Generalizing Homomorphic MACs for Arithmetic Circuits

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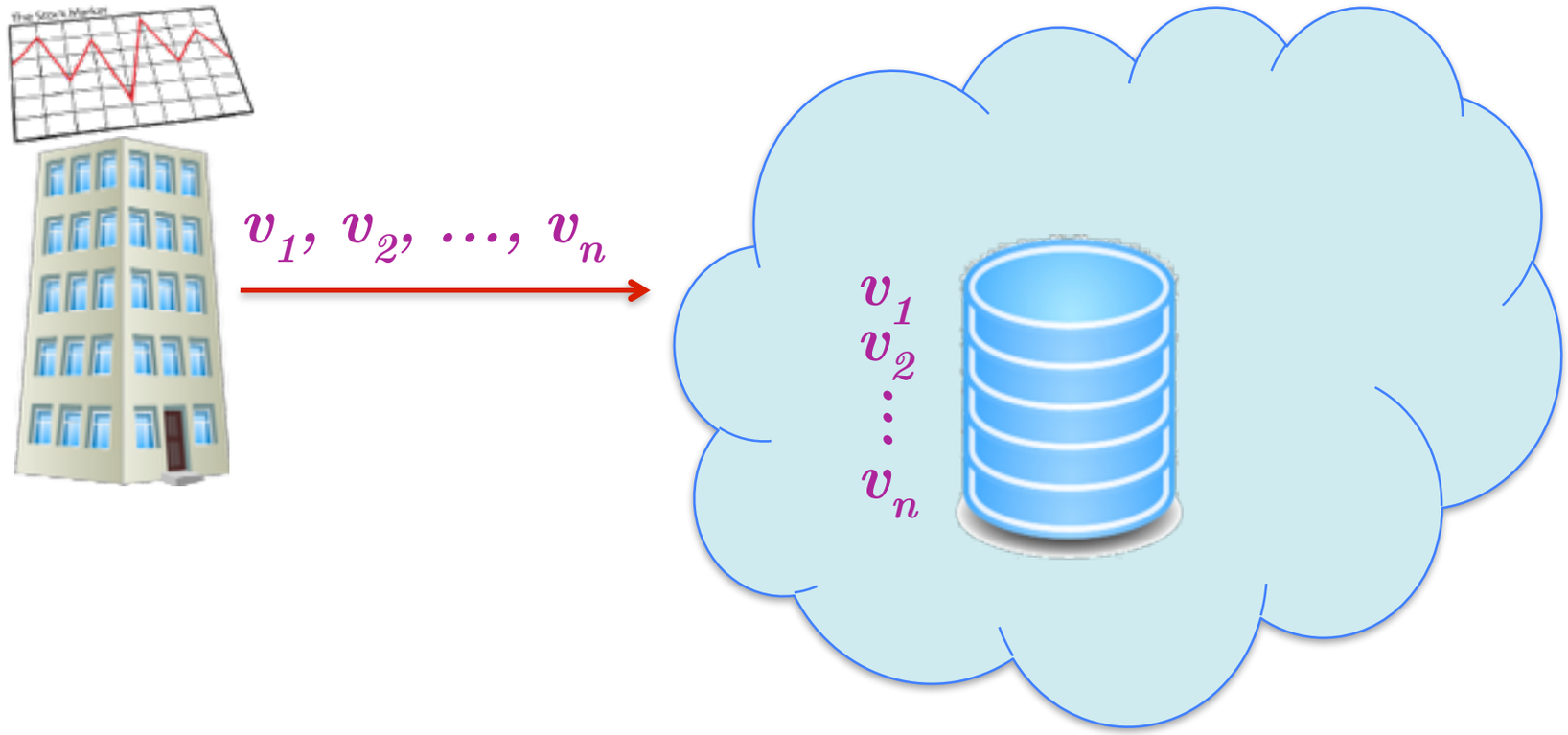
Outline

- Motivation
- Homomorphic MACs
  - Definition
  - Previous work
- Our results
- Summary & Open problems

Delegating Computations on Outsourced Data



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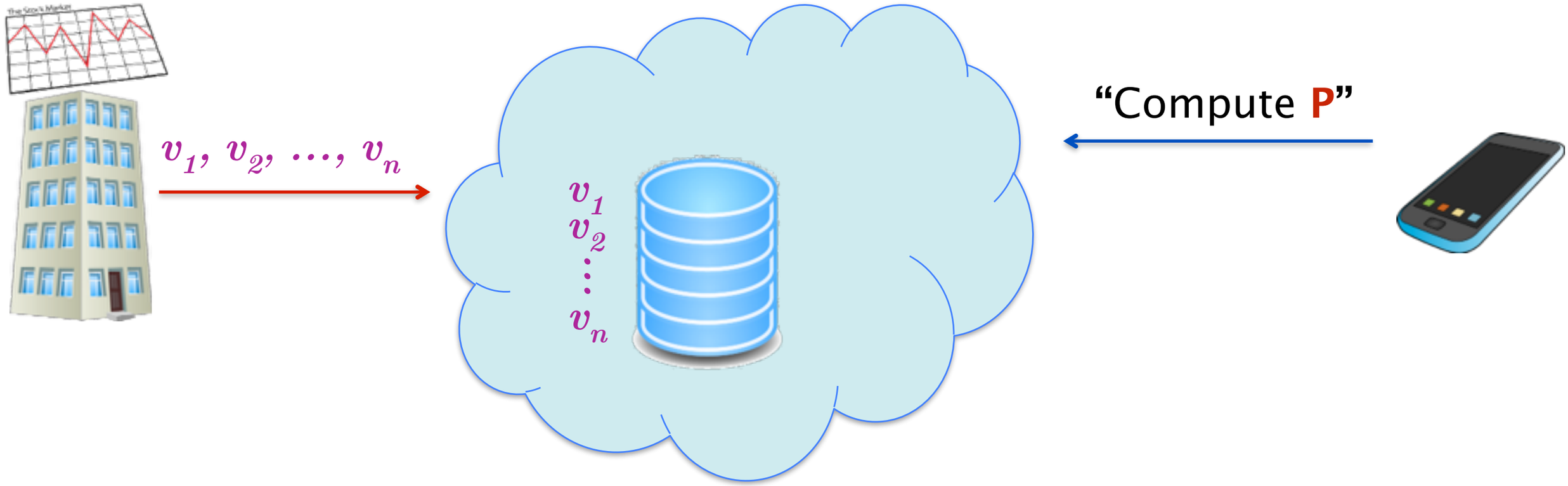

