Meet-in-the-Middle Preimage Attacks on Sponge-based Hashing[1]

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Abstract. The Meet-in-the-Middle (MitM) attack has been widely applied to preimage attacks on Merkle-Damgård (MD) hashing. In this paper, we introduce a generic framework of the MitM attack on sponge-based hashing. We find certain bit conditions can significantly reduce the diffusion of the unknown bits and lead to longer MitM characteristics. To find good or optimal configurations of MitM attacks, e.g., the bit conditions, the neutral sets, and the matching points, we introduce the bit-level MILP-based automatic tools on `Keccak`, `Ascon` and `Xoodyak`. To reduce the scale of bit-level models and make them solvable in reasonable time, a series of properties of the targeted hashing are considered in the modelling, such as the linear structure and CP-kernel for `Keccak`, the Boolean expression of Sbox for `Ascon`. Finally, we give an improved 4-round preimage attack on `Keccak-512/SHA3`, and break a nearly 10 years' cryptanalysis record. We also give the first preimage attacks on 3-/4-round `Ascon-XOF` and 3-round `Xoodyak-XOF`.

Keywords: MitM · Automatic Tool · `Keccak/SHA3` · `Ascon`· `Xoodyak`

1 Introduction

The Meet-in-the-Middle (MitM) attack proposed by Diffie and Hellman in 1977 [22] is a generic technique for cryptanalysis of symmetric-key primitives. The essence of the MitM attack is actually an efficient way to exhaustively search a space for the right candidate based on the birthday attack, i.e., dividing the

[1]The full version of the paper is available at *https://eprint.iacr.org/2022/1714*

whole space into two independent subsets (also known as neutral sets) and then finding matches from the two subsets. Suppose $E_K(\cdot)$ to be a block cipher whose size is n-bit such that $C = E_K(P) = F_{K_2}(F_{K_1}(P))$, where $K = K_1 \| K_2$ has n bits, and K_1 and K_2 are independent key materials of $n/2$ bits. For a given plaintext-ciphertext pair (P, C), a naive exhaust search attack needs a time complexity 2^n to find the key. However, the birthday-paradox based MitM attack computes independently $F_{K_1}(P)$ and $F^{-1}_{K_2}(C)$ with independent guesses of K_1 and K_2, and searches collision between $F_{K_1}(P)$ and $F^{-1}_{K_2}(C)$ to find the K with a time complexity about $2^{n/2}$. In the past decades, the MitM attack has been widely applied to the cryptanalysis on block ciphers [50,30,12,40] and hash functions [59,2,34]. In the meantime, various techniques have been introduced to improve the framework of MitM attack, such as internal state guessing [30], splice-and-cut [2], initial structure [59], bicliques [11], 3-subset MitM [12], indirect-partial matching [2,59], sieve-in-the-middle [15], match-box [32], dissection [24], differential-aided MitM [41,31,14], nonlinear constrained neutral words [28], etc. Till now, the MitM attack and its variants have broken MD4 [44,34], MD5 [59], KeeLoq [39], HAVAL [4,60], GOST [40], GEA-1/2 [7,1], etc.

At CRYPTO 2011 and 2016, several ad-hoc automatic tools [13,19] were proposed for MitM attacks. At IWSEC 2018, Sasaki [57] introduced MILP-based MitM attacks on GIFT block cipher. At EUROCRYPT 2021, Bao et al. [5] introduced the MILP-based automatic search framework for MitM preimage attacks on `AES`-like hashing, whose compression function is built from `AES`-like block cipher or permutations. At CRYPTO 2021, Dong et al. [28] further extended Bao et al.'s model into key-recovery and collision attacks. At CRYPTO 2022, Schrottenloher and Stevens [61] simplified the language of the automatic model and applied it in both classic and quantum settings. Bao et al. [6] considered the MitM attack in a view of the superposition states.

When applying to hash functions, most of the MitM attacks targeted on Merkle-Damgård [51,17] domain extender, whose compression function is usually built from a block cipher and PGV hashing modes [54], such as Davies-Meyer (DM), Matyas-Meyer-Oseas (MMO) and Miyaguchi-Preneel (MP). The goal of the MitM attack is to find a sequence of internal states which satisfy a closed computational path: there is a relation between the value before the first round and the value after the last round in previous applications of MitM attacks. For example, when attacking block-cipher based hashing modes (DM, MMO, MP, etc.), the closed computation path is computed across the first and last rounds via the feed-forward mechanism of the hashing modes. While when attacking block ciphers, the closed computation path is linked via an encryption/decryption oracle. As shown in Figure 1, one starts by separating the path in two chunks (splice-and-cut): the backward chunk and the forward chunk depending on different neutral sets. Both chunks form independent computation paths. One then finds a partial match between them at certain round.

When considering the new hashing mode, i.e., sponge-based hashing, there is no feed-forward mechanism anymore, e.g. `Keccak`. We have to try novel ways to build the so-called closed computational path for MitM attack. Most preim-

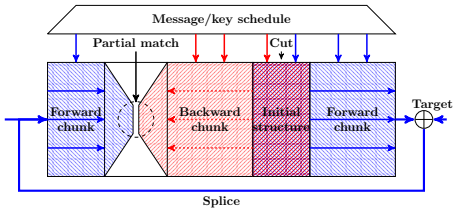


Fig. 1: The closed computation path of the MitM attack

age attacks on sponge-based hashing focus on the analysis on `Keccak/SHA-3` with linearization technique. At ASIACRYPT 2016, Guo et al. [35] introduced the *linear structure* technique to linearise several rounds of `Keccak` and derived upto 4-round preimage attacks. Later, Guo et al.'s attacks were improved by the cross-linear structure [46] at ToSC 2017 and *allocating approach* [45] at EUROCRYPT 2019. Further improvements in this line were proposed in [56,37,48,47]. At EUROCRYPT 2021, Dinur [23] gave the preimage attacks on `Keccak` by solving multivariate equation systems. Additionally, theoretical preimage attacks marginally better than exhaustive attacks were studied in [8,52].

At INDOCRYPT 2011, an MitM attack on 2-round `Keccak` is given by Naya-Plasencia et al. [53]. However, their MitM attack is different from what we are considering. In [53], after computing the inverse of one round `Keccak` from the target, partial internal states are known. Then, Naya-Plasencia et al. divide the message block into many independent parts and compute forward independently for each part until the known internal states, and filter the messages. Naya-Plasencia et al.'s attack is more like a divide-and-conquer and the birthday attack is not used.

Our Contributions.

We apply the birthday-paradox based MitM attack*, which has been widely used to attack Merkle-Damgård [51,17] hashing with PGV modes as well as block ciphers, to the sponge-based hash functions. Additionally, by applying bit conditions, the diffusion of the two neutral sets can be controlled and reduced, and therefore lead to longer MitM characteristics. Finally, we propose a generic MitM framework with conditions for sponge-based hash functions.

To apply our framework to `Keccak`, `Ascon-XOF` and `Xoodyak-XOF`, we have to search sound configurations for MitM attack, including the choice of the two neutral sets, the bit conditions, the matching points etc. As `Keccak`, `Ascon` are bit-level hashing, we introduce the bit-level MILP-based automatic tools to detect those configurations. Different from the previous byte-level MitM MILP models [5,28,61], the bit-level modelling usually leads to huge scale MILP models and makes it hard to solve in reasonable time. Therefore, we explore detailed

*The Demirci-Selçuk MitM attacks [18,29,20,21,10] are not considered in this paper, which is a quite different technique.

properties of dedicated ciphers to reduce the models. For `Keccak`, we apply the linear structures in starting states and CP-kernel properties in matching phase. For `Ascon`, the Boolean expressions of the Sbox are explored in the starting states and matching points. In previous modellings [5,28], cells depending on both two neutral sets are always regarded as useless and unknown in the MitM attack. The unknown cells can significantly reduce the number of known cells when propagating (somewhat like polluting), since any known cells will become unknown by operating with the unknown cells. Inspired by the *indirect-partial matching* technique [2,59], the cells depending on the additions of the two neutral sets are also useful and should not be regarded as unknown in the automatic searching models. Therefore, we introduce new constraints for those kinds of cells and reduce the polluting speed of the unknown cells.

At last, we derive a better 4-round preimage attack on `Keccak/SHA3-512` than Morawiecki et al.'s rotational cryptanalysis [52] at FSE 2013, that breaks their nearly 10 years' record. While previous preimage attack on `Keccak-512` with linear structure techniques [35] (including improvements with various techniques [48,36,56]) only reaches 3 rounds. For `Ascon-XOF`, the first 3-round and 4-round preimage attacks are given. For `Xoodyak-XOF`, the first 3-round preimage attack is given. A summary of the related results are given in Table 1.

Table 1: A Summary of the Attacks. Lin. Stru.: Linear Structure. MitM: MitM Attack. Diff.: Differential. †: this attack ignores the padding bits.

Target	Attacks	Methods	Rounds	Time	Memory	Ref.
`Keccak-512`	Preimage	Lin.Stru.	2	2^{384}	-	[35]
		Lin.Stru.	2	2^{321}	-	[56]
		Lin.Stru.	2	2^{270}	-	[48]
		Lin.Stru.	2	2^{252}	-	[36]
		Lin.Stru.	3	2^{482}	-	[35]
		Lin.Stru.	3	2^{475}	-	[56]
		Lin.Stru.	3	2^{452}	-	[48]
		Lin.Stru.	3	2^{426}	-	[36]
		Rotational	4	2^{506}	-	[52]
		MitM	4	$2^{504.58}$	2^{108}	Sect. 4.3
	Collision	Diff.	2	Practical	-	[53]
		Diff.	3	Practical	-	[25]
`Xoodyak-XOF`	Preimage	Neural	1	-	-	[49]
		MitM	3	$2^{125.06}$	2^{97}	Sect. 5.2
`Ascon-XOF`	Preimage	Cube-like	2	2^{103}	-	[27]
		MitM	3	$2^{120.58}$	2^{39}	Full Ver. [55]
		MitM	4	$2^{124.67}$	2^{54}	Sect. 6.2
		Algebraic†	6	$2^{127.3}$	-	[27]
	Collision	Diff.	2	2^{103}	-	[33]

Comparison to Schrottenloher and Stevens's MitM attack. At CRYPTO 2022, Schrottenloher and Stevens [61] introduced preimage attacks on `SPHINCS+-Haraka` [3], which is sponge-based hashing with permutation `Haraka` [43]. Their MitM attack computes from the two ends, i.e., the known inner part and the target, to the middle matching part. Combining with the guess-and-determine technique, they derived a 3.5-round (out of 5 rounds) quantum preimage attack on `SPHINCS+-Haraka`. As stated in [61, Section 3.1], their frameworks do not lead to interesting results on `Ascon` [27]. The reason may be that for `Keccak` or `Ascon` the inverse of one round is not as easy as `Haraka`. Our framework mainly uses the forward computation, and leads to novel results on both `Keccak` and `Ascon`.

2 Preliminaries

In the section, we give some brief descriptions of the Meet-in-the-Middle attack, the sponge-based hash function, the `Keccak-`f permutation, `Ascon-Hash` and `Ascon-XOF`, `Xoodyak` and `Xoodoo` permutation.

2.1 The Meet-in-the-Middle Attack

Since the pioneering works on preimage attacks on Merkle–Damgård hashing, e.g. `MD4`, `MD5`, and `HAVAL` [44,59,2,34], techniques such as *splice-and-cut* [2], *initial structure* [59] and *indirect-partial matching* have been invented to significantly improve the MitM approach. As shown in Figure 1, in the MitM attack, the compression function is divided at certain intermediate rounds (initial structure) into two chunks. One chunk is computed forward (named as forward chunk), and the other is computed backward (named as backward chunk). One of them is computed across the first and last rounds via the feed-forward mechanism of the hashing mode, and they end at a common intermediate round (partial matching point) and form a closed computation path of the MitM attack. In each of the chunks, the computation involves at least one distinct message word (or a few bits of it), such that they can be computed over all possible values of the involved message word(s) independently from the message word(s) involved in the other chunk (the distinct words are called neutral words). In the initial structure, the two chunks overlapped and the neutral words for both chunks appear simultaneously, but still, the computations of the two chunks on the neutral words are independent. The highlevel framework is in Figure 1, which can be divided into three configurations:

1. The chunk separation – the positions of initial structure and matching points.
2. The neutral sets – the selection on the two neutral sets (denoted as ■ or ■ sets), which determines the degree of freedom (DoF) for each chunk.
3. The matching – the deterministic relation used for matching, which determines the filtering ability (degree of matching, DoM).

After setting up the configurations, the basic attack procedure goes as follows.

