Strongly-Optimal Structure Preserving Signatures from Type II Pairings: Synthesis and Lower Bounds

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Abstract. Recent work on structure-preserving signatures studies optimality of these schemes in terms of the number of group elements needed in the verification key and the signature, and the number of pairing-product equations in the verification algorithm. While the size of keys and signatures is crucial for many applications, another important aspect to consider for performance is the time it takes to verify a given signature. By far, the most expensive operation during verification is the computation of pairings. However, the concrete number of pairings that one needs to compute is not captured by the number of pairing-product equations considered in earlier work.

To fill this gap, we consider the question of what is the minimal number of pairings that one needs to compute in the verification of structure-preserving signatures. First, we prove lower bounds for schemes in the Type II setting that are secure under chosen message attacks in the generic group model, and we show that three pairings are necessary and that at most one of these pairings can be precomputed. We also extend our lower bound proof to schemes secure under random message attacks and show that in this case two pairings are still necessary.

Second, we build an automated tool to search for schemes matching our lower bounds. The tool can generate automatically and exhaustively all valid structure-preserving signatures within a user-specified search space, and analyze their (bounded) security in the generic group model. Interestingly, using this tool, we find a new randomizable structure-preserving signature scheme in the Type II setting that is optimal with respect to the lower bound on the number of pairings, and also minimal with respect to the number of group operations that have to be computed during verification.

1 Introduction

Structure-preserving signatures [3] (SPS) are signature schemes defined over groups with a bilinear map in which messages, public keys and signatures are all

group elements, and the verification algorithm consists of evaluating so-called "pairing-product equations" (i.e., products of pairings of the aforementioned group elements). One of the main motivations of considering such specific signature schemes is that they are remarkably useful in the modular design of several cryptographic protocols, notably in combination with non-interactive zero-knowledge (NIZK) proofs of knowledge about group elements, and more specifically with the celebrated Groth-Sahai proof system [24]. In a nutshell, Groth-Sahai proofs allow one to prove knowledge of a set of group elements satisfying a certain pairing-product equation. For instance, by using SPS with Groth-Sahai proofs one can create a NIZK proof showing knowledge of a valid signature on some message (perhaps satisfying certain properties) without disclosing the message, the signature or both. This is only a basic example, though. Indeed, the combination of SPS with Groth-Sahai proofs has been shown to be a powerful tool for the modular design of several cryptographic protocols, such as blind signatures [3,20], group signatures [3,20,27], homomorphic signatures [10,26], oblivious transfer [22,14], tightly-secure encryption [25,1], and more.

Realization of SPS has been considered over the three possible bilinear groups settings introduced in the classification of Galbraith, Paterson and Smart [21]; the type of a pairing $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ depends on whether the two source groups are the same, i.e., $\mathbb{G}_1 = \mathbb{G}_2$ (*Type I*), or there is a one-way, efficiently computable homomorphism $\psi : \mathbb{G}_2 \to \mathbb{G}_1$ (*Type II*), or there is no known efficiently computable homomorphism in either direction between $\mathbb{G}_2$ and $\mathbb{G}_1$ (*Type III*). However, more recent work has focused on proving lower bounds on the complexity of SPS, and exhibiting optimal constructions that match lower bounds. The common measures of complexity adopted in all these works are the number of group elements in the public key, the number of group elements in the signature, and the number of pairing-product equations in the verification algorithm. Considering these measures, it has been shown in [4,8] that in both the Type I and the Type III settings SPS require at least 3 group elements in the signatures and 2 verification equations. However, the Type II setting has been shown to (surprisingly) deviate from these bounds: SPS in the Type II setting require at least 2 group elements in the signatures and admit a *single* verification equation [7]. Moreover, for SPS in the Type II setting, it has been shown that the lower bound for the number of group elements in the verification key is 2. Together with showing such lower bounds, these works [4,8,7] have proposed SPS schemes matching these (optimal) measures.

1.1 Our Contribution

We continue the study of the efficiency of SPS schemes by focusing on another important measure that, to the best of our knowledge, has not been considered in any previous work: the number of pairing computations that need to be performed by the verifier. Previous work [4,8,7] considers verifier efficiency only in terms of the number of pairing-product equations. Such a number, however, does not tell much about the number of pairings that the verifier needs to compute,

and thus about the concrete verification running time. So, considering that pairings are definitely the most expensive operation in this process, here we refine this question and ask what is the *minimal* number of pairings necessary in the verification of SPS, and in particular of schemes with optimal bandwidth (i.e., 2 elements in the signatures and 2 elements in the verification key). Indeed, even though having fewer elements in the public key and in the signature intuitively leads to fewer pairings, in practice it is unclear what is the minimal number of pairings that is needed.

In this paper we initiate this study focusing on the Type II setting, and our contribution is mainly twofold. First, we show lower bounds on the number of pairings necessary in the pairing-product verification equation. Second, we build a synthesis tool that automates the generation and security analysis of SPS schemes, and we leverage our tool in order to find new SPS schemes that match our new lower bounds and improve over previous work. In the following paragraphs, we discuss our contribution in more detail.

New Optimality Measures and Lower Bounds. First, we show lower bounds for the number of pairings in the pairing-product verification equation of SPS in the Type II setting. We prove that, when considering schemes that are already optimal with respect to previously considered measures (i.e., two group elements in the verification key, two group elements in the signature, and a single verification equation), *three* pairings are necessary for achieving security against chosen-message attacks, whereas *two* pairings are necessary for achieving security against random-message attacks.

More specifically, we refine our analysis and distinguish between, what we call, offline and online pairings. Informally speaking, *offline pairings* are pairings that involve only group elements in the public key or in the public parameters, whereas *online pairings* involve the message and/or elements of the signature. In other words, offline pairings are computed in every signature verification (when using the same verification key) and thus can be precomputed "offline" and be reused in an arbitrary number of verifications. In contrast, online pairings involve elements, such as the message and the signature, that inherently change every time, and thus must be computed "online". So, given this notion of online and offline pairings, we ask how many of the three necessary pairings can be computed offline. Such question is indeed quite relevant for practical purposes since online pairings are those that really matter (e.g., think of the case in which one verifies several signatures with the same verification key). We answer this question by proving that, for schemes secure against chosen-message attacks, among the three pairings, *at most one* can be precomputed, i.e., two online pairings are necessary. For schemes that are secure against random-message attacks, instead two online pairings are always necessary. We call schemes matching these bounds *strongly-optimal*.