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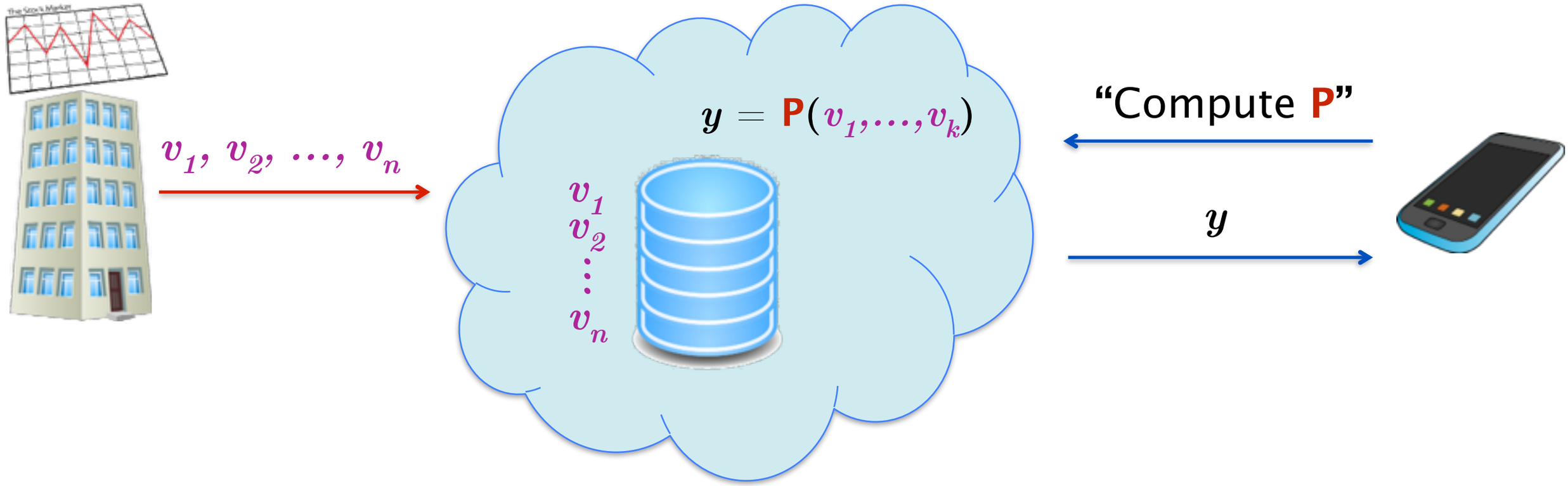

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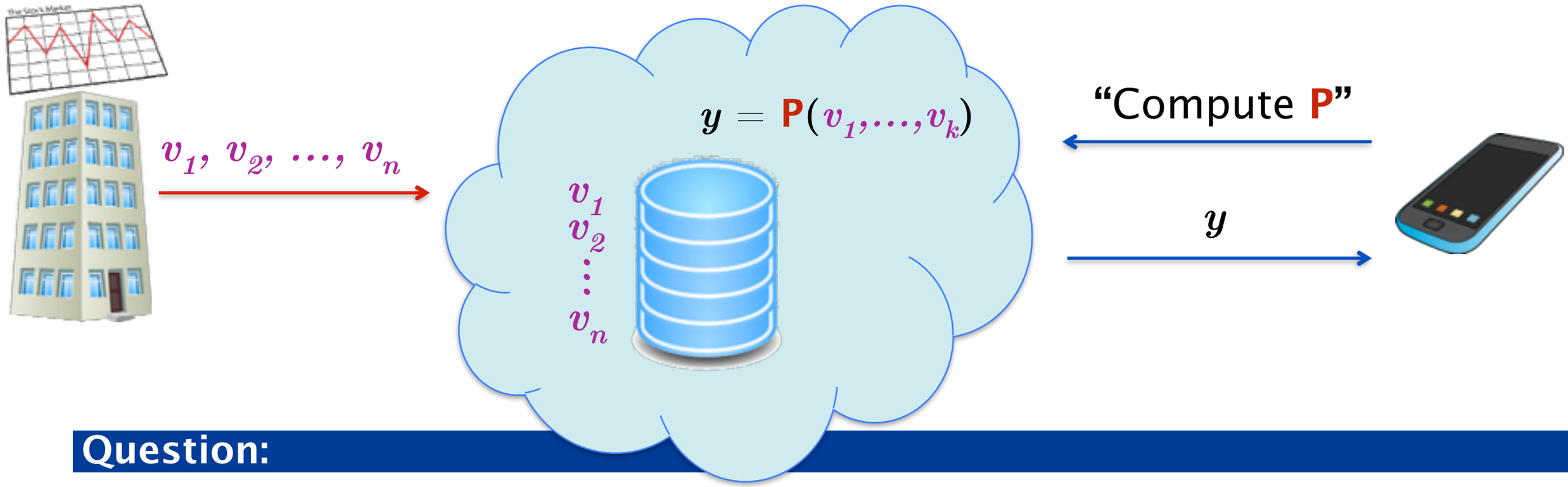