1. Choose constants for the initial structure.
2. For all 2^{d_1} values of ■ neutral set, compute backward from the initial structure to the matching points to generate a table L_1, whose indices are the values for matching, and the elements are the values of ■ neutral set.
3. Similarly, build L_2 for 2^{d_2} values of ■ neutral set with forward computation.
4. Check whether there is an m-bit match on indices between L_1 and L_2.
5. For the pairs surviving the partial match, check for a full-state match. Steps 1-5 will be repeated until we find a full match.

The attack complexity. Denote the size of the target by h, and the number of bits for the match by m. An MitM episode is performed with time $2^{\max(d_1,d_2)} + 2^{d_1+d_2-m}$ and the total time complexity of the attack is:

$$2^{h-(d_1+d_2)} \cdot (2^{\max(d_1,d_2)} + 2^{d_1+d_2-m}) \simeq 2^{h-\min(d_1,d_2,m)}. \quad (1)$$

To illustrate how the MitM attack works, we detail the 7-round attack on AES-hashing of Sasaki [58] in the Supplementary Material A in our full version paper [55] as an example.

2.2 The Sponge-based Hash Function

The sponge construction [9] shown in Figure 2 takes a variable-length message as input and produces a digest of any desired length. The b-bit internal state is composed of an outer part of r bits and an inner part of c bits, where r is the rate and c is the capacity. To evaluate the sponge function, one proceeds in three phases with an inner permutation f:

1. **Initialization:** Initialize the b-bit state with the given value (all 0's for `Keccak`) before proceeding the message blocks.
2. **Absorbing:** The message is padded and split into blocks of r bits. Absorb each r-bit block M_i by XORing into the internal state.
3. **Squeezing:** Produce the digest.

We named the hash functions with sponge construction as the sponge-based hash functions, e.g. `Keccak` [9], `Ascon` [27], `Xoodyak` [16], to name a few.

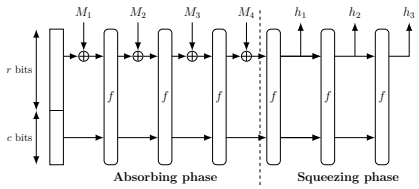


Fig. 2: The sponge construction

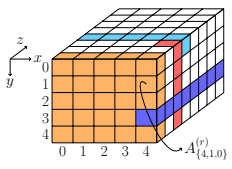


Fig. 3: The `Keccak` state

2.3 The Keccak-f Permutations

The Keccak hash function family [9] specifies 7 Keccak permutations, denoted Keccak-$f[b]$, where $b \in \{25, 50, 100, 200, 400, 800, 1600\}$ is the width of the permutation. In this paper, we focus on Keccak-$f[1600]$, where the state A is arranged as 5×5 64-bit lanes as depicted in Figure 3. Let $A^{(r)}_{\{x,y,z\}}$ denote the bit located at the x-th column, y-th row and z-th lane in the round r ($r \geq 0$), where $0 \leq x \leq 4,\ 0 \leq y \leq 4,\ 0 \leq z \leq 63$. For Keccak in the rest of this paper, all the coordinates are considered modulo 5 for x and y and modulo 64 for z. The function Keccak-$f[1600]$ consists of 24 rounds which consists of five operations $\iota \circ \chi \circ \pi \circ \rho \circ \theta$. Denote the internal states of round r as

$$A^{(r)} \xrightarrow{\theta} \theta^{(r)} \xrightarrow{\rho} \rho^{(r)} \xrightarrow{\pi} \pi^{(r)} \xrightarrow{\chi} \chi^{(r)} \xrightarrow{\iota} A^{(r+1)}.$$

Operations in each round are:

$$\begin{aligned}
\theta:&\quad \theta^{(r)}_{\{x,y,z\}} = A^{(r)}_{\{x,y,z\}} \oplus \sum\nolimits_{y'=0}^{4} (A^{(r)}_{\{x-1,y',z\}} \oplus A^{(r)}_{\{x+1,y',z-1\}}),\\
\rho:&\quad \rho^{(r)}_{\{x,y,z\}} = \theta^{(r)}_{\{x,y,z-\gamma[x,y]\}},\\
\pi:&\quad \pi^{(r)}_{\{y,2x+3y,z\}} = \rho^{(r)}_{\{x,y,z\}},\\
\chi:&\quad \chi^{(r)}_{\{x,y,z\}} = \pi^{(r)}_{\{x,y,z\}} \oplus (\pi^{(r)}_{\{x+1,y,z\}} \oplus 1) \cdot \pi^{(r)}_{\{x+2,y,z\}},\\
\iota:&\quad A^{(r+1)} = \chi^{(r)} \oplus RC_r,
\end{aligned} \tag{2}$$

where $\gamma[x, y]$'s are constants given in the Supplementary Material B in our full version paper [55], RC_r is round-dependent constant.

The Keccak and SHA3 Hash Function. The Keccak hash function follows the sponge construction. For Keccak$[r, c, d]$, the capacity is c, the bitrate is r and the diversifier is d. NIST standardized four SHA3-l versions ($l \in 224, 256, 384, 512$), where $c = 2l$ and $r = 1600 - 2l$. The only difference of Keccak and SHA3 is the padding rule. The padding rule for Keccak is padding the message with '10*1', which is a single bit 1 followed by the minimum number of 0 bits followed by a single bit 1, to make the whole length to a multiple of $(1600 - 2l)$. For SHA3, the message is padded with '0110*1'. However, the padding rule does not affect the final time complexity of our attack.

2.4 Ascon-Hash and Ascon-XOF

The Ascon family [27] includes the hash functions Ascon-Hash and Ascon-Hasha as well as the extendable output functions Ascon-XOF and Ascon-XOFa with sponge-based modes of operations.

Ascon Permutation. The inner permutation applies 12 round functions to a 320-bit state. The state A is split into five 64-bit words, and denote $A^{(r)}_{\{x,y\}}$ to be the x-th (column) bit of the y-th (row) 64-bit word, where $0 \leq y \leq 4$, $0 \leq x \leq 63$. The round function consists of three operations p_C, p_S and p_L. Denote the internal states of round r as $A^{(r)} \xrightarrow{p_S \circ p_C} S^{(r)} \xrightarrow{p_L} A^{(r+1)}$.

– **Addition of Constants p_C:** $A^{(r)}_{\{*,2\}} = A^{(r)}_{\{*,2\}} \oplus RC_r$.
– **Substitution Layer p_S:** For each x, this step updates the columns $A^{(r)}_{\{x,*\}}$ using the 5-bit Sbox. Assume the S-box maps $(a_0, a_1, a_2, a_3, a_4) \in \mathbb{F}_2^5$ to $(b_0, b_1, b_2, b_3, b_4) \in \mathbb{F}_2^5$, where a_0 is the most significant bit. The algebraic normal form (ANF) of the Sbox is as follows:

$$\begin{aligned} b_0 &= a_4a_1 + a_3 + a_2a_1 + a_2 + a_1a_0 + a_1 + a_0, \\ b_1 &= a_4 + a_3a_2 + a_3a_1 + a_3 + a_2a_1 + a_2 + a_1 + a_0, \\ b_2 &= a_4a_3 + a_4 + a_2 + a_1 + 1, \\ b_3 &= a_4a_0 + a_4 + a_3a_0 + a_3 + a_2 + a_1 + a_0, \\ b_4 &= a_4a_1 + a_4 + a_3 + a_1a_0 + a_1. \end{aligned} \tag{3}$$

– **Linear Diffusion Layer p_L:**

$$\begin{aligned} A^{(r+1)}_{\{*,0\}} &\leftarrow S^{(r)}_{\{*,0\}} \oplus (S^{(r)}_{\{*,0\}} \ggg 19) \oplus (S^{(r)}_{\{*,0\}} \ggg 28), \\ A^{(r+1)}_{\{*,1\}} &\leftarrow S^{(r)}_{\{*,1\}} \oplus (S^{(r)}_{\{*,1\}} \ggg 61) \oplus (S^{(r)}_{\{*,1\}} \ggg 39), \\ A^{(r+1)}_{\{*,2\}} &\leftarrow S^{(r)}_{\{*,2\}} \oplus (S^{(r)}_{\{*,2\}} \ggg 1) \oplus (S^{(r)}_{\{*,2\}} \ggg 6), \\ A^{(r+1)}_{\{*,3\}} &\leftarrow S^{(r)}_{\{*,3\}} \oplus (S^{(r)}_{\{*,3\}} \ggg 10) \oplus (S^{(r)}_{\{*,3\}} \ggg 17), \\ A^{(r+1)}_{\{*,4\}} &\leftarrow S^{(r)}_{\{*,4\}} \oplus (S^{(r)}_{\{*,4\}} \ggg 7) \oplus (S^{(r)}_{\{*,4\}} \ggg 41). \end{aligned}$$

`Ascon-Hash and Ascon-XOF`. The state A is composed of the outer part with 64 bits $A_{\{*,0\}}$ and the inner part 256 bits $A_{\{*,i\}}$ $(i = 1, 2, 3, 4)$. For `Ascon-Hash`, the output size is 256 bits, and the security claim is 2^{128}. For `Ascon-XOF`, the output can have arbitrary length and the security claim against preimage attack is $\min(2^{128}, 2^l)$, where l is the output length. In this paper, we target on `Ascon-XOF` with a 128-bit hash value and a 128-bit security claim against preimage attack.

2.5 `Xoodyak` and `Xoodoo` Permutation

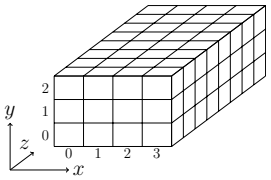


Fig. 4: Toy version of the `Xoodoo` state. The order in y is opposite to `Keccak`

Internally, `Xoodyak` makes use of the `Xoodoo` permutation [16], whose state (shown in Figure 4) bit denoted by $A^{(r)}_{\{x,y,z\}}$ is located at the x-th column, y-th

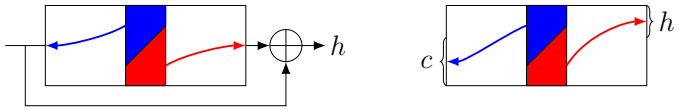
row and z-th lane in the round r, where $0 \le x \le 3,\ 0 \le y \le 2,\ 0 \le z \le 31$. For `Xoodoo`, all the coordinates are considered modulo 4 for x, modulo 3 for y and modulo 32 for z. The permutation consists of the iteration of a round function $R = \rho_{\text{east}} \circ \chi \circ \iota \circ \rho_{\text{west}} \circ \theta$. The number of rounds is a parameter, which is 12 in `Xoodyak`. Denote the internal states of the round r as

$$A^{(r)} \xrightarrow{\theta} \theta^{(r)} \xrightarrow{\rho_{\text{west}}} \rho^{(r)} \xrightarrow{\iota} \iota^{(r)} \xrightarrow{\chi} \chi^{(r)} \xrightarrow{\rho_{\text{east}}} A^{(r+1)}.$$

$$\begin{aligned}
\theta:&\quad \theta^{(r)}_{\{x,y,z\}} = A^{(r)}_{\{x,y,z\}} \oplus \sum\nolimits_{y'=0}^{2} (A^{(r)}_{\{x-1,y',z-5\}} \oplus A^{(r)}_{\{x-1,y',z-14\}}),\\
\rho_{\text{west}}:&\quad \rho^{(r)}_{\{x,0,z\}} = \theta^{(r)}_{\{x,0,z\}},\ \rho^{(r)}_{\{x,1,z\}} = \theta^{(r)}_{\{x-1,1,z\}},\ \rho^{(r)}_{\{x,2,z\}} = \theta^{(r)}_{\{x,2,z-11\}},\\
\iota:&\quad \iota^{(r)}_{\{0,0,z\}} = \rho^{(r)}_{\{0,0,z\}} \oplus RC_r,\ \text{where } RC_r \text{ is round-dependent constant},\\
\chi:&\quad \chi^{(r)}_{\{x,y,z\}} = \iota^{(r)}_{\{x,y,z\}} \oplus (\iota^{(r)}_{\{x,y+1,z\}} \oplus 1) \cdot \iota^{(r)}_{\{x,y+2,z\}},\\
\rho_{\text{east}}:&\quad A^{(r+1)}_{\{x,0,z\}} = \chi^{(r)}_{\{x,0,z\}},\ A^{(r+1)}_{\{x,1,z\}} = \chi^{(r)}_{\{x,1,z-1\}},\ A^{(r+1)}_{\{x,2,z\}} = \chi^{(r)}_{\{x-2,2,z-8\}}.
\end{aligned} \tag{4}$$

`Xoodyak` can serve as a `XOF`, i.e. `Xoodyak-XOF`, which offers arbitrary output length l. The preimage resistance is $\min(2^{128}, 2^l)$. We target on `Xoodyak-XOF` with output of 128-bit hash value and 128-bit absorbed message block.

