) \leftarrow \text{Gen}(1^k)$, and then generates shares as follows:
 - (a) Choose $\left\{ b_{1|1}^{(i)}, b_{2|1}^{(i)}, b_{3|1}^{(i)}, b_{1|2}^{(i)}, b_{2|2}^{(i)}, b_{3|2}^{(i)} \right\}_{i=0}^m$ uniformly at random.
 - (b) Choose $\hat{x}_3 \leftarrow \{0, 1\}$ and set $b_3^{(0)} = \text{maj}(x'_1, x'_2, \hat{x}_3)$. Set $b_{3|3}^{(0)} = b_3^{(0)} \oplus b_{3|1}^{(0)} \oplus b_{3|2}^{(0)}$.

\mathcal{S} then hands \mathcal{A} the public key pk , the values $\left\{ b_{1|1}^{(i)}, b_{2|1}^{(i)}, b_{3|1}^{(i)}, b_{1|2}^{(i)}, b_{2|2}^{(i)}, b_{3|2}^{(i)} \right\}_{i=0}^m$ (along with their appropriate signatures), and the value $b_{3|3}^{(0)}$ as the outputs of P_1 and P_2 from ShareGen.

4. If P_1 aborts execution of ShareGen, then \mathcal{S} extracts x''_2 from P_2 as its input to OR. It then sends $(1, x''_2)$ to the trusted party computing maj, outputs whatever \mathcal{A} outputs, and halts.
5. Otherwise, if P_1 does not abort, then \mathcal{S} picks a value i^* according to a geometric distribution with parameter $\alpha = \frac{1}{5}$.

In what follows, for ease of description, we will use x_1 and x_2 in place of x'_1 and x'_2 , keeping in mind that that \mathcal{A} could of course have used substituted inputs. We also ignore the presence of signatures from now on, and leave the following implicit in what follows: (1) \mathcal{S} always computes an appropriate signature when sending any value to \mathcal{A} ; (2) \mathcal{S} treats an incorrect signature as an abort; and (3) if \mathcal{S} ever receives a valid signature on a previously unsigned message (i.e., a *forgery*), then \mathcal{S} outputs fail and halts.

Also, from here on we will say that \mathcal{S} sends b to \mathcal{A} in round i if \mathcal{S} sends a value $b_{3|3}^{(i)}$ such that $b_{3|3}^{(i)} \oplus b_{3|1}^{(i)} \oplus b_{3|2}^{(i)} = b_3^{(i)} = b$.