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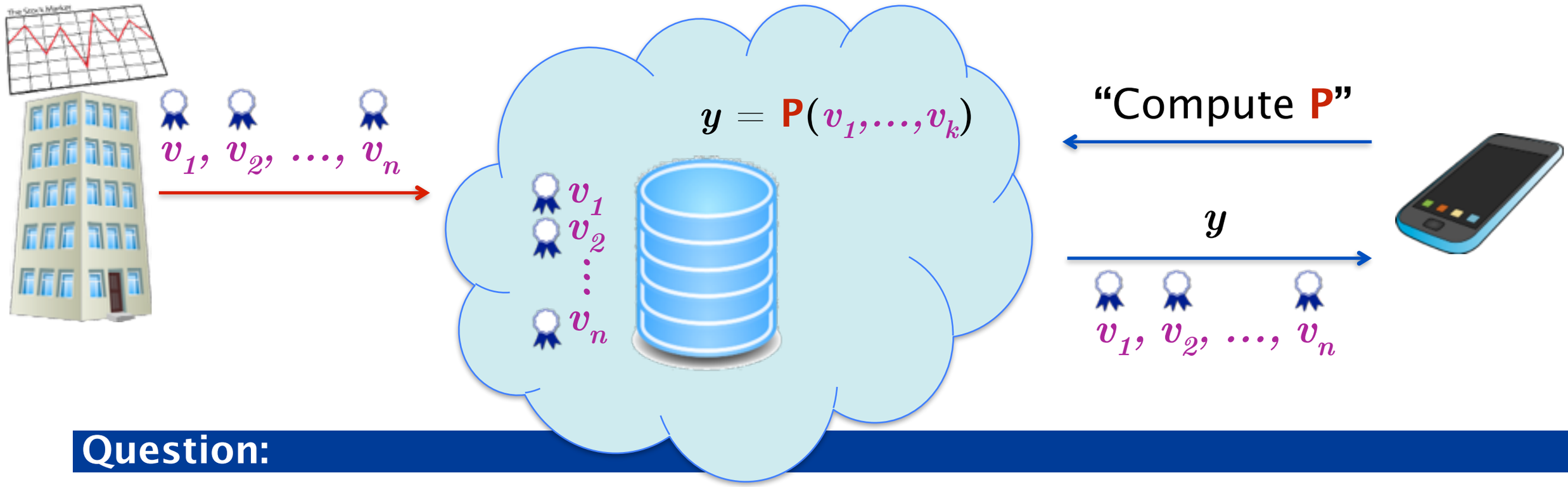

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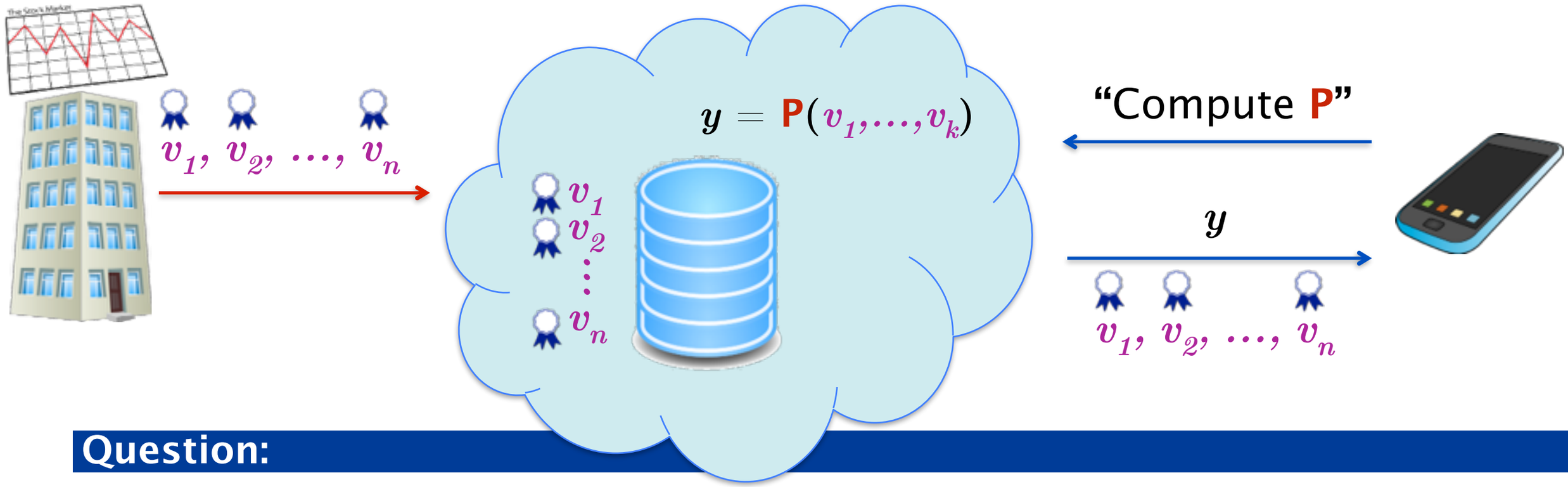

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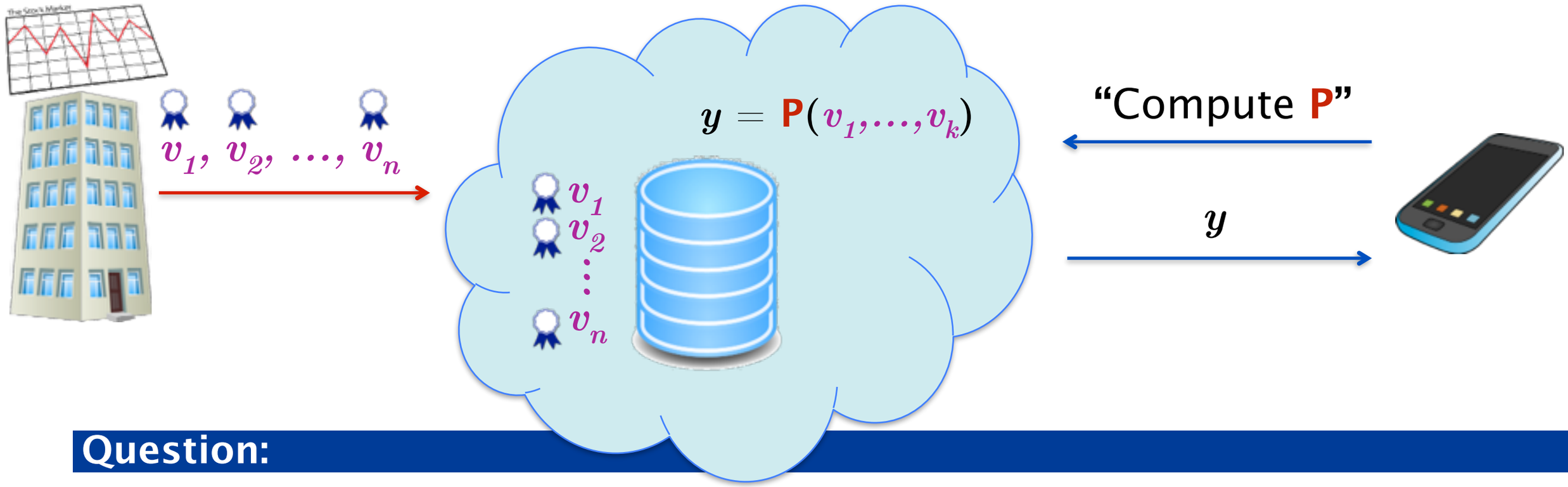