Once established these bounds, we address the question of constructing strongly-optimal SPS schemes. First, we consider schemes secure against chosen-message attacks: we look at the previous work (in the Type II setting) and observe that there already exists a strongly-unforgeable SPS matching our lower bounds [7].

Yet there is no known SPS scheme that is re-randomizable and allows for only two online pairings in verification. As discussed in [7], re-randomizable schemes are useful because one of the group elements in the signature is uniformly random. This property is convenient in some applications, e.g., anonymization protocols, as one of the signature elements can be revealed in the clear without leaking information on what was the original signature. So, as an additional contribution, in this paper we show a new re-randomizable SPS scheme that is strongly-optimal and improves over the re-randomizable scheme proposed in [7] by requiring one less online pairing. Then we take into consideration schemes secure against random-message attacks (RMA) for which there is no strongly-optimal candidate in the previous work. We fill this gap by showing a simple, strongly-optimal, RMA-secure SPS. We note that although random-message security is a weak notion, it has been shown useful in applications such as constructing adaptive oblivious transfer [22] and in a transformation for obtaining chosen-message secure SPS [1]. By using our strongly-optimal RMA-secure SPS scheme, all these applications can benefit of its improved efficiency.

Automated Synthesis of SPS. As emerges from the previous discussion, optimality results (at least in the single-dimensional form in which they have been developed so far) are insufficient in rich settings such as structure-preserving signatures where many meaningful measures of efficiency can be considered (e.g., verification time, key or signature size). Therefore, an attractive approach for achieving a broad range of optimality results is to perform an exhaustive search of valid SPS within user-defined parameters. In the second part of our work, we develop a synthesis tool that takes as input a user-defined budget, consisting of the number of pairings, group elements, etc. that can be used by the construction, and generates all possible expressions within this budget. Broadly speaking, our tool then uses an extension of the Generic Group Analyzer reported in [12], to generate, whenever it exists, a verification equation for the signature algorithm. Finally, our tool proves or disproves security of candidate schemes in the generic group model for the case when the adversary makes a bounded number of signing queries. Through this approach, we generate an exhaustive database, by exploring more than 2000 candidate SPS schemes, that can then be mined for different efficiency criteria. For instance, our database contains our new scheme with optimal number of online/offline pairings as well as the SPS schemes in the Type II setting that were previously proposed in [7]. Beyond its intrinsic interest, the database can also be used to validate or refute empirically new conjectures on SPS. For example, it is interesting to mention that our work on proving the new lower bounds was motivated by observing that among the schemes generated by our tool none of the ones secure against chosen-message attacks can be verified with only two pairings. More generally, our tool suggests the feasibility and interest to develop synthesis methods for structure-preserving cryptography. We believe that our methods can be extended to the Type I and Type III settings (however exhaustive search will be more difficult to attain because secure schemes must use two verification equations, which results in an exponential

growth in the search space), and to other forms of structure-preserving cryptography, such as structure-preserving commitments [3] and encryption [15].

1.2 Other Related Work

Structure-Preserving Signatures. While the notion of structure-preserving signature was first given by Abe et al. in [3], the first construction was proposed earlier by Groth [23], though its efficiency is far from being truly practical (it consists of hundreds of group elements). Green and Hohenberger [22] proposed SPS that are proved secure only against random-message attacks. Cathalo, Libert and Yung [16] constructed a scheme that is structure-preserving in a relaxed sense since it has a verification key which includes elements of the target group.

The study of lower bounds for SPS was put forward by Abe et al. [4] who showed that SPS in the Type III setting require at least three group elements in the signature and two pairing-product equations, and also proposed schemes matching these bounds that are only proven secure in the generic bilinear group model. Next, Abe et al. [5] refined the result in [4] considering schemes whose security can be proved under a non-interactive assumption using a black-box reduction. For this case they show that any scheme with only 3 elements in the signature cannot be proved secure under a non-interactive assumption. Optimal schemes in the symmetric (Type I) setting have been explored more recently by Abe et al. [8] who show that Type I SPS schemes require 3 elements in the signature and 2 verification equations (i.e., the same bounds as in Type III). Furthermore, the same work [8] proposes a general scheme that works in all three bilinear settings, and thus shows a Type II scheme with 3 elements in the signature and 2 verification equations. Finally, the recent work of Abe et al. [7] focused on the Type II setting and showed that in this setting the lower bounds are (surprisingly) different. Namely, Type II SPS schemes require 2 elements in the signature, a single verification equation, and 2 elements in the verification key (the latter being the first lower bound for the size of the verification key).

All the optimal schemes in [4,8,7] are proved secure directly in the generic bilinear group model. Another line of work investigated efficient SPS that can be proved secure under standard assumptions. Hofheinz and Jager [25] and Abe et al. [1,2] proposed schemes based on the decision linear assumption. The efficiency of these schemes, however, does not meet that of the schemes secure in the generic group model.

Finally, Chatterjee and Menezes [18] re-considered the result of Abe et al. [7] for Type II SPS in light of the current state-of-the-art implementations of Type II vs. Type III pairings. They start from the observation that implementations of Type III pairings are currently more efficient than the ones of Type II pairings. Then they note that Type II SPS, albeit optimal in terms of the number of signature elements and number of verification equations, are not as efficient as their Type III counterparts (i.e., the SPS schemes that can be obtained through

a semi-generic transformation from Type II to Type III [17]).[4] Although this is a valid point when considering concrete efficiency, we believe that the exploration of Type II SPS is still quite interesting. For example, they have a simpler structure that leads to a smaller search space when looking for new schemes. Yet, given a Type II scheme, one can always translate it to the Type III setting if concrete efficiency is a concern, e.g., using the approach from [17].

Computer-Aided Cryptography. In contrast to computer-aided tools for verifying cryptographic proofs, which have existed for some time, computer-aided tools for synthesizing new constructions are very recent. Barthe et al. [11] develop an automated tool, called ZooCrypt, for synthesizing padding-based encryption schemes; their tool uses a dedicated logic with an efficient proof search procedure to prove chosen-plaintext or chosen-ciphertext security, and an efficient method for finding attacks on insecure schemes. Because the search space for reasonably-sized constructions is small (about 10^6 well-typed schemes), simple trimming techniques are sufficient to cover the full search space efficiently. Malozemoff, Katz, and Green [28] develop an automated tool for proving security of modes of operations; the security of candidate schemes is proved using a type system, but no attack is exhibited for insecure schemes.