6. For round $i = 1, \dots, i^* - 1$, the simulator \mathcal{S} computes and then sends $b_3^{(i)}$ as follows:
- Select $\hat{x}_3 \leftarrow \{0, 1\}$ at random.
 - $b_3^{(i)} = \text{maj}(x_1, x_2, \hat{x}_3)$.
7. If P_1 aborts in round $i < i^*$, then \mathcal{S} sets $\hat{x}_2 = x_2$ and assigns a value to \hat{x}_1 according to the following rules that depend on the values of (x_1, x_2) and on the value of $b_3^{(i)}$:
- If $x_1 = x_2$, then \mathcal{S} sets $\hat{x}_1 = x_1$ with probability $\frac{3}{8}$ (and sets $\hat{x}_1 = \bar{x}_1$ otherwise).
 - If $x_1 \neq x_2$ and $b_3^{(i)} = x_1$, then \mathcal{S} sets $\hat{x}_1 = x_1$ with probability $\frac{1}{4}$ (and sets $\hat{x}_1 = \bar{x}_1$ otherwise).
 - If $x_1 \neq x_2$ and $b_3^{(i)} = x_2$, then \mathcal{S} sets $\hat{x}_1 = x_1$ with probability $\frac{1}{2}$ (and sets $\hat{x}_1 = \bar{x}_1$ otherwise).
- \mathcal{S} then finishes the simulation as follows:
- If $\hat{x}_1 \neq \hat{x}_2$, then \mathcal{S} submits (\hat{x}_1, \hat{x}_2) to the trusted party computing maj . Denote the output it receives from the trusted party by b_{out} . Then \mathcal{S} sets $b_1^{(i-1)} = b_{\text{out}}$, computes $b_{1|3}^{(i-1)} = b_1^{(i-1)} \oplus b_{1|1}^{(i-1)} \oplus b_{1|2}^{(i-1)}$, sends $b_{1|3}^{(i-1)}$ to P_2 (on behalf of P_3), outputs whatever \mathcal{A} outputs, and halts.
 - If $\hat{x}_1 = \hat{x}_2$, then \mathcal{S} sets $b_1^{(i-1)} = \hat{x}_1 = \hat{x}_2$, computes $b_{1|3}^{(i-1)} = b_1^{(i-1)} \oplus b_{1|1}^{(i-1)} \oplus b_{1|2}^{(i-1)}$, and sends $b_{1|3}^{(i-1)}$ to P_2 (on behalf of P_3). (We stress that this is done *before* sending anything to the trusted party computing maj .) If P_2 aborts, then \mathcal{S} sends $(0, 1)$ to the trusted party computing maj . Otherwise, it sends (\hat{x}_1, \hat{x}_2) to the trusted party computing maj . In both cases it outputs whatever \mathcal{A} outputs, and then halts.
- If P_2 aborts in round $i < i^*$, then \mathcal{S} acts analogously but swapping the roles of P_1 and P_2 as well as x_1 and x_2 .
- If both parties abort in round $i < i^*$ (at the same time), then \mathcal{S} sends $(0, 1)$ to the trusted party computing maj , outputs whatever \mathcal{A} outputs, and halts.
8. In round i^* :
- If $x_1 \neq x_2$, then \mathcal{S} submits (x_1, x_2) to the trusted party. Let $b_{\text{out}} = \text{maj}(x_1, x_2, x_3)$ denote the output.
 - If $x_1 = x_2$, then \mathcal{S} simply sets $b_{\text{out}} = x_1 = x_2$ *without querying the trusted party* and continues. (Note that in this case, $b_{\text{out}} = \text{maj}(x_1, x_2, x_3)$ even though \mathcal{S} did not query the trusted party.)
9. In rounds $i^*, \dots, m - 1$, the simulator \mathcal{S} sends b_{out} to \mathcal{A} .
- If \mathcal{A} aborts P_1 and P_2 simultaneously, then \mathcal{S} submits $(1, 0)$ to the trusted party (if he hasn't already done so in step 8a), outputs whatever \mathcal{A} outputs, and halts.
- If \mathcal{A} aborts P_1 (only), then \mathcal{S} sets $b_1^{(i-1)} = b_{\text{out}}$, computes $b_{1|3}^{(i-1)} = b_1^{(i-1)} \oplus b_{1|1}^{(i-1)} \oplus b_{1|2}^{(i-1)}$, and sends $b_{1|3}^{(i-1)}$ to P_2 (on behalf of P_3). Then:
- Case 1: $\mathbf{x}_1 \neq \mathbf{x}_2$.** Here \mathcal{S} has already sent (x_1, x_2) to the trusted party. So \mathcal{S} simply outputs whatever \mathcal{A} outputs and ends the simulation.
- Case 2: $\mathbf{x}_1 = \mathbf{x}_2$.** If P_2 does not abort, then \mathcal{S} sends (x_1, x_2) to the trusted party. If P_2 aborts, then \mathcal{S} sends $(0, 1)$ to the trusted party. In both cases \mathcal{S} then outputs whatever \mathcal{A} outputs and halts.

If \mathcal{A} aborts P_2 (only), then \mathcal{S} acts as above but swapping the roles of P_1, P_2 and x_1, x_2 . If \mathcal{A} does not abort anyone through round m , then \mathcal{S} sends (x_1, x_2) to the trusted party (if he hasn't already done so), outputs what \mathcal{A} outputs, and halts.

