

# Password-based Authenticated Key Exchange

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**Abstract.** *Authenticated Key Exchange* protocols enable several parties to establish a shared cryptographically strong key over an insecure network using various authentication means, such as strong cryptographic keys or short (*i.e.*, low-entropy) common secrets. The latter example is definitely the most interesting in practice, since no additional device is required, but just a human-memorable password, for authenticating the players.

After the seminal work by Bellare and Merritt, many settings and security notions have been defined, and many protocols have been proposed, in the two-user setting and in the group setting.

## 1 Introduction

Key exchange protocols are cryptographic primitives used to provide several users (two or more), communicating over a public unreliable channel, with a secure session key. This thus allows establishment of virtual secure channels over insecure networks, which is one of the main practical applications of cryptography. Bellare and Rogaway gave the first foundations in [13, 14], but password-based authentication required more work: in this setting, where the authentication means is a short secret chosen from a small set of possible values (a four-digit pin, for example), the brute-force method which consists in trying all the possible values in the dictionary succeeds after a rather small number of attempts. This attack is called *on-line dictionary attack* and is unavoidable. But its damages can be limited by a policy that invalidates or blocks the use of a password if a certain number of failed attempts has occurred, unless failures are undetectable [27].

This paper presents a brief survey on Password-based Authenticated Key Exchange (PAKE) protocols, with a presentation of some security models in Section 2, and relations to practice. Section 3 deals with some practical constructions.

## 2 Security Models

Bellare, Pointcheval and Rogaway [12], and Boyko, MacKenzie and Patel [16] first formalized security of Password-based Authenticated Key Exchange, in two different frameworks.

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## 2.1 Game-based Security

The former model [12], the so-called *Find-then-Guess* scenario, is in the indistinguishability-based framework where an adversary should not be able to get an advantage significantly greater than  $q_S/N$  (or at most  $\mathcal{O}(q_S)/N$  for some technicality reasons) in distinguishing a random session key from a real session key, if  $q_S$  is the number of active attacks and  $N$  the size of the dictionary. It has thereafter been improved to the *Real-or-Random* scenario [7]. More precisely, the adversary is given access to oracles: **Execute**-queries model passive attacks, **Send**-queries model active attacks, **Corrupt**-queries model corruptions with the leakage of long-term secrets, **Reveal**-queries model bad uses of session keys and thus the leakage of ephemeral secrets, and **Test**-queries model the semantic security of the session key with a real or random answer. In the Find-then-Guess scenario, only one **Test**-query can be asked, whereas in the Real-or-Random scenario many **Test**-queries can be asked with either always-real or always-random answers. The latter is clearly at least as strong as the former. But while both scenarios were known to be equivalent for encryption schemes [11], a linear loss in the number of **Test**-queries makes them quite different for PAKE, where the advantage should remain in  $\mathcal{O}(q_S)/N$ , whatever the number of **Test**-queries. We have then showed [7] that in this Real-or-Random scenario, **Reveal**-queries are not useful anymore, hence simplifying the security games.

## 2.2 Simulation-based Security

The latter model [16] is in the simulation-based framework, with an ideal functionality in which the adversary is allowed to check one password per session. This models on-line dictionary attacks. Excepted this test instance password, no information is leaked about the passwords and the session keys.

## 2.3 Universal Composability

In both above models, one formalized the fact that, with an active attack, the adversary can basically test one password, whereas passive eavesdropping does not (computationally) leak any information. The goal is essentially to rule out *off-line dictionary attacks* in which the adversary makes some active and passive attacks, and then makes an off-line brute-force attack on the dictionary. On-line brute-force attacks, which are unavoidable, should be the only possible way to have some information about the session keys, and thus many interactions with a real player are required.

However, there were still some limitations on the password distributions and for composition with other protocols, which were overcome by Canetti, Halevi, Katz, Lindell and MacKenzie [24]. They indeed provided an ideal functionality in the Universally Composable (UC) security framework [23], see Figure 1. This functionality also models on-line dictionary attacks with a **TestPwd**-query that can be asked once to each user in sessions. An important property is that passwords are chosen by the environment which then hands them to the parties

The functionality  $\mathcal{F}_{\text{PAKE}}$  is parameterized by a security parameter  $k$ . It interacts with an adversary  $\mathcal{S}$  and a set of parties  $P_1, \dots, P_n$  via the following queries:

