

# A Framework and Compact Constructions for Non-monotonic Attribute-Based Encryption

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# Summary of Our Results

- Non-monotonic KP-ABE schemes

- with shortest ciphertext length

- from the DBDH assumption

- with better (space) efficiency

- than previous scheme [OSW07]

- with completely unbounded

- attributes (for the first time)

**Constructed  
by our new  
framework**

- The first completely unbounded

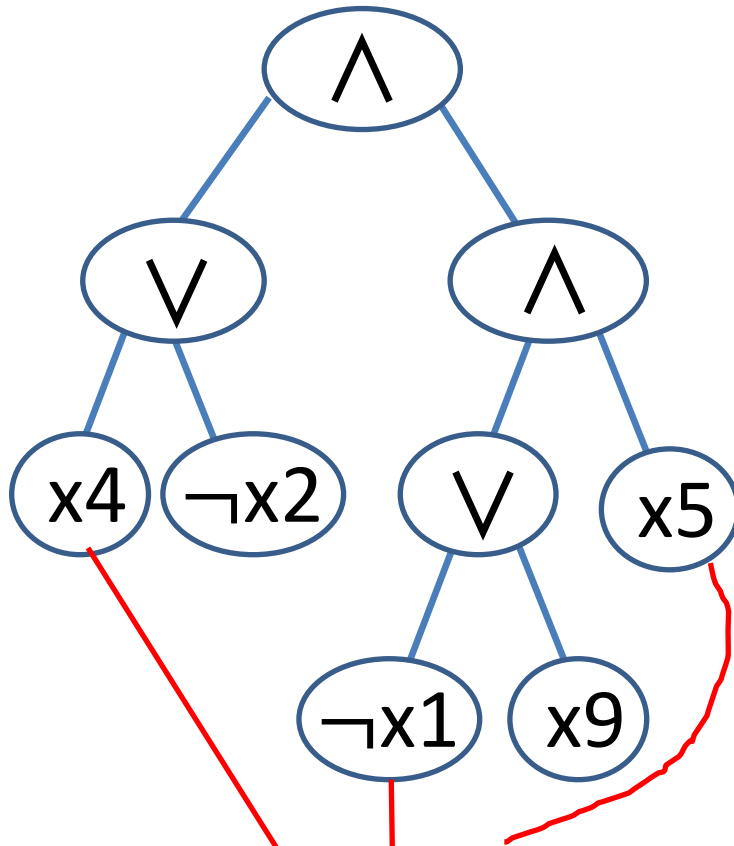
- non-monotonic CP-ABE scheme

# Definitions

# Access Tree

**Non monotone** Boolean formula

$$F = (x_4 \vee \neg x_2) \wedge ((\neg x_1 \vee x_9) \wedge x_5)$$



Satisfied leaves

$$S_1 = \{x_2, x_4, x_5\}$$

⇒ Satisfy the access tree