3 Meet-in-the-Middle Attack on Sponge-based Hashing



(a) MITM on DM (b) MITM on Sponge

Fig. 5: Differences in MitM attack on PGV and sponge hash functions

The essence of the Meet-in-the-Middle attack is actually an efficient way to exhaustively search a space for the right one based on the birthday attack. Taken the MitM attack on DM construction (Figure 5(a)) as an example, suppose the size of the internal state is n, the size of the output is h ($n \ge h$). In the perspective of exhaust search attack, one chooses a random internal state to verify if it leads to the given h-bit target. After searching a space of 2^h internal states, one will find the preimage. In the MitM attacks, as shown in Figure 5(a), the attacker starts from the internal state in the middle, which is divided into two independent forward and backward chunks (marked in red and blue, respectively). One computes the two chunks independently until the matching point to filter the wrong internal states. The details are given in Section 2.1.

When considering the sponge-based hashing, if we start from some similar internal states in the middle (as shown in Figure 5(b)) to search preimage with

the given target, the internal state has to satisfy not only the target in forward computation, but also the c-bit inner part in backward computation. In other words, we have to search a space with $2^{(h+c)}$ internal states to meet the target and the inner part. Taking the complexity of exhaustive search (i.e., 2^h) into consideration, it may be not a good idea to search a $(h+c)$-bit space even with MitM. For sponge-based hashing, we do not follow the conventional start-from-the-middle way to drive the MitM attack. We try to search a more compact space to find the preimage. In fact, we choose to search the r-bit outer part. With the known c-bit inner part, a search of h-bit subspace of the outer part (if $r > h$) with MitM method is enough to find the preimage. The highlevel framework of the MitM approach is shown in Figure 6 and highlighted in the green box. Since we start from the outer part and try to satisfy the h-bit target, only forward computations are involved. Like the MitM attack in Section 2.1, we need to specify the configurations: the two neutral sets of the outer part, the two independent forward computation chunks, the matching points. We may partially solve the inverse of the permutation from the h-bit target to get some internal bits. Thereafter, by forward computing the two independent chunks until those internal bits, the deterministic relations on the two neutral sets were established, which act as the matching point.

3.1 The Conditions in the MitM Attack

The key point of the MitM is to extend the number of rounds of the independent computation path for blue or red neutral words. For `Keccak`, we have $\chi : b_i = a_i \oplus (a_{i+1} \oplus 1) \cdot a_{i+2}$. Supposing a_{i+1} is blue neutral word and a_i is red neutral word, then b_i depends on both blue and red neutral words if $a_{i+2} = 1$, otherwise b_i only depends on the red neutral words a_i. That is what we called "conditions".

Setting conditions to control the characteristic can trace back to Wang et al.'s collision attacks with message modification techniques [62,63]. Then, conditions are applied to enhance the probability of the differentials, i.e., the conditional differential cryptanalysis [42]. Later, conditions are used to reduce the diffusion of the cube variables in dynamic cube attack [26] and conditional cube attacks [38]. In MitM attack, the conditions were used to build MitM attacks [59,2] on MD/SHA hashing with ARX structure. For modular addition $X + Y = Z$ $(X, Y, Z \in \mathbb{F}_2^{32})$, particularly the computation of i-th and $(i+1)$-th bits, assume that the carry from $(i-1)$-th bit to i-th bit is 0. Then, the $(i+1)$-th bit of Z is computed as $Z_{\{i+1\}} = X_{\{i+1\}} \oplus Y_{\{i+1\}} \oplus X_{\{i\}} \cdot Y_{\{i\}}$. When $X_{\{i+1\}}$ is blue neutral word and $Y_{\{i\}}$ is red neutral word, the idea of making $X_{\{i\}} = 0$ as a condition so that $Z_{\{i+1\}}$ is only affected by blue neutral word.

In this paper, we try to apply conditions to reduce the diffusion of the red/blue neutral words, and expect to find longer MitM characteristics. In our MitM attack on sponge-based hashing, the conditions usually depend on bits from both inner part (capacity) and outer part (rate). In order to modify certain conditions, we have to modify bits from the inner part. Therefore, as shown in Figure 6, we place the MitM attack in the processing of the last message block and modify the conditions determined by inner part by randomly changing the

first several message blocks (e.g. M_1 in Figure 6). Suppose there are μ conditions only determined by the inner part[†], the probability to find one right M_1 satisfying all the conditions is about $2^{-\mu}$.

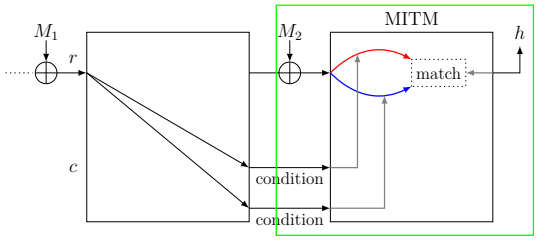


Fig. 6: Framework of the MitM attack on sponge-based hashing

Once we find one right M_1, we assign arbitrary values to all bits except those neutral bits for M_2. Then, an MitM episode is performed:

1. Suppose the two neutral sets of the outer part are of 2^{d_1} and 2^{d_2} values, respectively, which are marked by the ■ and ■ color. For each of 2^{d_1} values, compute forward to the matching points.
2. For each of 2^{d_2} values, compute forward to the matching points.
3. Compute backward with the known h-bit target to the matching points to derive an m-bit matching.
4. Filter states.

The complexity of the MitM episode is $2^{\max(d_1,d_2)}+2^{d_1+d_2-m}$, which actually checks $2^{d_1+d_2}$ M_2. In order to find a preimage of h, we have to repeat the episode for $2^{h-(d_1+d_2)}$ times. After the conditions in inner part are satisfied, suppose there are 2^{η} non-neutral bits in M_2 that provide 2^{η} MitM episodes. If $\eta+d_1+d_2<h$, we have to find $2^{h-(\eta+d_1+d_2)}$ M_1, which all satisfy the conditions in the inner part of the last permutation. The total time complexity is

$$2^{h-(\eta+d_1+d_2)}\cdot 2^{\mu}+2^{h-(d_1+d_2)}\cdot(2^{\max(d_1,d_2)}+2^{d_1+d_2-m}). \tag{5}$$

4 MitM Preimage Attack on `Keccak`

This section first gives some techniques and properties in previous preimage attacks on `Keccak`. Then we propose our MILP model for the MitM attack on `Keccak`. As an application, we mount a 4-round preimage attack on `Keccak`-512.

[†]Note that, if the conditions are determined by both outer part and inner part, then for given inner part, it is possible to change the message block (i.e., M_2 in Figure 6) to modify the conditions.

4.1 Preliminaries on Keccak

Most previous preimage attacks on Keccak/SHA3 are with the linearization technique. The *linear structure* technique allows to linearize the underlying permutation of Keccak for several rounds. In our attack, we also apply the *linear structure* technique proposed by Guo et al. [35] to linearize one round of Keccak, in order to speed up the search.

Linear Structure. We give an example to explain the *linear structure* technique. As shown in Fig. 7, the variables $v_{0,z}$ and $v_{1,z}$ $(0 \le z \le 63)$ are allocated as $A^{(0)}_{\{0,0,z\}} = v_{0,z}$, $A^{(0)}_{\{0,1,z\}} = v_{0,z} \oplus c_{0,z}$, $A^{(0)}_{\{2,0,z\}} = v_{1,z}$, $A^{(0)}_{\{2,1,z\}} = v_{1,z} \oplus c_{1,z}$, where $c_{0,z}$ and $c_{1,z}$ $(0 \le z \le 63)$ are constants. After the θ operation, the variables will not diffuse. After the π operation, any two variables are not adjacent in a row, and all outputs of $A^{(1)}$ are linear. To further reduce the diffusion of the variables in χ operation, one can add restricted constraints to the value of constant bits. For example, for the row $\pi^{(0)}_{\{*,0,z\}}$, setting two bit conditions $\pi^{(0)}_{\{1,0,z\}} = 0$ and $\pi^{(0)}_{\{4,0,z\}} = 1$, the other bits except $A^{(1)}_{\{0,0,z\}}$ in $A^{(1)}_{\{*,0,z\}}$ will be constants. Those conditions can be satisfied by modifying the message block and inner part.

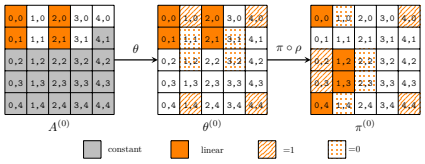


Fig. 7: The linear structure of 1-round Keccak-512

Properties of the Sbox χ. Guo et al. proposed several properties of the Sbox χ, which help to mount preimage attacks on Keccak [35]. Those properties can be also applied in our MitM attack, so we give a brief introduction in the following. Assume $\chi : \mathbb{F}_2^5 \to \mathbb{F}_2^5$ maps $(a_0, a_1, a_2, a_3, a_4)$ to $(b_0, b_1, b_2, b_3, b_4)$ as

$$b_i = a_i \oplus (a_{i+1} \oplus 1) \cdot a_{i+2}, \tag{6}$$

where all indices are modulo 5. The inverse operation χ^{-1} is

$$a_i = b_i \oplus (b_{i+1} \oplus 1) \cdot (b_{i+2} \oplus (b_{i+3} \oplus 1) \cdot b_{i+4}). \tag{7}$$

Property 1. [35] When there are three known consecutive output bits, two linear equations of the input bits can be constructed. E.g., assuming that (b_0, b_1, b_2) are known, two linear equations on (a_0, a_1, a_2, a_3) are constructed as

$$b_0 = a_0 \oplus (b_1 \oplus 1) \cdot a_2, \quad b_1 = a_1 \oplus (b_2 \oplus 1) \cdot a_3. \tag{8}$$

4.2 MILP Model of the MitM Preimage Attack on Keccak

In previous MILP models [5,28] of the MitM attack, each bit can take one of the four colors (■, ■, ■, and □). Generally, the □ bits depending on both ■ and ■,

are unknown and useless in the MitM attack. In our model, bits whose Boolean expression depending on the addition of ■ and ■ (not multiplied) can also be used in our MitM attack, which is known as *the indirect-partial matching technique* [59,2] and ignored by previous automatic models [5,28]. Therefore, we introduce another color, i.e., ■. In our MILP models, there are five colors (■, ■, ■, ■, and □). We first introduce a new efficient encoding scheme of those colors. Applying the *linear structure* technique, we can skip the first round and construct the model from the second round. Then we model the attribute propagation of the five colors over the five operations in each round of `Keccak`. We also model the matching phase with the CP-kernel for `Keccak`. With all above works, we build an automatic MILP model for the MitM preimage attack on `Keccak`.

Encoding Scheme. Since there are 5 colors to encode in the MILP model, the previous 2-bit encoding method [5,28] is not suitable and we introduce a new 3-bit encoding scheme, i.e., each bit is represented by three 0-1 variables $(\omega_0, \omega_1, \omega_2)$:

- `Gray` ■: $(1, 1, 1)$, global constant bits,
- `Red` ■: $(0, 1, 1)$, bits determined by ■ bits and ■ bits of starting state,
- `Blue` ■: $(1, 1, 0)$, bits determined by ■ bits and ■ bits of starting state,
- `Green` ■: $(0, 1, 0)$, bits determined by ■ bits, ■ bits and ■ bits, but the expression does not contain the product of ■ and ■ bits,
- `White` □: $(0, 0, 0)$, bits dependent on the product of ■ and ■ bits.

We set ω_1 to 0 for □ and to 1 for any other color (■, ■, ■, ■). So □ bit can be quickly detected by the value of ω_1. Then we set ω_0 to 1 for (■, ■) and to 0 for other color (■, ■, □). Similarly for ω_2.

Modelling the Starting State with the Linear Structure Technique. In the starting state, each of the 1600 bits takes one color of ■, ■ and ■. We allocate variables $\alpha_{\{x,y,z\}}$ and $\beta_{\{x,y,z\}}$ for the bit with index $\{x, y, z\}$, where $\alpha_{\{x,y,z\}} = 1$ if and only if the bit is ■ and $\beta_{\{x,y,z\}} = 1$ if and only if the bit is ■. Therefore, we can compute the initial DoF by $\lambda_{\mathcal{B}} = \sum \alpha_{\{x,y,z\}}, \lambda_{\mathcal{R}} = \sum \beta_{\{x,y,z\}}$. For `Keccak`, we apply the 1-round restricted linear structure as the example given in Section 4.1. Denote the starting state after XORing message block by $A^{(0)}$. The bits in $A^{(0)}_{\{0,0,z\}}$, $A^{(0)}_{\{0,1,z\}}$, $A^{(0)}_{\{2,0,z\}}$ and $A^{(0)}_{\{2,1,z\}}$ can be colored as ■ or ■. And the remaining bits of $A^{(0)}$ need to be ■. In order to control the diffusion of θ operation, $A^{(0)}_{\{0,0,z\}}$ and $A^{(0)}_{\{0,1,z\}}$ should be the same color and the $A^{(0)}_{\{0,0,z\}} \oplus A^{(0)}_{\{0,1,z\}}$ should be constant, which consumes one degree of freedom. Similarly for $A^{(0)}_{\{2,0,z\}}$ and $A^{(0)}_{\{2,1,z\}}$. Thereafter, the coloring pattern keeps the same over the first θ operation. Then with the conditions set in $\pi^{(0)}$ as introduced in Section 4.1, we can omit the first χ operation and construct the model from $A^{(1)}$ only considering the linear operation $\pi \circ \rho$ from $A^{(0)}$.