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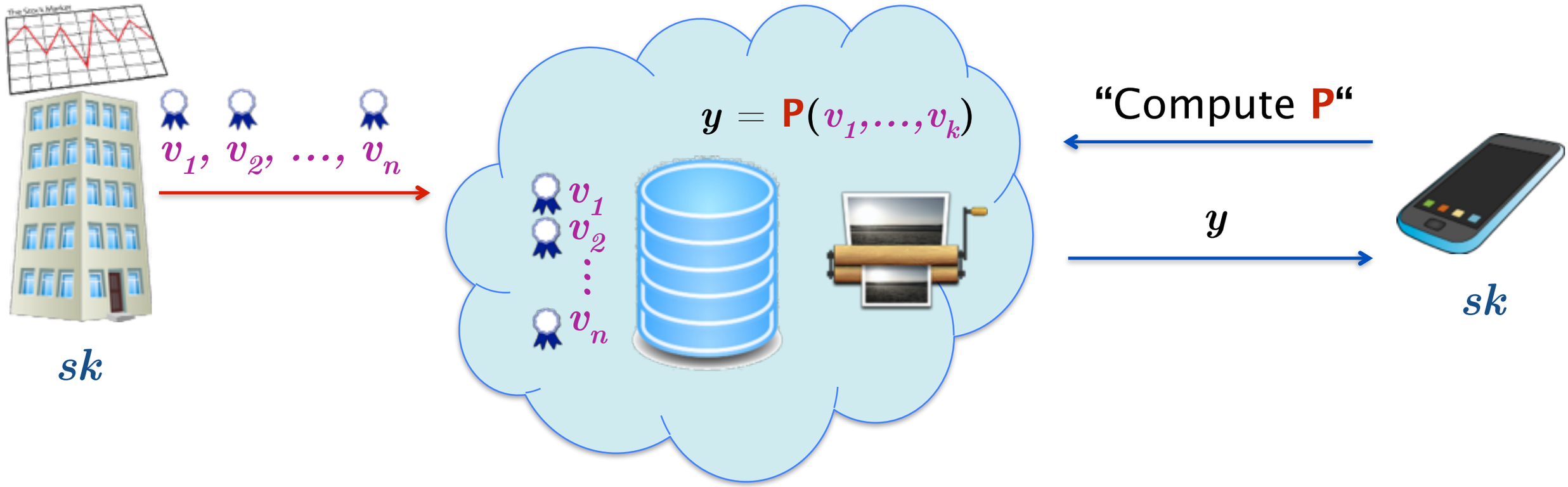

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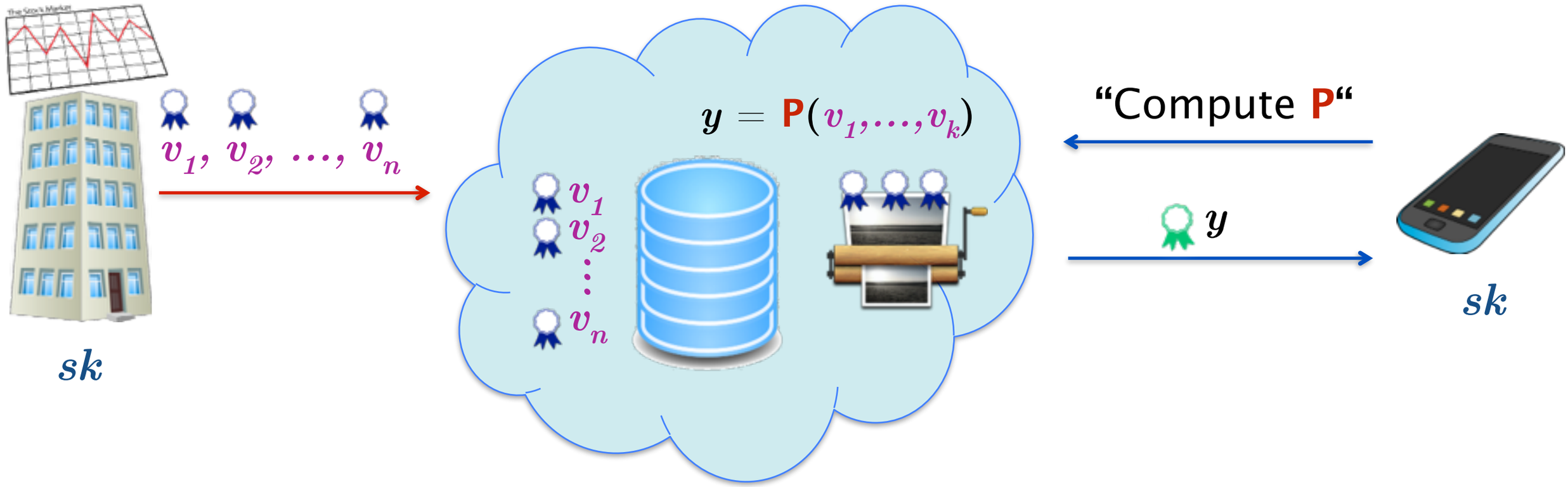