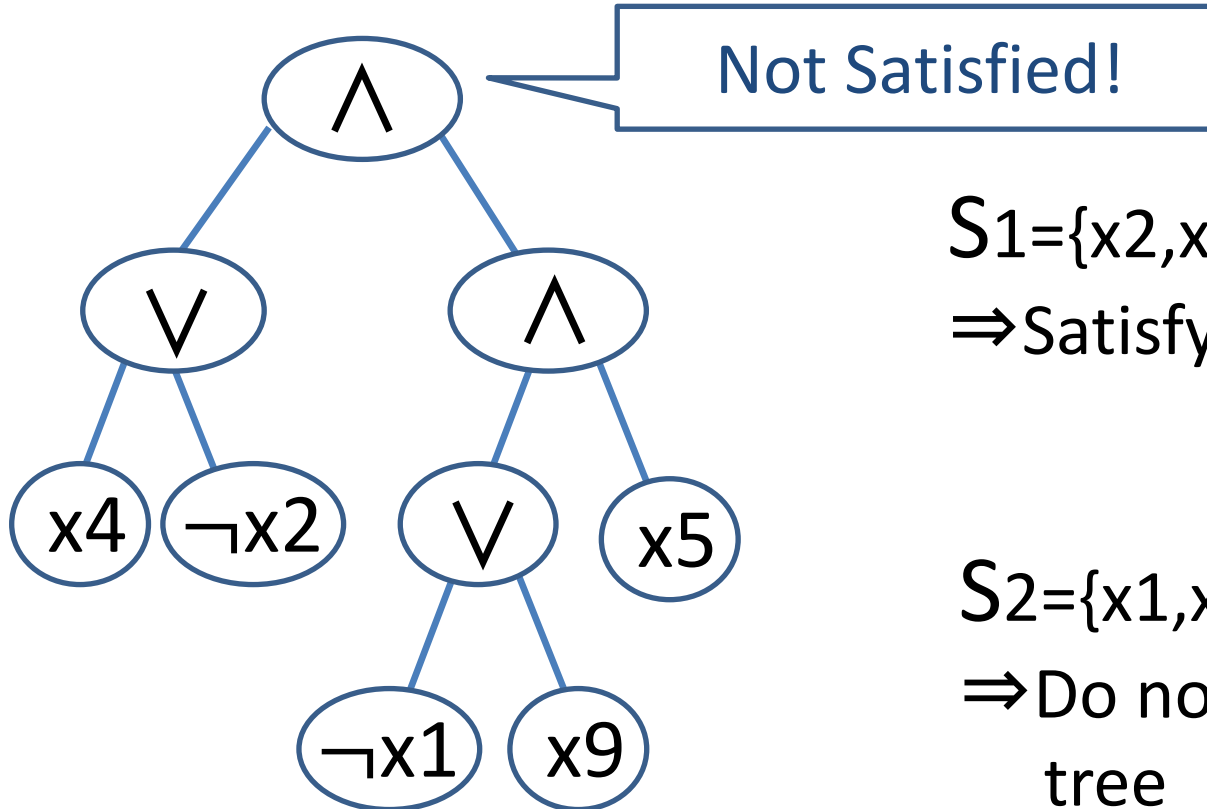
$$S_2 = \{x_1, x_5\}$$

⇒ Do not satisfy the access tree

# Access Tree

**Non monotone** Boolean formula

$$F = (x4 \vee \neg x2) \wedge ((\neg x1 \vee x9) \wedge x5)$$



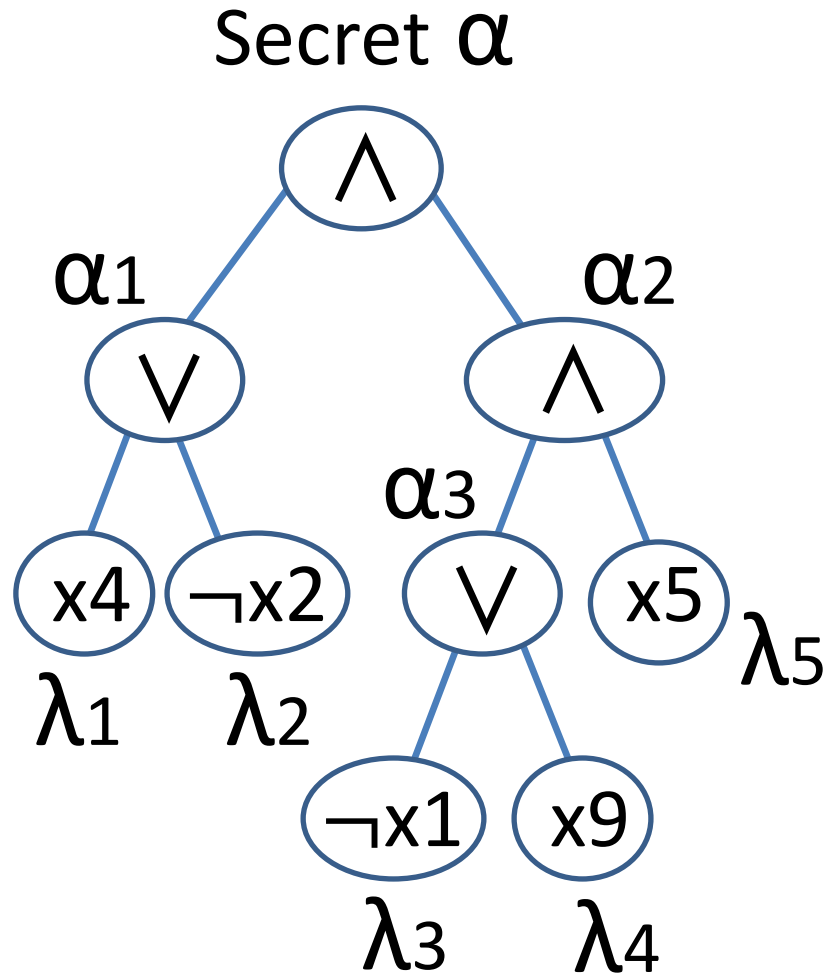
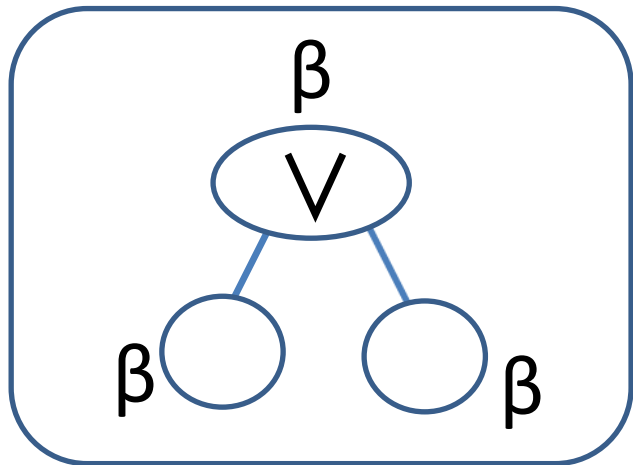
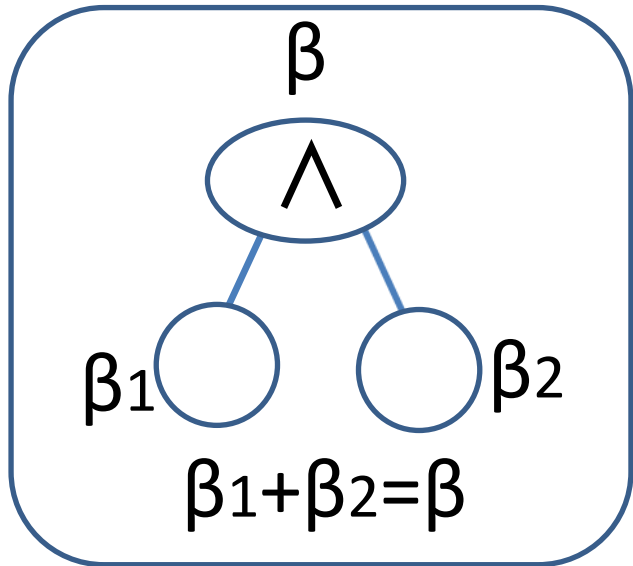
$$S1 = \{x2, x4, x5\}$$