Modelling the Attribute Propagation. The round function of `Keccak` consists of five operations θ, ρ, π, χ and ι. The linear operations ρ and π only change the position of each bit of the state. The operation ι can be ignored because it will not change the coloring pattern.

Modelling the θ operation. At first, we give the rule of `XOR` with an arbitrary number of inputs under the new coloring scheme. We name the rule of by `XOR-RULE`, which involves five rules:

1. `XOR-RULE-1`: If the inputs have (0,0,0) □ bit , the output is □.
2. `XOR-RULE-2`: If the inputs are all (1,1,1) ■ bits , the output is ■.
3. `XOR-RULE-3`: If the inputs have (1,1,0) ■ (≥ 1) and ■ (≥ 0) bits, the output will be ■ without consuming DoF, or ■ by consuming one DoF of ■.
4. `XOR-RULE-4`: If the inputs have (0,1,1) ■ (≥ 1) and ■ (≥ 0) bits, the output will be ■ without consuming DoF, or ■ by consuming one DoF of ■.
5. `XOR-RULE-5`: If the inputs have (0,1,0) ■ bits, or have at least two kinds of ■, ■ and ■ bits:
   (a) the output can be ■ without consuming DoF.
   (b) the output can be ■ (or ■) by consuming one DoF of ■ (or ■).
   (c) the output can be ■ by consuming one DoF of ■ and one DoF of ■.

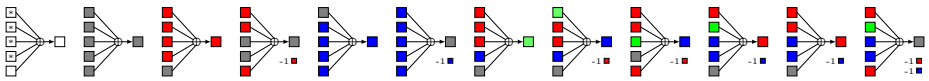


Fig. 8: `5-XOR-RULE` ("*" represents the bit can be any color)

We give some valid coloring patterns of 5 inputs of `XOR`, which are named by `5-XOR-RULE`, as shown in Figure 8. Similar to previous MitM attacks [5], we can use some new variables to identify which rule is applied in different cases. We define three 0-1 variables ν_i ($i \in \{0, 1, 2\}$), where $\nu_0 = 1$ if and only if all the ω_0's of the 5 input bits are 1, similar to the cases $i = 1, 2$. The above five rules can be represented by (ν_0, ν_1, ν_2):

1. $(\nu_0, \nu_1, \nu_2) = (*, 0, *)$, `XOR-RULE-1` is applied.
2. $(\nu_0, \nu_1, \nu_2) = (1, 1, 1)$, `XOR-RULE-2` is applied.
3. $(\nu_0, \nu_1, \nu_2) = (1, 1, 0)$, `XOR-RULE-3` is applied.
4. $(\nu_0, \nu_1, \nu_2) = (0, 1, 1)$, `XOR-RULE-4` is applied.
5. $(\nu_0, \nu_1, \nu_2) = (0, 1, 0)$, `XOR-RULE-5` is applied.

Taking $(\nu_0, \nu_1, \nu_2) = (1, 1, 0)$ as an example. $\nu_1 = 1$ means that all the ω_1's of the input bits are 1 and there is no □ bit. $\nu_0 = 1$ means that there only may have the ■ or ■. $\nu_2 = 1$ means that there must have ■ or ■ or □. Based on the above analysis, we can deduce that when $(\nu_0, \nu_1, \nu_2) = (1, 1, 0)$, there only have ■ and ■ and the number of ■ is greater than or equal to one, where `XOR-RULE-3`

is applied. Denote the output bit as $(\omega_0^O, \omega_1^O, \omega_2^O)$ and the consumed DoF of ■ bits and ■ bits are $(\delta_{\mathcal{R}}, \delta_{\mathcal{B}})$, we can derive

$$\begin{cases} \omega_0^O - \nu_0 \geq 0, & -\omega_0^O + \nu_1 \geq 0, \\ \omega_1^O - \nu_1 = 0, & \\ \omega_2^O - \nu_2 \geq 0, & -\omega_2^O + \nu_1 \geq 0, \end{cases} \qquad \begin{cases} \delta_{\mathcal{R}} - \omega_0^O + \nu_0 = 0, \\ \delta_{\mathcal{B}} - \omega_2^O + \nu_2 = 0. \end{cases} \tag{9}$$

In the θ operation, the expression of the output bit is XORing 11 input bits. If we directly compute the XORing value of the 11 input bits, we may double counting the consumption of DoF. For example, given (x, z), we compute $\theta^{(r)}_{\{x,0,z\}}$ and $\theta^{(r)}_{\{x,1,z\}}$ by $\theta^{(r)}_{\{x,y,z\}} = A^{(r)}_{\{x,y,z\}} \oplus \sum_{y'=0}^{4} (A^{(r)}_{\{x-1,y',z\}} \oplus A^{(r)}_{\{x+1,y',z-1\}})$. If bits in the common formula $\sum_{y'=0}^{4} (A^{(r)}_{\{x-1,y',z\}} \oplus A^{(r)}_{\{x+1,y',z-1\}})$ are only determined by ■ bits, and $A^{(r)}_{\{x,0,z\}}$, $A^{(r)}_{\{x,1,z\}}$ are ■ bits, we can let the summation bit of $\sum_{y'=0}^{4} (A^{(r)}_{\{x-1,y',z\}} \oplus A^{(r)}_{\{x+1,y',z-1\}})$ be ■ by consuming only one DoF of ■. Thereafter, the two output bits $\theta^{(r)}_{\{x,0,z\}}$, $\theta^{(r)}_{\{x,1,z\}}$ will be ■. However, if we directly set the two output bits $\theta^{(r)}_{\{x,0,z\}}$, $\theta^{(r)}_{\{x,1,z\}}$ to be ■ by the `XOR-RULE` individually, we have to consume 2 DoF of ■. To solve this problem, we depose the θ operation to three steps in our model, as described in the following expressions:

$$\begin{aligned} C^{(r)}_{\{x,z\}} &= A^{(r)}_{\{x,0,z\}} \oplus A^{(r)}_{\{x,1,z\}} \oplus A^{(r)}_{\{x,2,z\}} \oplus A^{(r)}_{\{x,3,z\}} \oplus A^{(r)}_{\{x,4,z\}}, \\ D^{(r)}_{\{x,z\}} &= C^{(r)}_{\{x-1,z\}} \oplus C^{(r)}_{\{x+1,z-1\}}, \\ \theta^{(r)}_{\{x,y,z\}} &= A^{(r)}_{\{x,y,z\}} \oplus D^{(r)}_{\{x,z\}}. \end{aligned}$$

At first, we compute the coloring pattern of $C^{(r)}_{\{x,z\}}$. Then, we compute the coloring pattern of $D^{(r)}_{\{x,z\}}$ and compute $\theta^{(r)}_{\{x,y,z\}}$ at last.

Modelling the χ operation. For the χ operation in the round 0, we add conditions to control the diffusion of the ■ or ■ and the first χ is omitted. For the χ operation from round 1, we build the `SBOX-RULE`. The χ operation maps $(a_0, a_1, a_2, a_3, a_4)$ to $(b_0, b_1, b_2, b_3, b_4)$. According to Equ. (6), $b_i = a_i \oplus (a_{i+1} \oplus 1) \cdot a_{i+2}$. Hence, for each output bit b_i, we determine its color by (a_i, a_{i+1}, a_{i+2}):

1. If there are □ bits in (a_i, a_{i+1}, a_{i+2}), the output is □.
2. If there are all ■ bits, the output is ■.
3. If there are only ■ (≥ 1) and ■ (≥ 0) bits, the output will be ■.
4. If there are only ■ (≥ 1) and ■ (≥ 0) bits, the output will be ■.
5. If there are ■, or more than two kinds of ■, ■ and ■ bits in (a_i, a_{i+1}, a_{i+2}):
   (a) if a_{i+1} and a_{i+2} are all ■ (or ■), the output is ■.
   (b) if a_{i+1} or a_{i+2} is ■, the output is ■.
   (c) if a_{i+1} and a_{i+2} are of arbitrarily two kinds of ■, ■, ■, the output is □.

The rules `SBOX-RULE` restrict the coloring pattern of $(a_i, a_{i+1}, a_{i+2}, b_i)$ to the subset of $\mathbb{F}_2^{12}$, which is described by the linear inequalities by using the convex hull computation. Some valid coloring patterns are shown in Figure 9.

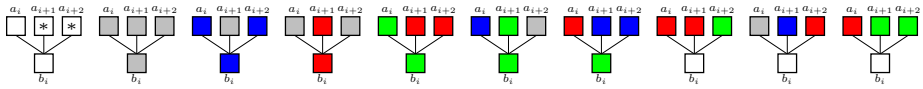


Fig. 9: `SBOX-RULE` for `Keccak` ("*" represents the bit can be any color)

Modelling the Matching Phase. Suppose the first 512 bits of $A^{(r+1)}$ are the hash value. In order to attack more rounds, we try to compute certain bits or relations in $A^{(r)}$ by the hash value in $A^{(r+1)}$, to act as the matching points.

Leaked linear relations of $A^{(r)}$. From the 512-bit hash, we know $A^{(r+1)}_{\{*,0,*\}}$ and the first 3 lanes of $A^{(r+1)}_{\{*,1,*\}}$. From $A^{(r+1)}_{\{*,0,*\}}$, we deduce $\pi^{(r)}_{\{*,0,*\}}$ from Equ. (7). Applying the inverse of the operations ρ and π to $\pi^{(r)}_{\{*,0,*\}}$, we can deduce

$$\theta^{(r)}_{\{x,x,z\}} = \pi^{(r)}_{\{x,0,z+\gamma[x,x]\}}, \forall\ 0 \le x \le 4, 0 \le z \le 63. \tag{10}$$

In addition, according to Equ. (8), two linear equations can be deduced from the first three bits of each row of $A^{(r+1)}_{\{*,1,*\}}$, which are given as

$$\begin{aligned} A^{(r+1)}_{\{0,1,z\}} &= \pi^{(r)}_{\{0,1,z\}} \oplus (A^{(r+1)}_{\{1,1,z\}} \oplus 1) \cdot \pi^{(r)}_{\{2,1,z\}}, \\ A^{(r+1)}_{\{1,1,z\}} &= \pi^{(r)}_{\{1,1,z\}} \oplus (A^{(r+1)}_{\{2,1,z\}} \oplus 1) \cdot \pi^{(r)}_{\{3,1,z\}}. \end{aligned}$$

Then applying the inverse of ρ and π, the linear equations are transformed to

$$\begin{aligned} A^{(r+1)}_{\{0,1,z\}} &= \theta^{(r)}_{\{3,0,z-\gamma[3,0]\}} \oplus (A^{(r+1)}_{\{1,1,z\}} \oplus 1) \cdot \theta^{(r)}_{\{0,2,z-\gamma[0,2]\}}, \\ A^{(r+1)}_{\{1,1,z\}} &= \theta^{(r)}_{\{4,1,z-\gamma[4,1]\}} \oplus (A^{(r+1)}_{\{2,1,z\}} \oplus 1) \cdot \theta^{(r)}_{\{1,3,z-\gamma[1,3]\}}. \end{aligned} \tag{11}$$

With the known value $\theta^{(r)}_{\{x,x,z\}}$ $(0 \le x \le 4, 0 \le z \le 63)$ by (10), we add the same known values (underlined) to both sides of (11), which are

$$\begin{aligned} &A^{(r+1)}_{\{0,1,z\}} \oplus \underline{\theta^{(r)}_{\{3,3,z-\gamma[3,0]\}}} \oplus \underline{(A^{(r+1)}_{\{1,1,z\}} \oplus 1)} \cdot \underline{\theta^{(r)}_{\{0,0,z-\gamma[0,2]\}}} \\ &= \theta^{(r)}_{\{3,0,z-\gamma[3,0]\}} \oplus \underline{\theta^{(r)}_{\{3,3,z-\gamma[3,0]\}}} \oplus \underline{(A^{(r+1)}_{\{1,1,z\}} \oplus 1)} \cdot (\theta^{(r)}_{\{0,2,z-\gamma[0,2]\}} \oplus \underline{\theta^{(r)}_{\{0,0,z-\gamma[0,2]\}}}) \\ &A^{(r+1)}_{\{1,1,z\}} \oplus \underline{\theta^{(r)}_{\{4,4,z-\gamma[4,1]\}}} \oplus \underline{(A^{(r+1)}_{\{2,1,z\}} \oplus 1)} \cdot \underline{\theta^{(r)}_{\{1,1,z-\gamma[1,3]\}}} \\ &= \theta^{(r)}_{\{4,1,z-\gamma[4,1]\}} \oplus \underline{\theta^{(r)}_{\{4,4,z-\gamma[4,1]\}}} \oplus \underline{(A^{(r+1)}_{\{2,1,z\}} \oplus 1)} \cdot (\theta^{(r)}_{\{1,3,z-\gamma[1,3]\}} \oplus \underline{\theta^{(r)}_{\{1,1,z-\gamma[1,3]\}}}). \end{aligned} \tag{12}$$