The above constitutes the full description of \mathcal{S} . Due to space limitations, the analysis of \mathcal{S} is given in the full version of this paper [7].

4 Completely-Fair Computation of Boolean OR

The protocol in the previous section enables completely-fair computation of 3-party majority; unfortunately, we were not able to extend the approach to the case of $n > 3$ parties. In this section, we demonstrate feasibility of completely-fair computation of a non-trivial function for an arbitrary number of parties n , any $t < n$ of whom are corrupted. Specifically, we show how to compute boolean OR with complete fairness.

The basic idea behind our protocol is to have the parties repeatedly try to compute OR *on committed inputs* using a protocol that is secure-with-designated-abort where only the lowest-indexed party can force an abort. (See the full version [7] for further discussion.) The key observation is that in case of an abort, the dishonest players only “learn something” about the inputs of the honest players in case all the malicious parties use input 0. (If any of the malicious players holds input 1, then the output is always 1 regardless of the inputs of the honest parties.) So, if the lowest-indexed party is corrupt and aborts the computation of the committed OR, then the remaining parties simply recompute the committed OR using ‘0’ as the effective input for any parties who have already been eliminated. The parties repeatedly proceed in this fashion, eliminating dishonest parties at each iteration. Eventually, when the lowest-indexed player is honest, the process terminates and all honest players receive (correct) output.

The actual protocol follows the above intuition, but is a bit more involved. A formal description of the protocol is given in Fig. 3, and the “committed OR” functionality is defined in Fig. 4.

Theorem 9 *Assume Com is a computationally-hiding, statistically-binding commitment scheme, and that $\pi_{\mathcal{P}}$ securely computes $\text{CommittedOR}_{\mathcal{P}}$ (with abort). Then the protocol of Fig. 3 computes OR with complete fairness.*

Proof. Let Π denote the protocol of Fig. 3. For simplicity we assume Com is perfectly binding, though statistical binding suffices. For any non-uniform, polynomial time adversary \mathcal{A} in the hybrid world, we demonstrate a non-uniform polynomial-time adversary \mathcal{S} corrupting the same parties as \mathcal{A} and running in the ideal world with access to an ideal functionality computing OR (with complete fairness), such that

$$\begin{aligned} & \left\{ \text{IDEAL}_{\text{OR}, \mathcal{S}}(x_1, \dots, x_n, k) \right\}_{x_i \in \{0,1\}, k \in \mathbb{N}} \\ & \stackrel{c}{\equiv} \left\{ \text{HYBRID}_{\Pi, \mathcal{A}}^{\text{CommittedOR}_{\mathcal{P}}}(x_1, \dots, x_n, k) \right\}_{x_i \in \{0,1\}, k \in \mathbb{N}} . \end{aligned}$$

Protocol 2

Inputs: Each party P_i holds input $x_i \in \{0, 1\}$, and the security parameter is k .

Computation:

1. Let $\mathcal{P} = \{P_1, \dots, P_n\}$ be the set of all players.
2. Each player P_i chooses random coins r_i and broadcasts $c_i = \text{Com}(1^k, x_i, r_i)$, where Com denotes a computationally-hiding, statistically-binding commitment scheme. If any party P_i does not broadcast anything (or otherwise broadcasts an invalid value), then all honest players output 1. Otherwise, let $\mathbf{c} = (c_1, \dots, c_n)$.
3. All players $P_i \in \mathcal{P}$ run a protocol $\pi_{\mathcal{P}}$ for computing $\text{CommittedOR}_{\mathcal{P}}$, with party P_i using $(x_i, r_i, \mathbf{c}_{\mathcal{P}})$ as its input where $\mathbf{c}_{\mathcal{P}} \stackrel{\text{def}}{=} (c_i)_{i:P_i \in \mathcal{P}}$.
4. If players receive \perp from the execution of $\text{CommittedOR}_{\mathcal{P}}$, they set $\mathcal{P} = \mathcal{P} \setminus \{P^*\}$, where $P^* \in \mathcal{P}$ is the lowest-indexed player in \mathcal{P} , and return to Step 3.
5. If players receive a set $\mathcal{D} \subset \mathcal{P}$ from the execution of $\text{CommittedOR}_{\mathcal{P}}$, they set $\mathcal{P} = \mathcal{P} \setminus \mathcal{D}$ and return to Step 3.
6. If players receive a binary output from the execution of $\text{CommittedOR}_{\mathcal{P}}$, they output this value and end the protocol.