Akinyele, Green, and Hohenberger [9] develop two synthesis tools for pairing-based cryptography. Their tool AutoGroup converts schemes in the Type I setting into schemes in the Type III setting, whereas their tool AutoStrong transforms an existentially unforgeable signature into a strongly unforgeable one, using SMT solvers to check whether the original signature satisfies a criterion allowing an efficient transformation. The idea of automatically transforming constructions from the symmetric to the asymmetric setting was further considered by Abe, Groth, Ohkubo and Tango [6], who develop an automated transformation of Type I protocols into Type III protocols.

Automating proofs in the generic group model has been recently considered by Barthe et al. [12] who propose a tool that enables to automatically analyze the validity of cryptographic assumptions in the generic (multilinear) group model. The tool developed in this work actually builds on the techniques of [12] in order to perform the security analysis of SPS in the generic bilinear group model.

Finally, in a concurrent and independent work, De Ruiter [19] recently proposed a tool for analyzing the security of structure-preserving signatures. The proposed tool provides a security analysis of SPS similar to the one we provide, even though the security arguments in [19] do not have a full formalization in the generic group model. Additionally, we note that our tool is not limited to the security analysis of SPS, but also includes the novel synthesis component which allows to automatically generate SPS schemes.

[4] One main issue that leads to such difference of performance here is that testing group membership in $\mathbb{G}_2$ (which is required in the signature verification) is significantly more expensive in Type II than in Type III groups.

2 Preliminaries

2.1 Bilinear groups

A *bilinear group description* is a tuple $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, \psi, G, H)$ where p is a prime number, $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$ are cylic groups of order p, G, H are generators of $\mathbb{G}_1$ and $\mathbb{G}_2$ respectively, $\psi\colon \mathbb{G}_2 \to \mathbb{G}_1$ is the homomorphism sending H to G (so that $\psi(H^x) = G^x$ for all $x \in \mathbb{Z}$), and $e\colon \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ is a nondegenerate bilinear pairing, meaning that $e(G, H)$ generates $\mathbb{G}_T$ and $e(G^x, H^y) = e(G, H)^{xy}$ for all $x, y \in \mathbb{Z}$.

A *bilinear group generator* $\mathcal{G}$ is an efficient algorithm which, on input of a security parameter 1^λ, returns a bilinear group description $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, \psi, G, H)$ with $p = 2^{\Omega(\lambda)}$ and efficient algorithms for computing group operations and the bilinear map e, and deciding equality of group elements and membership in the groups.

Furthermore, following Galbraith, Paterson and Smart [21], we say that the result is a Type I bilinear group if the homomorphism ψ is efficiently computable and efficiently invertible (in which case we can simplify identify $\mathbb{G}_1$ and $\mathbb{G}_2$), a Type II bilinear group if ψ is efficiently computable but not efficiently invertible (i.e. it is a one-way function) and a Type III bilinear group if ψ is neither efficiently computable nor invertible. This paper mainly focuses on the Type II setting.

Generic algorithms. In a bilinear group $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, \psi, G, H)$ generated by $\mathcal{G}$ we refer to deciding group membership, computing group operations in $\mathbb{G}_1$, $\mathbb{G}_2$ or $\mathbb{G}_T$, comparing group elements and evaluating the homomorphism or the bilinear map as the generic bilinear group operations. The signature schemes we construct only use generic bilinear group operations.

As is customary in the literature, we denote group elements in $\mathbb{G}_1$ and $\mathbb{G}_2$ by uppercase letters such as $M, R, S, V, W, \ldots$, and their discrete logarithms with respect to base G or H using the corresponding lowercase letters $m, r, s, v, w, \ldots$ In particular, for an element $X \in \mathbb{G}_2$, we have $\psi(X) = G^x$, and for $(Y, Z) \in \mathbb{G}_1 \times \mathbb{G}_2$, we have $e(Y, Z) = e(G, H)^{yz}$. Furthermore, we will often express pairings equations, such as $e(X, Y)^{a_1} = e(W, Z)^{a_2} \cdot e(T, H)^{a_3}$, as a quadratic polynomial involving the corresponding exponents, i.e., $a_1 xy = a_2 wz + a_3 t$.

2.2 Structure-preserving signature schemes

We study structure-preserving signature schemes (SPS) [3] on bilinear groups generated by group generator $\mathcal{G}$. We refer to the full version of this paper [13] for basic definitions about signature schemes. In a structure preserving signature scheme the verification key, the messages and the signatures consist only of group elements from $\mathbb{G}_1$ and $\mathbb{G}_2$ and the verification algorithm evaluates the signature by deciding group membership of elements in the signature, using

the homomorphism ψ and by evaluating pairing product equations, which are equations of the form:

$$\prod_i \prod_j e(X_i, Y_j)^{a_{ij}} = 1,$$

where $X_1, X_2, \ldots \in \mathbb{G}_1$, $Y_1, Y_2, \ldots \in \mathbb{G}_2$ are group elements appearing in PP, VK, M and Σ (where in the Type II setting it may hold $Y_i = \Psi(X_j)$ for some i, j) and the elements $a_{ij} \in \mathbb{Z}_p$ are constants stored in PP. More precisely:

Definition 1 (Structure-preserving signatures). *A signature scheme* (Setup, KeyGen, Sign, Verify) *is said to be structure-preserving with respect to some bilinear group generator* $\mathcal{G}$ *if*

- *PP consists of a bilinear group* $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, \psi, G, H)$ *generated by* $\mathcal{G}$ *and constants in* $\mathbb{Z}_p$*,*
- *the verification key consists of group elements in* $\mathbb{G}_1$ *and* $\mathbb{G}_2$*,*
- *the messages consist of group elements in* $\mathbb{G}_1$ *and* $\mathbb{G}_2$*,*
- *the signatures consist of group elements in* $\mathbb{G}_1$ *and* $\mathbb{G}_2$*,*
- *the signing algorithm only uses generic group operations,*[5] *and*
- *the verification algorithm only needs to decide membership in* $\mathbb{G}_1$ *and* $\mathbb{G}_2$*, use the homomorphism* ψ*, and evaluate pairing product equations.*

2.3 Known lower bounds on Type II SPS

A number of lower bounds on signature size, verification key size and the number of verification equations have been established for secure structure-preserving signature schemes and one-time signature schemes. In particular, Abe et al. [7] establish many such bounds in the Type II setting. As some of our results rely heavily on those bounds, we recall them below. Note that membership tests are not counted as "verification equations", although some of them may require an amortizable (aka offline) pairing computation in practical instantiations.