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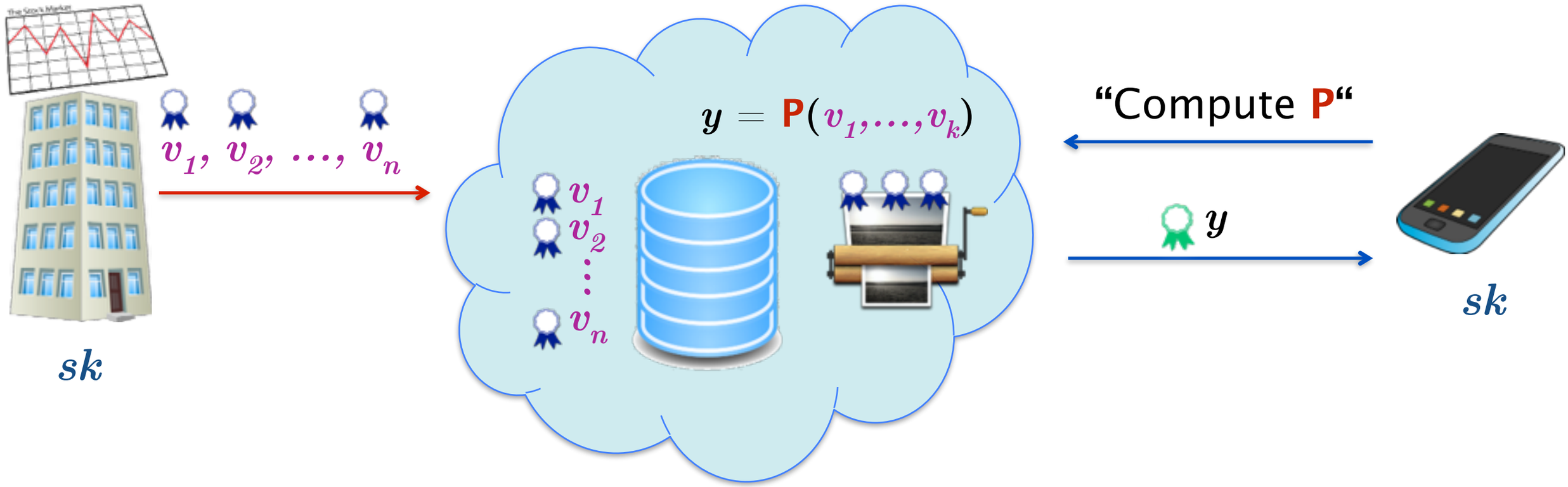

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Main Goals

- **Integrity**
  - ✓ Untrusted cloud must not be able to send incorrect y
  - Cloud cannot forge MACs.
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  - |🎗| << size of k input values.

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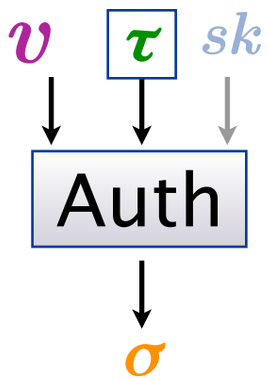

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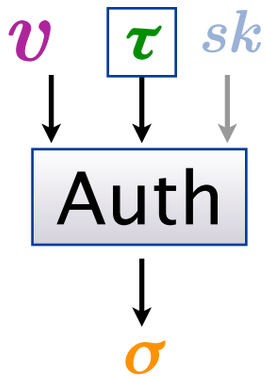

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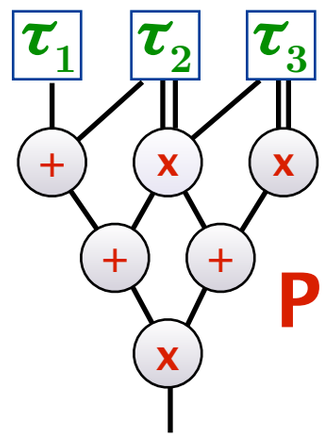

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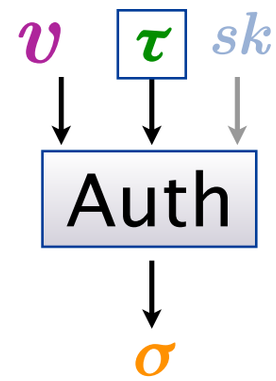

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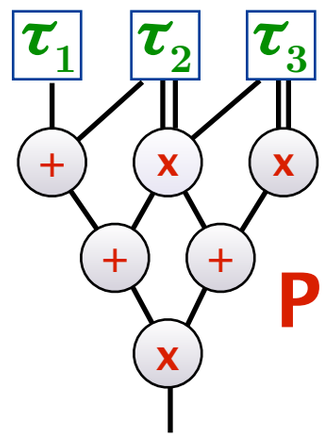

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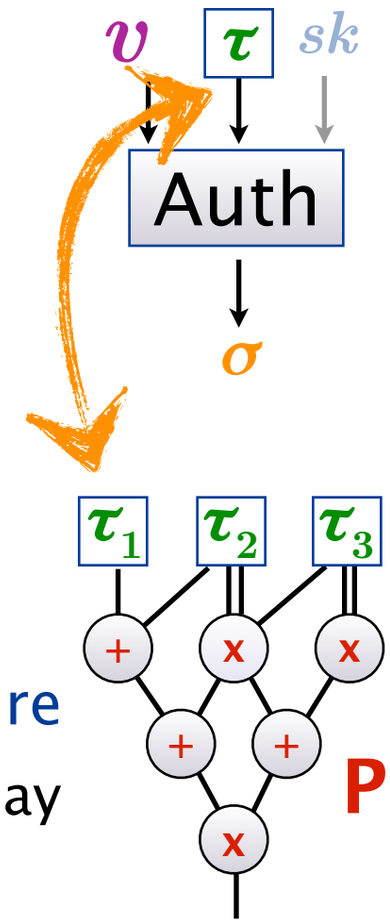

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Properties of Homomorphic MACs

- **Security:** …in 2 slides
- **Succinctness:** size of tags (returned by **Eval**) does not depend on the number of inputs of the computation
- **Composition:** authenticated outputs can be further used as inputs to other circuits

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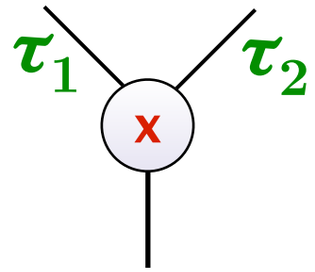

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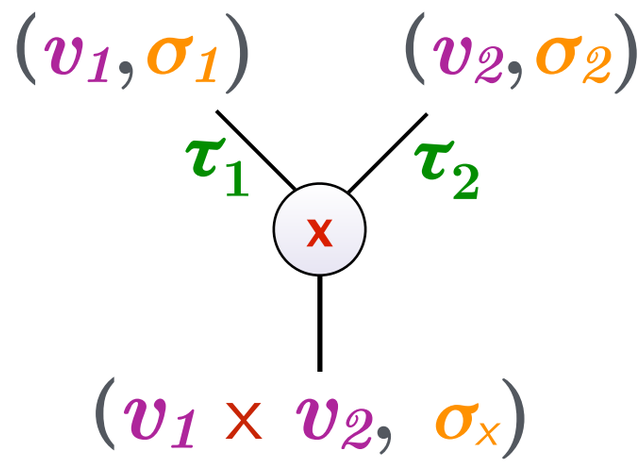