Fig. 3. A protocol computing OR for n players.

Applying the composition theorem of [9] then proves the theorem.

When no players are corrupt, security is trivial due to the assumed existence of a private broadcast channel. We now describe the execution of \mathcal{S} assuming a set $\mathcal{C} \neq \emptyset$ of corrupted parties:

1. Let $\mathcal{H} = \{P_1, \dots, P_n\} \setminus \mathcal{C}$ denote the honest players. Initialize $\mathcal{I} = \mathcal{C}$. Looking ahead, \mathcal{I} denotes the set of corrupted parties who have not yet been eliminated from the protocol.
2. \mathcal{S} invokes \mathcal{A} on the inputs $\{x_i\}_{i:P_i \in \mathcal{C}}$, the auxiliary input z , and the security parameter k .
3. For $P_i \in \mathcal{H}$, the simulator \mathcal{S} gives to \mathcal{A} a commitment $c_i = \text{Com}(1^k, x_i, r_i)$ to $x_i = 0$ using randomness r_i . \mathcal{S} then records the commitment c_i that is broadcast by \mathcal{A} on behalf of each party $P_i \in \mathcal{C}$. If any corrupted player fails to broadcast a value c_i , then \mathcal{S} submits 1's to the trusted party on behalf of all corrupted parties, outputs whatever \mathcal{A} outputs, and halts.
4. If $\mathcal{I} = \emptyset$, \mathcal{S} submits (on behalf of all the corrupted parties) 0's to the trusted party computing OR (unless it has already done so). It then outputs whatever \mathcal{A} outputs, and halts. If $\mathcal{I} \neq \emptyset$, continue to the next step.
5. \mathcal{S} sets $\mathcal{P} = \mathcal{H} \cup \mathcal{I}$ and obtains inputs $\{(r_i, x_i, \mathbf{c}^i)\}_{i:P_i \in \mathcal{I}}$ for the computation of $\text{CommittedOR}_{\mathcal{P}}$. For each $P_i \in \mathcal{P}$, the simulator \mathcal{S} computes the list of players \mathcal{D}_i that disagree with P_i (as in Fig. 4), using as the inputs of the honest parties the commitments defined in Step 3, and assuming that honest parties provide correct decommitments. Observe that if $P_i, P_j \in \mathcal{H}$ then $\mathcal{D}_i = \mathcal{D}_j \subseteq \mathcal{I}$. Let $\mathcal{D}_{\mathcal{H}} \subseteq \mathcal{I}$ be the set of parties that disagree with the honest parties.

CommittedOR \mathcal{P}

Inputs: The functionality is run by parties in \mathcal{P} . Let the input of player $P_i \in \mathcal{P}$ be (x_i, r_i, \mathbf{c}^i) where $\mathbf{c}^i = (c_j^i)_{j:P_j \in \mathcal{P}}$. The security parameter is k .

For each party $P_i \in \mathcal{P}$, determine its output as follows:

1. Say P_j *disagrees with* P_i if either (1) $c^j \neq \mathbf{c}^i$ or (2) $\text{Com}(1^k, x_j, r_j) \neq c_j^i$. (Note that disagreement is not a symmetric relation.)
2. Let \mathcal{D}_i be the set of parties who disagree with P_i .
3. If there exist any parties that disagree with each other, return \mathcal{D}_i as output to P_i . Otherwise, return $\bigvee_{j:P_j \in \mathcal{P}} x_j$ to all parties.