First, just as Type I and Type III SPS, Type II SPS for messages in $\mathbb{G}_1$ require two verification equations:

Lemma 1 ([7, Theorem 3]). *A structure-preserving signature scheme for messages in* $\mathbb{G}_1$ *must have at least two verification equations. This holds even for one-time signatures with security against random message attack.*

Since this paper will mostly focus on schemes with a single verification equation, we will therefore consider signatures on messages in $\mathbb{G}_2$, which can have a single verification equation. In that case, Abe et al. obtain a lower bound on verification key size, and show that all signature elements must be in $\mathbb{G}_2$.

[5] Technically, this condition was not required in the original definition of Abe et al. [3], but all known constructions satisfy this property and it is required for the proofs of most lower bounds to go through. Since an SPS scheme with a non-generic signing algorithm would be very unnatural and surprising, it seems appropriate to include genericity of the signer in the definition (see also the discussion in [7, §2.3]).

Lemma 2 ([7, Theorem 4 and Lemma 1]). *A structure-preserving signature scheme with a single verification equation must have at least two group elements in the verification key, and can have no non-redundant signature element in* $\mathbb{G}_1$*. This holds even for one-time signatures secure under random message attack.*

Finally, signatures in a secure Type II SPS scheme must consist of at least two elements (although that property does not hold for one-time signatures), and three elements for messages in $\mathbb{G}_1$ (idem).

Lemma 3 ([7, Theorem 5]). *An EUF-RMA-secure structure-preserving signature scheme must have at least* 2 *group elements for messages in* $\mathbb{G}_2$ *and at least* 3 *group elements for messages in* $\mathbb{G}_1$*.*

3 Lower Bounds on the Number of Pairings in the Type II Setting

In this section we show lower bounds for the number of pairings in the pairing-product verification equations of SPS in the Type II setting. In particular, in our analysis we consider SPS schemes that already match the lower bounds shown in [7], i.e., they have 2 group elements in the verification key, 2 group elements in the signature and the verification consists of a single pairing-product equation (as well as possible group membership tests).

To have a more refined and practically interesting analysis, we distinguish between pairings according to whether they can be precomputed from the public key or not. In the former case we call a pairing *offline* while in the latter case *online*.

3.1 Main result

Having defined the notion of online and offline pairings, we are now ready to state our main result. It shows that any optimal-size SPS scheme requires at least three pairings for verification, and two of these pairings must be online ones.

Theorem 1 (Main result). *Any EUF-CMA-secure structure-preserving signature scheme in the Type II setting with* 1 *verification pairing-product equation,* 2 *group elements in the verification key and* 2 *elements in the signature requires at least* 3 *pairings in the pairing-product equation, and at least* 2 *of them must be online pairings.*

To prove the theorem, we distinguish between three cases according to which groups the two elements V, W of the verification key belong to, i.e., (i) $V, W \in \mathbb{G}_2$, (ii) $V, W \in \mathbb{G}_1$, and (iii) $V \in \mathbb{G}_1, W \in \mathbb{G}_2$.

The first case is rather simple and is addressed in the following lemma which shows that there exists no such structure-preserving signature scheme.

Lemma 4. *There is no secure structure-preserving signature scheme in the Type II setting with a single verification equation and a verification key consisting entirely of elements of* $\mathbb{G}_2$.

Proof. We know by Lemma 2 and Lemma 3 that the signatures and messages all have to be in $\mathbb{G}_2$. Since all inputs are in $\mathbb{G}_2$ the scheme would also be secure in the Type I setting and must therefore be insecure since Type I SPS require two pairing product equations for security [8, Theorem 4]. □

The second case is somewhat more involved, and mainly addressed by the following lemma, proved in Section 3.3 below.

Lemma 5. *An EUF-CMA-secure structure-preserving signature scheme in the Type II setting with* 1 *verification pairing-product equation,* 2 *group elements* $V, W \in \mathbb{G}_1$ *in the verification key and* 2 *group elements in the signature requires at least* 3 *pairings in the pairing-product equation.*

That result establishes that 3 pairings are needed, so all that remains to show is that 2 of them must be online. This follows immediately from the following observation.

Lemma 6. *In a Type II structure-preserving signature scheme where all verification key elements are in* $\mathbb{G}_1$*, it is possible to compute all the offline pairings in a verification pairing-product equation using a single pairing evaluation. More generally,* $\ell + 1$ *pairing evaluations are sufficient if the verification key contains* ℓ *elements in* $\mathbb{G}_2$.

Proof. Indeed, if the verification key is $(V_1, \ldots, V_k, U_1, \ldots, U_\ell) \in \mathbb{G}_1^k \times \mathbb{G}_2^\ell$, then any product of offline pairings can be expanded into an expression of the form $\prod_{i,j} e(X_i, Y_j)^{c_{ij}}$ where Y_j runs through $H, U_1, \ldots, U_\ell$ (since these are the only elements of $\mathbb{G}_2$ in the verification key and the public parameters) and X_i runs through $G, V_1, \ldots, V_k, \psi(U_1), \ldots, \psi(U_\ell)$. By rewriting the product as:

$$\prod_j e\Big(\prod_i X_i^{c_{ij}}, Y_j\Big)$$

we can compute it with at most $\ell + 1$ pairing evaluations as required. □

Finally, to complete the proof of Theorem 1, we only need to prove it when the verification key consists of one element of $\mathbb{G}_1$ and one element of $\mathbb{G}_2$. This case, which is somewhat less interesting as such a scheme is less space efficient than when all key elements are in $\mathbb{G}_1$, but turns out to be more technically challenging, is dealt with in details in the full version of this paper [13].

3.2 Gaps in Bounds Between EUF-RMA and EUF-CMA-Security

Following Lemma 5, in the setting when $(V, W) \in \mathbb{G}_1^2$ the bound of Theorem 1 holds only for EUF-CMA-secure SPS schemes. This however is not the case in the

setting when the verification key is of the form $(V, W) \in \mathbb{G}_1 \times \mathbb{G}_2$: as discussed in the full version of this paper [13], the bound of Theorem 1 holds even for EUF-RMA-secure SPS schemes. In what follows we establish a slightly weaker general lower bound on the number of pairings in the single pairing-product equation of minimal EUF-RMA-secure SPS schemes in the setting when $(V, W) \in \mathbb{G}_1^2$.

