# Collision Attack on 5 Rounds of Grøstl 

Florian Mendel Vincent Rijmen Martin Schläffer


## KULEUVEN

The Grostl Hash Function

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■ SHA-3 finalist designed by Knudsen et al.

- iterative, Merkle-Damgård design principle
- wide-pipe construction, $2 n$-bit chaining value


## The Grøstl Compression Function



- Permutation based design
- $8 \times 8$ state and 10 rounds for Grøstl-256
- $8 \times 16$ state and 14 rounds for Grøstl-512


## The Grøstl-256 Round Transformations



- AES like round transformation

$$
r_{i}=M B \circ S H \circ S B \circ A C
$$

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## Existing Analysis of Grøstl I

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## Attacks on the Hash Function

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$\Rightarrow$ We will show collision attacks for up to 5 rounds of Grøstl

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- Similar to the attack on Grindahl
- Attack uses a new type of truncated differential trail spanning over more than one message block
- Starting with an (almost) arbitrary difference in the chaining variable
- Iteratively canceling the differences in the chaining variable
- Having only differences in one of the two permutations


## Equivalent Description of Grøst|

- To simplify the description of the attack we use an equivalent description of Grøstl

$$
\begin{aligned}
h_{0}^{\prime} & =M B^{-1}(\mathrm{IV}) \\
h_{i}^{\prime} & =P^{\prime}\left(M B\left(h_{i-1}^{\prime}\right) \oplus m_{i}\right) \oplus Q^{\prime}\left(m_{i}\right) \oplus h_{i-1}^{\prime} \quad \text { for } 1 \leq i \leq t \\
\text { hash } & =\Omega\left(M B\left(h_{t}^{\prime}\right)\right) \\
\text { with } h_{i} & =M B\left(h_{i}^{\prime}\right)
\end{aligned}
$$

- The last MixBytes transformation of the permutations $P$ and $Q$ are swapped with the XOR operation of the feed-forward


## Attack on 4 Rounds of Grøstl-256

- The core of the attack on 4 rounds are truncated differential trails for $P^{\prime}$ with only 8 active bytes at the output of round $r_{4}$

$$
64 \xrightarrow{r_{1}} 64 \xrightarrow{r_{2}} 8 \xrightarrow{r_{3}} 8 \xrightarrow{r_{4}} 8
$$

■ Using the rebound attack all the $2^{64}$ solutions for this truncated differential trail with a given/fixed difference difference at the input of $P^{\prime}$ can be found with complexity $2^{64}$ in time and memory


## Attack on 4 Rounds of Grøstl-256

- Choose some arbitrary $m_{1}, m_{1}^{*}$ to get a full active state in $h_{1}^{\prime}$
- Construct $2^{64}$ solutions for the truncated differential trail in $P^{\prime}$ to find a $m_{2}$ such that 8 bytes of the difference in $h_{2}^{\prime}$ are canceled



## Attack on 4 Rounds of Grøstl-256

- Construct $2^{64}$ solutions for a rotated variant of the truncated differential trail to cancel another 8 bytes of the difference in $h_{3}^{\prime}$



## Attack on 4 Rounds of Grøstl-256

- Repeat this in total 8 times until a collision has been found in $h_{9}^{\prime}$



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$\Rightarrow$ Collision attack for 4 rounds with complexity of $8 \cdot 2^{64}=2^{67}$

Extending the Attack to 5 Rounds

## Attack on 5 Rounds of Grøstl-256

- For the attack on 5 rounds we use truncated differential trails with only one active byte at the output of round $r_{3}$

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- Using the rebound attack all the $2^{8}$ solutions for this truncated differential with a given/fixed difference at the input of $P^{\prime}$ can be found with complexity $2^{64}$ in time and memory



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- Any of the $2^{8}$ solutions can be used to generate a new starting point for the next iteration, while keeping the same bytes inactive in chaining variable
$\Rightarrow$ Collision attack for 5 rounds with complexity of $8 \cdot 2^{64+56}=2^{123}$


## Summary

|  | rounds | complexity | memory |
| :---: | :---: | :---: | :---: |
| Grostl-256 | 3 | $2^{64}$ | - |
|  | 4 | $2^{67}$ | $2^{64}$ |
|  | 5 | $2^{123}$ | $2^{64}$ |

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for the collision attack on 4 rounds, and

$$
128 \xrightarrow{r_{1}} 64 \xrightarrow{r_{2}} 8 \xrightarrow{r_{3}} 2 \xrightarrow{r_{4}} 16 \xrightarrow{r_{5}} 16
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for the collision attack on 5 rounds
$\Rightarrow$ Collision attack on 4 and 5 rounds of Grøstl-512 with a complexity of $2^{131}$ and $2^{176}$

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|  | 5 | $2^{120}$ | $2^{64}$ |
|  | 3 | $2^{192}$ | - |
| Grøstl-512 | 4 | $2^{131}$ | $2^{64}$ |
|  | 5 | $2^{176}$ | $2^{64}$ |

## Thank you for your attention!