Fig. 4. Functionality CommittedOR \mathcal{P} , parameterized by a set \mathcal{P}

Let P^* be the lowest-indexed player in \mathcal{P} . If no parties disagree with each other, go to Step 6. Otherwise:

- (a) If $P^* \in \mathcal{I}$, then \mathcal{A} is given $\{\mathcal{D}_i\}_{i:P_i \in \mathcal{I}}$. If P^* aborts, then \mathcal{S} sets $\mathcal{I} = \mathcal{I} \setminus \{P^*\}$ and goes to Step 4. If P^* does not abort, then \mathcal{S} sets $\mathcal{I} = \mathcal{I} \setminus \mathcal{D}_{\mathcal{H}}$ and goes to Step 4.
 - (b) If $P^* \notin \mathcal{I}$, then \mathcal{A} is given $\{\mathcal{D}_i\}_{i:P_i \in \mathcal{I}}$. Then \mathcal{S} sets $\mathcal{I} = \mathcal{I} \setminus \mathcal{D}_{\mathcal{H}}$ and goes to Step 4.
6. \mathcal{S} computes the value $b = \bigvee_{P_i \in \mathcal{I}} x_i$.
- (a) If $b = 0$, and \mathcal{S} has not yet queried the trusted party computing OR, then \mathcal{S} submits 0's (on behalf of all the corrupted parties) to the trusted party and stores the output of the trusted party as b_{out} . \mathcal{S} gives b_{out} to \mathcal{A} (either as just received from the trusted party, or as stored in a previous execution of this step).
 - (b) If $b = 1$, then \mathcal{S} gives the value 1 to \mathcal{A} *without* querying the trusted party.
- \mathcal{S} now continues as follows:
- (a) If $P^* \in \mathcal{I}$ and P^* aborts, then \mathcal{S} sets $\mathcal{I} = \mathcal{I} \setminus \{P^*\}$ and goes to Step 4.
 - (b) If $P^* \notin \mathcal{I}$, or if P^* does not abort, then \mathcal{S} submits 1's to the trusted party if it has not yet submitted 0's. It outputs whatever \mathcal{A} outputs, and halts.

We refer the reader to the full version of this paper [7] for an analysis of the above simulation.

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A Proofs

A.1 Proof of Security for Majority With a Single Corrupted Party

Claim 10 *For every non-uniform, polynomial-time adversary \mathcal{A} corrupting P_1 and running Π in a hybrid model with access to ideal functionalities computing ShareGen (with abort) and OR (with complete fairness), there exists a non-uniform, poly-time adversary \mathcal{S} corrupting P_1 and running in the ideal world with access to an ideal functionality computing maj (with complete fairness), such that*

$$\begin{aligned} & \left\{ \text{IDEAL}_{\text{maj}, \mathcal{S}}(x_1, x_2, x_3, k) \right\}_{x_i \in \{0,1\}, k \in \mathbb{N}} \\ & \stackrel{s}{\equiv} \left\{ \text{HYBRID}_{\Pi, \mathcal{A}}^{\text{ShareGen}, \text{OR}}(x_1, x_2, x_3, k) \right\}_{x_i \in \{0,1\}, k \in \mathbb{N}} . \end{aligned}$$

Proof. Fix some polynomial-time adversary \mathcal{A} corrupting P_1 . We now describe a simulator \mathcal{S} that also corrupts P_1 and runs \mathcal{A} as a black box.

1. \mathcal{S} invokes \mathcal{A} on the input x_1 , the auxiliary input z , and the security parameter k .
2. \mathcal{S} receives input $x'_1 \in \{0, 1\}$ on behalf of P_1 as input to ShareGen .
3. \mathcal{S} computes $(sk, pk) \leftarrow \text{Gen}(1^k)$, and gives to \mathcal{A} the public key pk and values $b_{2|2}^{(0)}, b_{3|3}^{(0)}$, and $\left\{ b_{1|1}^{(i)}, b_{2|1}^{(i)}, b_{3|1}^{(i)} \right\}_{i=0}^m$ (along with their appropriate signatures) chosen uniformly at random.
4. If \mathcal{A} aborts execution of ShareGen , then \mathcal{S} sends 1 to the trusted party computing maj , outputs whatever \mathcal{A} outputs, and halts. Otherwise, \mathcal{S} picks a value i^* according to a geometric distribution with parameter $\alpha = \frac{1}{5}$. For simplicity in what follows, we ignore the presence of signatures and leave the following implicit from now on: (1) \mathcal{S} always computes an appropriate