Theorem 2. *Any EUF-RMA-secure structure-preserving signature scheme in the Type II setting with* 1 *verification pairing-product equation requires at least* 2 *pairings in the pairing-product equation, and both of them must be online pairings.*

Proof. It suffices to show that a Type II SPS scheme with a single verification equation (and which we can assume without loss of generality signs one-element messages) cannot be EUF-RMA-secure if the pairing-product equation consists of only one online pairing (and any number of offline pairings).

To see this, denote by $(S_1, \ldots, S_k)$ the signature vector (which is in $\mathbb{G}_2^k$ without loss of generality by Lemma 2), and observe that the pairing product equation must be of the form:

$$\prod_{i,j} e(X_i, Y_j) = e\Big(\psi(M)^{a_0} \cdot \prod_{i=1}^{k} \psi(S_i)^{a_i} \cdot Z, M^{b_0} \cdot \prod_{j=1}^{k} S_j^{b_j} \cdot T\Big)$$

where the pairings on the left-hand side are offline (and hence the X_i's and Y_j's do not depend on the message or the signature), and Z, T are elements which also do not depend on the message or the signature. But then we can do the change of variables:

$$(R', S') = \Big(\psi(M)^{a_0} \cdot \prod_{i=1}^{k} \psi(S_i)^{a_i}, M^{b_0} \cdot \prod_{j=1}^{k} S_j^{b_j}\Big)$$

and then (R', S') provides a two-element EUF-RMA-secure signature scheme whose verification equation is just:

$$\prod_{i,j} e(X_i, Y_j) = e(R' \cdot Z, S' \cdot T),$$

and in particular does not depend on the message: this is a contradiction. □

We see that the bounds given by Theorem 1 and Theorem 2 show a gap. Namely, there could exist an EUF-RMA-secure scheme with precisely two online pairings and no offline pairing. We confirm that both lower bounds are indeed tight, by providing an EUF-RMA-secure SPS with precisely two online pairings in Section 5.2.

3.3 Proof of Lemma 5

Proof. The proof proceeds by contradiction showing that having a scheme with only 2 pairings in the single pairing-product equation is impossible. Let us first

recall that in this setting we have a message $M \in \mathbb{G}_2$, verification keys $V, W \in \mathbb{G}_1$ and signature elements $R, S \in \mathbb{G}_2$. As usual, we denote their discrete logarithms by the corresponding lower case letters. We may write the general verification equation in terms of the discrete logarithms of M, R, S, V, W as follows:

$$\begin{aligned}(c_1m + c_2r + c_3s + c_4v + v_5w + c_6)(d_1m + d_2r + d_3s + d_4) = \\ (e_1m + e_2r + e_3s + e_4v + e_5w + e_6)(f_1m + f_2r + f_3s + f_4),\end{aligned} \tag{1}$$

where the products represent a pairing, the left factor in each product represent the element in $\mathbb{G}_1$ and the right factor the element in $\mathbb{G}_2$ of each pairing.

Now if we define the vectors $X_1 = (c_1, \ldots, c_6), X_2 = (e_1, \ldots, e_6), Y_1 = (d_1, \ldots, d_4), Y_2 = (f_1, \ldots, f_4)$ over $\mathbb{Z}_p$ and the matrix:

$$E = X_1^t Y_1 - X_2^t Y_2,$$

then the verification equation (1) can be rewritten as

$$(m, r, s, v, w, 1) \cdot E \cdot (m, r, s, 1)^t = 0.$$

A simple observation shows that if $\operatorname{Ker} E$ contains a vector $(x_1, \ldots, x_4)$, where $x_4 \neq 0$, then we may scale to $x_4 = 1$ and then

$$m = x_1,\ r = x_2,\ s = x_3$$

is a valid key-only attack forgery (since the kernel of E can be computed entirely from the public parameters). It follows that if the scheme is secure, then $\operatorname{Ker} E \subset \{(x_1, \ldots, x_4) \mid x_4 = 0\}$. However, this implies that the following system

$$\begin{cases} d_1m + d_2r + d_3s = -d_4 \\ f_1m + f_2r + f_3s = -f_4 \end{cases}$$

lacks a solution. Up to exchanging the roles of Y_1 and Y_2 without loss of generality, this implies that $Y_1 = cY_2 + (0, 0, 0, \lambda)$ for some constants c, λ, and hence:

$$E = X_1^t Y_1 - X_2^t Y_2 = X_1^t\big(cY_2 + (0,0,0,\lambda)\big) - X_2^t Y_2 = (\lambda X_1)^t \cdot (0,0,0,1) - (X_2 - cX_1)^t Y_2.$$

Therefore, after relabeling the coefficients, we may assume that $Y_1 = (0, 0, 0, 1)$ and the verification equation (1) can be rewritten as

$$\begin{aligned}c_1m + c_2r + c_3s + c_4v + c_5w + c_6 = \\ (e_1m + e_2r + e_3s + e_4v + e_5w + e_6)(f_1m + f_2r + f_3s + f_4),\end{aligned} \tag{2}$$

Now if $(c_4, c_5) = \lambda(e_4, e_5)$ or $(e_4, e_5) = \lambda(c_4, c_5)$, then we may replace the verification key by a single element $t = e_4v + e_5w$ or $t = c_4v + c_5w$, which is insecure by Lemma 2. It follows that

$$\det \begin{pmatrix} c_4 & c_5 \\ e_4 & e_5 \end{pmatrix} \neq 0$$

and we may do a linear change of variables

$$\begin{pmatrix} v' \\ w' \end{pmatrix} = \begin{pmatrix} c_4 & c_5 \\ e_4 & e_5 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} + \begin{pmatrix} c_6 \\ e_6 \end{pmatrix},$$

so that the verification equation (2) becomes, after renaming coefficients,

$$c_1 m + c_2 r + c_3 s + v = (e_1 m + e_2 r + e_3 s + w)(f_1 m + f_2 r + f_3 s + f_4). \quad (3)$$

Note that the vectors $(c_2, c_3), (e_2, e_3), (f_2, f_3)$ cannot all be collinear, because otherwise we may again compress the signature into one group element, which we already know is impossible.