According to the CP-kernel property [9] of operation θ, we deduce

$$\begin{aligned} \theta^{(r)}_{\{3,0,z-\gamma[3,0]\}} \oplus \theta^{(r)}_{\{3,3,z-\gamma[3,0]\}} &= A^{(r)}_{\{3,0,z-\gamma[3,0]\}} \oplus A^{(r)}_{\{3,3,z-\gamma[3,0]\}}, \\ \theta^{(r)}_{\{0,2,z-\gamma[0,2]\}} \oplus \theta^{(r)}_{\{0,0,z-\gamma[0,2]\}} &= A^{(r)}_{\{0,2,z-\gamma[0,2]\}} \oplus A^{(r)}_{\{0,0,z-\gamma[0,2]\}}, \\ \theta^{(r)}_{\{4,1,z-\gamma[4,1]\}} \oplus \theta^{(r)}_{\{4,4,z-\gamma[4,1]\}} &= A^{(r)}_{\{4,1,z-\gamma[4,1]\}} \oplus A^{(r)}_{\{4,4,z-\gamma[4,1]\}}, \\ \theta^{(r)}_{\{1,3,z-\gamma[1,3]\}} \oplus \theta^{(r)}_{\{1,1,z-\gamma[1,3]\}} &= A^{(r)}_{\{1,3,z-\gamma[1,3]\}} \oplus A^{(r)}_{\{1,1,z-\gamma[1,3]\}}. \end{aligned} \tag{13}$$

Combining (12) and (13), there are linear relations on bits in $A^{(r)}$ with the known hash value of $A^{(r+1)}$, which will be used in our matching points:

$$\begin{aligned} A^{(r)}_{\{3,0,z-\gamma[3,0]\}} \oplus A^{(r)}_{\{3,3,z-\gamma[3,0]\}} \oplus (A^{(r+1)}_{\{1,1,z\}} \oplus 1) \cdot (A^{(r)}_{\{0,2,z-\gamma[0,2]\}} \oplus A^{(r)}_{\{0,0,z-\gamma[0,2]\}}) \\ = A^{(r+1)}_{\{0,1,z\}} \oplus \theta^{(r)}_{\{3,3,z-\gamma[3,0]\}} \oplus (A^{(r+1)}_{\{1,1,z\}} \oplus 1) \cdot \theta^{(r)}_{\{0,0,z-\gamma[0,2]\}}, \end{aligned} \tag{14}$$

$$\begin{aligned} A^{(r)}_{\{4,1,z-\gamma[4,1]\}} \oplus A^{(r)}_{\{4,4,z-\gamma[4,1]\}} \oplus (A^{(r+1)}_{\{2,1,z\}} \oplus 1) \cdot (A^{(r)}_{\{1,3,z-\gamma[1,3]\}} \oplus A^{(r)}_{\{1,1,z-\gamma[1,3]\}}) \\ = A^{(r+1)}_{\{1,1,z\}} \oplus \theta^{(r)}_{\{4,4,z-\gamma[4,1]\}} \oplus (A^{(r+1)}_{\{2,1,z\}} \oplus 1) \cdot \theta^{(r)}_{\{1,1,z-\gamma[1,3]\}}. \end{aligned} \tag{15}$$

Observation 1 (Conditions in Matching Points of `Keccak`) *In (14), if four bits* $(A^{(r)}_{\{3,0,z-\gamma[3,0]\}}, A^{(r)}_{\{3,3,z-\gamma[3,0]\}}, A^{(r)}_{\{0,2,z-\gamma[0,2]\}}, A^{(r)}_{\{0,0,z-\gamma[0,2]\}})$ *in* $A^{(r)}$ *satisfy the following two conditions, there is a 1-bit filter:*

(1) There has no □ *in* $(A^{(r)}_{\{3,0,z-\gamma[3,0]\}}, A^{(r)}_{\{3,3,z-\gamma[3,0]\}}, A^{(r)}_{\{0,2,z-\gamma[0,2]\}}, A^{(r)}_{\{0,0,z-\gamma[0,2]\}})$.

(2) $(A^{(r)}_{\{3,0,z-\gamma[3,0]\}}, A^{(r)}_{\{3,3,z-\gamma[3,0]\}})$ *is of* (■,■), (■,■), (■,■), (■,■), *or* (■,■), *or opposite order .*

We introduce a binary variable $\delta_{\mathcal{M}}$ to represent whether there is a filtering. Similarly to the `XOR-RULE`, we add three 0-1 variables ν_i ($i \in \{0,1,2\}$), where $\nu_i = 1$ ($i = 0, 2$) if and only if all ω_i's of $(A^{(r)}_{\{3,0,z-\gamma[3,0]\}}, A^{(r)}_{\{3,3,z-\gamma[3,0]\}})$ are 1, and $\nu_1 = 1$ if and only if all ω_1's of $(A^{(r)}_{\{3,0,z-\gamma[3,0]\}}, A^{(r)}_{\{3,3,z-\gamma[3,0]\}}, A^{(r)}_{\{0,2,z-\gamma[0,2]\}}, A^{(r)}_{\{0,0,z-\gamma[0,2]\}})$ are 1. We can derive

$$\begin{cases} \nu_1 - \delta_{\mathcal{M}} \geq 0, \ -\nu_0 - \delta_{\mathcal{M}} + 1 \geq 0, \ -\nu_2 - \delta_{\mathcal{M}} + 1 \geq 0, \\ \nu_0 - \nu_1 + \nu_2 + \delta_{\mathcal{M}} \geq 0. \end{cases}$$

The Objective Function. Let $l_{\mathcal{R}}$, and $l_{\mathcal{B}}$ be the accumulated consumption of DoF of ■ and ■, i.e., $l_{\mathcal{R}} = \sum \delta_{\mathcal{R}}$ and $l_{\mathcal{B}} = \sum \delta_{\mathcal{B}}$. Therefore, we can get $\mathrm{DoF}_{\mathcal{R}} = \lambda_{\mathcal{R}} - l_{\mathcal{R}}$, $\mathrm{DoF}_{\mathcal{B}} = \lambda_{\mathcal{B}} - l_{\mathcal{B}}$. We also have $\mathrm{DoM} = \sum \delta_{\mathcal{M}}$. According to the time complexity given by Equ. (1), we need to maximize the value of $\min\{\mathrm{DoF}_{\mathcal{R}}, \mathrm{DoF}_{\mathcal{B}}, \mathrm{DoM}\}$ to find the optimal attacks. We introduce an auxiliary variable v_{obj}, impose the following constraints, and maximize v_{obj},

$$\{v_{obj} \leq \mathrm{DoF}_{\mathcal{R}}, v_{obj} \leq \mathrm{DoF}_{\mathcal{B}}, v_{obj} \leq \mathrm{DoM}\}. \tag{16}$$

4.3 MitM Preimage Attack on 4-Round `Keccak-512`

We follow the framework in Fig. 6 to perform the attack with (M_1, M_2) and place the MitM attack at M_2. We construct an MILP model for `Keccak-512` following Sect. 4.2. The code is in https://github.com/qly14/MITM-Preimage-Attack.git. By solving with our MILP model, we mount a 4-round MitM preimage attack on `Keccak-512`, as in Figure 10 and figures in the Supplementary Material B in our full version paper [55], which contains 3 additional symbols:

- ■, ■: there consumes one degree of freedom of ■ to let the bit be ■ or ■.
- ■: there consumes one degree of freedom of ■ to let the bit be ■.
- ■: ■ bits used for matching.

Conditions in the Linear Structure. State $A^{(0)}$ contains 16 ■ bits and 216 ■ bits. We take the similar strategy with [48] to get 1-round linear structure of `Keccak`. We introduce 116 binary variables $v = \{v_0, v_1, \cdots, v_{115}\}$ and 116 binary variables $c = \{c_0, c_1, \cdots, c_{115}\}$. Those variables v_i's and c_i's are placed at the $16 + 216 = 232$ ■ and ■ bits in $A^{(0)}$ as in Figure 10.For example, we set $A^{(0)}_{\{0,0,0\}} = v_0$ and $A^{(0)}_{\{0,1,0\}} = v_0 \oplus c_0$. When we choose $c \in \mathbb{F}_2^{116}$ to be arbitrary constant, the θ operation will act as identity with regard to the ■ and ■ bits in $A^{(0)}$. To reduce the diffusion of χ in round 0, we need to set some bits conditions in $\pi^{(0)}$ to be constants. According to (6), if a_{i+1} is ■ or ■, we need to set $a_{i+2} = 0$ and $a_i = 1$. So there are totally $232 \times 2 = 464$ conditions on $\pi^{(0)}$. After the inverse of $\rho \circ \pi$, we determine the positions of bit conditions in $\theta^{(0)}$. Taking the state $\theta^{(0)}_{\{*,*,1\}}$ as an example, there are six bit conditions as

$$\theta^{(0)}_{\{1,0,1\}} = 1,\ \theta^{(0)}_{\{1,2,1\}} = 0,\ \theta^{(0)}_{\{1,4,1\}} = 1, \theta^{(0)}_{\{3,1,1\}} = 0,\ \theta^{(0)}_{\{3,2,1\}} = 0,\ \theta^{(0)}_{\{4,4,1\}} = 1. \tag{17}$$

Above conditions can be converted to those on $A^{(0)}$. The bits in $A^{(0)}$ can be divided into two parts: the bits determined by the outer part (i.e., they can be modified by directly changing the absorbed M_2), and the bits determined by inner part. The equations in Equ. (17) can be transformed to

$$\begin{aligned}
&\boldsymbol{A^{(0)}_{\{1,0,1\}}} \oplus \boldsymbol{c_2} \oplus A^{(0)}_{\{0,2,1\}} \oplus A^{(0)}_{\{0,3,1\}} \oplus A^{(0)}_{\{0,4,1\}} \oplus \boldsymbol{c_1} \oplus A^{(0)}_{\{2,2,0\}} \oplus A^{(0)}_{\{2,3,0\}} \oplus A^{(0)}_{\{2,4,0\}} = 1,\\
&A^{(0)}_{\{1,2,1\}} \oplus \boldsymbol{c_2} \oplus A^{(0)}_{\{0,2,1\}} \oplus A^{(0)}_{\{0,3,1\}} \oplus A^{(0)}_{\{0,4,1\}} \oplus \boldsymbol{c_1} \oplus A^{(0)}_{\{2,2,0\}} \oplus A^{(0)}_{\{2,3,0\}} \oplus A^{(0)}_{\{2,4,0\}} = 0,\\
&A^{(0)}_{\{1,4,1\}} \oplus \boldsymbol{c_2} \oplus A^{(0)}_{\{0,2,1\}} \oplus A^{(0)}_{\{0,3,1\}} \oplus A^{(0)}_{\{0,4,1\}} \oplus \boldsymbol{c_1} \oplus A^{(0)}_{\{2,2,0\}} \oplus A^{(0)}_{\{2,3,0\}} \oplus A^{(0)}_{\{2,4,0\}} = 1,\\
&\boldsymbol{A^{(0)}_{\{3,1,1\}}} \oplus \boldsymbol{c_3} \oplus A^{(0)}_{\{2,2,1\}} \oplus A^{(0)}_{\{2,3,1\}} \oplus A^{(0)}_{\{2,4,1\}} \oplus \boldsymbol{A^{(0)}_{\{4,0,0\}}} \oplus A^{(0)}_{\{4,1,0\}} \oplus A^{(0)}_{\{4,2,0\}} \oplus A^{(0)}_{\{4,3,0\}} \oplus A^{(0)}_{\{4,4,0\}} = 0,\\
&A^{(0)}_{\{3,2,1\}} \oplus \boldsymbol{c_3} \oplus A^{(0)}_{\{2,2,1\}} \oplus A^{(0)}_{\{2,3,1\}} \oplus A^{(0)}_{\{2,4,1\}} \oplus \boldsymbol{A^{(0)}_{\{4,0,0\}}} \oplus A^{(0)}_{\{4,1,0\}} \oplus A^{(0)}_{\{4,2,0\}} \oplus A^{(0)}_{\{4,3,0\}} \oplus A^{(0)}_{\{4,4,0\}} = 0,\\
&A^{(0)}_{\{4,4,1\}} \oplus \boldsymbol{A^{(0)}_{\{3,0,1\}}} \oplus \boldsymbol{A^{(0)}_{\{3,1,1\}}} \oplus A^{(0)}_{\{3,2,1\}} \oplus A^{(0)}_{\{3,3,1\}} \oplus A^{(0)}_{\{3,4,1\}} \oplus \boldsymbol{c_0} \oplus A^{(0)}_{\{0,2,0\}} \oplus A^{(0)}_{\{0,3,0\}} \oplus A^{(0)}_{\{0,4,0\}} = 1,
\end{aligned} \tag{18}$$