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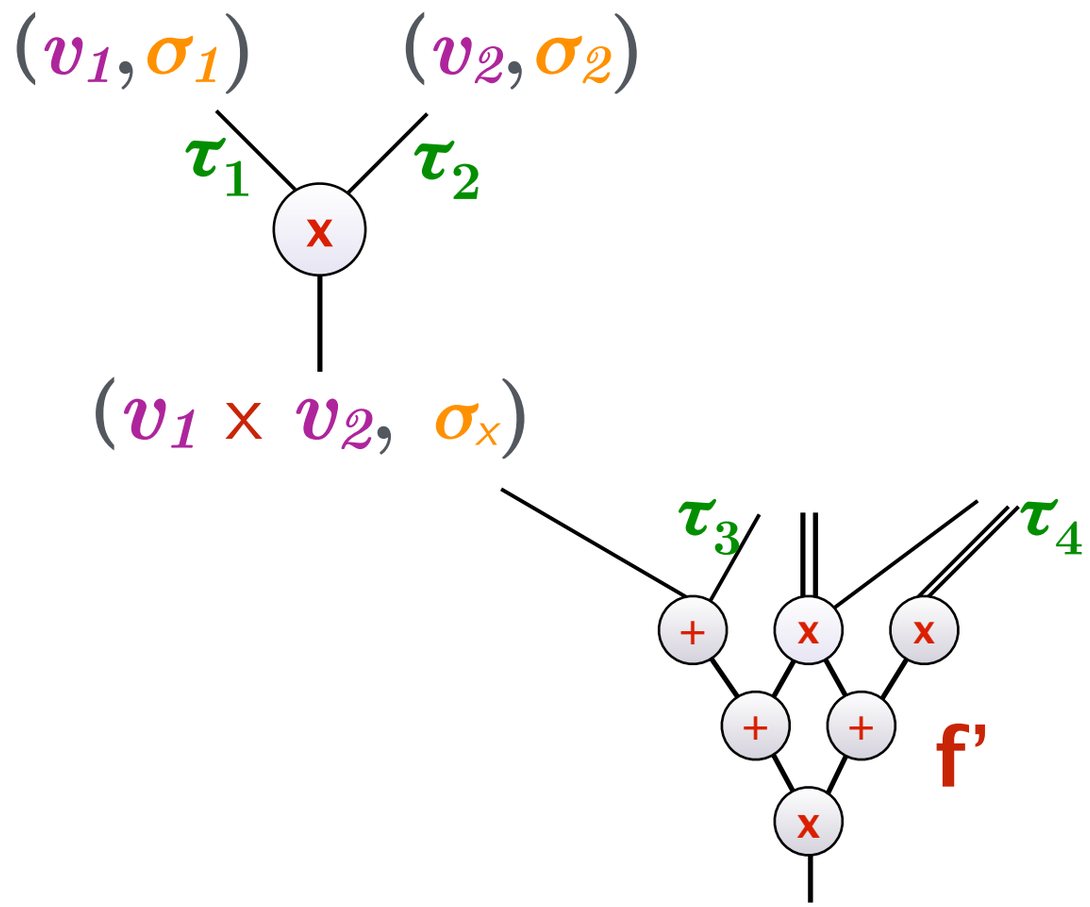

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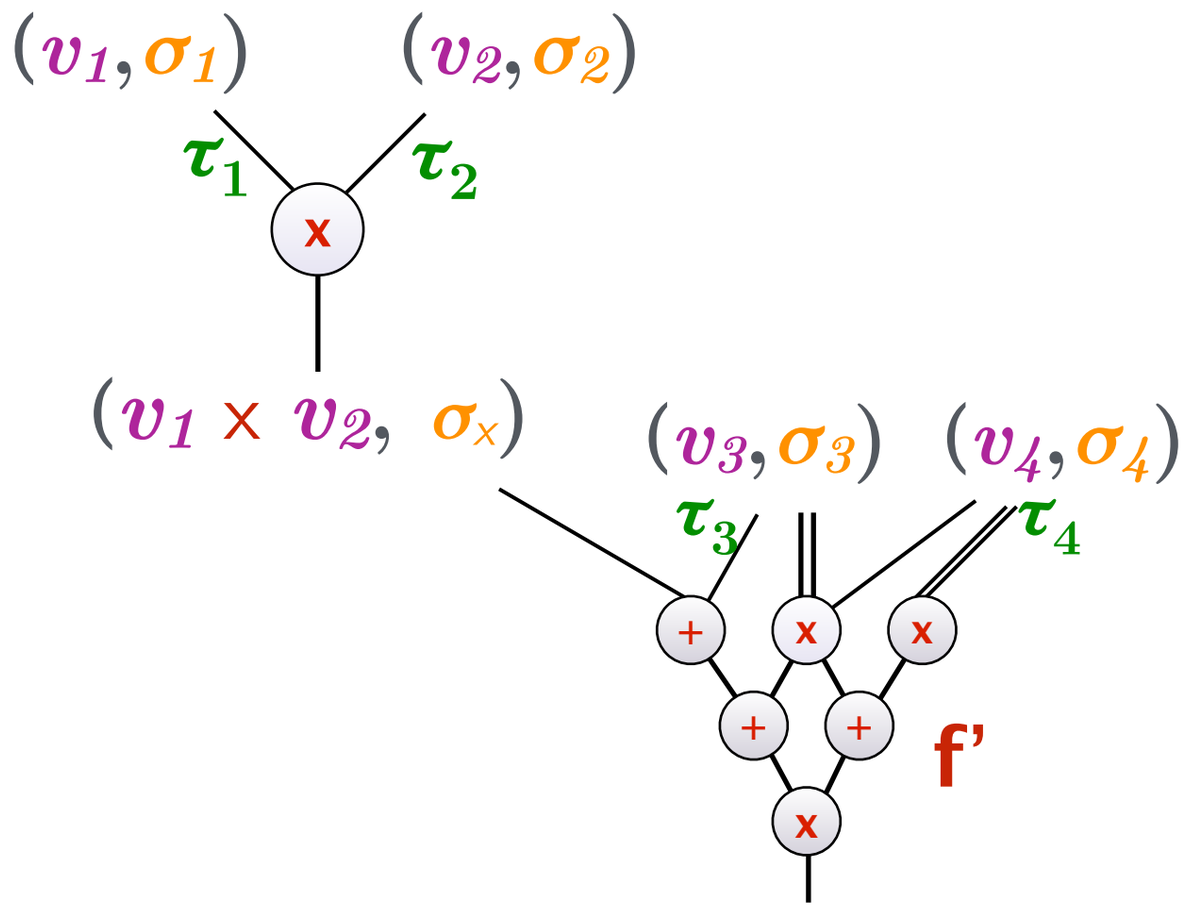