- $P_i$  asks for a **(NewSession, sid,  $P_i, P_j, pw$ )**: Send **(NewSession, sid,  $P_i, P_j$ )** to  $\mathcal{S}$ . If this is the first **NewSession**-query, or if this is the second **NewSession**-query and there is a record  $(P_j, P_i, pw')$ , then record  $(P_i, P_j, pw)$  and mark this record **fresh**.
  - $\mathcal{S}$  asks for a **(TestPwd, sid,  $P_i, pw'$ )**: If there is a record of the form  $(P_i, P_j, pw)$  which is **fresh**, then do:
    - If  $pw = pw'$ , mark the record **compromised** and reply with “correct guess”;
    - If  $pw \neq pw'$ , mark the record **interrupted** and reply with “wrong guess”.
  - $\mathcal{S}$  asks for a **(NewKey, sid,  $P_i, sk$ )**: If there is a record of the form  $(P_i, P_j, pw)$ , and this is the first **NewKey**-query for  $P_i$ , then:
    - If this record is **compromised**, or either  $P_i$  or  $P_j$  is **corrupted**, then output **(sid, sk)** to player  $P_i$ ;
    - If this record is **fresh**, and there is a record  $(P_j, P_i, pw')$  with  $pw' = pw$ , and a key  $sk'$  was sent to  $P_j$ , and  $(P_j, P_i, pw)$  was **fresh** at the time, then output **(sid, sk')** to  $P_i$ ;
    - In any other case, pick a new random key  $sk'$  of length  $k$  and send **(sid, sk')** to  $P_i$ .
- Either way, mark the record  $(P_i, P_j, pw)$  as **completed**.

**Fig. 1.** The PAKE Ideal Functionality  $\mathcal{F}_{\text{PAKE}}$

as inputs. This guarantees security even in the case where two honest players execute the protocol with two different passwords: the environment can emulate any distribution, mistypes of passwords and related passwords. Also note that allowing the environment to choose the passwords guarantees forward secrecy. This functionality mimics quite well some concrete requirements, but still, some leakage of information is not modeled, and could be exploited by a real-life adversary, whereas the ideal functionality does not allow it to the ideal-world adversary.

*Explicit Authentication.* With the above functionality, if neither party is corrupted, then they both end up with a uniformly-distributed session key, either the same key if the passwords are the same (success), or independent keys if the passwords are different (failure). Furthermore, the adversary learns nothing about the keys and the passwords, and even nothing about the status of the session (success or failure), but the users either. *Explicit authentication*, or mutual authentication modeled in [5], provides the players with a session key if and only if the passwords are the same, informing the adversary of success or not. This is an interesting additional feature, which is also more relevant in practice. In the real life, the adversary anyway learns whether the protocol succeeded or not, since in the latter case the communication stops.

Combined with the split functionality [10], it also allows to remove the **TestPwd**-query since the **NewKey**-query would reveal to the adversary whether

the passwords are the same or not, by leaking the success or failure status. The split functionality allows the adversary to split a session between users Alice and Bob into two sessions, one between Alice and the adversary trying to impersonate Bob, and a second one between Bob and the adversary trying to impersonate Alice. When the adversary plays with Alice, in case of success, this means it has guessed Alice’s password, which is similar to the `TestPwd`-query.

*Contributiveness.* In the  $\mathcal{F}_{\text{PAKE}}$  functionality, if one party is corrupted, or if the adversary successfully guessed the player’s password, the adversary is granted the right to fully determine the session key. Note that as soon as a party is corrupted, the adversary anyway learns the key, so one can think that nothing is lost by allowing it to fully determine it. But this is precisely the difference between *key agreement* and *key distribution* protocols.

In case of groups, this makes a huge difference. Hence the more recent functionality proposed by Abdalla, Catalano, Chevalier and Pointcheval [4] which

The functionality  $\mathcal{F}_{\text{GPAKE}}$  is parameterized by a security parameter  $k$ , and the parameter  $t$  of the contributiveness. It interacts with an adversary  $\mathcal{S}$  and a set of parties  $P_1, \dots, P_n$  via the following queries:

- $P_i$  asks for a **(NewSession, sid, Pid,  $P_i$ ,  $pw_i$ )**: If this is the first `NewSession`-query for  $P_i$ , where `Pid` is a set of at least two distinct identities containing  $P_i$ , record  $(\text{sid}, \text{Pid}, P_i, pw_i)$ , mark it `fresh`, and send  $(\text{sid}, \text{Pid}, P_i)$  to  $\mathcal{S}$ . Ignore any subsequent `NewSession`-queries with a different `Pid` set. If all the players involved in `Pid` have submitted their `NewSession`-queries, then record  $(\text{sid}, \text{Pid}, \text{ready})$  and send it to  $\mathcal{S}$ .
- $\mathcal{S}$  asks for a **(TestPwd, sid, Pid,  $P_i$ ,  $pw'$ )**: If there exists a record of the form  $(\text{sid}, \text{Pid}, P_i, pw_i)$  which is `fresh`:
  - If  $pw_i = pw'$ , mark the record `compromised` and reply with “correct guess”;
  - If  $pw_i \neq pw'$ , mark the record `interrupted` and reply with “wrong guess”.
- $\mathcal{S}$  asks for a **(NewKey, sid, Pid,  $sk$ )**: If there is a record of the form  $(\text{sid}, \text{Pid}, \text{ready})$ , then, denote by  $n_c$  the number of corrupted players, and
  - If all  $P_i \in \text{Pid}$  have the same passwords and  $n_c < t$ , choose  $sk' \in \{0, 1\}^k$  uniformly at random and store  $(\text{sid}, \text{Pid}, sk')$ .
  - If all  $P_i \in \text{Pid}$  have the same passwords but  $n_c \geq t$ , store  $(\text{sid}, \text{Pid}, sk)$ .
In both cases, for all  $P_i \in \text{Pid}$ , mark the record  $(\text{sid}, \text{Pid}, P_i, pw_i)$  `completed`. In any other case, store  $(\text{sid}, \text{Pid}, \text{error})$ , and for all  $P_i \in \text{Pid}$ , mark the record  $(\text{sid}, \text{Pid}, P_i, pw_i)$  `error`. When the key is set, report the result (either `error` or `completed`) to  $\mathcal{S}$ .
- $\mathcal{S}$  asks for a **(SendKey, b, sid, Pid,  $P_i$ )**: If  $P_i \in \text{Pid}$  and there is a recorded tuple  $(\text{sid}, \text{Pid}, \alpha)$  where  $\alpha \in \{0, 1\}^k \cup \{\text{error}\}$ , send  $(\text{sid}, \text{Pid}, \alpha)$  to  $P_i$  if  $b = 1$  or  $(\text{sid}, \text{Pid}, \text{error})$  if  $b = 0$ .
- $\mathcal{S}$  asks for a **(Corrupt, sid, Pid,  $P_i$ )**: If there is a recorded tuple  $(\text{sid}, \text{Pid}, P_i, pw_i)$ , then reveal  $pw_i$  to  $\mathcal{S}$ . If there also is a recorded tuple  $(\text{sid}, \text{Pid}, sk)$ , that has not yet been sent to  $P_i$ , then send  $(\text{sid}, \text{Pid}, sk)$  to  $\mathcal{S}$ .