Next, we look at the matrix

$$N = \begin{pmatrix} e_2 & e_3 \\ f_2 & f_3 \end{pmatrix}$$

and distinguish two cases depending on the determinant.

On one hand, if $\det N \neq 0$, then as before, a change of variables let us write the verification equation (3) in the form

$$c_1 m + c_2 r + c_3 s + v = (r + w)s.$$

Since m must be used in the verification equation, we know that $c_1 \neq 0$. An easy calculation then shows that if (m, r, s) is a triple satisfying the verification equation for the keys v, w, then so does $(m - (c_2 - s)/c_1, r + 1, s)$. This gives us a forgery unless $c_2 = s$ for a non-negligible set of signatures. However, if this happens, then s would be a redundant signature element. From Lemma 3 we know that the scheme must be insecure.

On the other hand, if $\det N = 0$, we have the two cases $(e_2, e_3) = \lambda(f_2, f_3)$ or $(f_2, f_3) = 0$. If $(f_2, f_3) = 0$, then

$$\det \begin{pmatrix} c_2 & c_3 \\ e_2 & e_3 \end{pmatrix} \neq 0$$

or otherwise $(c_2, c_3), (e_2, e_3), (f_2, f_3)$ would be collinear. It follows that the verification equation (3) reduces to

$$r + v = (s + w)(f_1 m + f_4).$$

and since $f_1 \neq 0$, as the message must be used, we may query $m_1 = -f_1^{-1} f_4$ getting back a signature (r_1, s_1), where $r_1 = -v$. Now make another query with $m_2 = f_1^{-1}(1 - f_4)$ to get back a signature (r_2, s_2). Then

$$r_2 + v = s_2 + w \Rightarrow w = r_2 + v - s_2 = r_2 - r_1 - s_2,$$

so with two *chosen-message* queries the attacker can transfer V, W to $\mathbb{G}_2$ and then we know the scheme cannot be secure (concretely, $(R_1, R_1 R_2^{-1} S_2) = (V^{-1}, W^{-1})$ is a valid signature on any message). Therefore, we must have that $(e_2, e_3) =$

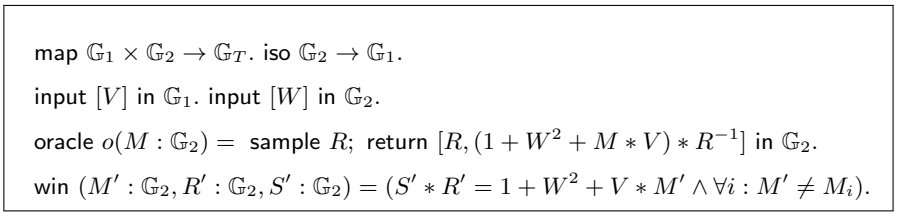

Fig. 1. Example of input for analyzing EUF-CMA security of an SPS scheme using our extended version of the GGA tool.

$\lambda(f_2, f_3)$. Again using the fact that $(c_2, c_3), (e_2, e_3), (f_2, f_3)$ are not collinear, we must have

$$\det \begin{pmatrix} c_2 & c_3 \\ f_2 & f_3 \end{pmatrix} \neq 0.$$

It follows that we may do a change of variables $s' = f_1 m + f_2 r + f_3 s + f_4$ and $r' = c_1 m + c_2 r + c_3 s$ and by the collinearity of (e_2, e_3) and (f_2, f_3) the verification equation (3) becomes of the form

$$r + v = (e_1 m + e_3 s + w + e_6) s$$

and now if (m, r, s) is a valid signature, then so is $(m + 1/e_1, r + s, s)$, which is a valid forgery, since m must be used in the verification equation and hence $e_1 \neq 0$. Also, note that in the latter case, the attack can be performed in the random-message security game. □

4 Synthesis of Schemes

Our tool[6] for the synthesis of SPS schemes consists of two components. The first component takes the description of a search space and generates all included SPS schemes. The second component classifies a given scheme by performing a proof and attack search.

4.1 Generation of Schemes

For our generation algorithm, we consider SPS schemes with generic *KeyGen* and *Sign* algorithms and assume all random values are sampled uniformly.

Our definition of an SPS scheme consists of

- the employed group type and the supported message space $\mathbb{G}_1^k \times \mathbb{G}_2^l$,
- the randomly sampled values $u_i \in \mathbb{Z}_p$ used in *KeyGen*,
- the verification keys $V_i = G^{f_i(\boldsymbol{u})} \in \mathbb{G}_1$ and $W_i = H^{g_i(\boldsymbol{u})} \in \mathbb{G}_2$,
- the randomly sampled values $r_i \in \mathbb{Z}_p$ used in *Sign*,

[6] available at `https://www.easycrypt.info/GGA`

– the signature elements $S_i = G^{s_i(\boldsymbol{u},\boldsymbol{r},\boldsymbol{m})} \in \mathbb{G}_1$ and $T_i = H^{t_i(\boldsymbol{u},\boldsymbol{r},\boldsymbol{m})} \in \mathbb{G}_2$, and

– the pairing-product equations used by *Verify*.

Here, f_i and g_i are arbitrary rational functions in the random variables $\boldsymbol{u}$. Similarly, s_i and t_i are rational functions in the random variables $\boldsymbol{u}, \boldsymbol{r}$ and the discrete logarithms $\boldsymbol{m}$ of the messages such that there exists a corresponding generic signing algorithm, i.e., S_i and T_i can be computed without knowing the discrete logarithms of the messages.

A search space description characterizes a finite set of SPS schemes and consists of (1) the group type, (2) the number of messages, verification key elements, and signature elements in $\mathbb{G}_1$ and $\mathbb{G}_2$, (3) the number of random values sampled in *KeyGen* and *Sign*, and (4) a description of the rational expressions that can be used for f_i, g_i, s_i, and t_i.

There are two ways to characterize the allowed rational expressions. First, the tool can take a set of Laurent polynomials with placeholders and allowed values for these placeholders, and generate all instances. Second, the tool accepts a set of constraints that specify bounds on the number of additions, the size of coefficients, and the degree of monomials. Then, it generates all Laurent polynomials that satisfy these constraints.