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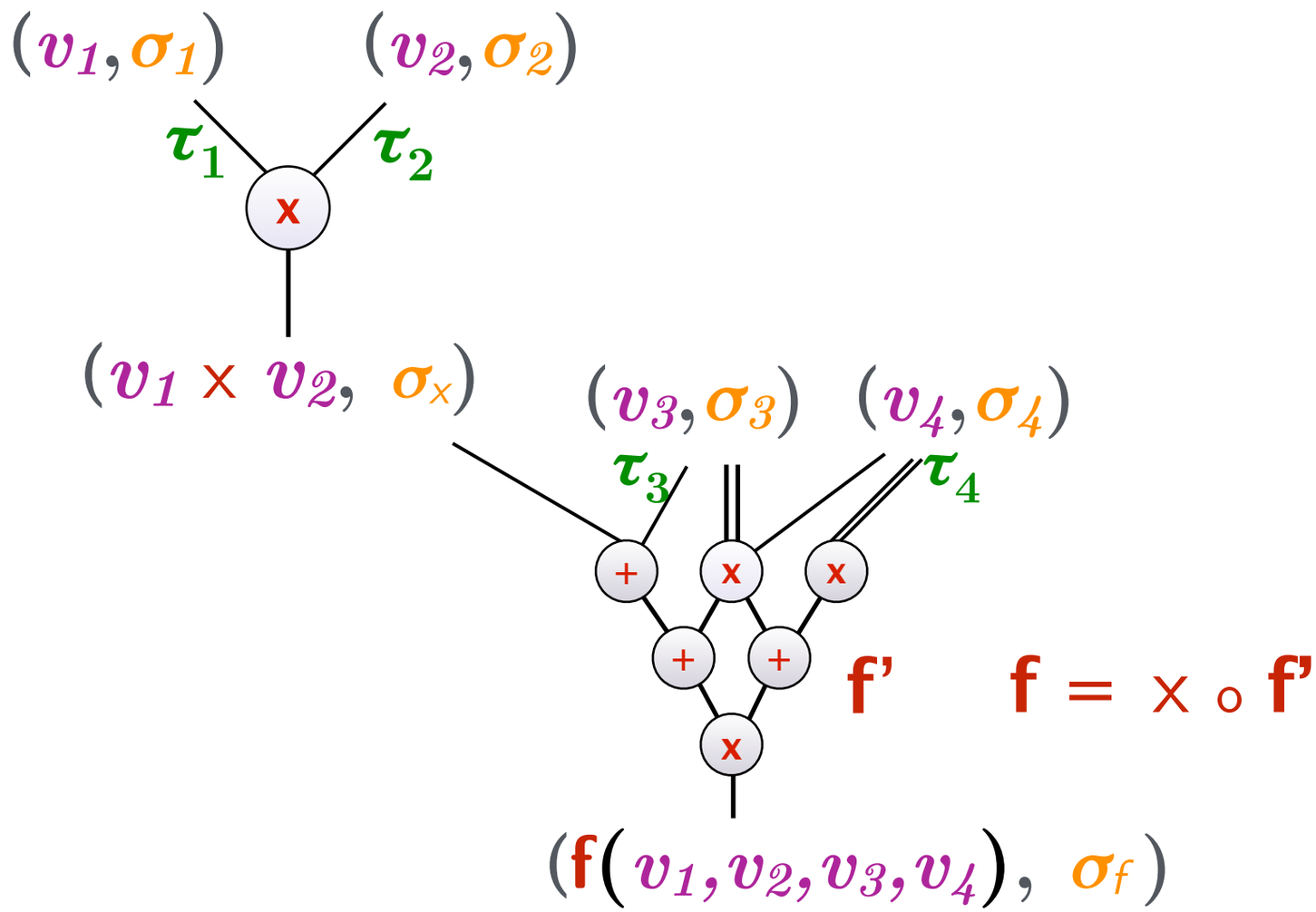