**Fig. 2.** The Contributory GPAKE Ideal Functionality  $\mathcal{F}_{\text{GPAKE}}$

provides the *contributiveness* property to Group Password-based Authenticated Key Exchange (GPAKE), see Figure 2. PAKE is a particular case of GPAKE with groups of size 2. The latter property allows the adversary to fully determine the session key only if it has corrupted enough players, more than a threshold. This threshold can even be maximal: as soon as a player is honest, if a common key is generated, it is uniformly distributed in an unpredictable way. This means that no player has a more important role, and so there is no player to corrupt in priority for the adversary. As explained above, and as done in [5], one can even remove TestPwd-queries, allowing the adversary to split the group into several subgroups, with sub-session-IDs, where the adversary plays the role of the other users.

### 3 Constructions

#### 3.1 Two-Party Password-based Authenticated Key Exchange

Bellovin and Merritt [15] proposed the first scheme, the so-called Encrypted Key Exchange (EKE), see Figure 3 for a sketch of the protocol, where  $\mathcal{E}$  is assumed to be an encryption scheme onto the group  $\mathbb{G}$ , sometimes modeled as an ideal cipher. A first security analysis has been provided in the indistinguishability-based framework, in the ideal-cipher model [12], followed by several proofs of variations [8, 18, 19], trying to reduce the need of ideal models but still keeping the initial efficiency of EKE. EKE has also been studied in the simulation-based framework, in the random-oracle model [16], followed by studies in the UC framework [3] with security against adaptive corruptions, but still in ideal models. Our “simple PAKE” protocols [8] are definitely the most efficient, with a random oracle only for extracting the session key, with a security analysis in the Find-then-Guess scenario, under the CDH assumption.

Katz, Ostrovsky and Yung [33] proposed the first practical scheme, but still less efficient than above schemes, in the standard model with a common reference string, followed by a generalization from Gennaro and Lindell (GL) [28, 29], using the power of smooth-projective hash functions [26], in the Find-then-Guess

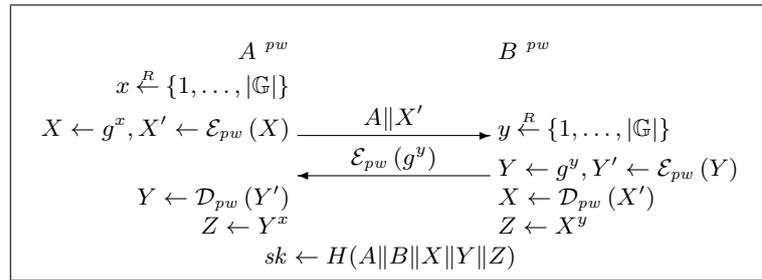


Fig. 3. Encrypted Key Exchange

scenario. Many variations [6, 24, 31, 34, 35] have thereafter been proposed, to get security in the UC framework, to improve round efficiency, or to rely on new assumptions.

Whereas the huge majority of the protocols rely on Diffie-Hellman assumptions, some efficient schemes have also been proposed on factoring-related assumptions [25, 30, 36, 37]. Besides the Secure Remote Password (SRP) protocol [39] and the Simple Password Exponential Key Exchange (SPEKE) protocol [32] that have been standardized, EKE-like and GL-like schemes are the two main streams, with security analyses in the UC framework.

### 3.2 Group Password-based Authenticated Key Exchange

For groups, while the first proposals were extensions of the group Diffie-Hellman key exchange [17, 20, 38], the Burmester and Desmedt construction [21, 22] became more appropriate, because of its constant number of rounds, independently of the size of the group. Several group password-based authenticated key exchange protocols have then been proposed [1, 2, 5, 9], essentially combining a two-party PAKE with the Burmester and Desmedt methodology.

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