⇒ Satisfy the access tree

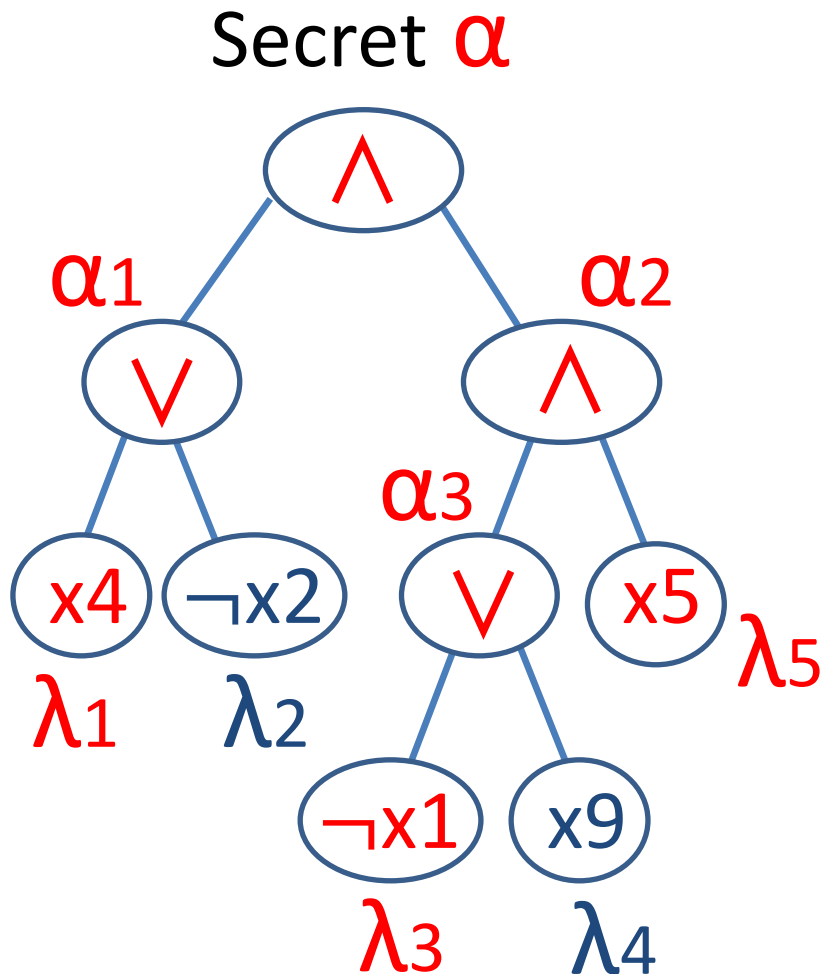
$$S2 = \{x1, x5\}$$

⇒ Do not satisfy the access tree

# Secret Sharing for Access Tree



# Property of the Secret Sharing Scheme



$$S = \{x_2, x_4, x_5\}$$

⇒ Satisfy the access tree

⇒ From shares corresponding to satisfied leaves ( $\lambda_1, \lambda_3, \lambda_5$ ), one can recover  $\alpha$ .

$$\text{i.e., } \alpha_1 = \lambda_1, \alpha_3 = \lambda_3$$

$$\alpha_2 = \alpha_3 + \lambda_5$$

$$\alpha = \alpha_1 + \alpha_2$$

If  $S'$  does not satisfy the access tree, one cannot recover  $\alpha$  from shares corresponding to satisfied nodes.

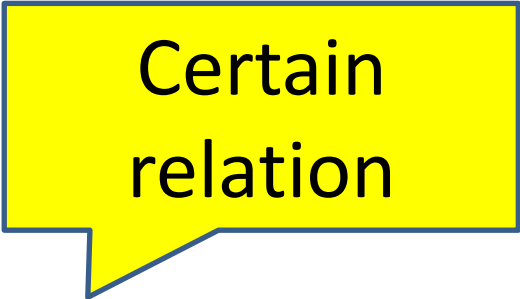
# Predicate Encryption (KEM version)

$\text{Setup}(1^\lambda) \rightarrow (\text{PK}, \text{MSK})$

$\text{KeyGen}(\text{MSK}, X) \rightarrow \text{SK}_X$

$\text{Enc}(\text{PK}, Y) \rightarrow (C_Y, K)$

$\text{Dec}(\text{PK}, Y, C_Y, \text{SK}_X) \rightarrow K$



Certain  
relation

Decryption is possible when  $R(X, Y) = 1$

KP-ABE, CP-ABE, spatial encryption etc. are all captured as a special case of predicate encryption by defining  $R$  appropriately.



# Non-monotonic Key Policy Attribute-Based Encryption

Attribute Space = {x1, x2, x3, ...}



Ciphertext  $C_s$

$S = \{x_3, x_{79}, x_{100}, x_{2000}\}$

Set of attributes



Secret Key  $SK_F$

$F = (\neg x_{44} \wedge x_{79}) \vee x_{101}$

Non-monotonic  
Boolean Formula

→  $(\neg x_{44} \wedge x_{79}) \vee x_{101}$   
 $= (1 \wedge 1) \vee 0 = 1$

→ Decryption is possible

# Two-mode Identity-Based Broadcast Encryption (TIBBE)

ID Space =  $\{x_1, x_2, x_3, \dots\}$



Ciphertext  $C_s$

e.g.,  $S = \{x_3, x_{79}, x_{100}, x_{2000}\}$

Set of IDs

## Two types of Keys



Type: IBBE

Secret Key  $SK_{IBBE, ID}$

can decrypt  $C_s$

Iff  $ID \in S$



Type: IBR

Secret Key  $SK_{IBR, ID}$

can decrypt  $C_s$

Iff  $ID \notin S$

# Our Framework to Construct Non-monotonic KP-ABE

# Our Framework to construct non-monotonic KP-ABE

- In [ALP11], conversion from **IBBE** with certain property **to monotonic KP-ABE** was given.
- We extend the result by [ALP11] and propose a conversion from **TIBBE** with certain property **to non-monotonic KP-ABE**.
  - Then, we construct various TIBBE schemes.