where $\boldsymbol{c_0} = \boldsymbol{A^{(0)}_{\{0,0,0\}}} \oplus \boldsymbol{A^{(0)}_{\{0,1,0\}}}, \boldsymbol{c_1} = \boldsymbol{A^{(0)}_{\{2,0,0\}}} \oplus \boldsymbol{A^{(0)}_{\{2,1,0\}}}, \boldsymbol{c_2} = \boldsymbol{A^{(0)}_{\{0,0,1\}}} \oplus \boldsymbol{A^{(0)}_{\{0,1,1\}}}, \boldsymbol{c_3} = \boldsymbol{A^{(0)}_{\{2,0,1\}}} \oplus \boldsymbol{A^{(0)}_{\{2,1,1\}}}$. Given an inner part, the 464 conditions on state $A^{(0)}$ will be a linear system of the $576 - 116 = 460$ variables of M_2 (marked by bold). We compute the rank of the coefficient matrix of the linear system is 250. In other words, through some linear transformations, there are 214 equations out of the total 464 only determined by the bits of inner part. For example, combining the 2nd and 3rd equations in Equ. (18), we can deduce an equation between two inner part bits $A^{(0)}_{\{1,2,1\}} \oplus A^{(0)}_{\{1,4,1\}} = 1$. We have to randomly test 2^{214} M_1 to compute the inner part satisfying the 214 equations. Then for a right inner part, there are $2^{460-250} = 2^{210}$ solutions of M_2, which make all the 464 equations hold. For each solution of M_2, the ■ and $c \in \mathbb{F}_2^{116}$ in the outer part will be fixed. Then with the fixed inner part, we can conduct the MitM episodes to filter states.

Consumed Degrees of Freedom. As shown in Figure 10, after adding 464 conditions and consuming 108 ■ and 8 ■ degrees of freedom in round 0, we can derive the coloring pattern in $A^{(1)}$. The remaining degrees of freedom for ■ and ■ are 108 and 8, respectively. We give two examples to explain the consumption of degrees of freedom in the computation from $A^{(1)}$ to $A^{(3)}$.

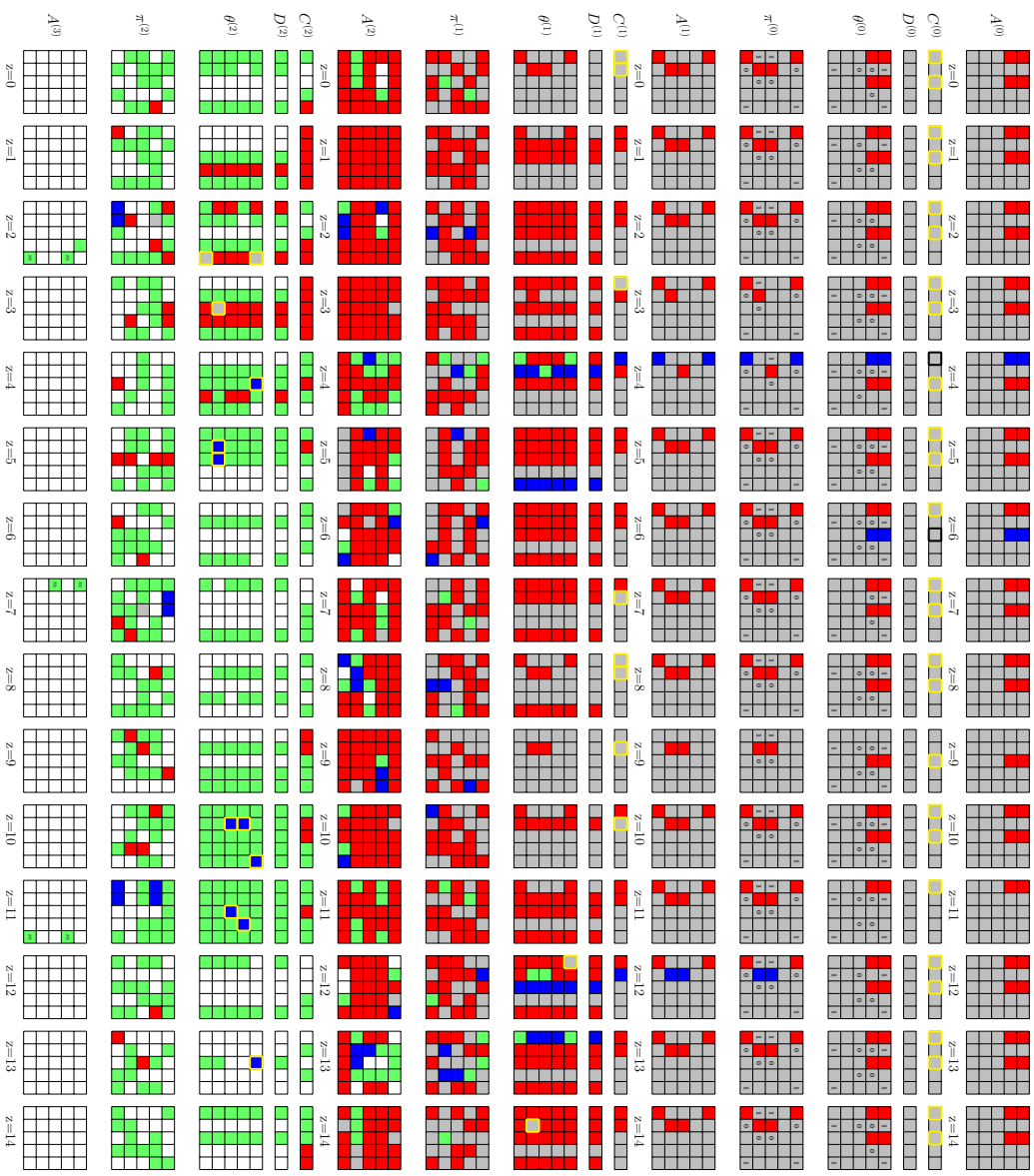


Fig. 10: The MitM preimage attack on 4-round Keccak-512 (part I)

1. For $\theta^{(1)}_{\{0,0,12\}}$ marked by ■ in Figure 11 (part of Figure 10), we set an equation of ■ to a constant, which means consuming one DoF of ■ to let $\theta^{(1)}_{\{0,0,12\}}$ be ■, as listed below:

$$\begin{aligned} &A^{(1)}_{\{0,0,12\}} \oplus A^{(1)}_{\{4,0,12\}} \oplus A^{(1)}_{\{4,1,12\}} \oplus A^{(1)}_{\{4,2,12\}} \oplus A^{(1)}_{\{4,3,12\}} \oplus A^{(1)}_{\{4,4,12\}} \\ &\oplus A^{(1)}_{\{1,0,11\}} \oplus A^{(1)}_{\{1,1,11\}} \oplus A^{(1)}_{\{1,2,11\}} \oplus A^{(1)}_{\{1,3,11\}} \oplus A^{(1)}_{\{1,4,11\}} = const., \end{aligned} \tag{19}$$

   where the bits marked by red are ■ and others marked by black are ■.
2. For $\theta^{(2)}_{\{1,3,5\}}$ marked by ■ in Figure 12, we set an equation of ■ to a constant, which means consuming one DoF of ■ to let $\theta^{(2)}_{\{1,3,5\}}$ be ■. Since there have

$$\begin{aligned} \theta^{(2)}_{\{1,3,5\}} &= A^{(2)}_{\{1,3,5\}} \oplus D^{(2)}_{\{1,5\}} = A^{(2)}_{\{1,3,5\}} \oplus C^{(2)}_{\{0,5\}} \oplus C^{(2)}_{\{2,4\}}, \\ C^{(2)}_{\{0,5\}} &= A^{(2)}_{\{0,0,5\}} \oplus A^{(2)}_{\{0,1,5\}} \oplus A^{(2)}_{\{0,2,5\}} \oplus A^{(2)}_{\{0,3,5\}} \oplus A^{(2)}_{\{0,4,5\}}, \\ C^{(2)}_{\{2,4\}} &= A^{(2)}_{\{2,0,4\}} \oplus A^{(2)}_{\{2,1,4\}} \oplus A^{(2)}_{\{2,2,4\}} \oplus A^{(2)}_{\{2,3,4\}} \oplus A^{(2)}_{\{2,4,4\}}. \end{aligned}$$

   We can set all the ■ involved to be constant:

$$\begin{aligned} &A^{(2)}_{\{1,3,5\}} \oplus A^{(2)}_{\{0,0,5\}} \oplus A^{(2)}_{\{0,1,5\}} \oplus A^{(2)}_{\{0,3,5\}} \oplus A^{(2)}_{\{0,4,5\}} \\ &A^{(2)}_{\{2,0,4\}} \oplus A^{(2)}_{\{2,1,4\}} \oplus A^{(2)}_{\{2,2,4\}} \oplus A^{(2)}_{\{2,3,4\}} \oplus A^{(2)}_{\{2,4,4\}} = const. \end{aligned} \tag{20}$$

   Then, we have $\theta^{(2)}_{\{1,3,5\}} = A^{(2)}_{\{0,2,5\}} \oplus const.$

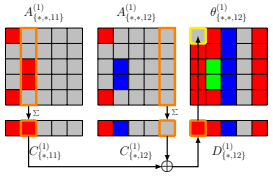


Fig. 11: Example (1) of consumed DoF

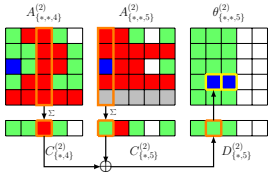


Fig. 12: Example (2) of consumed DoF

There are totally 100 ■ and ■ in internal states between $A^{(1)}$ and $\theta^{(2)}$, which means that the accumulated consumed degree of freedom of ■ is 100. Denote the 100-bit constants (i.e. constants such as (19) and (20)) as $c_{\mathcal{R}} \in \mathbb{F}_2^{100}$. At last, the numbers of remaining degrees of freedom for ■ and ■ are both 8 bits.

Matching Strategy with Green Bits. According to Obs. 1, we can use $A^{(3)}$ to count the number of matching equations, i.e. $m = 8$. We give an example matching Equ. (21) (marked by m in $A^{(3)}$ with $z = 2$ and $z = 41$ in Figure 10) according to Equ. (15), which satisfies the matching conditions of Obs. 1:

$$\begin{aligned} &A^{(3)}_{\{4,1,2\}} \oplus A^{(3)}_{\{4,4,2\}} \oplus (A^{(4)}_{\{2,1,22\}} \oplus 1) \cdot (A^{(3)}_{\{1,3,41\}} \oplus A^{(3)}_{\{1,1,41\}}) \\ &\oplus A^{(4)}_{\{1,1,22\}} \oplus \theta^{(3)}_{\{4,4,2\}} \oplus (A^{(4)}_{\{2,1,22\}} \oplus 1) \cdot \theta^{(3)}_{\{1,1,41\}} = 0. \end{aligned} \tag{21}$$

With known bits in $A^{(4)}$ and $\theta^{(3)}$, the left part of Equ. (21) can be written as Boolean expression (denoted as $f_{\mathcal{M}}$) on ■, ■ and ■ bits of the starting state. Therefore, denote $f_{\mathcal{M}} = f_{\mathcal{R}} \oplus f_{\mathcal{B}} \oplus f_{\mathcal{G}}$, where $f_{\mathcal{R}}$ only contains monomials on ■/■ bits, $f_{\mathcal{B}}$ contains monomials on ■/■ bits, and $f_{\mathcal{G}}$ contains monomials on ■ bits and constants. With the given ■ bits, we can compute the value $f_{\mathcal{R}} \oplus f_{\mathcal{G}} = f'_{\mathcal{M}}$ with forward computing `Keccak` permutation by setting all the ■ bits as 0. Similarly, we get $f_{\mathcal{B}} \oplus f_{\mathcal{G}} = f''_{\mathcal{M}}$ by setting ■ bits as 0. By setting all ■ and ■ bits as 0, we get $f_{\mathcal{G}} = f'''_{\mathcal{M}}$. Therefore, we compute $f_{\mathcal{R}} = f'_{\mathcal{M}} \oplus f'''_{\mathcal{M}}$ and $f_{\mathcal{B}} = f''_{\mathcal{M}} \oplus f'''_{\mathcal{M}}$. Then, we can derive the matching equation from Equ. (21) as $f_{\mathcal{R}} = f_{\mathcal{B}} \oplus f_{\mathcal{G}}$.