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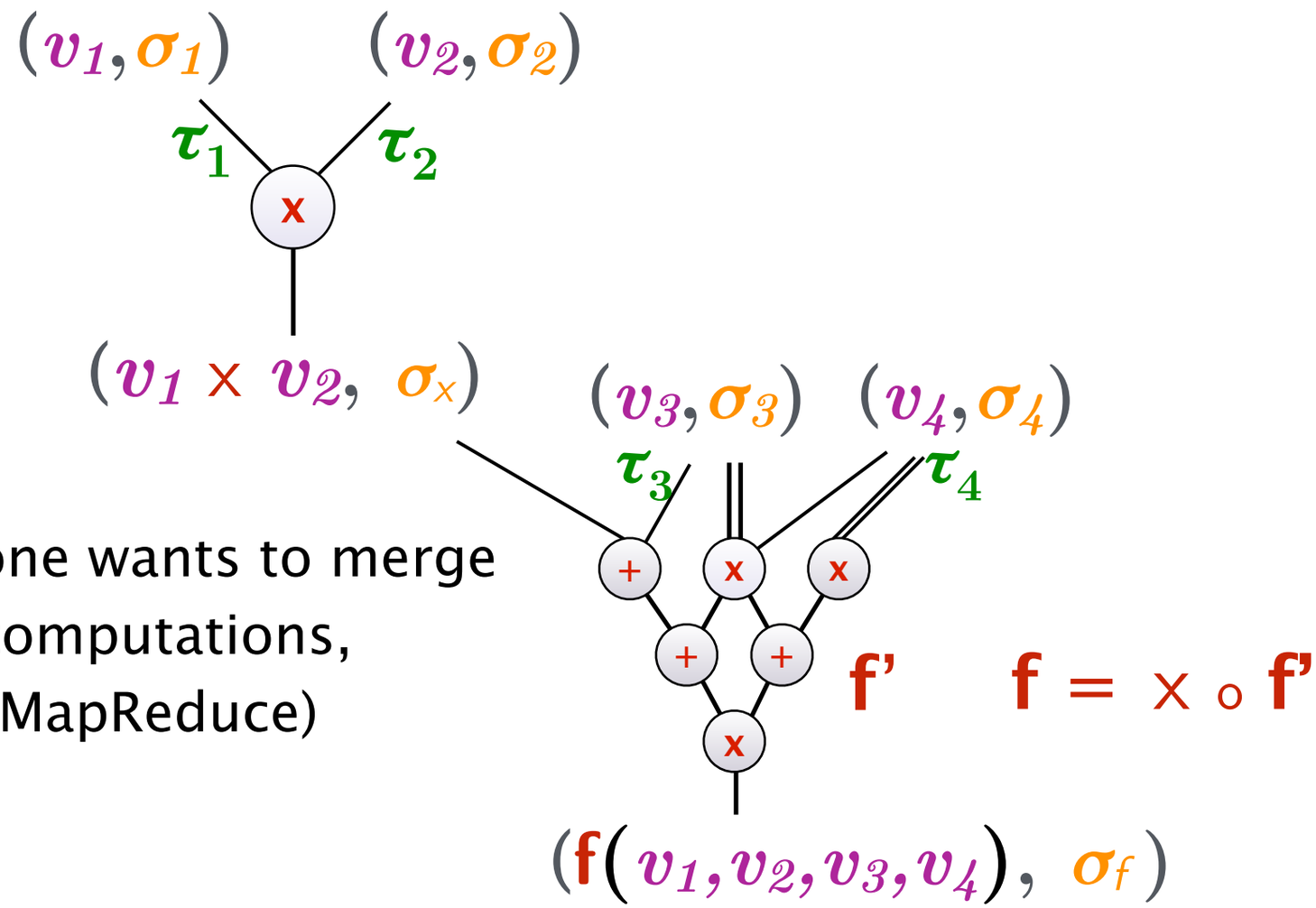

Very useful property if one wants to merge partially authenticated computations, e.g., for parallelization (MapReduce)

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Basic idea: nobody, without sk, can create a "valid" MAC

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Basic idea: additively homomorphic but not multiplicative homomorphic (similar to [BCIOP13]). Possible instantiations: Paillier, BV11.

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This Talk

Graded k-Linear maps

- **Gen**(1^λ, k) generates k groups of prime order p

$$G_1, G_2, \ldots, G_k$$

- with a collection of bilinear maps

$$e_{ij}: G_i \times G_j \to G_{i+j} : \quad e_{ij}(g_i^{\,a}, g_j^{\,b}) = g_{i+j}^{\,ab}$$

- **Notation:** $g \in G_1$, $g_i = e(g, \ldots, g)$ i times
- "Approximate" realizations via graded encodings [GGH13, CLT13]

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Our Homomorphic MAC

- **KeyGen(1^λ, D, k):**
  - Generate leveled k-linear groups of prime order p, e_{ij}: $G_i \times G_j \to G_{i+j}$
  - Take random generator g in G_1, sample $x, a \leftarrow Z_p$,
  - Compute g^{x^i}, g^{ax^i}, for i=1…D
  - Sample a seed K of a PRF F_K: $\{0,1\}^* \to Z_p$
  - sk=(K, g, x, a), ek=(g^a, {g^{x^i},g^{ax^i}}$_i$)
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- **Compress**(ek, $y(X)$): $\Lambda \leftarrow \prod^d_{i=1} (g^{x^i})^{yi} = g^{y(x)-y(0)}$
  - Similarly, compute $\Gamma \leftarrow g^{a[y(x)-y(0)]} = \Lambda^a$. Output σ=($y(0)$, Λ, Γ)

Our Homomorphic MAC cont'd

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- **CompositionEval(**ek, ϕ, $\sigma_1=(v_1, \Lambda_1, \Gamma_1)$, $\sigma_2=(v_2, \Lambda_2, \Gamma_2)$ **)**$\rightarrow \sigma=(v, \Lambda, \Gamma)$

(simplified description for ϕ single gate and elements in G_1)

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  - Basic idea: use the graded maps to compute $\phi(\Lambda_1, \ldots,\Lambda_n)\rightarrow\Lambda$, with deg(ϕ)≤k

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  - Homomorphic properties of the graded maps

Result

- **Security** under the (*D, k*)-**MDHI assumption**
  - Given (**g**, $\mathbf{g}^{x}$, …, $\mathbf{g}^{x\hat{}D}$) in $\mathbf{G}_1$, hard to compute $\mathbf{g}_k^{x\hat{}(Dk+1)}$ in $\mathbf{G}_k$
  - It can be shown hard in the generic multilinear group model, by extending the Uber assumption of [BBG05]
- **Theorem**. *If the (D,k)-MDHI assumption holds and* **F** *is a PRF, then the scheme is a secure homomorphic MAC with tags of size* $O(k^2)$ *and supports arithmetic circuits of degree* $\leq D$ *and composition circuits of degree* $\leq k$.

Comparison to other approaches

- Another approach to solve the problem is to leverage SNARKs
  - The homomorphic signature is a SNARK proof about the existence of valid signatures on the inputs
- However, by following this approach:
  - composition is achieved via recursive composition of proofs (proofs about validity of other proofs) [BCCT13]
  - function independence achieved via universal circuits
- Overall, less natural approach and likely to require non-falsifiable (knowledge) assumptions [GW11]
- In contrast, our solutions can be based on falsifiable assumptions

Conclusions & Open Problems

- We proposed new homomorphic MAC schemes
  - Based on encoding w/limited malleability
  - Multilinear maps – trading succinctness vs. composition
- **Main open questions:**
  - Can we achieve **Fully Homomorphic MACs with unbounded verification queries** ?
  - How about **Fully-Homomorphic Signatures?**

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Interesting observation: if we assume ideal compact k-linear maps with k<p exponential, our scheme is homomorphic for all circuits of bounded depth and secure against unbounded verification queries.

Thanks