**Remark:** Our conversion converts selectively secure TIBBE into selectively secure non-monotonic KP-ABE.

# Required Properties

We convert a TIBBE (KEM) with following property to a non-monotonic KP-ABE (KEM).

(\*)The form of KEM key  $K$  and a secret key for ID is

$$K = e(g, g)^{s\alpha}, \quad SK_{IBBE, ID} = (g^\alpha ***, ***)$$

$$SK_{IBR, ID} = (g^\alpha ***, ***)$$

where master secret key  $MSK = \alpha$ .

In the following, we construct KP-ABE scheme

$\{KP\text{-ABE.Setup, KP-ABE.KeyGen, KP-ABE.Enc, KP-ABE.Dec}\}$

out of TIBBE scheme (with the above property)

$\{TIBBE.Setup, TIBBE.KeyGen, TIBBE.Enc, TIBBE.Dec\}$

# Non-monotonic KP-ABE from TIBBE(1)

Universe of attribute = ID space  
(Thus, the resulting scheme has large universe)

$$\text{KP-ABE.Setup}(1^\lambda) = \text{TIBBE.Setup}(1^\lambda)$$

$$\text{KP-ABE.Enc}(\text{PK}, S, M) = \text{TIBBE.Enc}(\text{PK}, S, M)$$

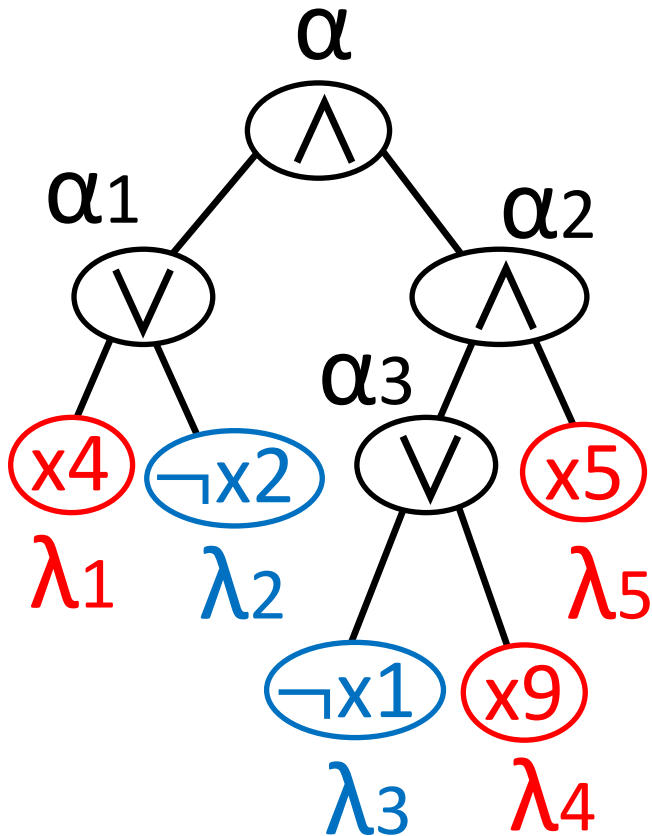
Set of attributes

Set of IDs

# Non-monotonic KP-ABE from TIBBE(2)

KP-ABE.KeyGen(MSK, F):

A Boolean formula



For leaf node w **positive attribute**:

$\text{TIBBE.KeyGen}(\text{IBBE}, x4, \lambda1) \rightarrow \text{SK}_{\text{IBBE},x4}$

ID

MSK

Also Generate  $\text{SK}_{\text{IBBE},x5}$  and  $\text{SK}_{\text{IBBE},x9}$ .