MitM Attack on 4-round `Keccak-512`. In our attack Algorithm 1, we first precompute inversely from $A^{(4)}$ to $A^{(3)}$, and derive 128 Boolean equations similar with Equ. (21). Among them, 8 equations act as the matching points in the MitM phase, and the other 120 equations are used to further filter the partial matched states.

Following the framework in Figure 6, we use two message blocks (M_1, M_2) to build the attack as Algorithm 1. In Line 5, once we find solutions of M_2, we can perform 2^{100} MitM episodes in Line 14 to 25 for each of the 2^{210} solutions. For each MitM episode, $2^8 \times 2^8$ internal states are exhausted. Suppose there needs 2^x possible values of M_1. To find a 512-bit target preimage, we need $2^{x-214+210+100+16} = 2^{512}$, i.e., $x = 400$. The steps of Alg. 1 are analyzed below:

- In Line 4, the time complexity is 2^{400} 4-round `Keccak` and $2^{400} \times 464^3$ bit operations to solve the linear system (the time to solve a system of n linear equations is about $O(n^3)$).
- In Line 9, the time complexity is $2^{400-214+210} \times \frac{1}{4} = 2^{394}$ 4-round `Keccak`.
- We describe the way to use Equ. (21) as the matching point. In the concrete `Keccak` attack, we can not set the 108 ■ bits to be 0 when computing $f_{\mathcal{B}} + f_{\mathcal{G}} = f''_{\mathcal{M}}$ and $f_{\mathcal{G}} = f'''_{\mathcal{M}}$. This is because, the actual size of the ■ and ■ neutral sets is both 2^8, not 2^{108}. The 100 consumed DoF of ■ bits of $A^{(1)}$ are used to make 100 internal bits (denoted as $c_{\mathcal{R}} \in \mathbb{F}_2^{100}$) to be ■/■, so the remaining ■ set of size 2^8 can be computed independently to the ■ set.
  We detail the method to derive similar matching equation like "$f_{\mathcal{R}} = f_{\mathcal{B}} \oplus f_{\mathcal{G}}$" or "$f_{\mathcal{B}} = f_{\mathcal{R}} \oplus f_{\mathcal{G}}$" for the two 2^8 ■/■ sets. With fixed ■ bits in $A^{(1)}$ and $c_{\mathcal{R}} \in \mathbb{F}_2^{100}$, there are 2^8 ■ bits stored in $U[c_{\mathcal{R}}]$, which is derived in Line 10 of Algorithm 1 following Dong et al.'s method [28]. Setting ■ in $A^{(1)}$ as 0, for each element of $U[c_{\mathcal{R}}]$, compute forward to the 8 matching equations (e.g. Equ. (21)) to get 8 $f'_{\mathcal{M}} = f_{\mathcal{R}} \oplus f_{\mathcal{G}}$. Randomly pick an element e of $U[c_{\mathcal{R}}]$ and set ■ in $A^{(1)}$ as 0, to compute the 8 matching equations $f'''_{\mathcal{M}} = f_{\mathcal{G}} + Const(e)$, where $Const(e)$ is determined by e. That is, for each of the 2^8 ■, compute $f''_{\mathcal{M}} = f_{\mathcal{B}} + f_{\mathcal{G}} + Const(e)$ by setting the 108 ■ $A^{(1)}$ bits as e. Therefore, we get $f''_{\mathcal{M}} + f'''_{\mathcal{M}} = f_{\mathcal{B}} = f_{\mathcal{R}} \oplus f_{\mathcal{G}} = f'_{\mathcal{M}}$ as filter. To dive into details, we refer the readers to Line 10 to Line 25. In Line 10, the time complexity is $2^{396+108} \times \frac{2}{4} = 2^{503}$ 4-round `Keccak`, since only two rounds from $A^{(1)}$ to $A^{(3)}$ are needed to compute to derive the ■/■ bits and the matching points.
- In Line 13, the time complexity is $2^{396+100} \times \frac{2}{4} = 2^{495}$ 4-round `Keccak`.

Algorithm 1: Preimage Attack on 4-round `Keccak-512`

Precompute inversely from the target to $A^{(3)}$, and derive 128 Boolean equations of similar form with Equ. (21)
`/* Among them, 8 Boolean equations act as the matching points in the MitM phase. The other 120 Boolean equations are used to further filter the partial matched states. */`
for 2^x *values of* M_1 **do**
  Compute the inner part of the 2nd block and solve the system of 464 linear equations
  if *the equations have solutions* `/* with probability of` 2^{-214} `*/`
  then
    for *each of the* 2^{210} *solutions of* M_2 **do**
      `/* With` $x = 400$`, there are` $2^{400-214+210} = 2^{396}$ `iterations */`
      Compute the ■ bits in $A^{(1)}$
      Traversing the 2^{108} values of ■ in $A^{(1)}$ while fixing ■ as 0, compute forward to determine 100-bit ■/■ bits (denoted as $c_{\mathcal{R}} \in \mathbb{F}_2^{100}$), and the 8-bit matching point, e.g., in (21), i.e., compute eight $f'_{\mathcal{M}} = f_{\mathcal{R}} \oplus f_{\mathcal{G}}$. Build the table U and store the 108-bit ■ bits of $A^{(1)}$ as well as the 8-bit matching point in $U[c_{\mathcal{R}}]$.
      `/* This method to solve the nonlinear constrained neutral words is borrowed from Dong et al. [28]. */`
      for $c_{\mathcal{R}} \in \mathbb{F}_2^{100}$ **do**
        Randomly pick a 108-bit ■ $e \in U[c_{\mathcal{R}}]$, and set ■ in $A^{(1)}$ as 0, compute to the matching point to get eight $f'''_{\mathcal{M}} = f_{\mathcal{G}} + Const(e)$
        for 2^8 *values in* $U[c_{\mathcal{R}}]$ **do**
          Restore the values of ■ of $A^{(1)}$ and the corresponding matching point (i.e., eight $f_{\mathcal{R}} \oplus f_{\mathcal{G}} = f'_{\mathcal{M}}$) in a list L_1 (indexed by matching point)
        end
        for 2^8 *values of* ■ **do**
          Set the 108-bit ■ in $A^{(1)}$ as e. Compute to the matching point to get eight $f''_{\mathcal{M}} = f_{\mathcal{B}} + f_{\mathcal{G}} + Const(e)$. Together with $f'''_{\mathcal{M}}$, compute $f_{\mathcal{B}} = f''_{\mathcal{M}} + f'''_{\mathcal{M}}$ and store ■ in L_2 indexed by matching point.
        end
        for *values matched between* L_1 *and* L_2 **do**
          Compute $A^{(3)}$ from the matched ■ and ■ bits
          if $A^{(3)}$ *satisfy the 120 precomputed Boolean equations* `/* Probability of` 2^{-120} `*/`
          then
            if *it leads to the given hash value* **then**
              Output the preimage
            end
          end
        end
      end
    end
  end
end

- In Line 15, this step is just to retrieve $U[c_{\mathcal{R}}]$ to restore it in L_1 with matching point as index. Suppose one access to the table is equivalent to one Sbox application. The time complexity is $2^{396+100+8} \times \frac{1}{4\times 320} = 2^{493.36}$ 4-round `Keccak`, since there are 4×320 Sboxes for 4-round `Keccak`.
- In Line 18, the time complexity is $2^{396+100+8} \times \frac{2}{4} = 2^{503}$ 4-round `Keccak`.
- In Line 21, the time is $2^{396+100+8+8-8} \times \frac{2}{4} = 2^{503}$ 4-round `Keccak`.
- In Line 22, $A^{(3)}$ is checked against the 120 Boolean equations precomputed in Line 1, which acts as a filter of 2^{-120}. After the filter, the time of the final check against the target h is $2^{396+100+8-120} = 2^{384}$ 4-round `Keccak`.

The total complexity is $2^{400} + 2^{400} \times 464^3 + 2^{394} + 2^{503} + 2^{495} + 2^{493.36} + 2^{503} + 2^{503} + 2^{384} \approx 2^{504.58}$ 4-round `Keccak`. The memory to store U is 2^{108}. We also give an experiment of 3-round `Keccak-512` in the Supplementary Material C in our full version paper [55].

Remark on padding rule. The last message block has at least 2-bit padding (i.e., '11') for `Keccak` and 4-bit padding (i.e., '0111') for SHA3. Therefore, we have $x = 402$ for `Keccak`-512 and $x = 404$ for SHA3-512. However, it only increases the negligible part $2^{400} \times 464^3$ in Line 4 to $2^{402} \times 464^3$ for `Keccak`-512 and $2^{404} \times 464^3$ for SHA3-512. Therefore the final time complexity is still $2^{504.58}$ 4-round `Keccak`-512 or SHA3-512 considering the padding. The memory is 2^{108}.

5 MitM Preimage Attack on `Xoodyak-XOF`

In this section, we list the differences in the MILP model with the model for `Keccak` in Section 4.2, and give an MitM preimage attack on 3-round `Xoodyak-XOF`.

5.1 MILP Model of the MitM Preimage Attack on `Xoodyak-XOF`

For `Xoodyak`, without the help of the *linear structure* technique, the situations of adding conditions to the state before the Sbox χ are more complex. We give the details of the condition rules and the matching rule for `Xoodyak` in the following.

Modelling the χ operation with conditions in Round 0. The Sbox χ of `Xoodyak` is different from that of `Keccak` at the sizes of inputs and outputs, which acts on a column $A^{(r)}_{\{x,*,z\}}$. Assume χ operation maps $(a_0, a_1, a_2) \in \mathbb{F}_2^3$ in to $(b_0, b_1, b_2) \in \mathbb{F}_2^3$ as $b_i = a_i \oplus (a_{i+1} \oplus 1) \cdot a_{i+2}$. In round 0, all the operations between the starting state $A^{(0)}$ and χ are linear, thus the inputs only have ■, ■, ■ and ■. We can add conditions on ■ bits to control the diffusion. The different rules from the `SBOX-RULE` for `Keccak` are listed below:

1. If there are two ■ bits and one ■/■/■ bit in (a_0, a_1, a_2), we can add one or two conditions to make one or two outputs to be ■. Without losing generality, suppose a_i is ■ or ■ or ■ and both a_{i+1} and a_{i+2} are ■ bits:
   (a) the color of b_i is always same with a_i.

(b) $a_{i+2}=1$, b_{i+1} will be ■; otherwise, the color of b_{i+1} will be same with a_i.
(c) $a_{i+1}=0$, b_{i+2} will be ■; otherwise, the color of b_{i+2} will be same with a_i.

2. If there is only one ■ bit and the other two are among (■, ■)/(■, ■)/(■, ■), we add conditions to reduce the number of ■ in the output. If a_i is ■:
(a) b_i is always be □.
(b) $a_i=0$, the color of b_{i+1} will be same with a_{i+1} and b_{i+2} will be ■.
(c) $a_i=1$, b_{i+1} will be ■ and the color of b_{i+2} will be same with a_{i+2}.
(d) without conditions on a_i, both b_{i+1} and b_{i+2} will be ■.

The rules can be described by a system of linear inequalities by using the convex hull computation. Some valid coloring patterns are shown in Figure 13.

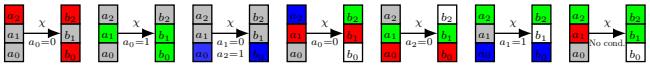


Fig. 13: Some valid coloring patterns with conditions for χ

Modelling the Matching Phase. Suppose the 128 bits in the top plane $A^{(r+1)}_{\{*,2,*\}}$ of $A^{(r+1)}$ is the hash value. We can easily compute the top plane of state $\chi^{(r)}_{\{*,2,*\}}$ by the inverse of ρ_{east}. Each bit of $\chi^{(r)}_{\{*,2,*\}}$ is computed by $b_2 = a_2 \oplus (a_0 \oplus 1) \cdot a_1$, where (a_0, a_1, a_2) comes from the input column of each Sbox in $\iota^{(r)}$. Hence, we deduce the deterministic relations of $\iota^{(r)}$ to count the DoMs.