signature when sending any value to \mathcal{A} ; (2) \mathcal{S} treats an incorrect signature as an abort; and (3) if \mathcal{S} ever receives a valid signature on a previously unsigned message, then \mathcal{S} outputs fail and halts.

5. \mathcal{S} now simulates the rounds of the protocol one-by-one: for $i = 1$ to $m - 1$, the simulator chooses random $b_{2|2}^{(i)}$ and $b_{3|3}^{(i)}$ and sends these to \mathcal{A} . During this step, an abort by \mathcal{A} (on behalf of P_1) is treated as follows:
 - (a) If P_1 aborts in round $i \leq i^*$, then \mathcal{S} chooses a random value \hat{x}_1 and sends it to the trusted party computing **maj**.
 - (b) If P_1 aborts in round $i > i^*$, then \mathcal{S} submits x'_1 to the trusted party computing **maj**.

In either case, \mathcal{S} then outputs whatever \mathcal{A} outputs and halts.

6. If P_1 has not yet aborted, \mathcal{S} then simulates the final round of the protocol. \mathcal{S} sends x'_1 to the trusted party, receives $b_{\text{out}} = \text{maj}(x'_1, x_2, x_3)$, and chooses $b_{1|2}^{(m)}$ and $b_{1|3}^{(m)}$ at random subject to $b_{1|2}^{(m)} \oplus b_{1|3}^{(m)} \oplus b_{1|1}^{(m)} = b_{\text{out}}$. \mathcal{S} then gives these values to \mathcal{A} , outputs whatever \mathcal{A} outputs, and halts.

Due to the security of the underlying signature scheme, the probability that \mathcal{S} outputs fail is negligible in k . Note that the view of P_1 is otherwise statistically close in both worlds. Indeed, until round m the view of P_1 is independent of the inputs of the other parties in both the real and ideal worlds. In round m itself, P_1 learns the (correct) output b_{out} in the ideal world and learns this value with all but negligible probability in the real world.

We therefore only have to argue that outputs of the two honest parties in the real and ideal worlds are statistically close. Clearly this is true if P_1 never aborts. As for the case when P_1 aborts at some point during the protocol, we divide our analysis into the following cases:

- If P_1 aborts the execution of **ShareGen** in step 4, then \mathcal{S} submits ‘1’ on behalf of P_1 to the trusted party computing **maj**. Thus, in the ideal world, the outputs of P_2 and P_3 will be $\text{maj}(1, x_2, x_3)$. In the real world, if P_1 aborts computation of **ShareGen**, the honest parties output $\text{OR}(x_2, x_3)$. Since $\text{maj}(1, x_2, x_3) = \text{OR}(x_2, x_3)$, their outputs are the same.
- If P_1 aborts in round i of the protocol (cf. step 5), then in both the real and ideal worlds the following holds:
 - If $i \leq i^*$, then P_2 and P_3 output $\text{maj}(\hat{x}_1, x_2, x_3)$ where \hat{x}_1 is chosen uniformly at random.
 - If $i > i^*$, then P_2 and P_3 output $\text{maj}(x'_1, x_2, x_3)$
 Since i^* is identically distributed in both worlds, the outputs of P_2 and P_3 in this case are identically distributed as well.
- If P_1 aborts in round m (cf. step 6), then in the ideal world the honest parties will output $\text{maj}(x'_1, x_2, x_3)$. In the real world the honest parties output $\text{maj}(x'_1, x_2, x_3)$ as long as $i^* \leq m - 1$, which occurs with all but negligible probability.

This completes the proof.