For leaf node w **negative attribute**:

$\text{TIBBE.KeyGen}(\text{IBR}, x2, \lambda2) \rightarrow \text{SK}_{\text{IBR},x2}$

ID

MSK

Also Generate  $\text{SK}_{\text{IBBE},x1}$

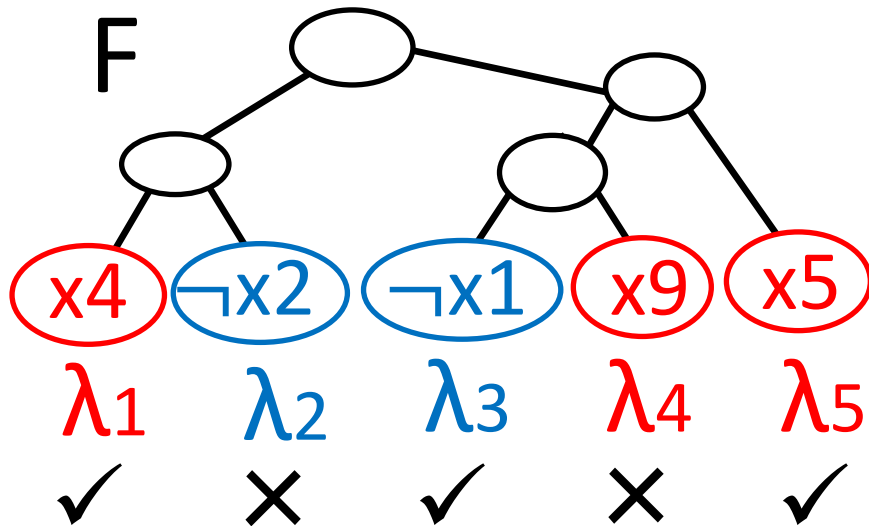
The final output is secret keys for all leaves:

$\text{SK}_F = \{ \text{SK}_{\text{IBBE},x4}, \text{SK}_{\text{IBR},x2}, \text{SK}_{\text{IBR},x1}, \text{SK}_{\text{IBBE},x5}, \text{SK}_{\text{IBBE},x9} \}$

# Non-monotonic KP-ABE from TIBBE(3)

KP-ABE.Dec( $C_s, SK_F$ ):

$S = \{x_2, x_4, x_5\}$



**Satisfied by S?**  
 $S = \{x_2, x_4, x_5\}$

For all satisfied leaves, compute partial decryption.

- $TIBBE.Dec(C_s, SK_{IBBE, x_4})$   
 $\rightarrow e(g, g)^{s\lambda_1}$  (recall  $x_4 \in S$ )
- $TIBBE.Dec(C_s, SK_{IBR, x_1})$   
 $\rightarrow e(g, g)^{s\lambda_3}$  (recall  $x_1 \notin S$ )
- Also compute  $e(g, g)^{s\lambda_5}$

Finally, compute  $K = e(g, g)^{s\alpha}$  from  $\{e(g, g)^{s\lambda_1}, e(g, g)^{s\lambda_3}, e(g, g)^{s\lambda_5}\}$ .

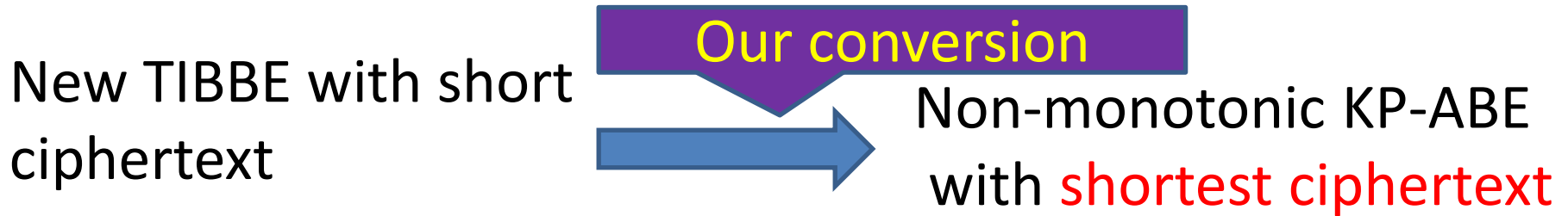


# Proposed Schemes and Comparison to Previous Schemes

# Our Proposed Schemes

- To construct non-monotonic KP-ABE schemes, we only need to construct TIBBE schemes.
- To obtain schemes with compact parameters, **we proposed various TIBBE schemes.**
- While construction of TIBBE seems to be much easier/simpler than non-monotonic KP-ABE, still, it is not trivial. (In fact, constructions of TIBBE schemes would be our main contribution rather than our semi-generic conversion.)