Observation 2 (Conditions in Matching Points of `Xoodyak`) *If (a_0, a_1, a_2) satisfy the following conditions, we say there is a 1-bit matching:*

1. *There is no □ bit in (a_0, a_1, a_2).*
2. *There is no the product of ■ and ■, concretely, (a_0, a_1) should not be (■, ■) or (■, ■) or (■, ■) or (■, ■), or opposite order.*
3. *In fact, (a_0, a_1, a_2) should be (■, ∗, ■) or (∗, ■, ■) or (■, ■, ■) or (■, ■, ■) or (■, ■, ■) or (■, ■, ■), where '∗' is ■ or ■ or ■ or ■. We exclude several cases such as (■, ■, ■), since it is a filter if ■ = 1, but not for ■ = 0.*

5.2 MitM Preimage Attack on 3-round `Xoodyak-XOF`

We also follow the framework in Figure 6 and perform the attack with two message blocks (M_1, M_2), where M_2 has two padding bits '10'. The MitM attack is placed in the 2nd block. Solving with our MILP model for `Xoodyak`, we get a 3-round MitM preimage attack, shown in the Supplementary Material D in our full version paper [55]. The starting state $A^{(0)}$ contains 4 ■ bits and 97 ■ bits. There are totally 53 conditions on ■ bits of $\iota^{(0)}$ (see Supplementary Material D in our full version paper [55]). In the computation from $A^{(0)}$ to $\iota^{(2)}$, the accumulated consumed degree of freedom of ■ is 93 and there is no DoF of ■ consumed. Therefore, $\text{DoF}_{\mathcal{B}} = 4$, $\text{DoF}_{\mathcal{R}} = 97 - 93 = 4$. The degree of matching is counted by the deterministic relations of $\iota^{(2)}$ according to Obs. 2, we get $\text{DoM} = 4$, where

$$\chi^{(2)}_{\{2,2,4\}} = \iota^{(2)}_{\{2,2,4\}} \oplus (\iota^{(2)}_{\{2,0,4\}} \oplus 1) \cdot \iota^{(2)}_{\{2,1,4\}},\ \chi^{(2)}_{\{1,2,10\}} = \iota^{(2)}_{\{1,2,10\}} \oplus (\iota^{(2)}_{\{1,0,10\}} \oplus 1) \cdot \iota^{(2)}_{\{1,1,10\}},$$
$$\chi^{(2)}_{\{1,2,19\}} = \iota^{(2)}_{\{1,2,19\}} \oplus (\iota^{(2)}_{\{1,0,19\}} \oplus 1) \cdot \iota^{(2)}_{\{1,1,19\}},\ \chi^{(2)}_{\{2,2,22\}} = \iota^{(2)}_{\{2,2,22\}} \oplus (\iota^{(2)}_{\{2,0,22\}} \oplus 1) \cdot \iota^{(2)}_{\{2,1,22\}}. \tag{22}$$

In the starting state $A^{(0)}$, there are 25 ■ bits which can be modified by changing M_2. The 53 conditions can form a linear system taking the 25 bits of M_2 as variables and the 256 bits inner part as constants. The rank of the coefficient matrix is 13. Therefore, we have to randomly test $2^{(53-13)} = 2^{40}$ M_1 to satisfy the 40 equations only determined by the inner part. Then for a right inner part, there are $2^{25-13} = 2^{12}$ solutions of M_2, which make all the 53 equations hold. The attack algorithm is similar to the attack on `Keccak` (see Supplementary Material D in our full version paper [55]). The total complexity is $2^{125.06}$ 3-round `Xoodyak-XOF`, and the memory to store U is 2^{97}.

6 MitM Preimage Attack on `Ascon-XOF`

In this section, we list the details of the MILP model for `Ascon-XOF` different from that for `Keccak`, and give an MitM preimage attack on 4-round `Ascon-XOF`.

6.1 MILP Model of the MitM Preimage Attack on `Ascon-XOF`

The Sbox of `Ascon` is much more complex than `Keccak`. In round 0, we add the conditions to the starting state to control the diffusion over the p_S operation.

Modelling the Starting State with Conditions. In the starting state $A^{(0)}$, the 64-bit outer part can be ■ or ■ bits, while the last 256-bit inner part is of ■. Assume p_S maps $(a_0, a_1, a_2, a_3, a_4)$ to $(b_0, b_1, b_2, b_3, b_4)$, where the a_0 is in the outer part and (a_1, a_2, a_3, a_4) are in the inner part. When a_0 is ■ and others are ■, all the output bits excluding b_2 should be ■ according to Equ. (3) (case 1 in Figure 14). However, if we add some conditions on (a_1, a_2, a_3, a_4), for example $a_1 = 1$ and $a_3 + a_4 = 1$, then according to Equ. (3), b_0 and b_3 can also be ■ (case 2 in Figure 14). We add conditions on the bits in the inner part of $A^{(0)}$ to control the diffusion of ■ and ■ over p_S. We name the rule by `CondSBOX-RULE`:

1. Condition on a_1: when $a_1 = 1$, b_0 is ■ and the color of b_4 is the same with a_0; when $a_1 = 0$, b_4 is ■ and the color of b_0 is the same with a_0.
2. Condition on $a_3 + a_4$: when $a_3 + a_4 = 1$, b_3 is ■; when $a_3 + a_4 = 0$, the color b_3 is the same with a_0.

Some valid coloring patterns of `CondSBOX-RULE` are shown in Figure 14.

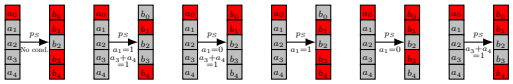


Fig. 14: Some valid red coloring patterns of `CondSBOX-RULE` (Similar to blue bits)

Modelling the Sbox of `Ascon`. We also build the Sbox operation without conditions, which is applied from round 1. According to Equ. (3), for each output bit b_i, we determine its color by all the five inputs $(a_0, a_1, a_2, a_3, a_4)$ and build the constraints independently. Taking b_0 as an example, $b_0 = a_4a_1 + a_3 + a_2a_1 + a_2 + a_1a_0 + a_1 + a_0$, we determine its color according to the following rules:

1. If there are □ bits in $(a_0, a_1, a_2, a_3, a_4)$, b_0 is □.
2. If there are all ■ bits, b_0 is ■.
3. If there are only ■ (≥ 1) and ■ (≥ 0) bits, b_0 will be ■.
4. If there are only ■ (≥ 1) and ■ (≥ 0) bits, b_0 will be ■.
5. If there are ■, or more than two kinds of ■, ■ and ■ bits in $(a_0, a_1, a_2, a_3, a_4)$:
   (a) if (a_0, a_1, a_2, a_4) are only ■ and ■ (or ■ and ■), b_0 is ■.
   (b) if a_1 is ■ or $(a_0,\ a_2,\ a_4)$ are ■, b_0 is ■.
   (c) if one of the three pairs (a_1, a_4), (a_1, a_2) and (a_0, a_1) is (■,■) or (■,■) or (■,■) or (■,■), or opposite order, b_0 is □.

Those rules for b_0 can be described by linear inequalities using the convex hull computation. The rules for b_1, b_2, b_3 and b_4 can be constructed by the same way.

Modelling the Matching Phase. We target on `Ascon-XOF` with a 128-bit hash value, which needs two output blocks. Suppose the 64-bit word $A^{(r+1)}_{\{*,0\}}$ of $A^{(r+1)}$ is the first 64-bit hash value of the first output block. We can easily compute the first 64 bits of state $S^{(r)}$ from the hash value by the inverse of p_L. Each bit of the first 64 bits of $S^{(r)}$ is computed by $b_0 = a_4a_1 + a_3 + a_2a_1 + a_2 + a_1a_0 + a_1 + a_0$ by Equ. (3), where $(a_0, a_1, a_2, a_3, a_4)$ comes from the inputs of each Sbox in $A^{(r)}$. Hence, we deduce the deterministic relations of $A^{(r)}$ to count the degree of freedom of matching.

Observation 3 (Conditions in Matching Points of `Ascon`) *If $(a_0, a_1, a_2, a_3, a_4)$ satisfy the following conditions, we say there is a 1-bit matching:*

1. *There is no □ bit in $(a_0, a_1, a_2, a_3, a_4)$.*
2. *There have ■ and ■ bits, or ■ bit in $(a_0, a_1, a_2, a_3, a_4)$.*
3. *There is no product of ■ and ■, concretely, (a_1, a_4) should not be (■,■) or (■,■) or (■,■) or (■,■), or opposite order, and same to (a_1, a_2) and (a_0, a_1).*

6.2 MitM Preimage Attack on 4-round `Ascon-XOF`

Applying the MILP model, we find a 4-round MitM preimage attack (see Supplementary Material E in our full version paper [55]). The starting state $A^{(0)}$ contains 4 ■ bits and 54 ■ bits. There are totally 44 conditions on ■ of $A^{(0)}$ (see Supplementary Material E in our full version paper [55]). In the computation from $A^{(0)}$ to $A^{(3)}$, the accumulated consumed degrees of freedom of ■ is 50 and there is no DoF of ■ consumed. Therefore, $\mathrm{DoF}_{\mathcal{B}} = 4$, $\mathrm{DoF}_{\mathcal{R}} = 54 - 50 = 4$. The four matching bit equations (DoM = 4) are derived by $A^{(3)}$ with Observation 3, which are:

$$\begin{cases} A^{(3)}_{\{15,4\}} \cdot A^{(3)}_{\{15,1\}} + A^{(3)}_{\{15,3\}} + A^{(3)}_{\{15,2\}} \cdot A^{(3)}_{\{15,1\}} + A^{(3)}_{\{15,2\}} + A^{(3)}_{\{15,1\}} \cdot A^{(3)}_{\{15,0\}} + A^{(3)}_{\{15,1\}} + A^{(3)}_{\{15,0\}} = S^{(3)}_{\{15,0\}}, \\ A^{(3)}_{\{25,4\}} \cdot A^{(3)}_{\{25,1\}} + A^{(3)}_{\{25,3\}} + A^{(3)}_{\{25,2\}} \cdot A^{(3)}_{\{25,1\}} + A^{(3)}_{\{25,2\}} + A^{(3)}_{\{25,1\}} \cdot A^{(3)}_{\{25,0\}} + A^{(3)}_{\{25,1\}} + A^{(3)}_{\{25,0\}} = S^{(3)}_{\{25,0\}}, \\ A^{(3)}_{\{47,4\}} \cdot A^{(3)}_{\{47,1\}} + A^{(3)}_{\{47,3\}} + A^{(3)}_{\{47,2\}} \cdot A^{(3)}_{\{47,1\}} + A^{(3)}_{\{47,2\}} + A^{(3)}_{\{47,1\}} \cdot A^{(3)}_{\{47,0\}} + A^{(3)}_{\{47,1\}} + A^{(3)}_{\{47,0\}} = S^{(3)}_{\{47,0\}}, \\ A^{(3)}_{\{57,4\}} \cdot A^{(3)}_{\{57,1\}} + A^{(3)}_{\{57,3\}} + A^{(3)}_{\{57,2\}} \cdot A^{(3)}_{\{57,1\}} + A^{(3)}_{\{57,2\}} + A^{(3)}_{\{57,1\}} \cdot A^{(3)}_{\{57,0\}} + A^{(3)}_{\{57,1\}} + A^{(3)}_{\{57,0\}} = S^{(3)}_{\{57,0\}}. \end{cases} \tag{23}$$

Following the framework in Figure 6, we choose (M_1, M_2) to make the 44 conditions hold, and perform the MitM attack on M_3. The attack algorithm is given in Supplementary Material E in our full version paper [55]. The time is $2^{124.67}$ 4-round `Ascon` and the memory is 2^{54}. In addition, we give a 3-round MitM preimage attack on `Ascon-XOF` with time of $2^{120.58}$ 3-round `Ascon` and memory of 2^{39} (see Supplementary Material E in [55]). An experiment on the MitM episode is given in Supplementary Material F in our full version paper [55].

7 Conclusion and Discussion

In this paper, we give the framework of the MitM attack on sponge-based hashing. To find good attacks, we build bit-level MILP based automatic tools for MitM attacks on `Keccak-512`, `Ascon-XOF`, and `Xoodyak-XOF`. Although the birthday-paradox MitM attack has been widely applied to block ciphers or MD-based hash functions since 1977, this is the first attempt to apply it to `Keccak`, etc. Our attacks lead to improved or first preimage attacks on reduced-round `Keccak-512`, `Ascon-XOF`, and `Xoodyak-XOF`.

Similar to previous preimage attacks [35,45], our attack on `Keccak` also uses the linearization-based techniques. In previous linearization-based preimage attacks [35,45], all the variables should not be multiplied with each other. In our MitM attack, the variables can be multiplied with each other within each set. For `Keccak`, to attack more rounds, we use a one-round linear structure to skip the MILP programming of the first round and accelerate the MILP model. It should be noted that the linear structure used in this paper is just a technique to accelerate the search. Moreover, by using the linear structure, the search only covers a small fraction of the whole space of the solutions, which may be not the optimal MitM attack at all. For other instances of `Keccak`, it is open problem to apply one or two-round linear structures in the search for MitM attacks.

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