Given a search space description and concrete polynomials for the verification keys and the signature elements, the tool can compute the (strongest) verification equation as follows. Using distinct variables $Z_1, Z_2, \ldots$ for all group elements in the verification keys, signature elements, and messages, enumerate all products over these variables that can be computed by applying the homomorphisms and the bilinear map. This yields a sequence of monomials $M_1, M_2, \ldots$ over the the variables Z_i denoting products in $\mathbb{G}_T$ that can be computed from the input of the verification algorithm using Ψ and e. To characterize the linear relations between the elements in $\mathbb{G}_T$ corresponding to the monomials M_i, we associate a rational expression F_i over $\boldsymbol{u}, \boldsymbol{r}, \boldsymbol{m}$ to M_i by evaluating the monomial for $Z_i := h_i(\boldsymbol{u}, \boldsymbol{r}, \boldsymbol{m})$ where h_i is the exponent of the group element associated with Z_i. Finally, we use linear algebra to compute a basis of the linear relations between the F_i and map them back to verification equations using M_i.

4.2 Proof and Attack Search

We classify generated schemes using a proof and attack search based on an extension of the generic group analyzer developed by Barthe et al. [12]. The generic group analyzer (GGA) is a tool that automatically analyzes cryptographic assumptions in generic group models. To analyze SPS schemes, we use GGA's support for the generic bilinear group model. Here, the adversary is given blackbox access (using handles) to elements in the groups $\mathbb{G}_1$, $\mathbb{G}_2$, and $\mathbb{G}_T$ and provided with oracles for performing the group operations and applying the bilinear map and the efficiently computable homomorphisms. The GGA tool also supports a restricted class of interactive assumptions that enable the analysis of signature schemes that sign messages in $\mathbb{Z}_p$, but does not support oracles that take group elements. To analyze such interactive assumptions, the GGA tool exploits that

the signing oracle queries are essentially non-adaptive. More concretely, since the signing oracle takes elements in $\mathbb{Z}_p$ and returns handles to group elements, the adversary can only use these returned handles to compute the forgery, but the arguments to signing oracle queries cannot depend on the results of earlier oracle queries. This allows the GGA tool to treat oracle return values like initially known values by using parameters to model the oracle arguments in $\mathbb{Z}_p$. Another reason why the GGA cannot be directly applied to most SPS schemes is that there is no support for Laurent polynomials, which are required to model signing algorithms that invert elements of $\mathbb{Z}_p$.

To overcome these limitations, we have extended the GGA tool with support for both features and our extension can now analyze assumptions, such as the one shown in Figure 1, that were out of scope of the original version. To support signing oracles that take handles to group elements, the adversary knowledge can contain polynomials with parameters that are used to model oracle arguments in $\mathbb{Z}_p$ and in the groups $\mathbb{G}_*$. The parameters introduced to model known group elements correspond to coefficients of linear combinations, i.e., we exploit that every known group element is a linear combination of initially known group elements, group elements returned by earlier oracle queries, or the result of applying a pairing or an isomorphism to such group elements. To compute a basis for all known group elements after q oracle queries, we recursively extend the knowledge after i queries with the results of an additional query, starting with the initial knowledge.

The definition in Figure 1 specifies the EUF-CMA security experiment for an SPS scheme in the Type II setting. The verification keys are specified in the second line, the signing algorithm is given in the third line, and the winning condition (including the verification equation) is given in the last line. Here, group elements are specified by giving their exponent polynomials and the variables V and W are assumed to be randomly sampled. For such an input and a bound on the number q of performed oracle queries, the tool either returns an attack or a proof that the scheme is q-EUF-CMA secure in the generic group model. We note that our tool can be invoked with any specified value of q, though in this specific setting of SPS it runs efficiently only with small values.

5 Synthesized Schemes

In this section we will present and discuss the SPS schemes that we obtained by searching using our tool described in Section 4.

5.1 A Summary of Our Search

We have performed an exhaustive search for Type II schemes with keys $V, W \in \mathbb{G}_1$, message $M \in \mathbb{G}_2$, and signature $T, S \in \mathbb{G}_2$ such that: V and W are random; $T = H^r \cdot U$ where r is random and U does not involve r; the exponent polynomials of S, i.e., $s(r, v, w, m)$, have coefficients in $\{0, 1\}$. The results of our search are presented in Table 1. We use "Proof" to denote that our tool could prove at

Search Space		Schemes		Results (for eq. cl.)		
Verification equation	First sig. elt.	total	eq. cl.	Noverif	Attack	Proof
$s = f(t, v, w, m)$	$T = H^r$	212	57	0	55	1
$s\,t = f(t, v, w, m)$	$T = H^r$	224	67	0	55	12
$s\,(t - w) = f(t, v, w, m)$	$T = H^{r+w}$	1344	774	651	103	14
$s\,w = f(t, v, w, m)$	$T = H^r$	224	126	0	120	3
		2004	1024	651	333	30

Table 1. Synthesis results for Type II with keys $V, W \in \mathbb{G}_1$, message $M \in \mathbb{G}_2$, and signature $T, S \in \mathbb{G}_2$. The value r in the exponent of T is always chosen as a random element in $\mathbb{Z}_p$.

least 2-EUF-CMA security. Among the SPS schemes that are found, we identify equivalent schemes according to the following notion: we say two schemes Σ and Σ' are in the same equivalence class if Σ can be obtained from Σ' by applying invertible affine transformations to the verification keys, the messages, and the signature elements. This implies the existence of reductions from the security of Σ to the security of Σ' and vice-versa. As a simple example, consider a scheme that is obtained from another scheme by first multiplying the message M with G and then applying the original signing algorithm. Overall, except for 10 of the 1024 analyzed schemes, our tool either finds an attack, proves at least 2-EUF-CMA security, or proves that there is no verification equation. For the 10 schemes, the tool either returned unknown or timed out[7].

5.2 New SPS Schemes

Among the schemes that we found using our tool, we highlight two of them that are of particular interest, as well as a counterpart of the first one in the Type III setting.

A Strongly-Optimal Randomizable SPS. The first scheme is an SPS which is randomizable and matches the lower bound of Theorem 1, i.e., it can be verified using one offline and two online pairings. This scheme improves over the previously known randomizable schemes (in particular over the one recently proposed in [7]) as the latter requires three online pairings. This new scheme is presented in Fig. 2, and its security, stated as follows, is proved in the full version of this paper [13].