# Our **First** Scheme and Comparison to Previous Scheme



Non-monotonic KP-ABE with compact ciphertext

Scheme	Ciphertext overhead (G)	Public key size (G,GT)	Secret key size (G)	# of pairing in Dec	Assumption
[ALP11]	<b>3</b>	<b>(2n+2,1)</b>	$(n+1)t$	<b>3</b>	n-DBDHE
[Ours]	<b>2</b>	<b>(n+1,1)</b>	$(n+1)t + t_2$	<b>2</b>	n-DBDHE

$n$ =maximum size of attribute set associated with a ciphertext

$t=t_1+t_2$ ,  $t_1$ =# of positive attribute in access policy

$t_2$ =# of negative attribute in access policy

# Our **Second** Scheme and Comparison to Previous Scheme

Our conversion

New TIBBE from DBDH  New non-monotonic KP-ABE **from DBDH**

Non-monotonic KP-ABE from DBDH

Scheme	Ciphertext overhead (G)	Public key size (G,GT)	Secret key size (G)	Assumption
[OSW07]	<b>2n-1</b>	(2n+2,0)	2t <sub>1</sub> +3t <sub>2</sub>	DBDH
[Ours]	<b>n+1</b>	<b>(n+2,1)</b>	2t <sub>1</sub> +3t <sub>2</sub>	DBDH

n=maximum size of attribute set associated with a ciphertext  
 t=t<sub>1</sub>+t<sub>2</sub>, t<sub>1</sub>=# of positive attribute in access policy  
 t<sub>2</sub>=# of negative attribute in access policy

# Unbounded KP/CP-ABE

Before going to our third and fourth scheme, we clarify what does “completely unbounded” means.

KP-ABE case

The case of CP-ABE is similar.



Secret key for Boolean formula  $F$

$F = ((\text{Att1} \vee \text{Att2}) \wedge \text{Att1}) \vee \text{Att2} \vee \text{Att1}$

Ciphertext  $C_s$  for  
 $S = \{\text{Att1}, \text{Att2}, \dots, \text{Att}_n\}$

Att1 appears 3 times.

- Is  $n = |S|$  unbounded?
- Is number of the same attribute appears in  $F$  unbounded?

# Our **Third** Scheme and Comparison to Previous Scheme

IBBE implicit in [RW13] +  
IBR proposed by [LSW10]



New unbounded TIBBE



**Our conversion**

First **completely unbounded**  
non-monotonic KP-ABE

Non-monotonic KP-ABE with unbounded set

Scheme	Unbounded set size for ciphertext?	Unbounded multi-use of the same attribute in F?	Security	Standard model?
[OT12]	<b>YES</b>	<b>No</b>	<b>Adaptive</b>	<b>YES</b>
[LSW10]	<b>YES</b>	<b>YES</b>	<b>Selective</b>	<b>No</b>
[Ours]	<b>YES</b>	<b>YES</b>	<b>Selective</b>	<b>YES</b>

# Our **Fourth** Scheme and Comparison to Previous Scheme

- While our KP-ABE schemes are constructed in a modular way, construction of our fourth scheme (CP-ABE) is more direct.

Monotonic unbounded CP-ABE[RW13]



First **non-monotonic completely unbounded** CP-ABE

Non-monotonic CP-ABE with unbounded set

Scheme	Unbounded set size for secret key?	Unbounded multi-use of the same attribute in F?	Security
[OT12]	<b>YES</b>	No	<b>Adaptive</b>
[Ours]	<b>YES</b>	<b>YES</b>	<b>Selective</b>

# Summary of Our Results (Again)

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**Constructed  
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- The first completely unbounded non-monotonic CP-ABE scheme