Collision Spectrum, Entropy Loss, T-Sponges and Cryptanalysis of GLUON-64

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Random functions



What happens when a random function is used to update the internal state of a cryptographic primitive?

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Known Results

CPS and Iterated (Pre)-Images

Applications to Cryptography

Application to GLUON-64

Conclusion

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For functions chosen uniformly at random among all the functions from ${\cal S}$ to itself (random mappings).

Using state shrinking/presence of trees



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- Shrinking of the state space of MICKEY observed by Hong and Kim (05), studied by Röck (08).

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Collision Probability Spectrum (CPS)



Definition (Collision Probability Spectrum)

We call *Collision Probability Spectrum* (CPS) of $g : S \to S$ the set $\{c_k\}_{k \ge 1}$

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Definition

The average number of non-zero roots is denoted κ and called *collision average*:

$$\kappa = \sum_{k \ge 1} \mathfrak{c}_k \cdot k - 1$$



Let $V_k = \{x_0 \in \mathcal{S}, g(x_0 + y) = g(x_0) \text{ has } k \text{ solutions}\}. \Rightarrow |V_k| = \mathfrak{c}_k \cdot |\mathcal{S}|$





















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Independence Assumption: In what follows, we assume that $x \in g(V_k)$ and $x \in V_j$ are independent for any k, j.

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 $|g^i(\mathcal{S})| \sim rac{|\mathcal{S}|}{i \cdot \kappa/2} \qquad \#\{ ext{ nodes in tree rooted in } g^i(\mathcal{S}) \} \sim rac{\kappa}{4} \cdot i^2$

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$Known\ {\rm CPS'}s$



Function	κ	$ \mathcal{S} / g^i(\mathcal{S}) $	tree size
MICKEY's update function	0.625	$2^{-1.7} \cdot i$	$2^{-2.7} \cdot i^2$
Random mapping	1	$2^{-1} \cdot i$	$2^{-2} \cdot i^2$
GLUON-64's update function	6.982	2 ^{1.8} · <i>i</i>	$2^{0.8} \cdot i^2$

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The T-sponge Construction





- c: capacity
- r: bitrate
- *m*₁, ..., *m_k*: Message
- *d*₁, ..., *d*_j: Digest
- g: random function

Flat Sponge Claim Revisited



If g is a function with collision average $\kappa,$ then finding collisions with Q queries succeeds with probability

$$\frac{Q^2}{2^{c+1}}\cdot\Big(1+\frac{\kappa-1}{2^r}\Big).$$

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Intuition: S has size 2^{c+r} . Collisions occur because of the "trimming" of the bitrate $(2^r/2^{c+r} = 2^c)$ and because of inner-collisions $(\kappa/2^{c+r})$.



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 $|g_{k_1}^{-i}(t)| \approx i \cdot \kappa/2$

 $|\text{Tree}| \approx i^2 \cdot \kappa/4$





















Finding element in $g_{k_1}^{-i}(t)$

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Finding element in $g_{k_1}^{-i}(t)$

 $\mathcal{C} \approx \frac{|\mathcal{S}|}{\kappa/2}$

Finding element in collision tree:

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- More?

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Description of GLUON-64



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Possible (with a SAT-solver) to enumerate the solutions of

$$(\rho^{10} \circ \operatorname{pad})(x + a) = (\rho^{10} \circ \operatorname{pad})(a)$$



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Thank you!