Theorem 3. *The signature scheme in Fig. 2 is EUF-CMA-secure in the generic bilinear group model.*

[7] we used a timeout of 30 seconds

This scheme can be translated to Type III groups using the transformation in [17], which essentially consists in "duplicating" R, i.e., to give $R = G^r \in \mathbb{G}_1$ and $T = H^r \in \mathbb{G}_2$, and adding a pairing-product equation to check that R, T have the same discrete logarithm, i.e., $e(R, H) = e(G, T)$. Such transformed scheme however requires one offline and four online pairings in the pairing-product equations. In what follows we propose a slightly different way to transform our scheme in the Type III setting which yields a solution requiring only three online pairings. The basic idea is that in the previous transformation T is not used in the first pairing-product equation, and its utility is to force the adversary to show that it knows the discrete log of R (or obtained (R, T) by applying a linear operation on a pair received by the challenger). We obtain the same functionality by letting the signer compute $T = H^{1/r}$. This allows us to test "equality of r between R and T" by checking $e(R, T) = e(G, H)$. The last pairing, however, involves only the generators and can thus be computed offline. A precise description of the resulting scheme is provided in Fig. 3, and the security result, stated below, is proved in the full version of this paper [13].

Theorem 4. *The signature scheme in Fig. 3 is EUF-CMA-secure in the generic bilinear group model.*

A Strongly-Optimal RMA-Secure SPS. Our second new SPS scheme, presented in Fig. 4, is secure against random-message attacks, and achieves the lower bound of only two pairings in the pairing-product equation (both necessarily online) for EUF-RMA-secure schemes, as stated in Theorem 2. In particular, it *beats* the lower bound of Theorem 1 that holds for EUF-CMA-secure schemes.

This scheme is also perfectly randomizable, with the simple randomization algorithm that sends a signature (R, S) on M to $(R \cdot H^t, S \cdot M^t)$ for some uniformly random t.

As an interesting note, we observe that the verification equation of this scheme is exactly the only possible one, according to our impossibility proof. Indeed, while our Lemma 5 holds for SPS schemes that are EUF-CMA-secure, the actual proof relies on EUF-RMA-security in all cases but one. For that particular case, in which we show a chosen-message attack, the verification equation is of the form $s + w = (r + v)(f_1 m + f_4)$ for some constants f_1, f_4, up to invertible linear transformations on the verification key and signature elements.

The security of this scheme, stated below, is proved in the full version of this paper [13].

Theorem 5. *The signature scheme in Fig. 4 is EUF-RMA-secure in the generic bilinear group model.*

6 Conclusion

In this work, we considered a new measure for the efficiency of SPS, that is, the number of pairings required in the verification equation. With respect to this

Setup(1^k)**:** return $PP = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, \psi, G, H) \leftarrow \mathcal{G}(1^k)$.
KeyGen(PP)**:** choose random $v, w \leftarrow \mathbb{Z}_p$ and return $VK = (V, W)$, $SK = (v, w)$ where $V = G^v$ and $W = G^w$.
Sign(PP, SK, M)**:** given $M \in \mathbb{G}_2$, choose a random $r \leftarrow \mathbb{Z}_p^*$ and return (R, S) where $R = H^r$ and $S = (M^v H^w)^{1/r}$.
Rerand$(PP, VK, M, (R, S))$**:** pick a random $\alpha \leftarrow \mathbb{Z}_p^*$ and compute a randomized signature (R', S') as $R' = R^\alpha$ and $S' = S^{1/\alpha}$.
Verify$(PP, VK, M, (R, S))$**:** accept if and only if $M, R, S \in \mathbb{G}_2$ and

$$e(\psi(R), S) = e(V, M) \cdot e(W, H).$$

Fig. 2. Our strongly-optimal re-randomizable SPS.

Setup(1^k)**:** return $PP = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, G, H) \leftarrow \mathcal{G}(1^k)$.
KeyGen(PP)**:** choose random $v, w \leftarrow \mathbb{Z}_p$ and return $VK = (V, W)$, $SK = (v, w)$ where $V = G^v$ and $W = G^w$.
Sign(PP, SK, M)**:** given $M \in \mathbb{G}_2$, choose a random $r \leftarrow \mathbb{Z}_p^*$ and return $(R, T, S) \in \mathbb{G}_1 \times \mathbb{G}_2^2$ where $R = G^r$, $T = H^{1/r}$ and $S = (M^v H^w)^{1/r}$.
Rerand$(PP, VK, M, (R, T, S))$**:** pick a random $\alpha \leftarrow \mathbb{Z}_p^*$ and compute a randomized signature (R', T', S') as $R' = R^\alpha$, $T' = T^{1/\alpha}$ and $S' = S^{1/\alpha}$.
Verify$(PP, VK, M, (R, T, S))$**:** accept if and only if $R \in \mathbb{G}_1$, $M, T, S \in \mathbb{G}_2$ and

$$e(R, S) = e(V, M) \cdot e(W, H) \quad \text{and} \quad e(R, T) = e(G, H).$$

Fig. 3. A re-randomizable SPS in Type III groups.

Setup(1^k)**:** return $PP = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, \psi, G, H) \leftarrow \mathcal{G}(1^k)$.
KeyGen(PP)**:** choose random $v, w \leftarrow \mathbb{Z}_p$ and return $VK = (V, W)$, $SK = (v, w)$ where $V = G^v$ and $W = G^w$.
Sign(PP, SK, M)**:** given $M \in \mathbb{G}_2$, choose a random $r \leftarrow \mathbb{Z}_p$ and return (R, S) where $R = H^r$ and $S = M^{r+v} H^{-w}$.
Verify$(PP, VK, M, (R, S))$**:** accept if and only if $M, R, S \in \mathbb{G}_2$ and

$$e(\psi(S) \cdot W, H) = e(R \cdot V, M).$$

Fig. 4. Our RMA-secure SPS with two pairings.

measure, we proved lower bounds and proposed new schemes matching these bounds in the Type II setting. In order to find schemes, we built a synthesis tool that automates the generation and security analysis of SPS. Currently, our methods only support security proofs with respect to a bounded number of signing queries. Developing new methods that support security proofs with respect to an unbounded number of queries for SPS is an interesting open problem that requires new techniques. Another direction left open by our work is to obtain similar results for the Type I and Type III setting. In contrast to the Type II setting, the Type I and Type III settings are more complex since they need more than one verification equation [4,8]. For synthesis, this causes a significant blowup of the search space, while for the minimality result, our proofs exploit that there is exactly one equation.

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